Double Barrier Options Valuation under Multifactor Pricing Models[1]

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Outline

1. Motivation

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Motivation

- Vast literature on the pricing of barrier options.
- However, main focus is on European-style contracts and single-factor option pricing models (e.g., GBM and CEV):
  - Merton (1973), Rubinstein and Reiner (1991) and Rich (1994) – single barrier European options under the GBM process;
  - Kunitomo and Ikeda (1992), Geman and Yor (1996), Sidenius (1998) and Pelsser (2000) – double barrier European options also under the GBM assumption;
  - Boyle and Tian (1999) and Davydov and Linetsky (2001, 2003) – double barrier European options but under a CEV diffusion.

Motivation

- Kuan and Webber (2003) price single- and double-barrier European-style options based on the first passage time density of the underlying asset price to the barrier level(s).
- Such optimal stopping time is recovered through the numerical solution of an integral equation that only involves the transition density function of the single model' state variable.
- However, Kuan and Webber (2003) limit their analysis to the GBM assumption and to one-factor interest rate models.
- Under the GBM, the approach offered by Kuan and Webber (2003) is less efficient than most of the analytical methods proposed in the literature.
- However, their approach should be more useful when applied to more general option pricing models.

Motivation

- Although the stochastic volatility framework offered by Heston (1993) is able to attenuate the *smile effect* associated to the log-normal assumption, very few attempts have been made to price analytically barrier options under this setup:
  - Lipton (2001) and Faulhaber (2002) propose two different methods to price continuously monitored and European-style double barrier options, but have to assume a zero drift for the underlying Itô process as well as the absence of correlation between the asset return and its volatility.
  - Griebsch and Wystup (2008) are able to avoid the previous two unrealistic assumptions, but only price discretely monitored barrier options through a multidimensional numerical integration approach that only remains efficient as long as the number of fixings is small.
- Gao, Huang and Subrahmanyam (2000) propose a quasi-analytical approach for pricing American-style barrier options, but they only focus on single barrier contracts under the GBM assumption.

Outline

Main Contributions

- The paper extends the pricing methodology developed by Kuan and Webber (2003) for European-style barrier options from a one-factor setup to a more general multifactor and Markovian financial model that is able to accommodate:
  - Stochastic volatility;
  - Stochastic interest rates;
  - Endogenous bankruptcy.
- Additionally, but not less importantly, the analysis is also extended to cope with the valuation of American-style barrier option contracts under the same general financial model.

Outline

A General Financial Model

- The pricing model adopted for the analytical valuation of barrier options can be understood as a jump to default extension–along the lines of Carr and Linetsky (2006)–of the multifactor and Markovian diffusion framework proposed by Detemple and Tian (2002).
- If one ignores the possibility of default, the proposed framework is viable not only to price equity options but also options on stock indices, on currencies and on commodities.
- It is assumed that the financial market is arbitrage-free and frictionless, and that trading takes place continuously on the time-interval $\mathcal{T} := [t_0, T]$, for some fixed time $T > t_0$.
- Uncertainty is represented by a complete probability space $(\Omega, \mathcal{G}, \mathbb{Q})$, where the equivalent martingale measure $\mathbb{Q}$ associated to the numéraire "money market account" is taken as given.

Model Description

- The (pre-default) underlying asset price S is described by the following time-inhomogeneous diffusion process (with killing) under the risk-neutral measure $\mathbb{Q}$:

$$\frac{dS_t}{S_t} = \left[r\left(Y,t\right) - q\left(Y,t\right) + \lambda\left(S,Y,t\right)\right]dt + \sigma\left(S,Y,t\right)dW_S^{\mathbb{Q}}\left(t\right), \quad (1)$$

where

  - $r\left(Y,t\right) \in \mathbb{R}$ is the riskless and short-term interest rate;
  - $q\left(Y,t\right) \in \mathbb{R}$ is the dividend yield;
  - $\lambda\left(S,Y,t\right) \in \mathbb{R}_+$ is the *hazard rate*;
  - $\sigma\left(S,Y,t\right) \in \mathbb{R}$ is the instantaneous volatility of asset returns; and
  - $W_S^{\mathbb{Q}}\left(t\right) \in \mathbb{R}$ is a standard Brownian motion.

Model Description

- Markovian model factors $Y \in \mathbf{D} \subseteq \mathbb{R}^m$:

$$dY_t = \mu(Y,t)\,dt + \gamma(Y,t) \cdot dW^{\mathbb{Q}}(t), \tag{2}$$

where $W^{\mathbb{Q}}(t) \in \mathbb{R}^n$ is a vector of n independent Brownian motions.

- The Wiener processes $\left\{W_S^{\mathbb{Q}}(u); t_0 \leq u \leq T\right\}$ and $\left\{W^{\mathbb{Q}}(u); t_0 \leq u \leq T\right\}$ are assumed to be correlated:

$$d\left\langle W_S^{\mathbb{Q}}, W_i^{\mathbb{Q}}\right\rangle(t) = \rho_i dt, \tag{3}$$

for $i = 1, \ldots, n$, with $|\rho_i| < 1$, and where $W_i^{\mathbb{Q}}(t)$ denotes the i^{th} element of vector $W^{\mathbb{Q}}(t)$.

- In opposition to Detemple and Tian (2002) or Nunes (2009a), but similarly to Carr and Linetsky (2006), the underlying asset price process can either diffuse or jump to default.

Model Description

- In the first case, bankruptcy occurs at the first passage time of the stock price to zero:

$$\tau_0 := \inf \{t > t_0 : S_t = 0\} . \tag{4}$$

- Alternatively, S can also jump to a *cemetery state* at the first jump time of a doubly-stochastic Poisson process $\tilde{\zeta}$ with intensity $\lambda (S, Y, t)$.
- Therefore, the time of default is simply given by

$$\zeta = \tau_0 \wedge \tilde{\zeta}. \tag{5}$$

Model Description

- The financial model described by equations (1) to (5) encompasses several well known option pricing models as special cases:
  - Taking $\lambda = 0$ and $\tau_0 = \infty$, while assuming that r, q, and σ are constant, then the standard GBM arises;
  - If $\lambda = 0$, $\tau_0 = \infty$, both r and q are constant, but $\sigma(S, Y, t) = \delta (S_t)^{\frac{\beta}{2}-1}$, for $\delta, \beta \in \mathbb{R}$, then the CEV model is obtained;
  - It can accommodate several stochastic interest rate processes–as in Nunes (2009a)–or stochastic volatility models–as, for instance, the Heston (1993) model–through the dependency of r and σ on Y;
  - Moreover, when r, q, λ, and σ do not depend on Y, then the jump to default extended diffusion process of Carr and Linetsky (2006, equation 2.1), and in particular the JDCEV model, follows.

Model Description

- Hence, the proposed financial model also allows the Carr and Linetsky (2006) original setup to be enlarged towards a multifactor formulation that can incorporate stochastic volatility and stochastic interest rates.
- Even though the proposed financial model considers explicitly the risk of default only associated to the underlying equity, it is easy to also accommodate the risk of default on the option' writer.
- This is specially relevant for the contracts under analysis since many barrier options are traded over-the-counter.
- Following, for instance, Jarrow and Turnbull (1995, equation 63), these *vulnerable options* can also be priced using our approach; it is only necessary to multiply our pricing solutions by the ratio between a risky and default-free discount factor.

Plain-Vanilla European Options

- Following, for instance, Carr and Linetsky (2006, equations 3.2 and 3.3), the time-t_0 price of a plain-vanilla European call (if $\phi = -1$) or put (if $\phi = 1$) on the underlying asset price S, with strike K, and maturity at time T, is equal to

$$v_{t_0}(S, Y, K, T; \phi) = v^0_{t_0}(S, Y, K, T; \phi) + v^D_{t_0}(S, Y, K, T; \phi), \quad (6)$$

- $v^0_{t_0}(S, Y, K, T; \phi)$ represents the time-t_0 price of the corresponding European standard option but conditional on no default until the maturity date T:

$$v^0_{t_0}(S, Y, K, T; \phi) := \mathbb{E}_{\mathbb{Q}}\left\{ \exp\left[-\int_{t_0}^{T} r(Y, l)\, dl \right] (\phi K - \phi S_T)^+ \mathbb{1}_{\{\zeta > T\}} \middle| \mathcal{G}_{t_0} \right\}. \quad (7)$$

Plain-Vanilla European Options

- $v_{t_0}^{D}(S, Y, K, T; \phi)$ corresponds to the recovery payment (of the strike K and at time T) associated to the European put contract:

$$v_{t_0}^{D}(S, Y, K, T; \phi) := (\phi K)^{+} \mathbb{E}_{\mathbb{Q}} \left\{ \exp\left[-\int_{t_0}^{T} r(Y, l)\, dl \right] \mathbb{1}_{\{\zeta \leq T\}} \middle| \mathcal{G}_{t_0} \right\}. \quad (8)$$

- The majority of the Markovian option pricing models proposed in the literature, under a no bankruptcy assumption (i.e. with $\lambda = 0$ and $\tau_0 = \infty$), provide efficient solutions for standard European-style contracts.
- Even under the risk of default, it is still possible to price European-style plain-vanilla options in closed-form.
- However, the analysis will now focus on the valuation of a European knock-out double barrier option with no rebate.

European Barrier Options

Definition 1

The time-T price of a unit face value and zero rebate European knock-out double barrier option on the asset price S, with strike K, lower barrier level L, upper barrier level U, and maturity at time T $(\geq t_0)$ is equal to

$$EKODB_T\left(S, Y, K, L, U; \phi\right) = \left(\phi K - \phi S_T\right)^+ \mathbb{1}_{\{\tau_{LU} > T\}}, \tag{9}$$

where $\phi = 1$ for a put option, $\phi = -1$ for a call option, and

$$\tau_{LU} := \inf\left\{u > t_0 : S_u \leq L \quad or \quad S_u \geq U\right\} \tag{10}$$

is the first passage time of the underlying asset price to one of the two barriers.

European Barrier Options

Proposition 1

$$EKODB_{t_0}(S, Y, K, L, U, T; \phi) = v^0_{t_0}(S, Y, K, T; \phi) - EKIDB^0_{t_0}(S, Y, K, L, U, T; \phi), \quad (11)$$

where function $v^0_{t_0}(S, Y, K, T; \phi)$ is given by equation (7),

$$\begin{aligned} & EKIDB^0_{t_0}(S, Y, K, L, U, T; \phi) \quad (12) \\ = \; & P(Y_{t_0}, T) \int_{t_0}^{T} SP(S_{t_0}, Y_{t_0}, u) \\ & \left[\int_{\mathbf{D}} \frac{v^0_u(L, Y, K, T; \phi)}{P(Y_u, T)} \mathbb{Q}^T \left(Y_u \in dY | S_u = L, Y_{t_0} \right) \right] \mathbb{Q}^T \left(\tau_L \in du | \mathcal{F}_{t_0} \right) \\ & + P(Y_{t_0}, T) \int_{t_0}^{T} SP(S_{t_0}, Y_{t_0}, u) \\ & \left[\int_{\mathbf{D}} \frac{v^0_u(U, Y, K, T; \phi)}{P(Y_u, T)} \mathbb{Q}^T \left(Y_u \in dY | S_u = U, Y_{t_0} \right) \right] \mathbb{Q}^T \left(\tau_U \in du | \mathcal{F}_{t_0} \right), \end{aligned}$$

European Barrier Options

Proposition 1

and

$$SP\left(S_{t_0}, Y_{t_0}, u\right) := \mathbb{E}_{\mathbb{Q}^T}\left\{ \exp\left[-\int_{t_0}^{u} \lambda\left(S, Y, l\right) dl\right] 1\!\!1_{\{\tau_0 > u\}} \middle| \mathcal{F}_{t_0} \right\} \tag{13}$$

is the probability (under the forward measure) of surviving beyond time $u > t_0$, while $\mathbb{Q}^T\left(\tau_B \in du \middle| \mathcal{F}_{t_0}\right)$ represents the probability density function of the first passage time τ_B, with $B \in \{L, U\}$, i.e.

$$\tau_L := \inf\left\{ u > t_0 : S_u \leq L, \sup_{t_0 \leq v \leq u} (S_v) < U \right\}, \tag{14}$$

$$\tau_U := \inf\left\{ u > t_0 : S_u \geq U, \inf_{t_0 \leq v \leq u} (S_v) > L \right\}. \tag{15}$$

First Passage Time Densities

Proposition 2

The two optimal stopping time densities $\mathbb{Q}^T\left(\tau_L \in du \middle| \mathcal{F}_{t_0}\right)$ and $\mathbb{Q}^T\left(\tau_U \in du \middle| \mathcal{F}_{t_0}\right)$ are the implicit solutions of

$$\begin{aligned} & F\left(t_0, S_{t_0}; u, L\right) \\ = & \int_{t_0}^{u} F(v, L; u, L)\,\mathbb{Q}^T\left(\tau_L \in dv \middle| \mathcal{F}_{t_0}\right) + \int_{t_0}^{u} F(v, U; u, L)\,\mathbb{Q}^T\left(\tau_U \in dv \middle| \mathcal{F}_{t_0}\right), \end{aligned} \tag{16}$$

and

$$\begin{aligned} & 1 - F\left(t_0, S_{t_0}; u, U\right) \\ = & \int_{t_0}^{u} [1 - F(v, L; u, U)]\,\mathbb{Q}^T\left(\tau_L \in dv \middle| \mathcal{F}_{t_0}\right) + \int_{t_0}^{u} [1 - F(v, U; u, U)]\,\mathbb{Q}^T\left(\tau_U \in dv \middle| \mathcal{F}_{t_0}\right), \end{aligned} \tag{17}$$

where

$$F(v, E_v; u, E_u) = \int_{\mathbf{D}} \mathbb{Q}^T\left(S_u \leq E_u \middle| S_v = E_v, Y_v\right) \mathbb{Q}^T\left(Y_v \in dY \middle| S_v = E_v, Y_{t_0}\right), \tag{18}$$

for any deterministic and real-valued spot levels E_v and E_u.

Plain-Vanilla American Options

Following, for instance, Nunes (2009b, equation 54), the time-t_0 price of a plain-vanilla American call (if $\phi = -1$) or put (if $\phi = 1$) on the underlying asset price S, with strike K, and maturity at time T, is equal to

$$V_{t_0}(S, Y, K, T; \phi) = V^0_{t_0}(S, Y, K, T; \phi) + V^D_{t_0}(S, Y, K, T; \phi), \tag{19}$$

where

$$V^0_{t_0}(S, Y, K, T; \phi) := \sup_{\tau \in \mathcal{S}} \left\{ \mathbb{E}_{\mathbb{Q}} \left[\exp\left(-\int_{t_0}^{T\wedge\tau} r(Y, l)\, dl \right) (\phi K - \phi S_{T\wedge\tau})^+ \mathbb{1}_{\{\zeta > T\wedge\tau\}} \middle| \mathcal{G}_{t_0} \right] \right\} \tag{20}$$

represents the time-t_0 standard American option value that is conditional on no default, and

$$V^D_{t_0}(S, Y, K, T; \phi) := (\phi K)^+ \mathbb{E}_{\mathbb{Q}} \left\{ \exp\left[-\int_{t_0}^{\zeta} r(Y, l)\, dl \right] \mathbb{1}_{\{\zeta \le T\}} \middle| \mathcal{G}_{t_0} \right\} \tag{21}$$

corresponds to the recovery payment (of the strike K and at the default time ζ) associated to the American put contract.

American Barrier Options

Proposition 3

$$AKIDB^0_{t_0}(S, Y, K, L, U, T; \phi) = EKIDB^0_{t_0}(S, Y, K, L, U, T; \phi) + EEPKIDB^0_{t_0}(S, Y, K, L, U, T; \phi), \tag{22}$$

where the function $EKIDB^0_{t_0}(S, Y, K, L, U, T; \phi)$ is given by equation (12),

$$\begin{aligned} & EEPKIDB^0_{t_0}(S, Y, K, L, U, T; \phi) \\ = \; & P(Y_{t_0}, T) \int_v^T SP(S_{t_0}, Y_{t_0}, u) \\ & \left[\int_{\mathbf{D}} \frac{EEP^0_u(E(Y, u), Y, K, T; \phi)}{P(Y_u, T)} \mathbb{Q}^T\left(Y_u \in dY | S_u = E(Y, u), Y_{t_0} \right) \right] \\ & \left\{ \int_{t_0}^u \left[\mathbb{Q}^T(\tau_e \in du | S_v = L) \mathbb{Q}^T\left(\tau_L \in dv | \mathcal{F}_{t_0}\right) \right. \right. \\ & \left. \left. + \mathbb{Q}^T(\tau_e \in du | S_v = U) \mathbb{Q}^T\left(\tau_U \in dv | \mathcal{F}_{t_0}\right) \right] \right\}, \end{aligned} \tag{23}$$

American Barrier Options

Proposition 3

the function $SP\left(S_{t_0}, Y_{t_0}, u\right)$ is defined by equation (13), and

$$EEP_u^0\left(E\left(Y,u\right),Y,K,T;\phi\right) := \left[\phi K - \phi E\left(Y,u\right)\right]^+ - v_u^0\left(E\left(Y,u\right),Y,K,T;\phi\right) \tag{24}$$

is the time-u ($> t_0$) early exercise premium associated to a plain-vanilla European option and valued at the optimal exercise boundary.

The implementation of Proposition 3 requires the knowledge of a new quantity: the probability density function of the first passage time to the early exercise boundary, which is such that

$$\begin{aligned} & \mathbb{1}_{\{\phi=-1\}} + \phi F\left(v,B;u,E\left(Y,u\right)\right) \qquad (25) \\ = & \int_v^u \left[\mathbb{1}_{\{\phi=-1\}} + \phi F\left(l,E\left(Y,l\right);u,E\left(Y,u\right)\right)\right] \mathbb{Q}^T\left(\tau_e \in dl \middle| S_v = B\right), \end{aligned}$$

where $B \in \{L, U\}$, and function $F(\cdot)$ is defined through equation (18), while $\phi = 1$ for a put option but $\phi = -1$ for a call option.

Outline

European-style knock-out double barrier call prices under the GBM assumption

σ	Strike price	KI1992	GY1996	S1998	P2000	Propositions 1 and 2: 16	32	64	128	256
					Panel A: $T - t_0 = 0.25$					
	95	4.3036	4.3036	4.3036	4.3036	4.3039	4.3037	4.3036	4.3036	4.3036
25%	100	2.4131	2.4131	2.4131	2.4131	2.4134	2.4132	2.4132	2.4131	2.4131
	105	1.0804	1.0804	1.0804	1.0804	1.0805	1.0805	1.0804	1.0804	1.0804
	95	0.9770	0.9770	0.9770	0.9770	0.9773	0.9770	0.9770	0.9770	0.9770
40%	100	0.5475	0.5475	0.5475	0.5475	0.5477	0.5476	0.5476	0.5475	0.5475
	105	0.2447	0.2447	0.2447	0.2447	0.2447	0.2447	0.2447	0.2447	0.2447
					Panel B: $T - t_0 = 0.50$					
	95	1.7038	1.7038	1.7038	1.7038	1.7046	1.7041	1.7039	1.7039	1.7038
25%	100	0.9703	0.9703	0.9703	0.9703	0.9708	0.9705	0.9704	0.9703	0.9703
	105	0.4418	0.4418	0.4418	0.4418	0.4421	0.4419	0.4418	0.4418	0.4418
	95	0.0878	0.0878	0.0878	0.0878	0.0880	0.0878	0.0878	0.0878	0.0878
40%	100	0.0492	0.0492	0.0492	0.0492	0.0494	0.0492	0.0492	0.0492	0.0492
	105	0.0220	0.0220	0.0220	0.0220	0.0223	0.0221	0.0220	0.0220	0.0220

American-style up-and-out put option prices under the GBM assumption

					Proposition 3			
S_{t_0}	σ	$T - t_0$	R1995	GHS2000	2d	3d	4d	5d
40.0	20%	0.25	5.0357	5.0360	5.0357	5.0356	5.0356	5.0356
40.0	20%	0.5	5.1881	5.1893	5.1860	5.1860	5.1860	5.1860
40.0	20%	0.75	5.3083	5.3095	5.3006	5.3005	5.3005	5.3005
40.0	20%	1	5.3861	5.3868	5.3679	5.3678	5.3678	5.3678
45.0	20%	0.25	1.5445	1.5448	1.5441	1.5441	1.5441	1.5441
45.0	20%	0.5	1.9375	1.9384	1.9369	1.9368	1.9368	1.9368
45.0	20%	0.75	2.1197	2.1204	2.1181	2.1180	2.1180	2.1180
45.0	20%	1	2.2151	2.2154	2.2121	2.2119	2.2119	2.2119
49.5	20%	0.25	0.1103	0.1103	0.1103	0.1102	0.1102	0.1102
49.5	20%	0.5	0.1613	0.1614	0.1614	0.1612	0.1612	0.1612
49.5	20%	0.75	0.1828	0.1828	0.1829	0.1826	0.1826	0.1826
49.5	20%	1	0.1936	0.1936	0.1937	0.1933	0.1933	0.1933
40.0	40%	0.25	5.9781	5.9778	5.9767	5.9767	5.9767	5.9767
40.0	40%	0.5	6.4285	6.4292	6.4238	6.4237	6.4237	6.4237
40.0	40%	0.75	6.6162	6.6171	6.6020	6.6017	6.6017	6.6017
40.0	40%	1	6.7054	6.7063	6.6758	6.6753	6.6753	6.6753
45.0	40%	0.25	2.7007	2.7010	2.7003	2.7002	2.7002	2.7002
45.0	40%	0.5	3.0368	3.0370	3.0350	3.0347	3.0347	3.0347
45.0	40%	0.75	3.1591	3.1594	3.1546	3.1540	3.1540	3.1540
45.0	40%	1	3.2145	3.2148	3.2036	3.2028	3.2028	3.2028
49.5	40%	0.25	0.2563	0.2563	0.2566	0.2563	0.2563	0.2563
49.5	40%	0.5	0.293	0.2930	0.2939	0.2934	0.2934	0.2934
49.5	40%	0.75	0.3059	0.3059	0.3075	0.3067	0.3067	0.3067
49.5	40%	1	0.3117	0.3117	0.3144	0.3132	0.3132	0.3132

European-style double barrier knock-out call prices under the CEV model

							Propositions 1 and 2				
K	U	L	β	δ	DL2001	DL2003	16	32	64	128	256
95	120	90	1	2.5	1.8805	1.8805	1.8814	1.8807	1.8805	1.8804	1.8805
95	120	90	0	2.5×10	2.0800	2.0808	2.0812	2.0804	2.0801	2.0800	2.0800
95	120	90	-2	2.5×10^3	2.5529	2.5528	2.5544	2.5533	2.5530	2.5528	2.5529
95	120	90	-4	2.5×10^5	3.1295	3.1295	3.1312	3.1301	3.1297	3.1295	3.1295
95	120	90	-6	2.5×10^7	3.8088	3.8088	3.8103	3.8094	3.8090	3.8088	3.8088
100	120	90	1	2.5	1.0958	1.0958	1.0964	1.0959	1.0958	1.0957	1.0958
100	120	90	0	2.5×10	1.2383	1.2383	1.2392	1.2386	1.2384	1.2383	1.2383
100	120	90	-2	2.5×10^3	1.5799	1.5799	1.5811	1.5803	1.5800	1.5799	1.5799
100	120	90	-4	2.5×10^5	2.0022	2.0022	2.0036	2.0027	2.0023	2.0022	2.0022
100	120	90	-6	2.5×10^7	2.5059	2.5059	2.5072	2.5064	2.5061	2.5059	2.5059
105	120	90	1	2.5	0.5126	0.5126	0.5130	0.5127	0.5126	0.5125	0.5126
105	120	90	0	2.5×10	0.5945	0.5945	0.5951	0.5947	0.5945	0.5945	0.5945
105	120	90	-2	2.5×10^3	0.7960	0.7960	0.7969	0.7964	0.7961	0.7961	0.7960
105	120	90	-4	2.5×10^5	1.0535	1.0535	1.0546	1.0539	1.0536	1.0535	1.0535
105	120	90	-6	2.5×10^7	1.3696	1.3697	1.3708	1.3701	1.3698	1.3697	1.3697

American-style double barrier knock-in put prices under the CEV model

K	U	L	β	δ	Standard European	European KIDB	Proposition 3			
							2d	3d	4d	5d
95	120	90	1	2.5	3.0297	3.0096	3.2691	3.2691	3.2692	3.2697
95	120	90	0	2.5×10	3.1094	3.0915	3.3344	3.3345	3.3345	3.3350
95	120	90	-2	2.5×10^3	3.2865	3.2723	3.4778	3.4778	3.4778	3.4782
95	120	90	-4	2.5×10^5	3.4982	3.4870	3.6441	3.6446	3.6446	3.6446
95	120	90	-6	2.5×10^7	3.7616	3.7529	3.8506	3.8506	3.8506	3.8515
100	120	90	1	2.5	4.7075	4.5521	5.0379	5.0379	5.0379	5.0379
100	120	90	0	2.5×10	4.7145	4.5717	5.0312	5.0312	5.0312	5.0312
100	120	90	-2	2.5×10^3	4.7436	4.6236	5.0258	5.0258	5.0258	5.0258
100	120	90	-4	2.5×10^5	4.7977	4.6976	5.0322	5.0322	5.0322	5.0322
100	120	90	-6	2.5×10^7	4.8867	4.8040	5.0538	5.0538	5.0538	5.0540
105	120	90	1	2.5	6.8961	6.4038	7.2445	7.2445	7.2446	7.2446
105	120	90	0	2.5×10	6.8194	6.3539	7.1699	7.1700	7.1701	7.1701
105	120	90	-2	2.5×10^3	6.6826	6.2677	7.0289	7.0291	7.0290	7.0291
105	120	90	-4	2.5×10^5	6.5681	6.2006	6.8973	6.8974	6.8974	6.8974
105	120	90	-6	2.5×10^7	6.4789	6.1555	6.7731	6.7732	6.7732	6.7732

Outline

Conclusions

Conclusions

- This paper extends the literature in two directions:
  - First, European-style (double) barrier options are priced under a multifactor and Markovian financial model that is able to accommodate stochastic volatility, stochastic interest rates and endogenous bankruptcy;
  - Second and more importantly, quasi-analytical pricing solutions are also proposed for American-style (double) barrier option contracts under the same general financial model.
- The proposed pricing solutions are shown to be accurate, easy to implement, and efficient.