Irreversible Investment in Oligopoly

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Introduction — Sequential irreversible investment

Consider a firm's problem to optimally expand production capacity under uncertainty:

- free choice of investment timing/scaling + irreversibility
  $\Rightarrow$ sequence of real options (on *marginal* investments)
- Pindyck (1988), Abel & Eberly (1996), Bertola (1998), Riedel & Su (2010)
- invest only at sufficiently *positive* NPV:
  "option value of waiting" [Dixit & Pindyck (1994)]

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  "option value of waiting" [Dixit & Pindyck (1994)]

Results hold only for monopolists:

- exercising a real option typically affects the underlying
- competition threatens option premia: preemption incentives

$\Rightarrow$ Strategic models of option exercise!

Introduction — Competitive models

Perfect competition:

- Leahy (1993)
  - continuum of investors $\rightarrow$ entry timing
  - 0 NPV investment
  - myopic entry is optimal
- Baldursson & Karatzas (1997)
  - general approach $\rightarrow$ same qualitative results
  - singular control problem (social planner)
    $\Rightarrow$ optimal stopping $\Rightarrow$ option exercise equilibrium conditions

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Oligopoly:

- Grenadier (2002)
  - symmetric n-player equilibrium
  - Markovian setting, analytically solvable example
  - increasing competition erodes option values

Introduction — Strategy types

Strategic effects depend on interaction opportunities:

- open loop strategies: actions depend only on exogenous data
- closed loop strategies: actions depend on current state
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Back and Paulsen (2009) clarify:

- *open loop* equilibrium — trigger $\bar{X}(q^i, q^{-i})$ only optimal for symmetric path $q^{-i} = (n-1)q^i$
- rigorous proof for same equilibrium
- technical issues severely complicate closed loop formulation

Introduction — Present paper

We take a general approach to the open loop strategy game:

- abstract underlying stochastics: non-Markovian, include jumps
- asymmetric initial capital stocks
- derive/isolate equilibrium conditions in terms of spot revenue only
- characterize investment behaviour/incentives

Stochastic game in continuous time

- $(\Omega, \mathcal{F}_\infty, (\mathcal{F}_t)_{t\geq 0}, \mathbf{P})$ filtered probability space satisfying usual conditions of right-continuity and completeness
- $n \in \mathbb{N}$ players with initial capital levels $(q^1, \ldots, q^n) \in \mathbb{R}^n_+$
- Strategy space of each player i is $\mathcal{A}(q^i)$

$$\mathcal{A}(q) \triangleq \{Q \text{ adapted, nondecreasing, left-cont., with } Q_0 = q \ \mathbf{P}\text{-a.s.}\}$$

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- Expected payoff from strategies $(Q^1, \ldots, Q^n) \in \prod_{i=1}^n \mathcal{A}(q^i)$

$$J^i(Q^i|Q^{-i}) \triangleq \mathbf{E}\left[\int_0^\infty \Pi(t, Q^i_t, Q^{-i}_t)\, dt - \int_0^\infty k_t\, dQ^i_t\right]$$

$$\tilde{Q} \triangleq \sum_{j=1\ldots n} Q^j \qquad Q^{-i} \triangleq \tilde{Q} - Q^i$$

Assumption 1

(i) For any $(\omega, t) \in \Omega \times [0, \infty)$, the mapping $(q^i, q^{-i}) \mapsto \Pi(\omega, t, q^i, q^{-i})$ is twice continuously differentiable. For $q^{-i} \in \mathbb{R}_+$ fixed, the partial derivative $\Pi_{q^i} \triangleq \partial \Pi / \partial q^i$ strictly decreases in q^i.

(ii) For $(q^i, q^{-i}) \in \mathbb{R}_+^2$ fixed, $(\omega, t) \mapsto \Pi(\omega, t, q^i, q^{-i})$ is progressively measurable.

(iii) For any $(Q^1, Q^2) \in \mathcal{A}(0)^2$, $\Pi(\omega, t, Q_t^1(\omega), Q_t^2(\omega))$ is $\mathbf{P} \otimes dt$-integrable.

(iv) The investment cost process (k_t) is a right-continuous supermartingale, strictly positive for $t \in \mathbb{R}_+$ and $k_\infty = 0$ $\mathbf{P}$-a.s.

Equilibrium

- Determining the best reply of player i to a given opponent investment process $Q^{-i} \in \mathcal{A}(q^{-i}), q^{-i} \in \mathbb{R}_+$, is an optimal control problem of the *monotone follower type* with value function

$$V(q^i, Q^{-i}) \triangleq \sup_{Q \in \mathcal{A}(q^i)} J(Q|Q^{-i})$$

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Definition

$(Q^{*_1}, \dots, Q^{*_n})$ is an open loop equilibrium if for all $i \in \{1, \dots, n\}$, $Q^{*_i} \in \mathcal{A}(q^i)$ and $J(Q^{*_i}|Q^{*_{-i}}) = V(q^i, Q^{*_{-i}})$.

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- Determine a best reply using literature on monotone follower problems; e.g. Bank (2005)
- Main problem is consistency in equilibrium

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Concerning the effect of opponent capital we make

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$$\Pi_{q^i q^i} + (n-1) \cdot \Pi_{q^i q^{-i}} < 0$$

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- Among the weakest sufficient conditions for uniqueness of equilibrium in the static Cournot game with payoff Π
- Implied by $\Pi_{q^i q^{-i}} < 0$ (strategic substitutes), sufficient for existence in the static game

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For asymmetric starting states we also need

Assumption 3

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$$\Pi(\omega, t, q^i, q^{-i}) = e^{-rt} P(X_t(\omega), q^i + q^{-i}) \cdot q^i$$

where inverse demand P decreases in supply and is affected by exogenous shocks (X_t)

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  where inverse demand P decreases in supply and is affected by exogenous shocks (X_t)
- With fixed aggregate capital, marginal revenue decreases in own capital

Equalizing equilibria

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- Only the currently smallest firms invest

Uniqueness

Theorem

Under Assumptions 1 and 3, any open loop equilibrium is an equalizing equilibrium.

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- Game inherits Cournot structure

Existence

Theorem

Under Assumptions 1–3, there exists for any $(q^1, \ldots, q^n) \in \mathbb{R}^n_+$ *an equalizing equilibrium of the game iff there exists an optimal control* $\hat{Q} \in \mathcal{A}(q^1)$ *for a particular auxiliary monotone follower problem. Then,* $Q^{*1} = \hat{Q}$.
An optimal control process exists if

$$\lim_{l \to \infty} \Pi_{q^i}(\omega, t, l, l) \leq 0 \text{ for all } (\omega, t) \in \Omega \times [0, \infty).$$

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- Any optimal control (resp. equilibrium) is unique due to concavity

Equilibrium characterization

Consider the "gradient"

$$\nabla J^i(Q^i|Q^{-i})_s \triangleq \mathbf{E}\left[\int_s^\infty \Pi_{q^i}(t, Q_t^i, Q_t^{-i})\, dt|\mathcal{F}_s\right] - k_s$$

Similar to Bertola (1998), Bank & Riedel (2001), any open loop equilibrium $(Q^{*1}, \dots, Q^{*n})$ is characterized by the first order conditions

$$\nabla J^i(Q^{*i}|Q^{*-i}) \leq 0 \text{ and } \int_0^\infty \nabla J^i(Q^{*i}|Q^{*-i})_s\, dQ_s^{*i} = 0, \; \mathbf{P}-\text{a.s.}$$

$(i = 1, \dots, n)$

$\rightarrow$ perfectly competitive equilibrium conditions
Baldursson & Karatzas (1997)

Equilibrium investment

Given Assumption 3, in *any* open loop equilibrium, firm i's capital follows

$$Q_t^{*i} = q^i \vee \sup_{0 \leq u < t} L_u$$

with an optional signal process L, *identical* for all firms.

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with an optional signal process L, *identical* for all firms.

- L_t: maximal capital level — facing current capital stocks — for which the opportunity cost of delaying marginal investment until any future stopping time τ is zero
- Assumptions $\Rightarrow$ monotonicity $\Rightarrow$ *myopic* investment optimal

Cournot competition

Consider Cournot spot competition:

$$\Pi(\omega, t, q^i, q^{-i}) = e^{-rt} P(X_t(\omega), q^i + q^{-i}) \cdot q^i$$

with $P_q < 0$ and process (X_t) satisfying Assumption 1

$\Rightarrow$ marginal revenue given by

$$\Pi_{q^i} = e^{-rt}\big(P(X_t(\omega), q^i + q^{-i}) + q^i \cdot P_q(X_t(\omega), q^i + q^{-i})\big)$$

- when firm size q^i decreases relative to market $q^i + q^{-i}$, investment externalities vanish
- option premia decrease by spot market Cournot effect, not explicit preemption

Explicit solutions

Inverse demand with constant elasticity and multiplicative shock:

$$P(x, q) = x \cdot p(q) \qquad p(q) = q^{-\frac{1}{\alpha}} \qquad X_t = e^{Y_t}$$

- $\alpha > 0$
- $(Y_t)_{t \geq 0}$ Lévy-process without negative jumps

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Proposition

If $\alpha > \frac{1}{n}$, the unique open loop equilibrium is

$$Q_t^{*i} = \sup_{0\leq u<t} \frac{1}{n}\kappa^\alpha X_u^\alpha \quad (i = 1 \ldots n)$$

with constant parameter κ.

- Investment in equilibrium whenever X sets a new record

Explicit solutions

For fixed $n \in \mathbb{N}$, κ is determined by

$$\kappa\Big(\frac{\alpha n}{\alpha n - 1}\Big) = \frac{\Phi^{-Y}(r)}{r\big(1 + \Phi^{-Y}(r)\big)} \triangleq \kappa_\infty,$$

where $\Phi^{-Y}(r)$ is the Laplace exponent of $-Y$ at r.

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where $\Phi^{-Y}(r)$ is the Laplace exponent of $-Y$ at r.

- Aggregate capital $Q^* = n \cdot Q^{*_i} = \sup_{0 \le u < t} \kappa^\alpha X_u^\alpha$ increases in n
- Earlier investment with stronger competition
- Option values diminish

Perfect Competition

We can pass to the limit:

- continuum of firms, each earning revenue flow

$$e^{-rt}e^{X_t}P(q) = \lim_{n\to\infty} \Pi_{q^i}\left(\omega, t, n^{-1}q, (n-1)\,n^{-1}q\right)$$

after entry at cost k_τ, where q is aggregate capital

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  after entry at cost k_τ, where q is aggregate capital
- In equilibrium, aggregate capital

$$Q_t^\infty = \sup_{0\le u<t} \kappa_\infty^\alpha X_u^\alpha$$

  solves a social planner's problem;
  cf. Baldursson & Karatzas (1997)
- Firms enter at zero NPV, no delay