Components of bull and bear markets: bull corrections and bear rallies

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Presentation Outline

- INTRO: some questions that motivated us to pursue this topic
- TRADITIONAL: Ex post dating algorithms (filters) for bull and bear markets
- NEW: probability model for the distribution of aggregate stock returns
  - focused to identify bear market rallies and bull market corrections
- OUTLINE:
- Proposed model structure
- Bayesian estimation & model comparison
- Some results concerning implied characteristics of market dynamics
  - Characteristics of market trends and subtrends implied by parameter estimates
  - Identification of turning points
- Applications: predicting turning points and VaR
- Recent market conditions

Motivation: Questions

- Are there low frequency trends in stock returns?
  - Can we identify trends or cycles in aggregate stock returns?
  - Can they be used to improve investment decisions?
- Are two 'regimes' adequate to capture the dynamics?
  - What are typical characteristics of bull and bear market regimes?
  - Is it useful to model intra-regime dynamics?
- Is it useful to invest in a probabilistic approach?
  - What is the probability that this a bull market rather than a bear rally?
  - What is the probability of moving from a bull correction into a bear?
- Is it useful to use information in the entire distribution of returns?
  - Do investors use both return & and risk to identify the state or regime?
  - How do bear rallies and bull markets differ?

Some Contributions:

We propose a new 4-state Markov-switching (MS) model for stock returns that:

- Allows bull and bear regimes to be unobserved and stochastic
- Accommodates bear rally and bull correction states within regimes
  - Bear and bear rally states govern the bear regime
  - Bull and bull correction states govern the bull regime
- Captures heterogeneous intra-regime dynamics
  - Allow for bear rallies and bull corrections without a regime change
  - Realized bull and bear regimes can be different over time
  - Conditional autoregressive heteroskedasticity in a regime
- Probability statements on regimes and future returns available
  - What is the probability of a bear market rally at time t?
  - What is the probability of a transition from a rally to a bull market?
- Can forecast (both market states and returns) out-of-sample
  - Out-of-sample forecasts useful for market timing
  - Conditional VaR predictions are sensitive to market regimes

Some Other Applications of Markov-Switching Models

Applications of MS models to stock returns include, *among many others*:

- Regime switching in equilibrium asset pricing models: Cecchetti, Lam, and Mark (1990), Kandel and Stambaugh (1990), Gordon and St. Amour (2000), Calvet and Fisher (2007), Lettau, Ludvigson and Wachter (2008), Guidolin and Timmermann (2008)
- Relate business cycles and stock market regimes: Hamilton and Lin (1996)
- Duration dependence in stock market cycles: Maheu and McCurdy (2000a), Lunde and Timmermann (2004)
- Regime switching for joint nonlinear dynamics of stock and bond returns: Guidolin and Timmermann (2006, 2007)
- Implications of nonlinearities due to regimes switches for asset allocation and/or predictability of returns: Turner, Startz and Nelson (1989), van Norden and Schaller (1997), Chauvet and Potter (2000), Maheu and McCurdy (2000b), Perez-Quiros and Timmermann (2001), Ang and Bekaert (2002a), Guidolin and Timmermann (2005)
- Interest rates: Garcia and Perron (1996), Ang and Bekaert (2002b)

Data

- Daily capital gain return
  - 1885 -1925 capital gain returns from Schwert (1990)
  - 1926-2008 CRSP S&P 500, vwretx
  - 2009-2010 Reuters, SPTRTN on SPX
- Convert to daily continuously compounded returns
- Compute weekly return as Wed to Wed
- Compute weekly RV_t as sum of intra-week daily squared returns
- Scale by 100

Table: Weekly Return Statistics (1885-2010)[a]

N	Mean	standard deviation	Skewness	Kurtosis	J-B[b]
6498	0.085	2.40	-0.49	11.2	18475.5*

[a] Continuously compounded returns

[b] Jarque-Bera normality test: p-value = 0.00000

New MS-4 Model allowing Bull Corrections and Bear Rallies

MS-4

$$\begin{aligned} r_t | s_t &\sim N(\mu_{s_t}, \sigma^2_{s_t}) \\ p_{ij} &= p(s_t = j | s_{t-1} = i), \quad i = 1, ..., 4, \;\; j = 1, ..., 4. \end{aligned}$$

Terminology & Identification:

- States refer to s_t and are identified by:

$$\begin{aligned} \mu_1 &< 0 \text{ (bear state)}, \\ \mu_2 &> 0 \text{ (bear rally state)}, \\ \mu_3 &< 0 \text{ (bull correction state)}, \\ \mu_4 &> 0 \text{ (bull state)}; \\ \sigma^2_{s_t} & \quad \text{No restriction} \end{aligned}$$

- Regimes combine states as follows:
  - $s_t = 1, 2$ bear regime
  - $s_t = 3, 4$ bull regime

MS-4 Model allowing Bull Corrections and Bear Rallies, cont.

$$\text{Transition matrix } P = \begin{pmatrix} p_{11} & p_{12} & 0 & p_{14} \\ p_{21} & p_{22} & 0 & p_{24} \\ p_{31} & 0 & p_{33} & p_{34} \\ p_{41} & 0 & p_{43} & p_{44} \end{pmatrix}$$

The unconditional probabilities associated with P are defined as π_i, $i = 1, ..., 4$

We impose the following conditions on long-run returns in each regime[1],

$$\begin{aligned} E[r_t|\text{bear regime}, s_t = 1, 2] &= \frac{\pi_1}{\pi_1 + \pi_2}\mu_1 + \frac{\pi_2}{\pi_1 + \pi_2}\mu_2 < 0 \\ E[r_t|\text{bull regime}, s_t = 3, 4] &= \frac{\pi_3}{\pi_3 + \pi_4}\mu_3 + \frac{\pi_4}{\pi_3 + \pi_4}\mu_4 > 0. \end{aligned}$$

[1] Since investors cannot identify states with probability 1, modeling investors' expected returns at each point is beyond the scope of this paper. Regimes or states may have negative expected returns for some period for a variety of reasons such as changes in risk premiums due to learning following breaks, different investment horizons, etc.

Bayesian Estimation

MS-K

$$
\begin{aligned}
r_t|s_t &\sim N(\mu_{s_t}, \sigma^2_{s_t}) \\
p_{ij} &= p(s_t = j | s_{t-1} = i), \quad i = 1, ..., K, \;\; j = 1, ..., K.
\end{aligned}
$$

- 3 groups of parameters $M = \{\mu_1, ..., \mu_K\}$, $\Sigma = \{\sigma^2_1, ..., \sigma^2_K\}$, and the elements of the transition matrix P
- $\theta = \{M, \Sigma, P\}$ and given data $I_T = \{r_1, ..., r_T\}$
- Augment the parameter space to include the states $S = \{s_1, ..., s_T\}$
- Conditionally conjugate priors
  $\mu_i \sim N(m_i, n^2_i)$, $\sigma^{-2}_i \sim G(v_i/2, s_i/2)$ and
  each row of P follows a Dirichlet distribution

Bayesian Estimation

- Gibbs sampling from the full posterior $p(\theta, S|I_T)$ by sequentially sampling
  - $S|M, \Sigma, P$
    - Joint draw of S following Chib (1996) (forward-backward smoother)
  - $M|\Sigma, P, S$
    - Standard linear model results
  - $\Sigma|M, P, S$
    - Standard linear model results
  - $P|M, \Sigma, S$
    - Dirichlet draw
- Drop any draws that violate identification constraints

Bayesian Estimation

- Discard an initial set of draws to remove any dependence from startup values,
- Remaining draws $\{S^{(j)}, M^{(j)}, \Sigma^{(j)}, P^{(j)}\}_{j=1}^{N}$ are collected
- Simulation consistent estimates can be obtained as sample averages of the draws.

$$\frac{1}{N}\sum_{j=1}^{N}\mu_k^{(j)} \xrightarrow{N\to\infty} E[\mu_k|I_T], \quad \frac{1}{N}\sum_{j=1}^{N}\sigma_k^{(j)} \xrightarrow{N\to\infty} E[\sigma_k|I_T]$$

- Byproduct of estimation is smoothed state estimates

$$p(\widehat{s_t = i|}I_T) = \frac{1}{N}\sum_{j=1}^{N}\mathbf{1}_{s_t=i}(S^{(j)})$$

for $i = 1, ..., K$.

- Forecasts and estimates account for parameter and regime uncertainty

Model Comparison

- Marginal likelihood for model $\mathcal{M}_i$ is defined as

$$p(r|\mathcal{M}_i) = \int p(r|\mathcal{M}_i, \theta) p(\theta|\mathcal{M}_i) d\theta$$

  - $p(\theta|\mathcal{M}_i)$ is the prior and

$$p(r|\mathcal{M}_i, \theta) = \prod_{t=1}^{T} f(r_t | I_{t-1}, \theta)$$

is the likelihood which has S integrated out according to

$$f(r_t|I_{t-1}, \theta) = \sum_{k=1}^{K} f(r_t|I_{t-1}, \theta, s_t = k) p(s_t = k|\theta, I_{t-1}).$$

Bayes Factors

Chib (1995) estimate of the marginal likelihood

$$\widehat{p(r|\mathcal{M}_i)} = \frac{p(r|\mathcal{M}_i, \theta^*)p(\theta^*|\mathcal{M}_i)}{p(\theta^*|r, \mathcal{M}_i)}$$

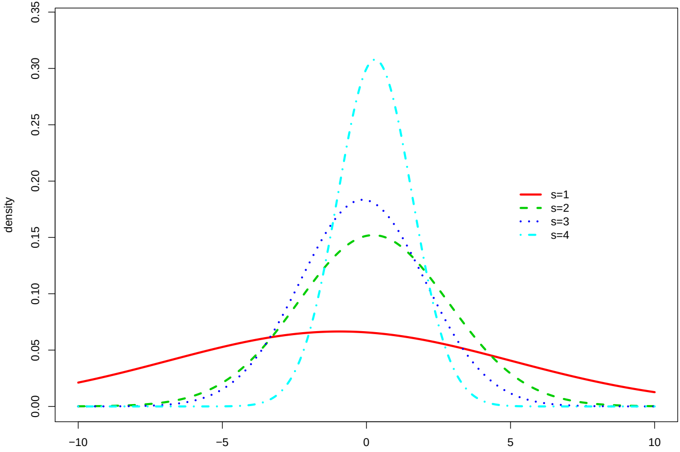
where θ^* is a point of high mass in the posterior pdf.
A log-Bayes factor between model $\mathcal{M}_i$ and $\mathcal{M}_j$ is defined as

$$\log(BF_{ij}) = \log(p(r|\mathcal{M}_i)) - \log(p(r|\mathcal{M}_j)).$$

Kass and Raftery (1995) suggest interpreting the evidence for $\mathcal{M}_i$ versus $\mathcal{M}_j$ as

- $0 \leq \log(BF_{ij}) < 1$ not worth more than a bare mention
- $1 \leq \log(BF_{ij}) < 3$ positive
- $3 \leq \log(BF_{ij}) < 5$ strong
- $\log(BF_{ij}) \geq 5$ very strong

State Densities



MS-4-State Model Posterior Estimates

	mean	95% DI
μ_1	-0.94	(-1.50, -0.45)
μ_2	0.23	(0.04, 0.43)
μ_3	-0.13	(-0.31, -0.01)
μ_4	0.30	(0.22, 0.38)
σ_1	6.01	(5.41, 6.77)
σ_2	2.63	(2.36, 3.08)
σ_3	2.18	(1.94, 2.39)
σ_4	1.30	(1.20, 1.37)
μ_1/σ_1	-0.16	(-0.25, -0.07)
μ_2/σ_2	0.09	(0.02, 0.17)
μ_3/σ_3	-0.06	(-0.14, -0.01)
μ_4/σ_4	0.23	(0.17, 0.31)

$$\text{Transition matrix } P = \begin{pmatrix} 0.921 & 0.076 & 0 & 0.003 \\ 0.015 & 0.966 & 0 & 0.019 \\ 0.010 & 0 & 0.939 & 0.051 \\ 0.001 & 0 & 0.039 & 0.960 \end{pmatrix}$$

Unconditional State Probabilities

	mean	95% DI
π_1	0.070	(0.035, 0.117)
π_2	0.157	(0.073, 0.270)
π_3	0.304	(0.216, 0.397)
π_4	0.469	(0.346, 0.579)

- Unconditional prob of bear $\pi_1 + \pi_2 = 0.227$
- Unconditional prob of bull $\pi_3 + \pi_4 = 0.773$

Some Posterior Regime Statistics for Bear Markets

	MS-2	MS-4
variance from Var$(E[r_t\|s_t]\|s_t = 1,2)$	0.00	0.31
variance from $E[$Var$(r_t\|s_t)\|s_t = 1,2]$	19.6	16.1
skewness	0	-0.42
kurtosis	3	5.12

Analogous results for bull markets are in the paper

Posterior Statistics for Regimes and States

	posterior mean
Bear mean	-0.13
Bear duration	77.8
Bear cumulative return	-9.94
Bear stdev	4.04
Bull mean	0.13
Bull duration	256.0
Bull cumulative return	33.0
Bull stdev	1.71
s=1: cumulative return	-12.4
s=2: cumulative return	7.10
s=3: cumulative return	-2.13
s=4: cumulative return	7.88
s=1: duration	13.5
s=2: duration	31.2
s=3: duration	17.9
s=4: duration	27.2

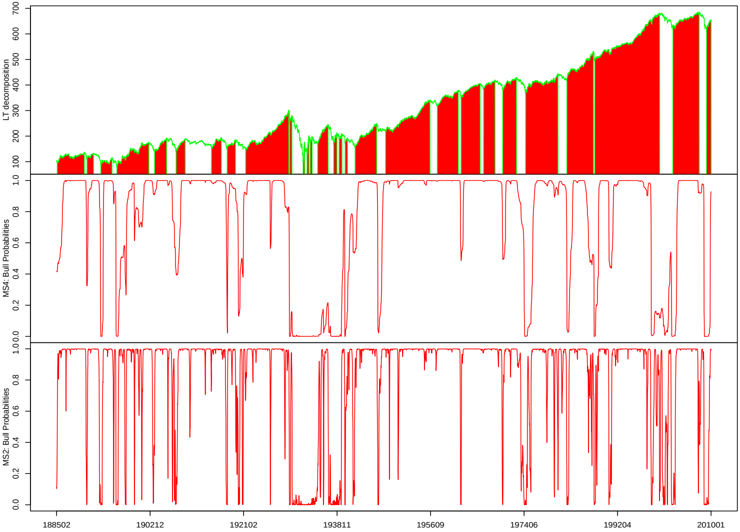
Posterior statistics for various population moments

Log Marginal Likelihoods: Alternative Models

Model	$\log f(Y \mid \text{Model})$	log-Bayes Factor
Constant mean with constant variance	-14924.1	1183.7
Constant mean with 4-state i.i.d variance	-14256.7	516.3
MS-2-state mean with 4-state i.i.d. variance	-14009.5	269.1
MS-2-state mean with coupled MS 2-state variance	-13903.3	162.9
MS-4-state mean with coupled MS 2-state variance	-13849.9	109.5
MS-4-state mean with coupled MS 4-state variance	-13740.4	

- MS-4 strongly dominates all alternatives

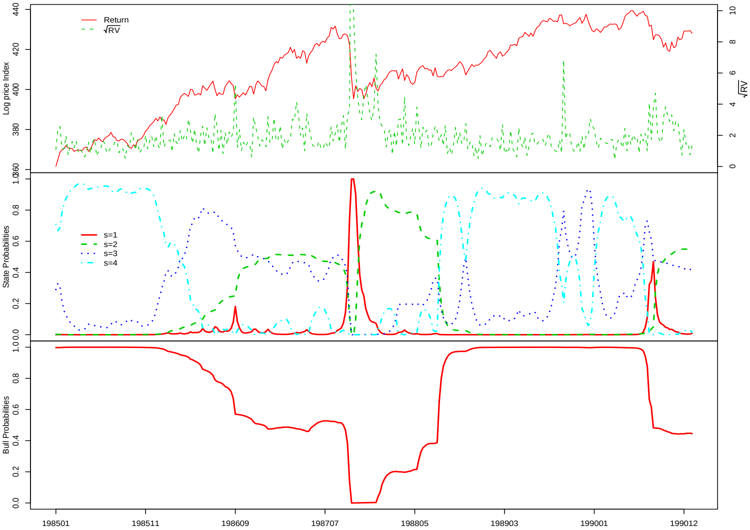
LT dating algorithm, MS-4 and MS-2 Smoothed Probabilities



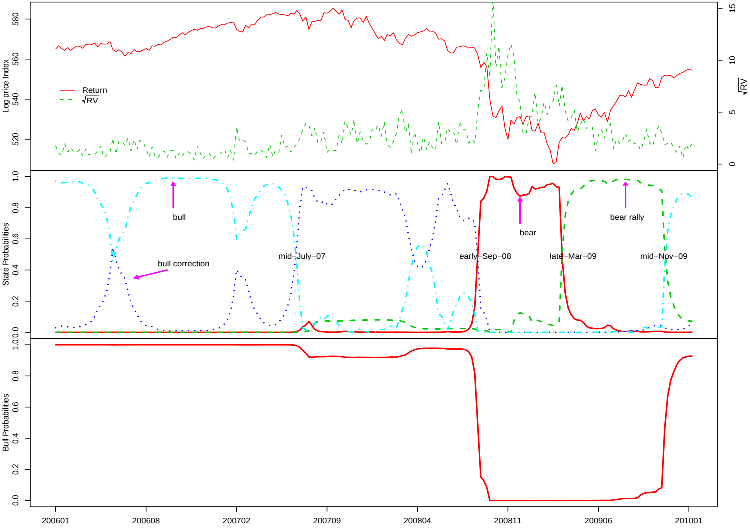
MS-4, 1927-1939

MS-4, 1980-1985

MS-4, 1985-1990



MS-4, 2006-2010



Summary

- Sorting of states and regimes is precise
- Bull and bear regime are heterogeneous
  - Bear regimes feature recurrence of states 1 (bear) and 2 (bear rally)
  - Bull regimes feature recurrence of states 3 (bull correction) and 4 (bull)
- Most turning points occur through bear rally or bull correction
  - $p(s_t = 2 | s_{t+1} = 4, s_t = 1 \text{ or } 2) = 0.9342$
  - $p(s_t = 3 | s_{t+1} = 1, s_t = 3 \text{ or } 4) = 0.8663$
- Asymmetric transitions both within regimes and between regimes
  - Bull corrections revert to bull more often than bear rallies bounce back to bear

Predictive Density of Returns

The predictive density for future returns based on current information at time $t-1$ is computed as

$$p(r_t|I_{t-1}) = \int f(r_t|\theta, I_{t-1})p(\theta|I_{t-1})d\theta$$

which involved integrating out both state and parameter uncertainty using the posterior distribution $p(\theta|I_{t-1})$. From the Gibbs sampling draws $\{S^{(j)}, M^{(j)}, \Sigma^{(j)}, P^{(j)}\}_{j=1}^{N}$ based on data I_{t-1} we approximate the predictive density as

$$p\widehat{(r_t|I_{t-1})} = \frac{1}{N}\sum_{i=1}^{N}\sum_{k=0}^{K} f(r_t|\theta^{(i)}, I_{t-1}, s_t = k)p(s_t = k|s_{t-1}^{(i)}, \theta^{(i)})$$

where $f(r_t|\theta^{(i)}, I_{t-1}, s_t = k)$ follows $N(\mu_k^{(i)}, \sigma_k^{2(i)})$ and $p(s_t = k|s_{t-1}^{(i)}, \theta^{(i)})$ is the transition probability.

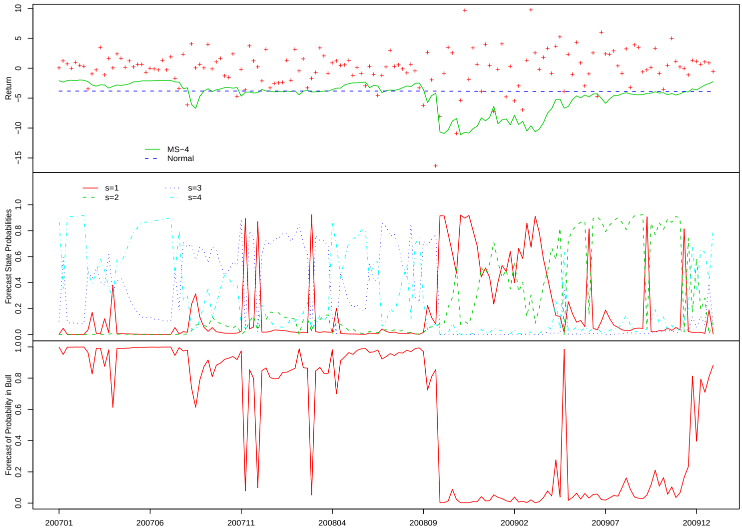
Value-at-Risk (VaR)

- $\text{VaR}_{(\alpha),t}$ is 100α percent quantile for the distribution of r_t given I_{t-1}.
- Compute $\text{VaR}_{(\alpha),t}$ from the predictive density MS-4 model as

$$p(r_t < \text{VaR}_{(\alpha),t} | I_{t-1}) = \alpha.$$

- Given a correctly specified model, the prob of a return of $\text{VaR}_{(\alpha),t}$ or less is α.
- Comparison with $N(0, s^2)$ where s^2 is the sample variance using I_{t-1}.

Out-of-Sample VaR and Probability of Bull



Recent State of the Aggregate Market

Data to close of Jan. 20, 2010

- $p(s_t = 1|I_t) = 0.0008$ bear
- $p(s_t = 2|I_t) = 0.0714$ bear rally
- $p(s_t = 3|I_t) = 0.0633$ bull correction
- $p(s_t = 4|I_t) = 0.8645$ bull

Following week, transition to a bull market correction

Summary

- Propose a new 4-state Markov-switching (MS) model for stock returns
- Offers richer characterizations of market dynamics
  - Two states govern the bear regime
  - Two states govern the bull regime
- Heterogeneous intra-regime dynamics
  - Allow for bear rallies and bull corrections without a regime change
  - Realized bull and bear regimes can be different over time
  - Conditional autoregressive heteroskedasticity in a regime
- Probability statements on regimes and future returns available
- Our model strongly dominates other alternatives
- Estimated bull and bear regimes match traditional sorting algorithms
- Bull corrections and bear rallies empirically important
- Out-of-sample forecasts of turning points
- VaR predictions sensitive to market regimes