Discrete Dynamic Strategies in Affine Models

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Joint work with F. Angelini

Setting the problem

- Hedging a contingent claim H with maturity T with a strategy $\vartheta_{t_k}, k = 0, \ldots, N-1$ in a risky asset S.
- Start from a value c to hedge the payoff H. The hedging error of the strategy is

$$\varepsilon(\vartheta, c) = H - c/P(0,T) - \sum_{k=1}^{N} \vartheta_{t_k} \Delta \bar{S}_{t_k}.$$

- The goal is to compute mean and variance of $\varepsilon(\vartheta, c)$ for given H, c, ϑ

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General Setting

- Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \leq t < \infty}, P)$ be a probability space. $X = (X_t)$ is a time-homogeneous affine process with state space $D \subset \mathbb{R}^d$
- $y = \ln(S)$ is one component of X.
- Other possible components of X
  - ... stochastic volatility
  - ... stochastic interest rate

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Affine Processes

- A time - homogeneous Markov process X is affine if

$$E_t\left[\exp\left(u \cdot X_T\right)\right] = \exp\left(\alpha(u,t,T) + \beta(u,t,T) \cdot X_t\right)$$

  where $\alpha(u,t,T)$, $\beta(u,t,T)$ satisfy a set of Riccati equations and are analytic on a domain $U \subset \mathbb{C}^d$, for $t \in [0,T]$
- SV model: $X = (y,v)$, $u = (u_1,u_2)$ and $\beta = (u_1, \beta_2)$ (Heston)
- Levy: $X = (y)$, $\beta = u$. (BS and ...)
- SIR model: $X = (r)$, β solves Riccati (Vasicek, CIR)
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Integral representation of payoffs

- Write the payoff of a contingent claim written on S, maturity T, as

$$H = \int_{R-i\infty}^{R+i\infty} e^{zy_T} p(z) dz$$

where $y = \ln(S)$

- $$H = (S_T - K)^+ = \frac{1}{2\pi i} \int_{R-i\infty}^{R+i\infty} e^{zy_T} \frac{K^{1-z}}{z(z-1)} dz$$

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Hedging Strategies

- If Q is a pricing measure, P_t the price at time t of a contingent claim is
$$P_t = E_t^Q[H]$$
(assume here $r = 0$ deterministic)
- If the payoff H has an integral representation
$$P_t = E_t^Q[\int_{R-i\infty}^{R+i\infty} e^{zy_T} p(z)dz]$$
- Using Fubini
$$\begin{aligned} P_t &= \int_{R-i\infty}^{R+i\infty} E_t^Q[e^{z1_y \cdot X_T}]p(z)dz \\ &= \int_{R-i\infty}^{R+i\infty} \exp\left(\bar{\alpha}(z1_y, t, T) + \bar{\beta}(z1_y, t, T) \cdot X_t\right) p(z)dz \end{aligned}$$

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Hedging Strategies in SV

- Consider hedging strategy ϑ of the form

$$\vartheta_{t_k} = \int_{R-i\infty}^{R+i\infty} \vartheta_{t_k}(z) p(z) dz,$$

with

$$\vartheta_{t_k}(z) = \exp\left(A(z, t_k) + B_1(z, t_k) y_{t_k} + B_2(z, t_k) v_{t_k}\right)$$

Examples for Heston Model

- Model Delta Δ_t^H: $A = \ln(z) + \bar{\alpha}(z1_y, t, T)$, $B_1 = z - 1$ $B_2 = \bar{\beta}_2(z1_y, t, T)$ Why?
- Continuous Time Local Optimal Strategy

$$\Theta_t^* = \Delta_t^H + \frac{\rho\sigma}{S_t}\mathcal{V}_t^H$$

$A = \ln(z + \rho\sigma\beta_2(z1_y, t, T)) + \alpha(z1_y, t, T)$, $B_1 = z - 1$
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- BS Delta, with constant volatility σ
$A = \ln(z) + \bar{\alpha}^{bs}(z1_y, t, T)$, $B_1 = z - 1$, $B_2 = 0$
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$$\sigma_t^2 = \frac{1}{T-t}E_t\int_t^T v_s ds$$

$A = \ln(z) + something$, $B_1 = z - 1$, B_2

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Hedging error

- The hedging error of such a strategy for a claim w.i.r. can be written as

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\varepsilon(\vartheta, c) &= H - c - \sum_{k=1}^{N} \vartheta_{t_k} \Delta S_k = \\
&= \int_{R-i\infty}^{R+i\infty} \left(e^{z y_T} - \sum_{k=1}^{N} \vartheta_{t_k}(z) \Delta S_k \right) p(z) dz - c
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Expected value of hedging error

- $$E[e^{zy_T}] = \phi(z1_y, X_0, 0, T)$$

- $$\begin{aligned} E[\vartheta_{t_k}(z)\Delta S_k] &= e^A E\left[e^{(z-1)y_{t_{k-1}}+B_2 v_{t_{k-1}}}(e^{y_{t_k}} - e^{y_{t_{k-1}}})\right] \\ &= e^A(\phi_2((z-1, B_2), (1,0), X_0, 0, t_{k-1}, t_k) - \\ &\quad \phi_2((z, B_2), X_0, 0, t_{k-1})) \end{aligned}$$

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Main results

- Semi-explicit formulas for expected value and variance of hedging error, for any number of trading dates, for any claim w.i.r. and any hedging strategy of the described form in affine models
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$$E[\varepsilon(\vartheta,0)] = \int_{R-i\infty}^{R+i\infty} e(z)p(z)dz$$
$$E[\varepsilon(\vartheta,0)^2] = \int_{R-i\infty}^{R+i\infty}\int_{R-i\infty}^{R+i\infty} V(y,z)p(y)p(z)dydz,$$

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Heston model

$$dy_t = (\mu - \frac{1}{2}v_t)dt + \sqrt{v_t}dW_t^1$$
$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dW_t^2$$

with $d < W_t^1, W_t^2 >= \rho dt$
$v_0 = 0.05$, $\mu = 0$, $\theta = 0.05$, $\kappa = 3$, $\sigma = 0.5$
$y_0 = \log(S_0) = \log(100)$.
Feller condition $2\kappa\theta > \sigma^2$ does not hold!

Heston model

- European ATM call options with maturity $T = 0.5$
- model Delta (delta),
- Black-Scholes Delta with expected volatility (deltabsev),
- variance-optimal in continuous time ($\theta^{\star}$),
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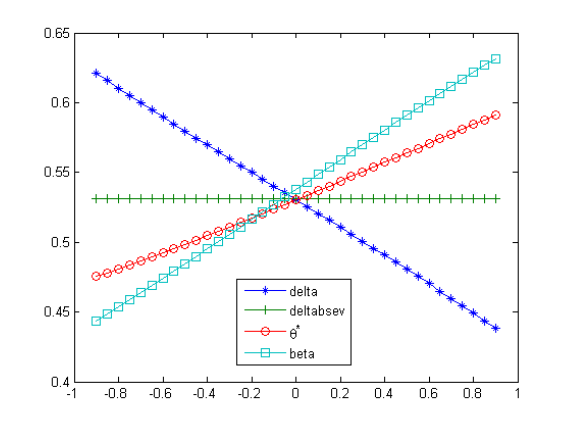
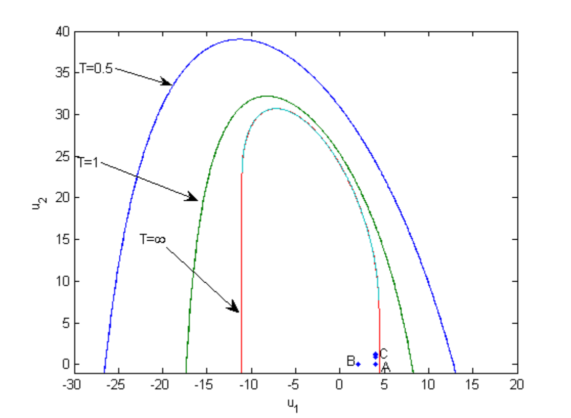
Hedge ratios as functions of ρ



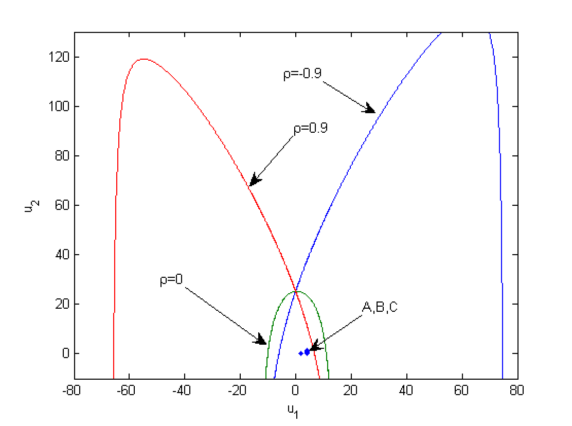
Back to Intro

Moments in Heston model

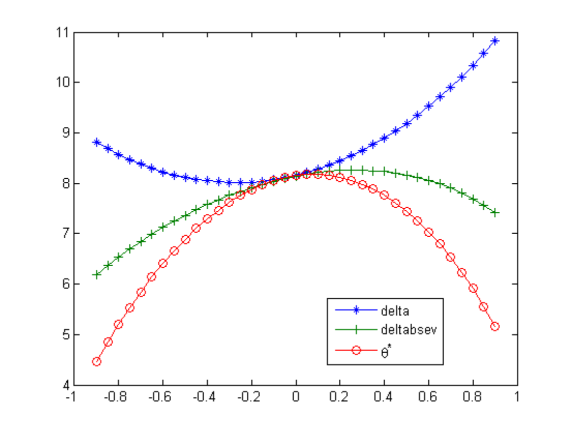
$$M(T) = \left\{(u_1, u_2) \in \mathbb{R}^2 \mid E\left[e^{u_1 y_T + u_2 v_T}\right] < \infty\right\}$$



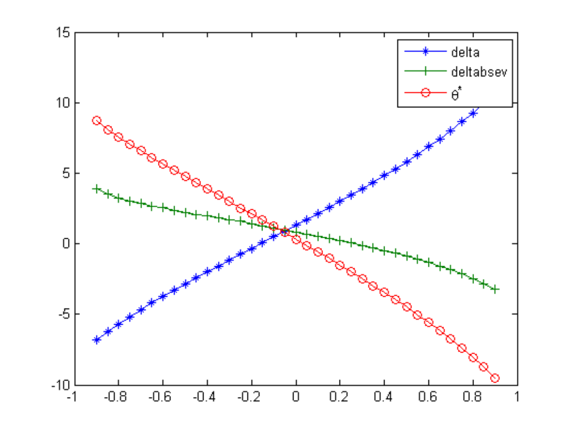
$M(1)$ and ρ



Variances of hedging strategies as functions of ρ



Sensitiveness of variance as functions of ρ



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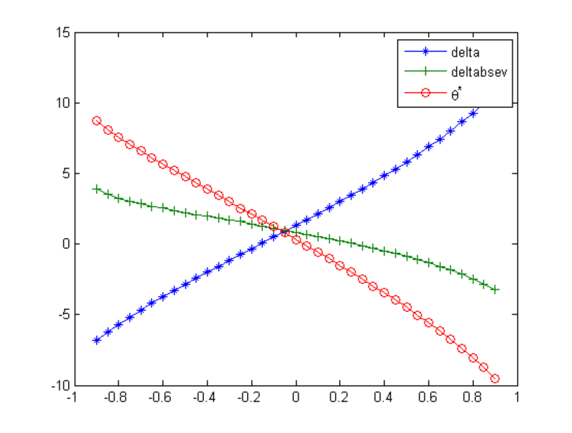
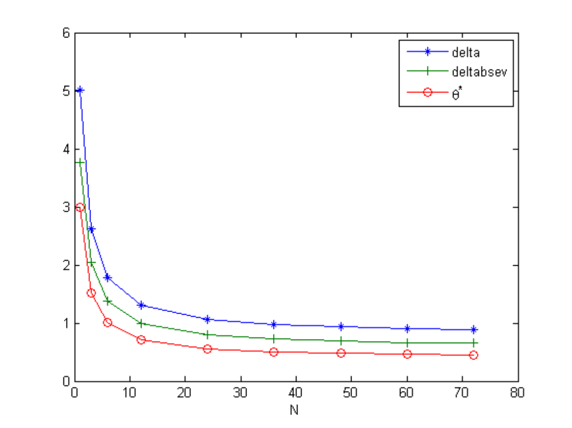


Variances as functions of the number of hedging intervals N



Conclusions

- An efficient way to compute moments of hedging errors of different type of strategies for claims w.i.r. and for a wide class of models
- A measure for the performances of hedging strategies in different settings, for instance under model mispecification
- In the paper: Proofs, formulas, CIR, comparisons to Monte Carlo...
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Technical Conditions

- $$2 1_y \in U \cap \mathbb{R}^d \Rightarrow \bar{S}_t \in L^2(P)$$

  DFS (2003)

- $$2R1_y \in U \cap \mathbb{R}^d$$
  $$\Rightarrow E\left[e^{2Ry_T}\right] < \infty$$
  $$\Rightarrow H \in L^2(P)$$

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Back

Previous results

- Hubalek, Kallsen and Krawczyk (2006), compute optimal variance in Levy case, discrete and continuous time
- Cerny, Kallsen (2008), Kalssen, Pauwels (2009) and Kallsen, Vierthauer (2009) compute optimal variance in SV models (Heston) in continuous time
- Angelini, Herzel (2009) compute variance for sub-optimal strategies in Levy processes in discrete time
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- The price at time t

$$P_t = \int_{R-i\infty}^{R+i\infty} \exp\left(\bar{\alpha}(z1_y, t, T) + \bar{\beta}(z1_y, t, T) \cdot X_t\right) p(z)dz$$

SV model $\bar{\beta}(z1_y, t, T) = (z, \bar{\beta}_2(z1_y, t, T))$

- The Delta at time t

$$\Delta_t = \int_{R-i\infty}^{R+i\infty} ze^{-y_t} \exp\left(\bar{\alpha}(z1_y, t, T) + \bar{\beta}(z1_y, t, T) \cdot X_t\right) p(z)dz$$
$$= \int_{R-i\infty}^{R+i\infty} z \exp\left(\bar{\alpha}(z1_y, t, T) + (\bar{\beta}(z1_y, t, T) - 1_y) \cdot X_t\right) p(z)dz$$

Back

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