Transition from Rotating Waves to Modulated Rotating Waves on a Sphere

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It is now believed that the presence of spiral waves and scroll waves in the cardiac tissue is the cause of the cardiac arrhythmia that can lead to ventricular fibrillation. There is a well-developed mathematical theory for planar spiral waves that appear in excitable media modelled by reaction-diffusion systems on $\mathbb{R}^2$ with Euclidean symmetry.
The heart can be approximated with a sphere as well. Therefore, in this talk, we consider one-parameter dependent reaction-diffusion systems on a sphere in $\mathbb{R}^3$, which are equivariant under the group $SO(3)$ of all rigid rotations. Two main types of spatial-temporal patterns that can appear in such systems are rotating waves (equilibria in a co-rotating frame) and modulated rotating waves (periodic solutions in a co-rotating frame).
The transition from rotating waves to modulated rotating waves on a sphere is explained by a supercritical Hopf bifurcation from a rotating wave, $SO(3)$-symmetry and finite-dimensional center manifold reduction. Using properties of the adjoint representations of $SO(3)$ and its Lie algebra $so(3)$, and of the exponential map $e\colon so(3) \to SO(3)$, as well as Baker-Campbell-Hausdorff formula in $so(3)$, we obtain reduced differential equations on $so(3)$ and then, formulae for primary frequency vectors and the associated periodic parts of the modulated rotating waves obtained by a supercritical Hopf bifurcation from a rotating wave. As a consequence, there are three types of motions on a sphere for the tips of these modulated rotating waves. These types of motions are visualized using Maple.
For a resonant Hopf bifurcation from a rotating wave in two-parameter dependent $SO(3)$-equivariant reaction-diffusion systems on a sphere, we obtain a branch of modulated rotating waves with primary frequency vectors orthogonal to the frequency vector of the rotating wave undergoing Hopf bifurcation.