THE FIELDS INSTITUTE
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ABSTRACTS 1.2

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On one Roiter's conjecture and partial order relation forms (25-30)

Let $(M, \leq)$ is finite partially ordered set (poset) with the set of elements $M = \{m_1, \ldots, m_n\}$ and $\chi : M \times M \mapsto \{0; 1\}$ is characteristic function of partial order relation on the M, i.e. $\chi(m_i, m_j) = 1$ iff $m_i \leq m_j$. Then, following to [2] we can introduce the quadratic form of partial order relation by

$$\chi_M(x) = \chi_M(x_1, \ldots, x_n) = \sum_{i,j=1}^{n} \chi(m_i, m_j) x_i x_j$$

which we call characteristic form for poset $(M, \leq)$.

As well as for Tits form of posets (introduced by Yu. Drozd — cf. [1]) the criterions for finite (or tame) representation type of poset may be formulated in the terms of properties of χ_M, exactly, in the terms [2] of the norm of relation $||(M, \leq)|| = \min\{\chi_M(x) | x \in K\}$ where $K = \{x | \sum_{i=1}^{n} x_i = 1 \ \ \textit{and the all} \ \ x_i \geq 0\}$. Namely, $(M, \leq)$ has finite (resp., tame) representation type iff $||(M, \leq)|| > \frac{1}{4}$ (resp., iff $||(M, \leq)|| \geq \frac{1}{4}$).

Following to [3] the posets $(M, \leq)$ is said to be *ρ-exact* iff its norm may be achived only on exact vectors (i.e. vectors with non-zero coordinates) from the simplex K. For example, posets from classical lists of critical and hypercritical posets (given by M.Kleiner and L.Nazarova respectively — cf. [1]) are ρ-exact.

Also, as in [3] the poset S is said to be *fence* if S is a union of t nonintersective chains (i.e full ordered subposets) $Z_1, \ldots, Z_t$, where $||Z_i|| \geq 2 \ \ (i = \overline{1, t}), \ \ t > 1; \ \ \min(Z_i) < \max(Z_{i+1}), \ \ i = \overline{1, t-1}$ and there are t¿no other comparisons between elements of different chains.

In [3] A.V.Roiter found all fencies (that he called *uniform*) which may be ρ-exact and formulated conjecture, asserting that finite poset is ρ-exact if and only if it is disjoint union (i.e. cardinaly sum) of some chains and (or) some uniform fencies.

In this talk the finite partially ordered sets for which its (oriented) Hasse graph does not containe a round (for example, if the last one is a simple connected) are considered. For such sets the canonical Z-equivalence between characteristic form of the partial order relation and of the Tits form of corresponding Hasse graph is constructed and is proved. As a corollary, in this case we find a very simple and effective criterions for positive and for non-negative definiteness of characteristic forms. Z-equivalence theorem is obtained as a special case of the more general and deep analogous fact justified for arbitrary finite partially ordered set M (namely, asserting the canonical Z-equivalence of certain bilinear form (which is constructed by the natural way from corresponding Hasse graph $\Gamma = \Gamma(M)$) to the non-symmetrical bilinear Tits form of the graph Γ^0 which is anti-isomorphic to the last one).

Using these results the full and explicit description of ρ-exact partial ordered sets (introduced by A.V.Roiter) is obtained. Thus we prove (see [4], [5]) the above-mentioned Roiter's conjecture from [3].

References

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