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Stable endomorphism algebras of modules over special biserial algebras

This is joint work with A. Zimmermann.

Let A be a k -algebra with k an algebraically closed field. By $D^b(A)$ we denote the derived category of bounded complexes of finite-dimensional A -modules.

The stable endomorphism algebra $\underline{\text{End}}_A(M)$ of an A -module M is defined as the endomorphism algebra $\text{End}_A(M)$ modulo all endomorphisms which factor through projective A -modules.

Our aim is to study stable endomorphism algebras of modules without self-extensions over one of the main classes of tame algebras, the so-called special biserial algebras. Our main result is the following:

Theorem. *Let A be a special biserial algebra, and let M be an A -module. If $\text{Ext}_A^1(M, M) = 0$, then $\underline{\text{End}}_A(M)$ is a gentle algebra.*

Gentle algebras are a narrow subclass of the class of special biserial algebras, so one should regard this outcome as very surprising.

This theorem has the following consequence. By $T[i]$ we denote the usual shift of a complex T in $D^b(A)$ by i degrees.

Corollary. *Let A be a finite-dimensional gentle algebra, and let T be a complex in $D^b(A)$. If $\text{Hom}_{D^b(A)}(T, T[1]) = 0$, then $\text{End}_{D^b(A)}(T)$ is a gentle algebra. In particular, any algebra B , which is derived equivalent to A , is gentle, and (up to Morita equivalence) there are only finitely many such algebras B .*