

WEAK HOPF ALGEBRAS AND SINGULAR SOLUTIONS OF QUANTUM YANG-BAXTER EQUATION

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ABSTRACT. We investigate a generalization of Hopf algebra $\mathfrak{sl}_q(2)$ by weakening the invertibility of the generator K , i.e. exchanging its invertibility $KK^{-1} = 1$ to the regularity $K\overline{K}K = K$. This leads to a weak Hopf algebra $w\mathfrak{sl}_q(2)$ and a J -weak Hopf algebra $v\mathfrak{sl}_q(2)$ which are studied in detail. It is shown that the monoids of group-like elements of $w\mathfrak{sl}_q(2)$ and $v\mathfrak{sl}_q(2)$ are regular monoids, which supports the general conjecture on the connection between weak Hopf algebras and regular monoids. Moreover, from $w\mathfrak{sl}_q(2)$ a quasi-braided weak Hopf algebra \overline{U}_q^w is constructed and it is shown that the corresponding quasi- R -matrix is regular $R^w \hat{R}^w R^w = R^w$.

1. INTRODUCTION

The concept of a weak Hopf algebra as a generalization of a Hopf algebra was introduced and its characterizations and applications were studied in other works of the author's. A k -bialgebra¹ $H = (H, \mu, \eta, \Delta, \varepsilon)$ is called a *weak Hopf algebra* if there exists $T \in \text{Hom}_k(H, H)$ such that $id * T * id = id$ and $T * id * T = T$ where T is called a *weak antipode* of H . This concept also generalizes the notion of the left and right Hopf algebras.

The first aim of this concept is to give a new sub-class of bialgebras which includes all of Hopf algebras such that it is possible to characterize this sub-class through their monoids of all group-like elements. It was known that for every regular monoid S , its semigroup algebra kS over k is a weak Hopf algebra as the generalization of a group algebra.

The second aim is to construct some singular solutions of the quantum Yang-Baxter equation (QYBE) and research QYBE in a larger scope. On this hand, a quantum quasi-double $D(H)$ for a finite dimensional cocommutative perfect weak Hopf algebra with invertible weak antipode was built and it was verified that its quasi- R -matrix is a regular solution of the QYBE. In particular, the quantum quasi-double of a finite Clifford monoid as a generalization of the quantum double of a finite group was derived.

In this paper, we will construct two weak Hopf algebras in the other direction as a generalization of the quantum algebra $\mathfrak{sl}_q(2)$. We show that $w\mathfrak{sl}_2(q)$ possesses a quasi- R -matrix which becomes a singular (in fact, regular) solution of the QYBE, with a parameter q . In this reason, we want to treat the meaning of $w\mathfrak{sl}_q(2)$ and

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¹In this paper, k always denotes a field.

its quasi- R -matrix just as $\mathfrak{sl}_q(2)$. It is interesting to note that $w\mathfrak{sl}_q(2)$ is a natural and non-trivial example of weak Hopf algebras.

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