THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

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Prime ideals in orbit algebras (25-30)

We study the geometry of exceptional curves (in the sense of Lenzing) over arbitrary fields. Exceptional curves are the parameter curves for separating tubular families for tame hereditary algebras and for canonical algebras Λ (in the sense of Ringel and Crawley-Boevey). We restrict to *homogeneous* exceptional curves, that is, Λ is a tame hereditary bimodule algebra. We show that a large class of these curves admits (non-commutative) projective coordinate algebras R where each prime ideal of height one is principal, generated by a homogeneous normal element. Let $\mathbb{X}$ be a homogeneous exceptional curve, and assume that there is an automorphism on $\mathrm{coh}(\mathbb{X})$ acting transitively on the set of isoclasses of line bundles and fixing the points. Such an automorphism exists in many cases, namely, if the underlying tame bimodule is of type $(1,4)$ or $(4,1)$ or non-simple of type $(2,2)$. Fix some line bundle L, define $L(n) = \sigma^n(L)$ for each $n \in \mathbb{Z}$ and – as variation of the preprojective algebras in the sense of Baer, Geigle and Lenzing – let R be the orbit algebra $\bigoplus_{n\in\mathbb{Z}} \mathrm{Hom}(L, L(n))$. Then the correspondence between points and prime ideals is given by universal exact sequences:

Theorem: Let S be a simple sheaf concentrated in the point $x \in \mathbb{X}$. Let $e = e(x)$ be the multiplicity, let $f = \deg(S)$ the degree of S, $d = ef$ and let

$$0 \longrightarrow L \xrightarrow{\pi} L(d) \longrightarrow S^e \longrightarrow 0$$

be the S-universal exact sequence of L. (1) The homogeneous element π is normal, that is, $R\pi = \pi R$. (2) $P = R\pi$ is a homogeneous prime ideal. (3) P is a completely prime if and only if $e = 1$. Each homogeneous prime ideal in R of height one is of this form.
As application we give a criterion when R is almost commutative in the sense that homogeneous elements commute up to multiplication with units. We discuss further properties of these orbit algebras and present explicit examples.