THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

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The irreducuble Components of Lusztig's
Nilpotent Variety and Crystal Bases (50-60)

Let Q be a quiver without loops. We denote the quantized version of the enveloping algebra of the negative part of the corresponding Kac-Moody-Lie algebra by U^-. Lusztig has defined varieties $\mathcal{R}(\Pi(Q);d)_0$, also called Lusztig's nilpotent varieties, consisting of nilpotent representations of the preprojective algebra $\Pi(Q)$ of Q of dimension vector d. It was shown by Kashiwara and Saito ([1]) that the irreducible components of the various $\mathcal{R}(\Pi(Q);d)_0$, where d runs through all possible dimension vectors of Q, form the crystal of U^-. The principal aim of this talk is to compute the number of irreducible components of $\mathcal{R}(\Pi(Q);d)_0$ using so- called nilpotent class representations (nc-representation) of Q with dimensions vector d. Informally a nc- representation assigns to Q and d certain nilpotent classes, so that the generic nc-representations are in natural bijection with the irreducible components of $\mathcal{R}(\Pi(Q);d)_0$. Furthermore we mention certain applications: the number of irreducible components in the intersection of a nilpotent class with the strictly upper- trinagular matrices (in the general linear group) can be computed using nc-representations (see [2] and [3]). Finally we relate $\mathcal{R}(\Pi(Q);d)_0$ for Q affine and d the imaginary root to the exceptional locus in the Kleinian singularity (see also [CS]).

References: [1] M. Kashiwara, Y. Saito: Geometric construction of crystal bases. Duke Mat. J. 89 (1997), no. 1, 9 - 36 [2] W. Borho, R. MacPherson: Partial resolution of nilpotent varieties. Analysis and topology on singular spaces, II, III (Luminy 1981), 23 - 74, Asterisque 101 - 102, Soc. Math. France, 1983 [3] N. Spaltenstein: Classes Unipotentes et Sous- groupes de Borel. LNM 946, Springer Verlag 1982 [4] H. Cassens, P. Slodowy: On Kleinian singularities and quivers. Singularities (Oberwolfach 1996), 263 - 288, Progr. Math. 162, Birkhäuser, 1998