Mathematical Ethics:
A Problem Based Approach

Douglas Henrich
Iroquois Ridge High School

November 2, 2011

Contents

Chapter 1

Introduction

1.1 Abstract

This paper addresses mathematics instruction based on the dual concepts of mathematical ethics and ethical mathematics. The paper develops an understanding of mathematical ethics based on the necessity of social responsibility and student engagement. Using a problem-based model the somewhat controversial conclusion is reached that the only inherent ethics within mathematics is the requirement of consistency.

1.2 Introduction

Before I became a teacher I worked for almost twenty-five years within the policing and private security sector. At times, I had to arrest individuals and was always troubled by the question, “What has led an individual to make the wrong choices in life that has led them to this moment where I now have to arrest them?” After I left policing and security to become a high school teacher, I continued to ask the same related questions, “What leads students to make the wrong choices in their learning and classroom behavior?” and “What leads students to make the right choices in their learning and classroom behavior?” The answer to these question led me back to university where I began taking philsophy courses and started formally studying ethics. I introduced ethics into the curriculum of my Grade 10, 11 and 12 Computer Engineering courses and am now prepared to do the same for the mathematics courses that I teach.

1.3 Ethics

The study of ethics focuses on three main areas:

1. **Deontology (DUTY):** People will make the right choice because they feel that it is their duty. For example, when you are hired by an employer, you show up to work and do the required work because you have signed an employment contract and have agreed to do the work. It is your duty. Or, a fireman will enter a burning building to save someone because it is a duty, or obligation, that they owe that person.

   Duty is the necessity of acting from respect for rules of agreement.

2. **Conseqentialism** People will make the right choice based on their fear of punishment or a promise of a reward. For example, if you don't show up for work you could be fired. If you do show up for work you will be paid for what you do.

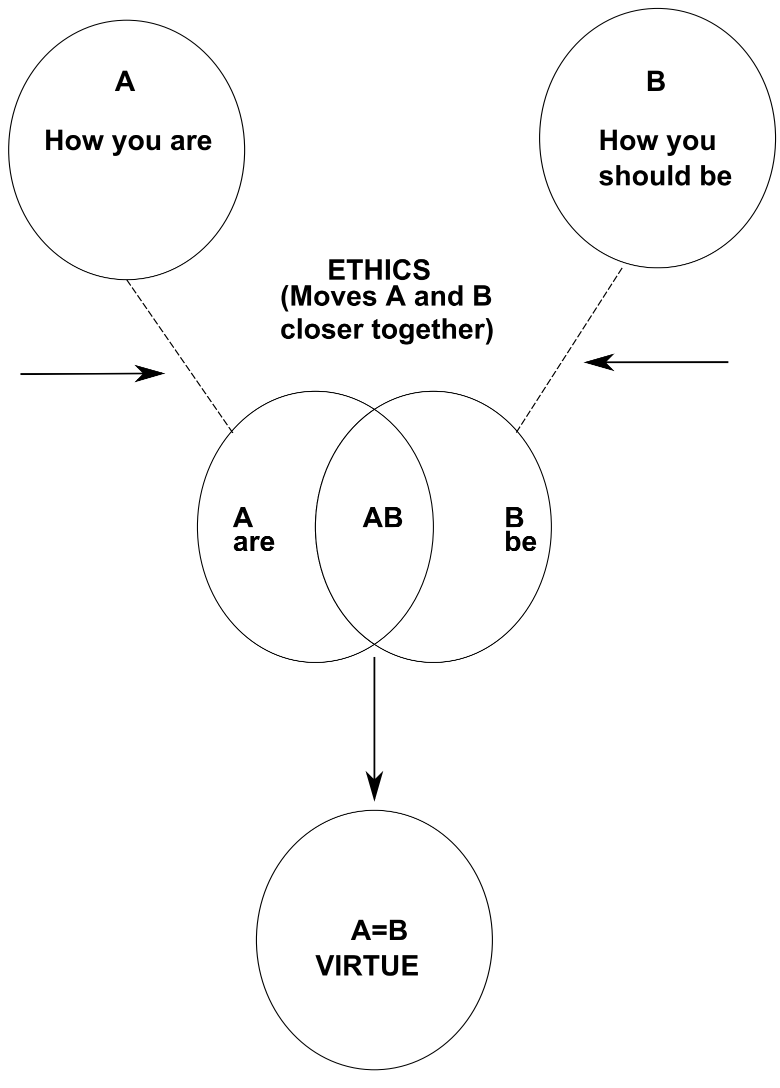
3. **Virtue Theory -** People will make the right choice by modeling their behavior after someone they respect or admire. This works, as long as you choose the right person to model your behavior after.

In general, people will more likely make the right choice if they are in an environment where others around them are also making the right choice.

1.4 Ethics in the Classroom

Ethics in the Classroom
I have successfully incorporated ethics within the curriculum of Grade 10, 11, and 12 Computer Engineering. The first assignment for Grade 11 Computer Engineering is to create a Code of Ethics for the class. I then have it placed onto a plaque with all of the student names affixed to the plaque. It is mounted and becomes a permanent fixture of the classroom. The students realize that the plaque affirms their legacy as Computer Engineering students. See Contact Information and go to my website –>ethics–>Philosophical Ethics–>Ethics_Paper_Our_Voices_2011.doc

1.5 The Role of Ethics

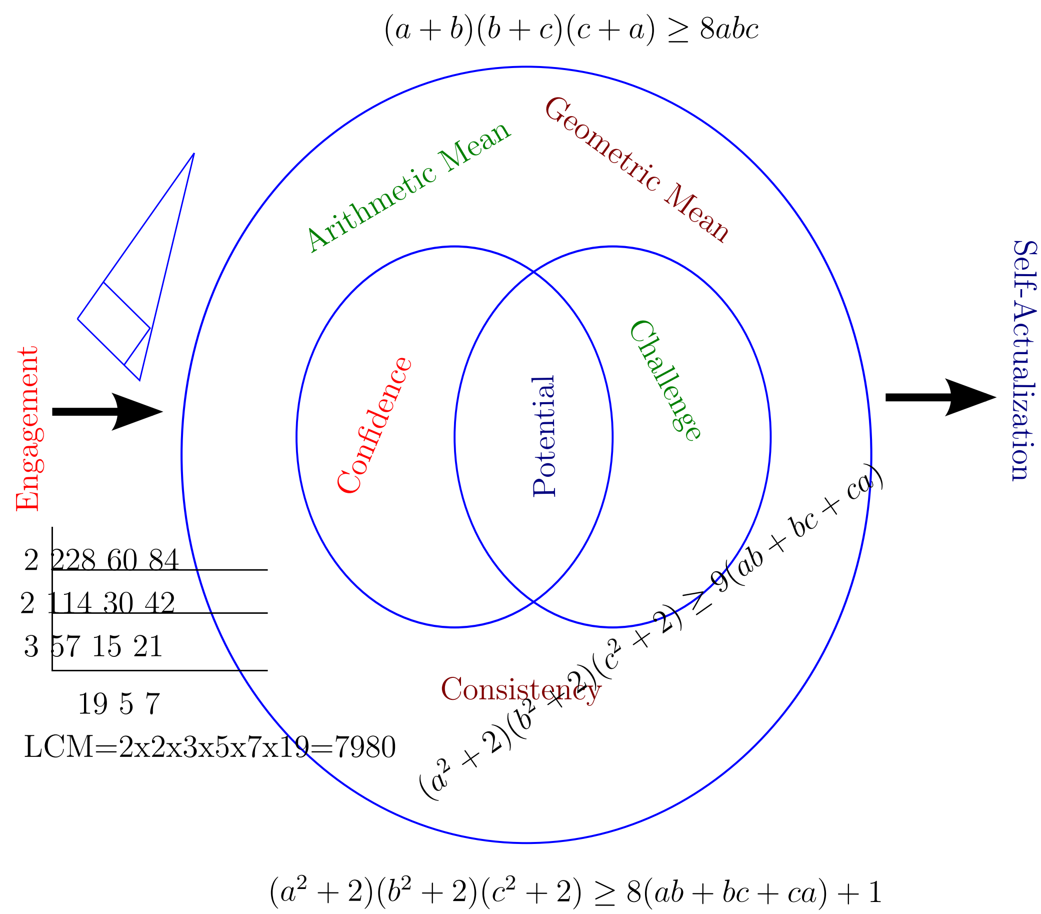


Chapter 2

Ethical Mathematics

2.1 How We Learn

The Value of Consistency



2.2 Ethics in Mathematics

As humans, we are instinctively drawn to patterns. Indeed, some would argue that it is our affinity to patterns that defines our humanness. Life could not exist without the stability provided by the earth and sun and the delicate balance between the biotic and abiotic environment. Mathematics arose out of our desire to find patterns to explain what we see around us. In some sense, mathematics was a tool that allowed us to take pattern making to a higher level.

The famous Indian mathematical prodigy, Srinivasa Ramanujan (1887-1920) was initially hampered in his discoveries because he used his own system of mathematical nomenclature that was unfamiliar to other mathematicians. Many thought that his early work was gibberish. It was only after some of the world's top mathematicians saw the internal consistency within Ramanujan's work that he went on to become one of the greatest mathematical genius's of the twentieth century.

The value of consistency was demonstrated to me last year by a Grade 9 math student who introduced me to an intriguing method of finding the Lowest Common Multiple (LCM) of numbers. This was a method that his father had shown him.

$$\begin{array}{r|rrr} 2 & 228 & 60 & 84 \\ \hline 2 & 114 & 30 & 42 \\ \hline 3 & 57 & 15 & 21 \\ \hline & 19 & 5 & 7 \end{array}$$

LCM=2x2x3x5x7x19=7980

This method was consistent with the factor tree method that I used but was organized into a much more efficient information block. The pattern and level of consistency inherently appealled to me. This is the method I now teach Grade 9 students when finding the LCM.

2.3 Fun With Math

Over the summer I was preparing enrichment material for the clustered calculus class that I will be teaching this year. I came across this fascinating math problem which is Problem 5 of the 2004 Asian-Pacific Math Olympiad. Show that:

$$(a^2+2)(b^2+2)(c^2+2) \geq 9(ab+bc+ca); a, b, c \geq 0$$

This problem absorbed me. My wife and I were visiting New York City in August. She was touring the Metropolitan Museum of Modern Art, and I was sitting in the lobby working on this problem. Needless to say, this did not go over well with her. After several days, I was able to show:

$$(a^2+2)(b^2+2)(c^2+2) \geq 8(ab+bc+ca)+1; a, b, c \in \Re$$

Hint:

$$\tfrac{1}{2}(abc+1)^2 + \tfrac{1}{2}(abc-1)^2 + 2[(a-b)^2+(b-c)^2+(a-c)^2] + \cdots$$
$$\cdots + 2[(ab-1)^2+(bc-1)^2+(ac-1)^2] \geq 0$$

Along the way, I was reminded of the inequality relationship between the Arithmetic Mean and the Geometric Mean:

$$\frac{x_1+x_2+x_3+\cdots+x_n}{n} \geq (x_1x_2x_3\cdots x_n)^{\frac{1}{n}}$$

Using this, I was easily able to show:

$$(a+b)(b+c)(c+a) \geq 8abc$$

But I was no further along in proving the original inequality. By the end of the summer, I had given up so I went online and found the solution. (See Bibliographic item 3). My point here is that knowing the internal consistency of math, I persisted and along the way discovered a lot more meaningful mathematics than had I just looked up the solution right away. This is the spirit of mathematics that we need to inspire witin our students.

Chapter 3

The Ladder Box Problem

3.1 Doin the Math

Let's Do the Math:

Given the mathematical expression: $x^4 + 2x^3 - 98x^2 + 2x + 1 = 0$
Solve for $x \in \Re$ and factor the expression over the Real numbers.

In this context, would this problem immediately engage you and would you really care whether you find a solution or not? For most of us, no it wouldn't. We would need to see the social context of the problem before it engages us. Our challenge is to restate this problem so that the same learning objective is met but in a manner that engages the student to the extent that they will persist until they find a solution.

3.2 Learning Objectives

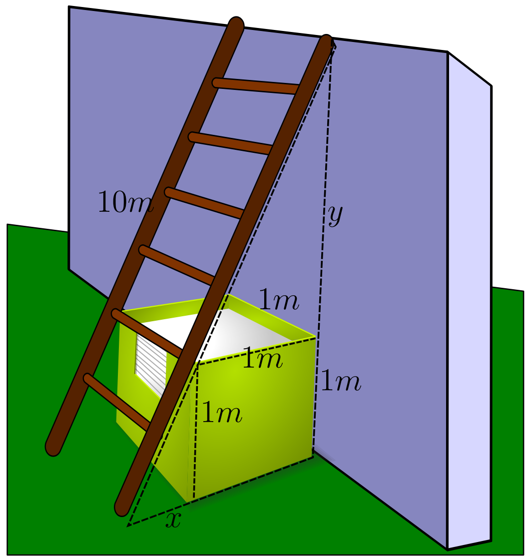
The Ladder Box Problem

Learning Obectives

- To think "outside the box" (pun intended).
- To use mathematics to model real world problems.
- To show the interplay between algebra and geometry.
- To foster mathematical diligence (i.e. don't give up until you have tried everything - and even then, just put the problem aside and try it again later).

I use this material as an assignment in my Grade 12 clustered calculus class. I have used it as a problem of the week in my Grade 9 Academic math class and my Grade 11U math class. As well, I have introduced it to my Grade 10, 11 and 12 computer engineering classes. Each grade level approaches the problem at a different entry point. The math students focus on the algebraic approach while the computer engineering students will search online looking for mathematical software to solve the quartic equation. The calculus students will often use the graph of the quartic as a starting point.

The Ladder Box Problem



3.3 Problem Statement

PROBLEM:

A 10m ladder leans against a wall just touching the edges of a 1m x 1m x 1m box pushed against the wall. Find the distance that the bottom of the ladder is from the box.

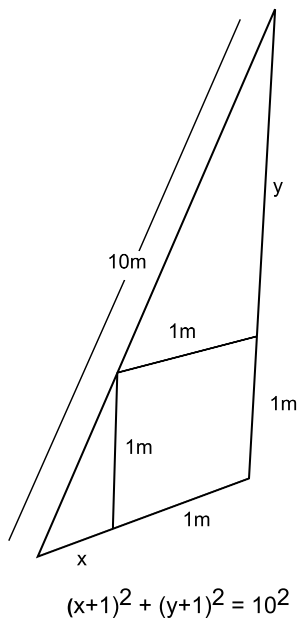
3.4 Assignment

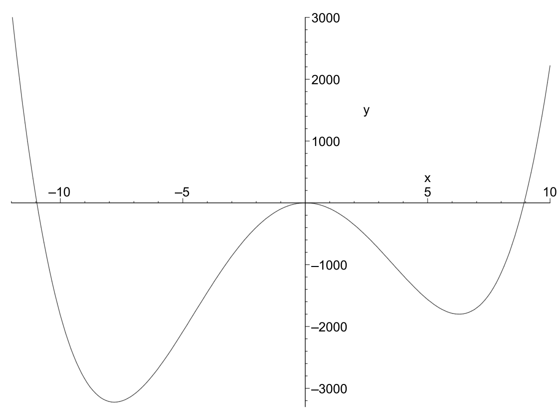
Answer the following questions:

1. How many unique solutions would you expect to find? Why?
2. Using x to represent the distance from the bottom of the ladder to the side of the box and using y to represent the distance of the top of the ladder to the top of the wall, draw a two-dimensional diagram of the physical situation.
3. Using the Pythagorean Theorem, state an equation involving x and y.
4. Using similar triangles, find a simple relation between x and y.
5. Using substitution show that x must satisfy the quartic equation: $x^4 + 2x^3 - 98x^2 + 2x + 1 = 0$.
6. Using mathematical software, or the general solution to the quartic, solve for x to three decimal places.
7. Either by hand, or by using mathematical software, sketch the graph of the quartic over the restricted domain: $-12 \leq x \leq 10$ and the restricted range: $-3300 \leq y \leq 3000$.
8. Solve for x in exact form.
9. Factor the quartic equation $x^4 + 2x^3 - 98x^2 + 2x + 1 = 0$ over the Real numbers as a product of quadratics.

3.5 Solutions to the Ladder Box Problem

Solutions To The Ladder Box Problem





1. There will be two unique solutions for x. From the diagram, we see that there are two possible positions for the ladder, one with the bottom of the ladder very close to the box and the other with the bottom of the ladder far away from the box.

2. See diagram at the top of this page.

3. $(x+1)^2 + (y+1)^2 = 10^2$

4. Using similar triangles, find a simple relation between x and y.
   $xy = 1$

Solutions To The Ladder Box Problem (Continued)

5. Using substitution show that x must satisfy the quartic equation: $x^4 + 2x^3 - 98x^2 + 2x + 1 = 0$.

$$
\begin{aligned}
(x+1)^2 + (y+1)^2 &= 10^2 && (3.1)\\
y &= \frac{1}{x} && (3.2)\\
(x+1)^2 + \left(\frac{1}{x} + 1\right)^2 &= 100 && (3.3)\\
(x+1)^2 + \left(\frac{x+1}{x}\right)^2 &= 100 && (3.4)\\
x^2 + 2x + 1 + \left(\frac{x^2+2x+1}{x^2}\right) &= 100 && (3.5)\\
x^4 + 2x^3 + x^2 + x^2 + 2x + 1 &= 100x^2 && (3.6)\\
\therefore x^4 + 2x^3 - 98x^2 + 2x + 1 &= 0 && (3.7)
\end{aligned}
$$

6. Using Maple or the TI83 Graphing Calculator we can approximate the two solutions for x as:
$x \approx 0.112m$ or $x \approx 8.938m$

7. Sketch the graph of the quartic over the restricted domain: $-12 \leq x \leq 10$ and the restricted range: $-3300 \leq y \leq 3000$. See sketch on top of the previous page.

Solutions To The Ladder Box Problem (Continued)

8. Solve for x in exact form.

$$
\begin{aligned}
(x+1)^2+(y+1)^2 &= 10^2 && (3.8)\\
xy &= 1 && (3.9)\\
x^2+2x+1+y^2+2y+1 &= 100 && (3.10)\\
x^2+2(1)+y^2+2(x+y)-100 &= 0 && (3.11)\\
x^2+2xy+y^2+2(x+y)-100 &= 0 && (3.12)\\
(x+y)^2+2(x+y)-100 &= 0 && (3.13)
\end{aligned}
$$

$$x+y=\frac{-2\pm\sqrt{2^2-4(-100)}}{2} \quad (3.14)$$

$$x+y=\frac{-2+\sqrt{404}}{2} : x,y>0 \quad (3.15)$$

$$\therefore x+y=-1+\sqrt{101} \quad (3.16)$$

$$x+\frac{1}{x}=-1+\sqrt{101} \quad (3.17)$$

$$\therefore x^2+(1-\sqrt{101})x+1=0 \quad (3.18)$$

$$\therefore x_1=\frac{-1+\sqrt{101}+\sqrt{98-2\sqrt{101}}}{2} \text{ or} \quad (3.19)$$

$$x_2=\frac{-1+\sqrt{101}-\sqrt{98-2\sqrt{101}}}{2} \quad (3.20)$$

9. Factor the quartic equation $x^4+2x^3-98x^2+2x+1=0$ over the Real numbers.

   Using the factor theorem and polynomial division we have:
   $\therefore (x^2+(1-\sqrt{101})x+1)(x^2+(1+\sqrt{101})x+1)=0$

Chapter 4

An Ethical Problem

4.1 Standard Optimization Problem

Problem Description XYZ Corporation produces a commercial product that is in great demand by consumers on a national basis. Unfortunately, near the plant where it is produced there is a large population of dove tailed turtles who are adversely affected by contaminants from the plant. XYZ has a filtering process that is expensive and any increase in filtering effectiveness reduces their profit. Dove tailed turtles are not a protected species hence there are no environmental rules regulating XYZ's level of contaminants. Clearly, no filtering at all would maximize XYZ's profitability.

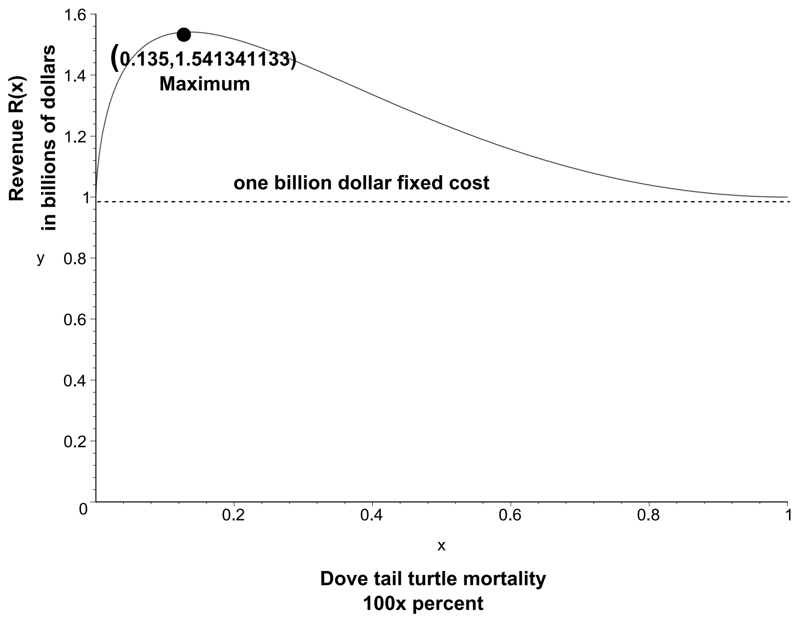
However, a local environmental group monitors XYZ's contaminant level and maintains a website showing the percentage mortality rate of the dove tailed turtles due to XYZ's contaminants. XYZ has noticed that the higher the mortality percentage, the less items are bought and the lower their profitability. Their Marketing Department and Research Group has established the following Revenue Function, R(x), as a function of Dove Tail Turtle Mortaility expressed as decimal between 0 to 1 representing mortality percentage:

$R(x) = 1 + x(lnx)^2$; $0 < x \leq 1$, $1 < y < 1.6$
$y = R(x)$ is expressed in billions of dollars and represents the revenue generated. Since XYZ has fixed operating costs of one billion dollars, the profit function, $P(x)$ is given by $P(x) = R(x) - 1$

Problem Statement:

1. Sketch the graph of $R(x) = 1 + x(lnx)^2$; $0 < x \leq 1$, $1 < y < 1.6$
2. Find the optimal dove tail turtle mortality rate percentage that will maximize revenue.
3. State what the maximum profit will be.

4.2 Optimization Problem Solution



1. Sketch the graph of $R(x) = 1 + x(lnx)^2$; $0 < x \leq 1$, $1 < y < 1.6$
   See above graph.

2. Find the optimal dove tail turtle mortality rate percentage that will maximize revenue.
   $R'(x) = lnx(lnx + 2)$
   Setting $R'(x) = 0$ and solving for x gives an optimal value of $x = e^{-2} \approx 0.135$ or 13.5 % mortality rate

3. State what the maximum profit will be.
   $R(e^{-2}) = 1 + \frac{4}{e^2} \approx 1.541341133$
   $P(x) = R(x) - 1 \approx 0.541341133$
   Thus their will be a maximum profit of $541,341,133.00

I find this problem somewhat obscene. Hopefully, if this problem where presented to students in this form, they would also object to the premise of the problem. Let's restate this problem in a fashion that inspires social responsibility.

Optimal Social Responsibility

You have been hired as the mathematical consultant for XYZ Corporation. There problem is as stated in 4.1. They have asked you to help them optimize their profit and request that you:

1. Sketch the graph of $R(x) = 1 + x(lnx)^2$; $0 < x \leq 1$, $1 < y < 1.6$
2. Find the optimal dove tail turtle mortality rate percentage that will maximize revenue.
3. State what the maximum profit will be.

As a successful graduate of MCV4U1, you easily do the math and come up with the solution shown on 4.2. But don't collect your money yet. You also understand that mathematics is an intentional human activity and carries with it a certain social responsibility. Are there any ethical considerations that you would bring to your client's attention? (See answer below)

XYZ is adopting a utilitarian philosophy in that they think that their actions are good as long as they optimize profits for their shareholders. They must realize, however, that returns must also be optimized for their stakeholders, i.e. any entity affected by their actions which, in a global environment, includes the dove tailed turtles. I am sure that the turtles would find a 13.5% mortality rate unacceptable regardless of the profits. While we realize that the turtles are not moral agents, environmentalists can act on the turtle's behalf and must also be considered stakeholders.

As well, $R(x)$ has inherent boundary conditions that can change over time. Who knows what the cumulative effect of the contaminants will be on the biotic environment. Right now, it is only affecting the turtles. At higher levels, over time, it could also start affecting humans. XYZ would then find themselves potentially facing multibillion dollar lawsuits which would offset any short term profits that they may have made at the expense of the turtle population.

An analogy, would be tobacco companies during the first half of the twentieth century. They are now facing the threat of multibillion dollar lawsuits. Is XYZ Corporation prepared to risk the health of future generations of children? Or, more importantly to them, are they prepared to face potential lawsuits? As well, since they have been formally advised of the potential problems that their continued actions may produce if they ignore your advice, their liability has been significantly increased. Of course, you are bound by a confidentiality agreement and cannot make your findings known to the media.

Chapter 5

Mathematical Ethics

5.1 Is Mathematics Value Free?

There are really two fundamentally opposite schools of thought when it comes to teaching mathematics. The first is the **traditionalist** or **back-to-basics approach** which claims that we teach mathematics so that students can master a specific body of mathematical knowledge and skills and then evaluate them on the basis of how complete their mastery is. Philosophically, we can think of this as "mathematical absolutism".

Reform oriented educators place understanding as the primary goal of mathematics education and see **constructivism** as the philosophical basis of mathematics education. Stemhagen (Doin the Math, p. 63) argues that the traditionalist approach is of little value in establishing a human context for mathematics. On the other hand, the constructivist focus on child-centred learning, differentiated instruction and on the individual's construction of mathematical knowledge, "..can lead to neglect of other contextual factors, such as social and environmental factors." (Doin the Math, p. 64).

Stemhagen argues that mathematics is an intentional human activity and that, " we need to acknowledge that mathematics itself is fundamentally historically, socially, and culturally situated. Rather than simply finding ethical uses for mathematics, we need to teach mathematics in a way that recognizes that it is not different in kind than other enterprises (particularly ethical ones)." ("Doin the Math" p. 64).

In other words, mathematics can no longer be considered value free. Just like a gun has an implied valence or intent to serve as a means to kill, mathematics is embedded within a social and cultural context and, necessarily, must also have a valence. Our challenge is to identify the values of mathematics and identify its valence.

Atweh, (*Socially Response-able Mathematics Education: Implications of an Ethical Approach*, p. 274) states that although mathematics education has began to recognize sociocultural perspectives, "..there is a noted absence of discussion of *ethics* as it relates to the discipline."

5.2 Subjectivity vs. Objectivity

My basic thesis in this discussion is that when we consider mathematics objectively, the only implicit ethics is one of consistency. Our problem with this, however, is that mathematics is, indeed, an intentional human activity and in some sense must relate to our subjective understanding of the world we live in. For example, no matter how hard we try, we cannot envision a point or a line. We can create constructive models but will never be able to actually see a point or a line. That never stopped Euclid from developing a consistent geometry, however. Non-Euclidean Geometry did not detract from this consistency but merely changed the boundaries.

Too often, we teach mathematics to students as if it were static and absolute with everything defined as a procedure. To shock students out of this mindset, I will often ask them to define one. This is not an easy task and is actually impossible. Try to define one yourself. See what you come up with. I then go on and say to students, "Well if you can't define one, how could you possibly expect to define two?" I then ask them to explain to me how the operations on numbers are consistent and is it possible to use something without being able to explain or define it, as long as the results are consistent. Believe me, this can be a spirited discussion. What we are really addressing is the essential ethics of mathematics: consistency.

Historically, mathematics has always had a role in quantifying philisophic thought as it relates to logic, aesthetics, and ethics. Indeed, Aristotle used mathematics to develop formal rules of logic to assist in analyzing arguments. Mathematics was thought to bring order to the ethics field and Pythagoras saw the entire universe related to number and believed that justice is represented by square numbers. Perhaps string theory may prove him right after all.

How does this look on a subjective level when we "do the math" and take it into the "real world"? Atweh suggests that "..school mathematics should support student response-ability not only to *read* the world but also to *transform* the world." (p. 274)

Chapter 6

Conclusion

Does this mean that you should change the way you teach mathematics? Not at all. Atweh suggests that students should be "...engaged in meaningful and authentic "real world" problems that develops both mathematical capability and also develops an understanding of the social world and how to contribute to its transformation." (Atweh, p. 274) That's not change, that's just good pedagogy.

Engineering, medicine and law have always had an ethics component built into their body of knowledge. Since 2008 and the fallout of Enron, most major business schools now incorporate ethics into their curriculum. As well, most computer science and computer engineering disciplines require that students have some exposure to ethics.

Perhaps mathematics has been viewed as being "value free" for too long and it is time that we acknoledge that there is a role for ethical imperatives within mathematics. This won't happen overnight, however, and any change will certainly be incremental.

Chapter 7

Author Profile

7.1 Douglas Henrich

Douglas Henrich has been a Mathematics, Computer Engineering and Computer Science teacher at Iroquois Ridge High School, Oakville, Ontario for the last eight years. He lives in Burlington with his wife Marianne, who is a vision itinerant teacher working with blind and low-vision students, and their cat, Archie. Prior to becoming a teacher, Doug spent over twenty-five years working within the police and private security sector and is a former Peel Regional Police Officer. Doug is trained as a CPTED Level I Practitioner (Crime Prevention Through Environmental Design), Use of Force Theory Instructor and has worked as a Security Analyst/Security Consultant where he conducted both CPTED Reviews and facilitated Focus Groups for a wide variety of clients. Doug has worked as a Law and Security instructor at Conestoga College, Kitchener and has served as the Vice-Chair on their Law & Security/Police Foundations Advisory Panel. Doug is also a sessional instructor at Sheridan College, Oakville teaching Information Ethics to 4th year students in the Bachelor of Applied Information Science program. As well, Doug is the author of Patrol Procedures for Private Security Professionals (Pearson, 2001) and is currently working on a second book with a tentative title of, Practical Ethics: Developing a Social Conscience

7.2 Contact Information

1. website: http://chatt.hdsb.ca/~henrichd
   Go to ethics–>mathematical ethics

2. e-mail: henrichd@hdsb.ca

3. phone: (905) 845-0012

4. fax: (905) 845-0013

Bibliography

[1] "Types of Ladder Box Problems". Jurgen Koller. Retrieved October 23, 2011, from http://www.mathematische-basteleien.de/ladder.htm#Ladder%20Box%20Problem. 2004.

[2] "XVI Asian Pacific Mathematics Olympiad, March, 2004". Retrieved from http://www.mmjp.or.jp/competitions/APMO/files/apmo2004_prb.pdf

[3] "Asian Pacific Mathematics Olympiad Solutions: 1989 to 2004". Jacob Tsimerman. Retrieved from http://www.scribd.com/doc/37657884/Asain-Pacific-Mathematical-Olympiad-1989-2009-R-SOPHORN-doc

[4] "The Ladder Problem". A. Donald. Retrieved October 23, 2011, from http://www.adonald.btinternet.co.uk/Ladder/Ladder.html. 2011.

[5] "Another Ladder Problem". Jim Loy. Retrieved October 23, 2011, from http://www.jimloy.com/puzz/ladder0.htm. 2003.

[6] “A Mathematical Approach to Ethics”’. George David Birkhoff. Lecture delivered at the Rice Institute, March 6, 7, and 8, 1940.

[7] “Socially Response-able Mathematics Education: Implications of an Ethical Approach”’. Bill Arweh and Kate Brady. Eurasia Journal of Mathematics, Science and Technology Education, 2009, 5(3), pp 267-276.

[8] “Doin’ the Math: On Meaningful Mathematics Ethics Connections”. Kurt Stemhagen. The Montana Mathematics Enthusiast, ISSN 1551-3440, Vol 5 No. 1 pp. 59 -66. 2008. The Montana Council of Teachers of Mathematics & Information Age Publishing.

[9] “Ladder” The Open Clip Art Library. 2011. Retrieved October 23, 2011, from `http://www.openclipart.org/detail/155425/%C3%89chelle-contre-un-mur-by-mathafix`.

[10] “Notebox” The Open Clip Art Library. 2011. Retrieved October 23, 2011, from `http://www.openclipart.org/detail/153625/notebox-by-chrisdesign`.