

On-line Mathematics: Visions and Opportunities, Issues and Challenges, and Recommendations

White Paper based on
The Fields Institute for Research in Mathematical Sciences'
Mathematics On-line Working Meeting
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Introduction

Forty years ago groups of mathematics educators were experimenting with and advocating for the computer delivery of mathematics lessons (Suppes, 1965). In these early projects, computer assisted instruction [CAI] packages were delivered from large mainframe computers to elementary grade pupils sitting at teletype terminals located in their schools. The vast technological gulf between the university based developers of such systems and the school teachers who would use them and the public in general meant that the promised immanent educational revolution failed to appear.

The technological landscape has changed drastically during the passing four decades. Today the network connected personal computer has become a common household appliance and schools have classroom computers linked to the World Wide Web [Web]. Sophisticated software makes computer communication and information access easy and appealing for users of all ages. Surfing the Web, visiting chat rooms, and communicating by e-mail are becoming major recreational activities for the "net generation" (Tapscott, 1998).

This new technical reality has encouraged some to again predict a coming educational revolution, but future directions are not clear. On the small-scale side of the picture, individual teachers are using the Web to provide their classes with access to current information and experiences that lie outside the classroom (Heide & Stilborne, 1999). On a larger-scale, educational institutions - school boards, colleges, and universities - are beginning to convert regular courses for delivery via the Web, and some authors are predicting the demise of these traditional education providers in the face of competition from agencies and corporations dedicated to the production of Web-based courses (Downes, 2000).

Noting the wide range of predictions related to the on-line delivery of education, in early 2001 the Fields Mathematics Education Forum established a working group to explore the issues and produce a statement concerning the teaching and learning of mathematics via the Web. As part of this exercise the working group organized a symposium, the Mathematics On-line Working Meeting, November 15-17, 2001. This report is the outcome of the deliberations of the 60 (?) mathematics educators who attended the meeting.

History

The Fields Institute, named after the Canadian mathematician John Charles Fields, was established in 1992 as a research centre for the mathematical sciences. The Institute, with primary funding from the Ontario Ministry of Training, Colleges and Universities and the federal Natural Sciences and Engineering Research Council [NSERC], provides visiting mathematicians with on-going highly focussed mathematics research seminars and opportunities to investigate the industrial and business applications of their work. Included in the Institute's mandate is the support of mathematics education at all ages, from the beginning of schooling through to graduate and post-doctoral study. To further this goal, the Institute, since early 1997, has coordinated and sponsored the Fields Mathematics Education Forum. Here in regular meetings, mathematics educators from the elementary and secondary school systems, colleges, and universities meet to exchange ideas and develop proposals for the improvement of teaching and learning in the discipline. The Forum has come to play a significant role in Ontario education, organizing symposia and workshops addressing key issues and contributing to the development of the province's new secondary school mathematics curriculum (Ontario Ministry of Education, 2000; Ontario Ministry of Education and Training, 1999).

Context: Change in Mathematics Education

Since the early 1980s there has been an on-going debate with repeated calls for change in mathematics teaching and learning at both the school and university levels (National Council of Teachers of Mathematics [NCTM], 1980; Schoenfeld, 1983). This change program, which is most fully developed in the Principles and Standards for School Mathematics (NCTM, 2000), has been led in North America by the National Council of Teachers of Mathematics [NCTM] and the Mathematical Association of America [MAA], and locally in Ontario by the Ontario Association for Mathematics Education [OAME], the Ontario Mathematics Coordinators' Association [OMCA], and the Ontario Colleges Mathematics Association [OCMA]. With participants drawn from the active members of these associations, the debates and positions of the Fields Mathematics Education Forum reflect the trends of the larger change movement. In 1997 a Fields Institute sponsored workshop issued a white paper, Mathematics Education for the 21st Century (Langford, Long & McDougall, 1997), which called for a curriculum that delivered "a context-rich, problems-based environment, showing the relevance of mathematics to other disciplines" (p. 14). Mathematical modelling would be a central theme of the workshop's proposed new program and students would have "opportunities to pose their own problems and solve them through investigations and project-based learning" (p. 14). The white paper set out "the integration of information technologies into the curriculum as a priority" (p. 15) and in calling for every classroom to have "access to information technology resources on the Internet" (p. 16) signalled an interest in mathematics on-line.

Continuing discussion has refined and strengthened the Forum's image of mathematics learning, and this vision was taken as the starting point for discussions at the Mathematics On-line Working Meeting. As an initial task, participants were presented with the following vision statement:

We want students to be immersed in a mathematics culture that gives them an opportunity to: a) learn, use, and refine inquiry, investigation, experimentation and problem solving processes, and b) develop the tools/skills/habits of a life-long learner, learn significant concepts and procedures (with understanding) that they can then use in an integrated, authentic fashion to conduct inquiries, experiments, investigations and problems.

and were asked to identify the characteristics of on-line environments that would support such mathematics learning.

Context: On-line Learning

The recent flurry of research reports, government and corporate statements, and popular press articles concerning the present and future state of on-line learning suggest that there is divergence in opinions concerning both the nature and the viability of the enterprise. Teachers who employ the Internet in their lessons do so by integrating Web-based activities into their on-going curricular program. Their students use information accessed via the Web in larger classroom-based projects and to a lesser degree use the Internet for inter-school collaborative activities and for publishing personal work (Becker, 1999). On-the-other-hand task forces involving leaders from government, business, and education (Ontario Ministry of Education/Ontario Ministry of Training, Colleges and Universities, 2000, June; Web-based Education Commission, 2000) see a much larger enterprise with full courses being delivered via the Web. University generated reports suggest use of the Web could have positive financial

returns and reduce the cost pressures being experienced by tertiary level educational institutions (Task Force on Learning Technologies, 2000). In many ways, differing opinions on the best directions for on-line learning reflect the larger debates concerning the balance between direct instruction and constructivist approaches. Research (Becker, 1999) shows that Internet use is greatest for teachers who regularly employ constructivist practices such as those advocated by the NCTM, OAME, and the Fields Mathematics Education Forum (Langford, Long & McDougall, 1997). Unfortunately it appears that the developers of Websites do not take as strong a constructivist position. A survey of 436 Websites designed to support learning in mathematics, science, and technology (Mioduser, Nachmias, Lahav & Oren, 2000) found that "a traditional, hierarchical, highly structured, and directed instruction mode still prevails. Only 28% of the sites support inquiry-based learning" (p. 62).

The advocates for large-scale course delivery via the Web claim that Web-based education "holds extraordinary promise" (Web-based Education Commission, 2000, p. iii), but recent research suggests there are issues to be addressed. Schollie (2001) found that although 95% of Alberta students, parents, and teachers participating in on-line schooling were satisfied or very satisfied with the quality of education delivered, in fact, in mathematics, these pupils scored below regular school students on standardized achievement tests.

On-line course development is not a simple task and within the Ontario university system, movement to Web-based courses has been slow. Of the 17 distance education mathematics courses listed in the Ontario Council for University Lifelong learning [OCULL] (2001) course directory, only 3 are offered on-line. Universities are finding that, with the high costs of course development and maintenance, Web-based programs are not the revenue generating ventures anticipated. During 2001 both the Massachusetts Institute of Technology (MIT, 2001, April 4) and New York University (N.Y.U., 2001, November 30) announced that they were abandoning their for-profit programs to market on-line courses.

Innovative teachers have developed very productive and educationally sound mathematics projects integrating on-line activities, but movement from this starting point to wider and more extensive programs such as full course delivery is not a simple step. Mathematics educators need to think about, discuss, and clearly describe the types of Web-based mathematics learning environments required. The Fields Institute Mathematics On-line Working Meeting is one step in this direction.

Working Meeting Schedule

Thursday November 15, 2001

7:30 PM Public Forum

Ann Heide, University of Ottawa
Active Learning in the Digital Age Classroom

Dr. Jonathan Borwein FRSC, Simon Fraser University
Collaborative On-line Mathematics: Wishing and Hoping

9:30 PM Reception

Friday November 16, 2001

8:30 - 9:00 Coffee

9:00 - 9:30 Welcome and Introduction

9:30 - 10:30 Working Groups - Mixed Instructional Levels

10:30 - 10:50 Break

10:50 - 12:00 Working Groups - Mixed Instructional Levels

12:00 - 1:00 Lunch Break

1:00 - 1:30 Debriefing of Morning Session & Review of Afternoon Activities

1:30 - 3:10 Working Groups - Instructional level

3:10 - 3:30 Break

3:30 - 4:20 Presentations and Posters - first session

4:20 - 5:00 Presentations and Posters - second session

5:00 Reception

Saturday November 17, 2001

8:30 - 9:00 Coffee

9:00 - 10:30 Working Groups - Instructional level

10:30 - 10:50 Break

10:50 - 11:40 Presentations from Working Groups

11:40 Next steps

Discussion Questions and Tasks

Visions and Opportunities (Friday morning)-Mixed Instructional Level Groups

We want students to be immersed in a mathematics culture that gives them an opportunity to: a) learn, use, and refine inquiry, investigation, experimentation and problem solving processes, and b) develop the tools/skills/habits of a life-long learner, learn significant concepts and procedures (with understanding) that they can then use in an integrated, authentic fashion to conduct inquiries, experiments, investigations and problems.

1. What characteristics should an on-line course possess in order to support the above vision?
2. What current 'best' classroom practices and new practices do we wish to see manifested in on-line courses?
3. What is the classroom/on-line experience that we are trying to create?
4. What on-line teaching/learning tools/technologies do we need to implement our vision? How may these be used to extend mathematics teaching and learning beyond the physical classroom and possibly improve how teachers teach and how students learn mathematics?

Issues and Challenges (Friday afternoon) - Instructional Level Groups

1. What are the issues and challenges around pedagogy?
2. What are the issues and challenges around implementation?
3. What have we learned about on-line learning environments for the teaching of mathematics? What are some of the exemplary practices in existing on-line learning initiatives (i.e. use of technology, use of resources)?
4. How do we ensure students get an opportunity to interact effectively with the subject matter, their teacher, and co-learners, witness/ learn from the model behaviour of others reflect on/communicate their actions/experiences/learning, and assess their knowledge and skills in an effective, ongoing, and authentic fashion?
5. How do we overcome outmoded perceptions of teaching, learning, outmoded perceptions of subjects such as mathematics, outmoded views of the roles of teachers and students, outmoded views of what constitutes an effective learning task/environment?
6. How do we accommodate different learning styles of students, how can we obtain the tools and proper environment for this ideal learning and delivery"

Recommendations (Saturday morning) - Instructional Level Groups

1. Where should we be five years from now?
2. What do we need to do to get there? (i.e., communication, further forums, research, development projects)?

This paper, which reflects the consensus of the Working Meeting participants, is organized using the three themes employed to structure our discussions: Visions and Opportunities, Issues and Challenges, and Recommendations.

Visions and Opportunities

Our pedagogical vision is that students construct a solid understanding of mathematics and a confidence that mathematics makes sense as a whole rather than in isolated bits. This is possible when students are immersed in a mathematics learning culture that gives them the opportunity to:

- Learn, use and refine inquiry, investigation, experimentation and problem solving processes;
- Understand significant mathematics concepts, procedures and relationships that they can then use in an integrated, authentic fashion.

Such a vision of mathematics pedagogy has been persistent in mathematics education research and championed, in part or in whole, under such reform banners as discovery learning, problem solving and, most recently, constructivism. This vision is currently embedded to a significant degree and in various forms in mathematics curriculum documents across Canada and the United States. It seems rather obvious that this vision should guide us in the design on on-line mathematics teaching and learning. However, this vision has also been elusive in attempts to implement it on a large scale.

On-line learning offers the opportunity to expand the boundaries and methods of mathematics education. Presently, the landscape of on-line mathematics learning is changing quickly. Our vision is that the design, development and implementation of on-line mathematics education is well thought out and care is taken that on-line mathematics education does not set an unintended and possibly an undesired pedagogical direction. Well-designed on-line mathematics learning will require clear learning goals and cooperation between people who have technical, mathematical and pedagogical experience and expertise.

Below we outline our visions for on-line mathematics education in greater detail. The visions are organized around the four commonplaces of education, namely, the student, the teacher, the subject matter (mathematics) and the learning environment.

Student

- Students need opportunities to experience thinking and attitudes associated with mathematical activity. Some of this will be possible through appropriate interactive multimedia experiences. The on-line human interactions – between teacher and students, and among students – will play a key role in the mathematics thinking and attitudes that are modeled and encouraged.
- Many students associate computers with games and immediate feedback and gratification. This will not be the case with open-ended on-line explorations where there is not a game to be won and there is no immediate feedback of correct or incorrect answers. This is a re-training issue for students that we envision to be explicitly addressed in on-line learning. Part of the solution is appropriate introductory activities that accustom students to think mathematically on-line.
- Students need opportunities to check their understanding. Feedback and assessment should be built into on-line activities.
- Students need opportunities to problem solve and express their math thinking and ideas. On-line design needs to take this into account.

Teacher

- Our goal is not to create teacher-proof on-line learning where it is assumed that the embedded instructional design will function as ‘designed’ regardless of the teacher.

- The role of the teacher in the on-line environment is crucial. The teacher's role will have significant impact on the nature of the learning environment, what students attend to mathematically, how the on-line resources are used by students, and what they learn and what they value.
- We envision the teacher playing a key role in creating an on-line learning culture that values and encourages students' sense making and construction of mathematical knowledge.
- Well-designed on-line mathematics learning will be educational for both students and teachers and may act as pedagogical models that may affect and change a teacher's practice in the direction of the pedagogical goal outlined above. For example, appropriately designed multimedia investigations of mathematics concepts and relationships will help teachers gain new understandings of the mathematics they teach and what is important to attend to mathematically. Such on-line learning can form the basis for the mathematical development of teachers to broaden their understanding of mathematics concepts and relationships and mathematics activity.
- The role of the teacher is crucial for creating a feeling of community and facilitating collaborative learning.

Learning Environment

- The design of learning should be modular. It should be possible to access and use the modules in many different ways. For example, a module may be used in a fully on-line course, in a partially on-line course, and as a supplement to a classroom-based course.
- Although there will be some need to create fully on-line courses, we envision on-line learning modules being used as supplements and enhancements of classroom-based courses. This may be especially the case for elementary and secondary school applications.
- Instructional design should take priority over technological form.
- We should not try to do too much. The design focus should be on what is most important for mathematics and how to do it well. What is possible technologically may not be what is desirable pedagogically.
- Simplicity should be a focus of design – technology should not become an obstacle when engaging in on-line learning.
- On-line learning should allow for both group learning and individual learning.
- There should be various forms of discussion possible – private, public, synchronous, asynchronous.
- There is a need to consider a variety of communication tools – discussion, whiteboard, audio, video – and their possible advantages in an on-line learning environment.
- Where appropriate, various forms of on-line collaboration should be available so the on-line experience is that of a community of learners rather than individuals doing and learning mathematics in isolation. In some cases, where on-line experiences are enhancements of regular classroom settings, this community may be created mostly in the classroom settings. In other cases, on-line discussions and sharing should be used. In the latter cases, appropriate on-line tools (allowing for text, symbolic and visual representations) will be necessary.
- On-line support, feedback and assessment should be timely. Some of this may be automated. However, the role of the teacher and peers in this process is crucial for creating a feeling of community and facilitating collaborative learning.
- On-line activities should be flexible. We should avoid lock-step learning. Students should have some choices based on ability and interest. Teachers should have some choice to use a variety of strategies in their goal to help students construct a solid understanding of

mathematics. We need to allow for opportunities for personal expression in learning and teaching.

- We can and should provide a variety of approaches on-line, in some cases in ways that we cannot in the classroom.
- We should allow flexible entry and flexible paths to completion.
- An on-line course should change with participant input. This means that the amount of course material would be considerably more extensive than the amount visible or used at any given time. As participants engage with course materials and with each other, the nature and scope of the course would change to reflect:
 - The interests of the participants
 - The level of expertise of the current group
 - Progress made to-date by individuals
 - Learning styles prevailing in the group
- Opportunities for tailoring the course to reflect the needs of individuals or to respond to a shift across a group of participants
- On-line activities should allow for exploration of concepts and relationships. This seems quite appropriate for a technologically based course or module. Multiple representations can be linked and explored dynamically, for example.
- There should be careful consideration and application of technologies and how they may best benefit mathematics education. For example, web based technology may facilitate communication, interactivity, illustrating/simulating concepts that would be difficult to illustrate in other media, and using electronic memory for diagnosis and tracking, to give a few examples.
- On-line learning design should not be a reproduction of a textbook.
- The interface should be intuitive, transparent, and user-friendly.
- The language used in an on-line learning environment should be invitational.
- There should be an efficient archival resource – e.g., of resources and past discussions.
- On-line tools should be compatible, so that the user does not have to carry out complex technical tasks to make communication possible.
- There should be access to mathematical objects that can be manipulated by students and teachers. It should be easy to communicate mathematically.
- There should be links to other resources beyond the course site – to provide contexts for problems and give access to data.
- There should be a significant help section: e.g., glossary, technical help, program help.

Mathematics

- The primary focus of mathematics activity should be on problem solving. Learning should be situated in activity that is authentic for the discipline of mathematics.
- The content and context of mathematics should engage the learner. There is a meaningful context in which mathematics is explored and understood.
- The activities should address various learning styles and needs.
- There should be a balance of on-line experiences, including demonstration, performance and experimentation.
- Students should be motivated to think mathematically. Care needs to be taken to ensure that the focus of on-line activity is on mathematics.

- On-line learning can provide interactive, exploration-based mathematics experiences that help teachers and students see mathematics differently and broaden their understanding of mathematics concepts and of mathematics activity.
- Mathematics is the study of relationships. On-line technology appears naturally advantageous in this regard in that hypertext is good at delineating relationships.
- Initial implementation should address key areas of mathematics and topics that may not be well understood (i.e., functions, rational numbers, and integers, to name a few). Consideration should be given to topics that best lend themselves to the representations possible on-line.

Issues and Challenges

The meeting began with a stimulating discussion of our vision and opportunities for mathematics on-line education. In the next session our focus was switched to the “reality check” of identifying issues and challenges needed in order to attain our vision. The make-up of the groups was also changed at this time to participants who were all at the same level of instruction. Each group’s conclusions portrayed some unique and also some over-lapping perspectives. We found that there are many issues and challenges facing teachers, students and administrators in choosing some type of on-line learning. A broad issue is the perception that certain fundamental aspects of good mathematical pedagogy and curriculum will be changed when this is not necessary or even desired by educational experts. The greatest challenge is to ensure that on-line environments provide students with quality learning experiences. The supporting educational system has to be prepared to change and adapt to the demands that will be created with this type of learning.

The points the participants identified as issues and challenges are organized in areas of the student, the teacher, the learning environment and the subject matter of mathematics. Many of the points are applicable for all on-line learning environments and others are unique to only mathematics education.

Student

Our objective is to have on-line learning as one of the ways for students to become successful in learning mathematics. This creates many challenges.

1. Issue: Students display many different learning styles when learning mathematics. A visual learning style is emphasized in on-line learning whereas tactile manipulation for kinesthetic learning is difficult for students to experience in this environment. Challenge: As on-line learning is not as holistic as face-to-face learning it is a challenge to accommodate the students’ needs.
2. Issue: On-line learning is dependent on student skills such as the ability to read well and be technologically proficient. In mathematics, algebraic graphing software and spreadsheet software are used extensively depending on the age of the student. Working in groups but through the medium of technology is a necessary capacity used extensively in on-line learning. Challenge: In order to be successful students will require mastery of certain skills before taking on an on-line course. This presents a challenge for elementary and secondary schools to ensure that their graduates are prepared for on-line learning.
3. Issue: Students want choice in the different versions of on-line learning. They may want to work at their own rate or they may require parts of courses - “what I need when I need it”. Challenge: Many different types of on-line learning challenge the role of teachers, the instructional design of the courses or modules, and the administrative support for such initiatives.
4. Issue: Students need ready and equal access to the Internet network in on-line learning. Challenge: This presents demands on our educational technology system to ensure that students are all treated fairly in their availability to the Network.
5. Issue: The mathematics content the students learn may be undermined in the midst of all this change to a different approach to learning. Challenge: The mathematics curriculum must be upheld and be paramount in ensuring quality mathematical learning.
6. Issue: In this type of environment, there are barriers to student contact with peers and the teacher. Challenge: Educators must ensure that students’ communication skills and access to others are not diminished.

7. Issue: Students desire a safe and secure environment in taking a fully on-line course. Challenge: A student's personal vulnerability needs to be taken into consideration through the design and the student's involvement in any type of on-line course.

Teacher

The teacher's role changes to more of a facilitator and the degree of change varies depending on the type of on-line learning. Teachers need to maintain their professionalism of quality fundamental pedagogical practices in teaching mathematics on-line.

1. Issue: It is paramount that an on-line mathematics course be thoroughly organized and developed ahead of time. Challenge: More emphasis is put on certain aspects of the teacher's practice such as the advance preparation and complete lesson plans.
2. Issue: Assessment strategies are altered to fit on-line learning in mathematics. Challenge: Consideration has to be given to the challenges presented by assessment through technology while maintaining quality evaluation. Also, it must be taken into consideration when developing assessment by technology that this can result in more opportunities for students to cheat.
3. Issue: A sharing of information with other teachers becomes even more important in this new environment. Challenge: Teachers need support in seeking advice and gathering examples of best practices from experienced role models.
4. Issue: It is essential to have research into matters that arise for discussion such as the ideal class size, the teacher's role and the use of textbooks. Challenge: Many teachers need guidance in the actual act and type of research such as action research where they learn as they teach and observe.
5. Issue: Instructional design in a technological environment takes on a new role in remote teaching. Challenge: The teacher needs new skills and knowledge for this task.
6. Issue: The issues involved in the intellectual property become even more important in such a teaching environment. Challenge: Teachers need guidelines as to the actual ownership and copyright of their own developed material.
7. Issue: The teacher wants to feel safe and secure in teaching fully on-line modules or courses. Challenge: Protocols for students and teachers need to be developed for on-line relationships.
8. Issue: In this change to the educational environment, teachers are in need of great assistance. Challenge: The teachers' unions and professional bodies need to be involved and proactive in supporting and protecting teachers.
9. Issue: There could be an issue with society's perception that the role of students and their teachers has completely changed. Challenge: All those involved in education must be united in the message to society that the interaction between teachers and students is still fundamental to quality learning of mathematics.

Learning Environment

A partnership of experts in discipline, technology, pedagogy and design is required for this new learning environment. There is a great need for commitment from the supporting structure of administration.

1. Issue: Meaningful material and necessary tools such as learnware need the support of time and funding to be developed properly. Challenge: Educators must be ensured that adequate financial support and resources is available in order for them to develop these quality tools and material.

2. Issue: There is a monopoly of certain on-line technological tools that may influence the choice of pedagogy. Challenge: Educators must ensure that good mathematical pedagogy is paramount in any decision to adopt a technological tool.
3. Issue: There is a need for educators to share and learn from each other in this environment. Challenge: Administration is asked to promote and facilitate these initiatives which involve learning from your peers.
4. Issue: Students require ease in navigating through courses. Challenge: There are demands on the technology support staff to ensure that the technology is user-friendly.
5. Issue: Decisions are made as to the degree to which on-line learning is included in a course. Challenge: It is imperative that a concerted effort is made to involve all the relevant educational professionals in the degree to how much on-line learning is involved in any educational pursuit.
6. Issue: Given the time constraints in any educational environment, educators are pressured to make quick decisions on the timing and the extent of on-line learning. Challenge: There is a need for input from all educators in deciding when and how to include on-line learning in the course.

Mathematics

The educational system needs to stay true to the subject matter of mathematics. The pedagogy should be in control not technology or the pursuit of efficiency.

1. Issue: Rich learning tasks are important in developing mathematical concepts. Challenge: On-line technology tools need to support exploration and discussion in mathematics.
2. Issue: Students should be able to easily communicate mathematics electronically. Challenge: The emphasis is on collaborative communication systems and easily manipulated mathematical objects in the on-line learning environment.
3. Issue: Best practices in a classroom may not adapt easily to on-line learning. Challenge: The on-line environment must absorb only those actions of the classroom which fit quality mathematical learning, teaching and content.

Recommendations

Earlier in this paper, we described our vision and opportunities for on-line mathematics education as developed at the on-line mathematics conference. We focused on opportunities for students, teachers, mathematics as a subject, and the learning environment. In many cases, these visions model good face-to-face teaching practices with facilitation and on-line collaborative experiences. In this section, we outline the recommendations proposed by the conference participants.

Each public education level (elementary, secondary, college and university) discussed the issues and then provided recommendations for on-line mathematics learning. The following section will identify the recommendations by the following categories: teacher, student, mathematics, learning environment, implementation, and research. The next section will summarize the common recommendations among the groups for on-line mathematics learning.

Student

It is recommended that we:

- provide software for elementary students
- could build a continuum of student on-line experiences, from elementary to post-secondary
- colleges need policies that ensure they maintain intellectual ownership for students' research projects
- colleges need to have appropriate use policies for the use of technology

Teacher

It is recommended that we:

- organize on-line professional development for teachers. A graduated series of developmental work for teachers needs to be created and teachers can pick the level that they are comfortable
- provide administrative support for teachers' professional development
- provide on-line teacher learning experiences about good mathematics. If teachers have a positive experience using the on-line programs themselves; there will be an easier transition for using that in their classrooms
- important to provide experiences to teachers to become an effective moderator
- Faculties of Education should teach teacher candidates about moderating learning in an e-learning environment and in a classroom.
- boards should provide time for implementing these professional development experiences during the school week
- faculty of education students, as part of their course work, may help develop and update a public database of screened on-line resources
- could train teachers using on-line courses so they are familiar and comfortable with the technology
- Faculty of education experience may be partly on-line
- provide staff development (pre-service and in-service) related to on-line learning
- colleges create opportunities for dialogue among current practitioners to discuss what does and doesn't work in relation to this new format
- colleges create opportunities to share the information with other faculty such as technology information, curriculum, best practices, maybe informal faculty meetings that deal with issues
- colleges require appropriate teaching facility, availability, hardware, need physical plant to

support this new modality

- colleges should provide courses for faculty on on-line learning and bring in experts to assist with development, design and move course from f2f to on-line

Learning Environment

It is recommended that we:

- each school board needs a committee that deals with technical support for on-line learning. They should develop protocols and provide information to superintendents, technology technicians, coordinators, principals/vice principals, and school board members/trustees
- development of programs should always include the technology expert
- create a committee on assessment on-line professional development to explore what tools are being used to communicate understanding of mathematics.
- need to take the time to create quality learning environments as poor instructional models entrenched in on-line modules may be difficult to undo
- software developers may find it difficult to start fresh and may be inclined to build on their initial investments despite shortcomings
- should use a modular approach where good modules should be of value to students, teachers and parents
- should encourage multiple approaches to developing a solid understanding and allow for teacher and student choice
- need to develop a set of criteria to evaluate on-line learning packages and to guide users in the selection or development of on-line learning experiences (criteria to focus on the quality of the educational experience such on-line learning packages support)
- colleges need to inform public of new format of delivery both on-line, partial on-line through providing information, research, and marketing campaigns for public, parents, students, and other interested parties such as high schools, colleges, and universities must participate
- colleges courses should be offered in 3 formats and need to be consistent. Students should be able to move from one format to another with little difficulty
- technology assistance such as technologists to provide assistance to move to on-line college courses
- perceive the continued existence of a physical university - providing on-line enhancements to courses such as on-line labs & tutorials, collaborative peer-to-peer help, collaborative mathematical exploration, on-line modules for students lacking particular skills & concepts, and inter-university sharing of specialized senior courses

Mathematics

It is recommended that we:

- create exemplary practices with school districts
- colleges need to have appropriate mathematics software
- should provide support for a variety of initiatives and experiments in on-line course enhancement with a focus on the doing of mathematics. Commitment for experimentation from university administrations
- focus needs to be on and must be seen to be on mathematics and not technology.
- need linked software such that on-line mathematical objects can be collaboratively manipulated
- need a system that translates captured graphics output from one program into the mathematical

objects they represent to permit further manipulation by other participants using the same or different software.

Implementation

It is recommended that we:

- promotion is necessary by an organization such as Fields Institute
- create a strand of OAME conferences and leadership conference to involve more teachers
- provide some recognition such as getting certificates and recognition of their participation in this pioneering stage of on-line teaching in mathematics
- create a committee to evaluate software. Coordinate with other organizations to evaluate software and design introductory lessons using the software
- school boards can work together to create on-line learning activities
- coordinate Implementation suggestions for teachers such as what things need to enable on-line learning to take place
- Disseminate white paper to Ministry of Education, Industry, School districts and OAME, OMCA, CMS
- create Fields on-line mathematics subcommittees to investigate database of web-based mathematics resources, On-line modules
- discuss at a Fields working meeting reports of sub-committees, examples of on-line teaching resources, Federal government sponsorship and samples of what is happening
- colleges need to create opportunities for access to technicians/technologists to generate a common level of understanding
- colleges need to create opportunities for union involvement with regard to on-line teaching, recognition, workload, equity
- colleges advocate need to be proactive, sit on committees, integrate selves into full college life and be knowledgeable,
- colleges should support on-line learning with time, money and additional resources for on-line learning
- colleges should acknowledge that on-line learning is teaching and requires the same level of expertise, knowledge, dedication, professionalism, etc
- investigate cost benefit for conducting on-line university courses of the form the group envisions could cost more than traditional delivery because faculty involvement and monitoring would be greater.
- need common university software standards, protocols, and Open source code so that experiments can modify and adapt applications to particular needs and aims.

Research

It is recommended that we:

- provide opportunities for school boards to get research on on-line learning
- conduct a further Fields forum, perhaps in the format of the CMESG conference, with some presentations, working groups and infusion of research about on-line mathematics teaching/learning, with follow-up groups
- gather and disseminate information from teachers using on-line mathematics teaching/learning environments
- Fields should conduct the next forum with concrete examples to guide discussion, development of criteria and protocols, with computers, the on-line materials, the students and

teachers

- should build a public database of web-based resources starting with school districts that have already done this
- need research that allows us to characterize current typical and exemplary on-line learning practices.
- colleges work with interested groups who have done on-line learning

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