Empirical Assessment of an Intertemporal Option Pricing Model with Latent Variables

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First version: September 2000
This version: January 2002

- **We estimate preference parameters from option prices**
- **How come?**
  - **Preference-free formulas**
  - **Option prices should be informative about preference parameters**
  - **Are they? Evidence from SP 500 option prices**
- **Other motivations:**
  - **Current debate on extracting preference parameters from option prices**
  - **Macrofinance: Are preference parameters reasonable?**

- **Considerable divergence between risk-neutral distributions estimated from option prices after 1987 crash and conditional distributions estimated from time series of returns.**

  - **implied volatility extracted from at-the-money options differs substantially from realized volatility over the lifetime of the option.**
  - **risk neutral distributions feature a substantial negative skewness (revealed by asymmetric implied volatility curves when plotted against moneyness).**
  - **shape of volatility curves change over time; skewness is time-varying.**

- **One possible explanation of divergence: existence of time-varying risk premia.**

- **In a jump-diffusion model proposed by Bates (1996), Pan (2000) shows that the addition of both volatility and jump risk premia allows to fit well the joint time series of S&P 500 spot and option price data.**

- **The model can explain well the changing shapes of the implied volatility curves over time.**

- **The skewed patterns are largely due to investors' aversion to jump risks.**

- **But in this approach investors have different risk attitudes towards the diffusive return shocks, volatility shocks and jump risks.**

- **In a nonparametric framework, Jackwerth (2000), Aït-Sahalia and Lo (2000) and Engle and Rosenberg (1999) uncover the risk-aversion function implied by the comparison between the objective and the risk-neutral distributions.**

- **Jackwerth (2000) finds that the preferences are oddly shaped, with marginal utilities increasing in some parts.**

- **However, implied-tree and kernel methodologies used to recover the risk-neutral and the subjective probabilities are not likely to separate neatly the preferences from the probabilities.**

- **Risk aversion changes with time horizon**

- **These results underline the potential importance of investors' preferences for options prices**

- **But question of knowing if option prices are compatible with reasonable preferences largely unanswered.**

- **We propose a utility-based option pricing model with stochastic volatility and jump features.**

- **The model is cast within the recursive utility framework of Epstein and Zin (1989).**

- **Disentangling respective roles of discounting, risk aversion and intertemporal substitution might be important for option pricing**

- **An option contract will naturally be affected by the value of time as well as the price of risk associated with the underlying asset.**

- **We derive an option pricing formula which generalizes the Black and Scholes (1973) and the Hull and White (1987) and Heston (1993) stochastic volatility formulas.**

- **An essential feature of this generalized option pricing formula is that it is not in general preference-free.**

- **In so-called preference-free formulas, preference parameters are eliminated from the option pricing formula through the observation of the bond price and the stock price.**

- **In fact, preference parameters are hidden in the observed stock and bond prices.**

- **In our model, the bond pricing formula and the stock pricing formula provide two dynamic restrictions.**

- **The key assumption: presence of an unobservable state variable driving the fundamentals (consumption and dividends) of the economy.**

- **Interplay between preferences and latent factors that affect the stochastic discount factor have been explored recently.**

- **Garcia, Luger and Renault (2001): the option pricing model we estimate in this paper can reproduce the various patterns observed in implied volatility curves as well as changing skewness over time.**

- **David and Veronesi (1999): option prices are affected by investors' beliefs about the drift of a firm's fundamentals.**

- **In particular, they emphasize how investors' beliefs and their degree of risk aversion affect stock returns and hence option prices.**

Outline of presentation

1. **An intertemporal Option Pricing Model with Latent Variables**
2. **Pricing Formulas for Bonds and Stocks**
3. **A Generalized Option Pricing Formula**
4. **Estimation of the Option Pricing Model**
5. **Calibrating the Model for Practical Option Pricing**

- **Recursive utility framework proposed by Epstein and Zin (1989)**

- **Euler condition valid for any asset j:**

$$E[\beta^{\gamma}(\frac{C_{t+1}}{C_t})^{\gamma(\rho-1)} M_{t+1}^{\gamma-1} R_{j,t+1} | J_t] = 1$$

where M_{t+1} represents the return on the market portfolio, $R_{j,t+1}$ the return on any asset j, and $\gamma = \frac{\alpha}{\rho}$.

- **Coefficient or relative risk aversion is $1-\alpha$, elasticity of intertemporal substitution is $1/(1-\rho)$.**

- **Payoff of the market portfolio is the total endowment of the economy C_t.**

- **Return on the market portfolio M_{t+1}:**

$$M_{t+1} = \frac{P_{t+1}^M + C_{t+1}}{P_t^M}.$$

with $\lambda_t = \frac{P_t^M}{C_t}$, we obtain for Euler condition for the market:

$$\lambda(J)^\gamma = E\left[\beta^\gamma \left(\frac{C_{t+1}}{C_t}\right)^{\gamma\rho} (\lambda(J_{t+1}) + 1)^\gamma \, |J_t = J\right].$$

- **Similarly, we will be looking for a solution $\varphi_t = \varphi(J_t) = \frac{S_t}{D_t}$ to the stock pricing equation:**

$$\varphi(J) = E\left[\beta^\gamma \left(\frac{C_{t+1}}{C_t}\right)^{\gamma\rho-1} \left(\frac{\lambda_{t+1}+1}{\lambda_t}\right)^{\gamma-1} \varphi(J_{t+1})\frac{D_{t+1}}{D_t} \, |J_t = J\right].$$

- **The dynamic behavior of rates of return:**

$$LogM_{t+1} = Log\frac{\lambda(J_{t+1}) + 1}{\lambda(J_t)} + Log\frac{C_{t+1}}{C_t}, \textbf{and}$$

$$LogR_{t+1} = Log\frac{S_{t+1}}{S_t} = Log\frac{\varphi(I_{t+1})}{\varphi(I_t)} + \log\frac{D_{t+1}}{D_t},$$

is determined by the joint probability distribution of the stochastic process (X_t, Y_t, J_t) **where:** $X_t = Log\frac{C_t}{C_{t-1}}$ **and** $Y_t = Log\frac{D_t}{D_{t-1}}$.

A pricing model conditional on latent state variables

- **We define this dynamics through a stationary vector-process of state variables U_t so that:**

$$J_t = \vee_{\tau \leq t}[X_\tau, Y_\tau, U_\tau].$$

- **We want state variables to be exogenous and stationary and to subsume all temporal links between the variables of interest (X_t, Y_t).**

Assumption 1.: The fundamentals (X, Y) do not cause the state variables U in the Granger sense or equivalently, given assumption 2 below, the conditional probability distribution of (X_t, Y_t) given $U_1^T = (U_t)_{1 \leq t \leq T}$ coincides, for any $t = 1, ..., T$, with the conditional probability distribution given $U_1^t = (U_\tau)_{1 \leq \tau \leq t}$.

Assumption 2.: The pairs $(X_t, Y_t)_{1 \leq t \leq T}, t = 1, ..., T$ are mutually independent knowing $U_1^T = (U_t)_{1 \leq t \leq T}$.

Assumption 3:

$$\begin{pmatrix} X_{t+1} \\ Y_{t+1} \end{pmatrix} |\mathbf{U}_t^{t+1} \sim \aleph \left[\begin{pmatrix} m_{Xt+1} \\ m_{Yt+1} \end{pmatrix}, \begin{bmatrix} \sigma^2_{Xt+1} & \sigma_{XYt+1} \\ \sigma_{XYt+1} & \sigma^2_{Yt+1} \end{bmatrix} \right],$$

where $m_{Xt+1} = m_X(U_1^{t+1}), m_{Yt+1} = m_Y(U_1^{t+1}), \sigma^2_{Xt+1} = \sigma^2_X(U_1^{t+1}),$

$\sigma_{XYt+1} = \sigma_{XY}(U_1^{t+1}), \sigma^2_{Yt+1} = \sigma^2_X(U_1^{t+1})$**.**

- **In other words, these mean and variance covariance functions are time-invariant and measurable functions with respect to** U_t^{t+1}, **which includes both** U_t **and** U_{t+1}.

Under assumptions 1 and 2 we have:

$$P_t^M = \lambda(U_t) C_{t,} \qquad S_t = \varphi(U_t) D_t,$$

where $\lambda(U_t)$ **and** $\varphi(U_t)$ **are respectively defined by :**

$$\lambda(U_t)^\gamma = E\left[\beta^\gamma \left(\frac{C_{t+1}}{C_t}\right)^{\gamma\rho} (\lambda(U_{t+1})+1)^\gamma \,|U_t\right],$$

and

$$\boldsymbol{\varphi}(\mathbf{U}_t) = \mathbf{E}\left[\beta^\gamma \left(\frac{C_{t+1}}{C_t}\right)^{\gamma\rho-1} \left(\frac{\lambda(U_{t+1})+1}{\lambda(U_t)}\right)^{\gamma-1} (\varphi(U_{t+1})+1)\frac{D_{t+1}}{D_t} \,|U_t\right].$$

$$LogM_{t+1} = Log\frac{\lambda(U_{t+1})+1}{\lambda(U_t)} + X_{t+1}, \textbf{and}$$

$$LogR_{t+1} = Log\frac{\varphi(U_{t+1})+1}{\varphi(U_t)} + Y_{t+1}.$$

The Pricing of Bonds

- **Given the Euler condition and assumptions 1, 2 and 3, the price of a bond delivering one unit of the good at time T,** $B(t,T)$, **is given by the following formula:**

$$B(t,T) = E_t\left[\beta^{\gamma(T-t)}\left(\frac{C_T}{C_t}\right)^{\alpha-1}\prod_{\tau=t}^{T-1}\left[\frac{(1+\lambda(U_1^{\tau+1})}{\lambda(U_1^{\tau})}\right]^{\gamma-1}\right].$$

- **Therefore, it can be written as:**

$$B(t,T) = E_t[\widetilde{B}(t,T)].$$

where:

$$\widetilde{B}(t,T) = \beta^{\gamma(T-t)}a_t^T(\gamma)\exp((\alpha-1)\sum_{\tau=t}^{T-1} m_{X\tau+1} + \frac{1}{2}(\alpha-1)^2\sum_{\tau=t}^{T-1}\sigma_{X\tau+1}^2),$$

with: $a_t^T(\gamma) = \prod_{\tau=t}^{T-1}\left[\frac{(1+\lambda(U_1^{\tau+1})}{\lambda(U_1^{\tau})}\right]^{\gamma-1}$.

- **For von-Neuman preferences ($\gamma = 1$) the term premium is proportional to the square of the coefficient of relative risk aversion (up to a conditional stochastic volatility effect).**

- **Even without any risk aversion ($\alpha = 1$), preferences still affect the term premium through the non-indifference to the timing of uncertainty resolution ($\gamma \neq 1$).**

- **Important sub-case where the term premium will be preference-free:**

$$\widetilde{B}(t,T) = B(t,T) = E_t \prod_{\tau=t}^{T-1} B(\tau, \tau+1).$$

- **Noticing that $\widetilde{B}(t,T) = \prod_{\tau=t}^{T-1} \widetilde{B}(\tau, \tau+1)$, this will occur as soon as $\widetilde{B}(\tau, \tau+1) = B(\tau, \tau+1)$, that is when $\widetilde{B}(\tau, \tau+1)$ is known at time τ.**

- **If and only if the mean and variance parameters $m_{X\tau+1}$ and $\sigma_{X\tau+1}$ depend on $U_\tau^{\tau+1}$ only through U_τ.**

The Pricing of Stocks

- **By a recursive argument:**

$$E_t\left[\beta^{\gamma(T-t)}a_t^T(\gamma)b_t^T\left(\frac{C_T}{C_t}\right)^{\alpha-1}\frac{D_T}{D_t}\right]=1,$$

with: $b_t^T=\prod_{\tau=t}^{T-1}\frac{(1+\varphi(U_1^{\tau+1}))}{\varphi(U_1^{\tau})}$.

- **Using the conditional log-normality assumption 3, we obtain:**

$$E_t\left[\widetilde{B}(t,T)b_t^T\exp(\sum_{\tau=t+1}^{T}m_{Y\tau}+\frac{1}{2}\sum_{\tau=t+1}^{T}{\sigma^2}_{Y\tau}+(\alpha-1)\sum_{\tau=t+1}^{T}\sigma_{XY\tau})\right]=1.$$

- **With the definitional equation:**

$$E[\frac{S_T}{S_t}|U_1^T] = \frac{\varphi(U_1^T)}{\varphi(U_1^t)} \exp(\sum_{\tau=t+1}^{T} m_{Y\tau} + \frac{1}{2}\sum_{\tau=t+1}^{T} \sigma_{Y\tau}^2),$$

- **A useful way of writing the stock pricing formula is:**

$$E_t\left[Q_{XY}(t,T)\right] = 1,$$

where:

$$Q_{XY}(t,T) = \widetilde{B}(t,T) b_t^T \frac{\varphi(U_1^t)}{\varphi(U_1^T)} \exp((\alpha - 1)\sum_{\tau=t+1}^{T} \sigma_{XY\tau}) E[\frac{S_T}{S_t}|U_1^T].$$

- **To understand the role of the factor $Q_{XY}(t,T)$, it is useful to notice that it can be factorized:**

$$Q_{XY}(t,T) = \prod_{\tau=t}^{T-1} Q_{XY}(\tau,\tau+1),$$

- **Important particular case where $Q_{XY}(\tau,\tau+1)$ is known at time τ and therefore equal to one.**

- **Neither the conditional means and variances of X_t or Y_t at time t nor the covariance σ_{XYt} depend on U_t.**

- **Since we also have $\widetilde{B}(\tau, \tau+1) = B(\tau, \tau+1)$, we can express the conditional expected stock return as:**

$$E\left[\frac{S_T}{S_t}|U_1^T\right] = \frac{1}{\prod_{\tau=t}^{T-1} B(\tau, \tau+1)} \frac{1}{b_t^T} \frac{\varphi(U_1^T)}{\varphi(U_1^t)} \exp((1-\alpha) \sum_{\tau=t+1}^{T} \sigma_{XY\tau}).$$

- **For pricing over one period (t to $t+1$) :**

$$E\left[\frac{S_{t+1} + D_{t+1}}{S_t}|U_1^t\right] = \frac{1}{B(t, t+1)} \exp[(1-\alpha)\sigma_{XYt+1}],$$

- **Very close to a standard conditional CAPM equation (and unconditional in an iid world), which remains true for any value of the preference parameters α and ρ.**

- **The stochastic setting which produces this CAPM relationship will also produce most standard option pricing models.**

- We arrive at the generalized Black-Scholes and Hull and White formula for pricing options

$$\frac{p_t}{S_t} = E_t \left\{ Q_{XY}(t,T) F(d_1) - \frac{K\tilde{B}(t,T)}{S_t} F(d_2) \right\},$$

where:

$$d_1 = \frac{Log\left[\frac{S_t \; Q_{XY}(t,T)}{K\tilde{B}(t,T)}\right]}{(\sum_{t=t+1}^{T} s_{Yt}^2)^{1/2}} + \frac{1}{2}\left(\sum_{t=t+1}^{T} s_{Yt}^2\right)^{1/2}$$

$$d_2 = d_1 - \left(\sum_{t=t+1}^{T} s_{Yt}^2\right)^{1/2}$$

$$\tilde{B}(t,T)=b^{g(T-t)} a_t^T(g)\exp\left((a-1)\sum_{t=t+1}^{T} m_{Xt} + \frac{1}{2}(a-1)^2\sum_{t=t+1}^{T} s_X^2 X_t\right),$$

with:

$$a_t^T(g)=\prod_{t=t}^{T-1}\left[\frac{1+l(U_1^{t+1})}{l(U_1^t)}\right]^{g-1}, \quad l_t(\cdot)=\frac{P_t^M}{C_t}.$$

$$Q_{XY}(t,T)=\tilde{B}(t,T)\, b_t^T \exp\left((a-1)\sum_{t=t+1}^{T} s_{XYt}\right) E\left[\frac{S_T}{S_t}\middle| U_1^T\right]$$

with:

$$b_t^T = P_{t=t+1}^{T} \frac{1+j(U_1^t)}{j(U_1^t)}, \quad j_t(\cdot)=\frac{S_t}{D_t}$$

- **Our equilibrium-based option pricing formula does not preclude incompleteness.**

- **Points out in which cases this incompleteness will invalidate the preference-free paradigm, i.e. when the conditions** $Q_{XY}(t,T) = 1$ **and** $\widetilde{B}(t,T) = \prod_{\tau=t}^{T-1} B(\tau, \tau+1)$ **are not fulfilled.**

- **In this case preference parameters appear explicitly in the option pricing formula through** $\widetilde{B}(t,T)$ **and** $Q_{XY}(t,T)$.

A MARKOV-CHAIN PROCESS FOR THE STATE VARIABLES

$$X_t = m_x(U_t) + \sigma_x(U_t)\,\varepsilon_{X_t}$$

$$Y_t = m_y(U_t) + \sigma_y(U_t)\,\varepsilon_{Y_t}$$

Process $\{U_t\} \equiv$ 2 state discrete first-order Markov Chain.

$$p_{ij} = \Pr\left(U_t = j \,|U_{t-1} = i\right) \qquad\qquad i, j = 1, 2$$

Unconditional probabilities

$$\pi_1 = \frac{1 - p_{22}}{2 - p_{11} - p_{22}}$$

$$\pi_2 = 1 - \pi_1$$

$$m_x(U_t) = \alpha_0 + \alpha_1 U_t \qquad\qquad \sigma_x(U_t) = (\sigma_0 + \sigma_1 U_t)$$

Informational Content of Option Prices on preference parameters:

Issue = to gauge the accuracy of estimators of preference parameters based on option prices **and** the GBS model.

→ **Monte Carlo experiments** on data simulated according to the GBS MODEL:

→ given values of the parameters for these experiments:

$p_{11} = 0.9$ $p_{22} = 0.6$

$m_{X_1} = 0.0015$ $m_{Y_1} = m_{Y_2} = 0$

$m_{X_2} = -0.0009$ $s_{Y_1} = 0.02$

$s_{X_1} = s_{X_2} = 0.003$ $s_{Y_2} = 0.12$

Moments of stock returns

$$E[r_t] = \sum_{i=1}^{2}\sum_{j=1}^{2}\pi_i p_{ij}\left(\log\frac{\varphi_j+1}{\varphi_i}+m_{Yj}\right),$$

$$Var[r_t] =$$

$$\sum_{i=1}^{2}\sum_{j=1}^{2}\pi_i p_{ij}\left[\left(\log\frac{\varphi_j+1}{\varphi_i}\right)^2+2m_{Yj}\left(\log\frac{\varphi_j+1}{\varphi_i}\right)+m_{Yj}^2+\sigma_{Yj}^2\right]-\mathbf{E}[\mathbf{r}_t]^2,$$

$$Cov[r_t, r_{t-1}] =$$

$$\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\pi_i p_{ij} p_{jk}\left[\left(\log\frac{\varphi_j+1}{\varphi_i}\right)^2+m_{Yj}^2\right]\left[\left(\log\frac{\varphi_k+1}{\varphi_j}\right)^2+m_{Yk}^2\right]-E[r_t]^2.$$

Moments for option prices of different moneynesses and maturities

$$E\left[\frac{\pi_t}{K}\right] = \sum_{i=1}^{2}\pi_i C_t(U_t=i,\kappa,\tau)$$

Tables I, II and III: Descriptive statistics for the method of moments estimator of preference parameters. The moments used in the estimation are the mean, the variance and the autocovariance of the respective series. For options, we also used the mean of the three options with different moneyness. The true values are $\rho = -10$, $\alpha = -5$ and $\beta = 0.95$ for the preferences and $p_{11} = 0.9$, $p_{22} = 0.6$, $m_{X1} = 0.0015$, $m_{X2} = -0.0009$, $\sigma_{X1} = \sigma_{X2} = .003$, $m_{Y1} = m_{Y2} = 0$, $\sigma_{Y1} = 0.02$, $\sigma_{Y2} = 0.12$ and $\rho_{XY} = 0.6$ for the endowment process. The results are reported for options with maturity of one period . The results are based on 1000 replications of the experiment.

Table I

Options Prices (time series)	ρ	α	β
Mean	-10.1585	-4.6162	0.9445
Median	-10.2131	-4.7979	0.9445
Std Err	1.0524	1.8975	0.0093
RMSE	1.0638	1.9350	0.0108

Options Prices (across moneyness)	ρ	α	β
Mean	-10.1421	-4.6770	0.9504
Median	-10.2171	-4.7927	0.9500
Std Err	1.0117	1.2921	0.0159
RMSE	1.0212	1.3312	0.0159

Table II

Stock Returns	ρ	α	β
Mean	-11.0711	-2.4557	0.9950
Median	-10.9812	-1.8966	0.9955
Std Err	1.0457	1.6153	0.0035
RMSE	1.4965	3.0134	0.0451

Table III

Price-dividend ratio	ρ	α	β
Mean	-10.5537	-3.5051	0.9501
Median	-10.0003	-4.9861	0.9497
Std Err	1.2742	2.1530	0.0017
RMSE	1.3887	2.6202	0.0017

2nd Experiment: Joint estimation of the structural parameters

12 unknown structural parameters

$$\begin{cases} P_{11}, P_{22}, m_{x1}, m_{x2}, s_{x1}, s_{x2}, s_{y1}, s_{y2}, r_{xy} \\ \left(m_{y1} = m_{y2} = 0 \quad \text{"known"} \right) \\ a, b, g \end{cases}$$

12 moments:

$$E r_t 1 Var\, r_t, Cov(r_t, r_{t-1})$$

$$E\left[\frac{GBS(x_i)}{K_i} \right] \text{ for:}$$

→ 3 moneynesses

→ maturities 1, 2, 3

Is there evidence of preference parameters in

S&P 500 Option Prices?

- **We estimate the model every day with a 3-month window lag and the moment conditions used in the simulation study**

- **Based on estimates, we forecast each day the pricing error for all options the next day.**

- **We report results by maturity categories:**

Categories:	**Short,**	**Medium,**	**and long**
	$(\tau < 60)$	$(60 < \tau < 180)$	$(\tau > 180)$

for three models: GBS, EU, SV.

Table IV: Descriptive statistics for the joint estimation of the structural parameters by the method of moments. The true values are $\rho = -10$, $\alpha = -5$ and $\beta = 0.95$ for the preferences and $p_{11} = 0.9$, $p_{22} = 0.6$, $m_{X1} = 0.0015$, $m_{X2} = -0.0009$, $\sigma_{X1} = \sigma_{X2} = .003$, $m_{Y1} = m_{Y2} = 0$, $\sigma_{Y1} = 0.02$, $\sigma_{Y2} = 0.12$ and $\rho_{XY} = 0.6$ for the endowment process. The results are based on 1000 replications of the experiment.

Table IV

	β	ρ	α	p_{11}	p_{22}	ρ_{XY}
Mean	0.9164	-10.0517	-4.9728	0.8983	0.5916	0.5954
Median	0.9504	-9.9903	-5.0177	0.9010	0.5983	0.5997
Std Err	0.1119	1.4381	1.3672	0.0507	0.0749	0.0980
RMSE	0.1168	1.4383	1.3667	0.0507	0.0753	0.0981
	m_{X1}	m_{X2}	σ_{X1}	m_{Y1}	σ_{Y1}	σ_{Y2}
Mean	0.0520	0.0500	0.0068	-0.0780	0.0462	0.1849
Median	0.0013	-0.0052	0.0031	-0.0088	0.0193	0.1249
Std Err	1.0176	0.8822	0.0267	0.5529	0.3704	0.2028
RMSE	1.0183	0.8832	0.0269	0.5581	0.3711	0.2128

Table V: Yearly Means and Standard Errors of Weekly Estimated Preference Parameters from S&P 500 Option Price Data over the Period 1991-1995

Table V

GBS Model					
	ρ	γ	β	CRRA	EIS
1991	-0.2048 (0.0904)	-1.6637 (0.9144)	0.9397 (0.0372)	0.6885 (0.0987)	0.8342 (0.0564)
1992	-0.0936 (0.0400)	-1.9975 (0.4171)	0.9783 (0.0180)	0.8201 (0.0646)	0.9156 (0.0321)
1993	-0.2007 (0.0737)	-2.4294 (1.1218)	0.9413 (0.0380)	0.5509 (0.1269)	0.8358 (0.0494)
1994	-0.2110 (0.1211)	-1.7369 (0.6011)	0.9142 (0.0437)	0.6706 (0.1366)	0.8334 (0.0778)
1995	-0.1963 (0.1504)	-1.8744 (0.7700)	0.9029 (0.0377)	0.6884 (0.1559)	0.8466 (0.0870)
1991-1995	-0.1812 (0.1114)	-1.9406 (0.8458)	0.9353 (0.0444)	0.6838 (0.1478)	0.8532 (0.0710)

Expected Utility Model			
	ρ	β	CRRA
1991	-8.7505 (1.7685)	0.9513 (0.0229)	9.7505 (1.7685)
1992	-6.2337 (3.7156)	0.8401 (0.1259)	7.2337 (3.7156)
1993	-4.9742 (1.8897)	0.9710 (0.0275)	5.9742 (1.8897)
1994	-5.1044 (7.0187)	0.8321 (0.1026)	6.1044 (7.0187)
1995	-5.7259 (6.1479)	0.8172 (0.1230)	6.7259 (6.1479)
1991-1995	-6.1590 (4.8260)	0.8824 (0.1130)	7.1590 (4.8260)

Table VI: Yearly Relative Pricing Errors for Short, Medium and Long-Term Call Options Averaged Over Moneyness. GBS refers to the generalized Black-Scholes formula in (3.5);EU to the same formula special case where the parameter γ is equal to 1; SV to the stochastic volatility formula (special case of (3.5) with $Q_{XY}(t,T)=1$).

Table VI

Relative Errors				Absolute Errors			
Short-Term	GBS	EU	SV	Short-Term	GBS	EU	SV
1991 (3132)	0.8588	1.4995	1.5798	1991(3132)	3.1444	4.4779	4.8473
1992 (2928)	1.3303	1.8417	1.9287	1992(2928)	3.6726	4.2741	5.2431
1993 (2921)	1.7720	1.7636	1.7769	1993(2921)	4.2028	3.8674	4.2968
1994 (3365)	1.4821	1.9350	2.3282	1994(3365)	3.1141	3.8733	4.4483
1995 (4022)	1.4664	1.3508	2.1910	1995(4022)	4.0907	4.2658	5.6873

Relative Errors				Absolute Errors			
Medium-Term	GBS	EU	SV	Medium-Term	GBS	EU	SV
1991 (2187)	0.3436	0.7731	0.7669	1991 (2187)	2.8921	3.9251	4.4258
1992 (2379)	0.7215	1.1348	1.2831	1992 (2379)	3.3759	4.6117	5.4437
1993 (2163)	1.2042	1.3287	1.3471	1993 (2163)	4.4210	4.4138	4.9754
1994 (2897)	1.2097	1.5967	1.9032	1994 (2897)	3.6488	4.4388	4.9771
1995 (2991)	0.8658	0.9799	1.4150	1995 (2991)	4.5432	5.2378	6.4743

Relative Errors				Absolute Errors			
Long-Term	GBS	EU	SV	Long-Term	GBS	EU	SV
1991 (694)	0.0036	0.1946	0.2374	1991 (694)	2.5882	3.0367	3.4266
1992 (538)	-0.0170	0.2246	0.2128	1992 (538)	3.0306	3.9870	3.3401
1993 (492)	0.2138	0.2969	0.2278	1993 (492)	2.5911	2.9982	2.8856
1994 (910)	-0.0006	0.0864	0.2543	1994 (910)	3.5838	4.4165	3.5591
1995 (1053)	0.1212	0.2417	0.5201	1995 (1053)	3.3501	4.4264	4.4793

Calibrating the Model for Practical Option Pricing

New specification for dividend volatility

$$\sigma_Y(U_t = j) = \delta_{0j} + \delta_{1j}\sigma_t^* \sqrt{(T-t)},$$

for $j = 1, 2$**.**

Estimation method: for a given maturity $(\mathbf{T}-\mathbf{t})$, we minimized

$$\frac{1}{M_{S_t/K}} \sum_{\tau=t-h}^{t} \left[E\left[GBS\left(U_t, \frac{S_t}{K}, (T-t), \sigma_t^* \right) \right] - \pi_\tau \left(\frac{S_t}{K}, (T-t) \right) \right]^2,$$

We impose constraints:

$$E_t \left[Q_{XY}(t,T) \right] = 1$$

$$E_t \left[\tilde{B}(t,T) \right] = \exp(-r(T-t))$$

Table VII: Yearly Means and Standard Errors of Daily Estimated Parameters for the Fundamentals and State Variable Processes from S&P 500 Option Price Data over the Period 1991-1995.

Table VII

λ_1	λ_2	φ_1	φ_2	p_{11}	p_{22}	ρ_{XY}
8.1182	10.6924	12.5700	18.8356	0.9758	0.8078	-0.3178
(0.6675)	(0.8453)	(0.7349)	(1.5631)	(0.0243)	(0.1266)	(0.5120)
	m_{X1}	m_{X2}	σ_X	m_Y	σ_{Y1}	σ_{Y2}
	-0.3216	0.0623	0.0202	-0.0688	0.0365	0.1139
	(0.1654)	(0.2270)	(0.0427)	(0.0076)	(0.0176)	(0.0802)

Table VIII: Conditional Pricing with Implied Volatility. Yearly Relative and Absolute Errors for Short-Term Call Options Averaged Over Moneyness. GBS refers to the generalized Black-Scholes formula in (3.5); EU to the same formula special case where the parameter γ is equal to 1; SV to the stochastic volatility formula (special case of (3.5) with $Q_{XY}(t,T) = 1$); BS refers to the ad hoc BS model.

Table VIIIA (one-day ahead forecast)

Relative Errors				
Short-Term	GBS	EU	SV	BS
1991 (3132)	0.0068	0.0078	0.0573	-0.0065
1992 (2928)	0.0212	0.0214	0.0728	0.0022
1993 (2921)	0.0221	0.0216	0.0775	-0.0034
1994 (3365)	0.0886	0.0888	0.1914	0.0473
1995 (4022)	0.0626	0.0611	0.1619	0.0092

Absolute Errors				
Short-Term	GBS	EU	SV	BS
1991 (3132)	0.9223	0.9214	1.0630	0.8019
1992 (2928)	0.7828	0.7829	0.8834	0.6899
1993 (2921)	0.7441	0.7456	0.8540	0.6616
1994 (3365)	0.6991	0.6987	0.8763	0.5959
1995 (4022)	0.9637	0.9656	1.2545	0.6802

Table VIIIB (five-day ahead forecast)

Relative Errors		
Short-Term	GBS	BS
1991 (3085)	0.032	0.015
1992 (2861)	0.017	0.002
1993 (2871)	0.017	0.009
1994 (3329)	0.087	0.044
1995 (3969)	0.068	0.0085

Absolute Errors		
Short-Term	GBS	BS
1991 (3085)	1.0432	0.9335
1992 (2861)	0.8226	0.7438
1993 (2871)	0.8151	0.7259
1994 (3329)	0.8642	0.7370
1995 (3969)	1.1004	0.7635

Table IX: Yearly Means and Standard Errors of Weekly Estimated Preference Parameters from S&P 500 Option Price Data over the Period 1991-1995

Table IX

GBS Model			
	ρ	γ	β
1991	-0.9010 (0.3821)	-0.8324 (0.3887)	0.9150 (0.0135)
1992	-0.9522 (0.5600)	-0.4948 (0.4557)	0.8704 (0.0512)
1993	-0.3631 (0.2426)	-2.9782 (1.2942)	0.9448 (0.0082)
1994	-0.6221 (0.4469)	-1.8325 (0.8712)	0.9471 (1.0620)
1995	-0.3040 (0.0941)	-1.2201 (0.3075)	0.9526 (0.0086)

Expected Utility Model		
	ρ	β
1991	-1.5242 (2.5058)	0.9804 (0.0198)
1992	0.1664 (1.3060)	0.9620 (1.5749)
1993	-1.1387 (1.1143)	0.9458 (0.0140)
1994	-2.0040 (1.3927)	0.9871 (0.0066)
1995	-2.2802 (1.7051)	0.9681 (0.0008)

Conclusion

- **Preferences matter for option pricing**
- **Option prices help distinguish between the expected and the non-expected utility models.**
- **The estimates we obtain for the preference parameters are quite reasonable.**
- **Out-sample performance of equilibrium-based models is in line with the ad-hoc BS model.**
- **In our method preference parameters enter consistently in the equilibrium pricing of all assets.**
- **Possible extensions: other specifications for preferences or different distributions for the state variable**