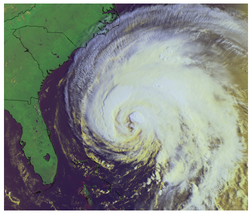
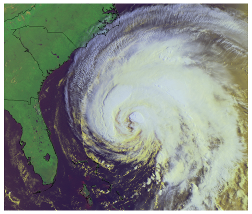
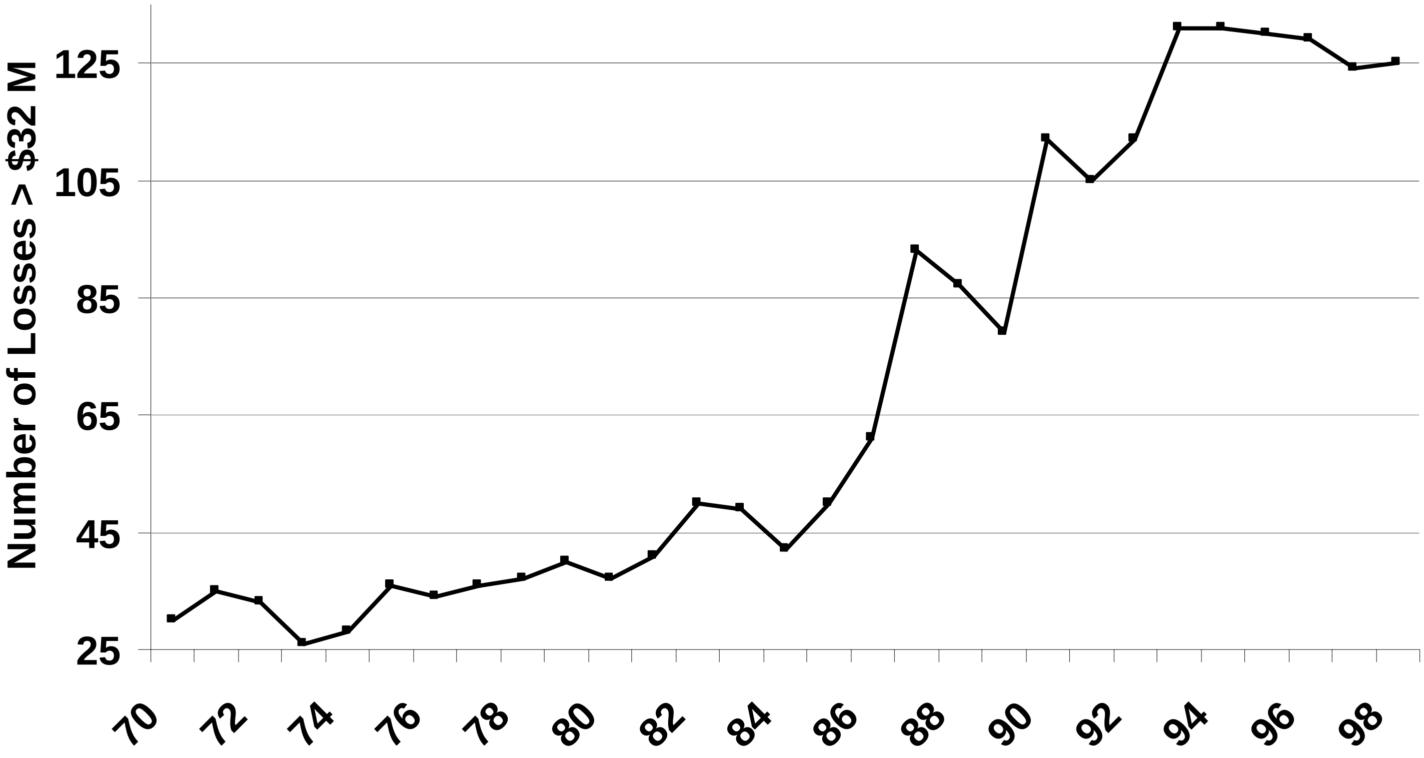


A Statistical Approach To Pricing Catastrophic Loss (CAT) Securities

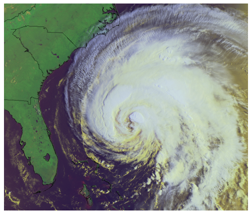
J. David Cummins
University of Pennsylvania
Christopher Lewis
U.S. Office of Federal Housing Enterprise Oversight
Richard D. Phillips
Georgia State University

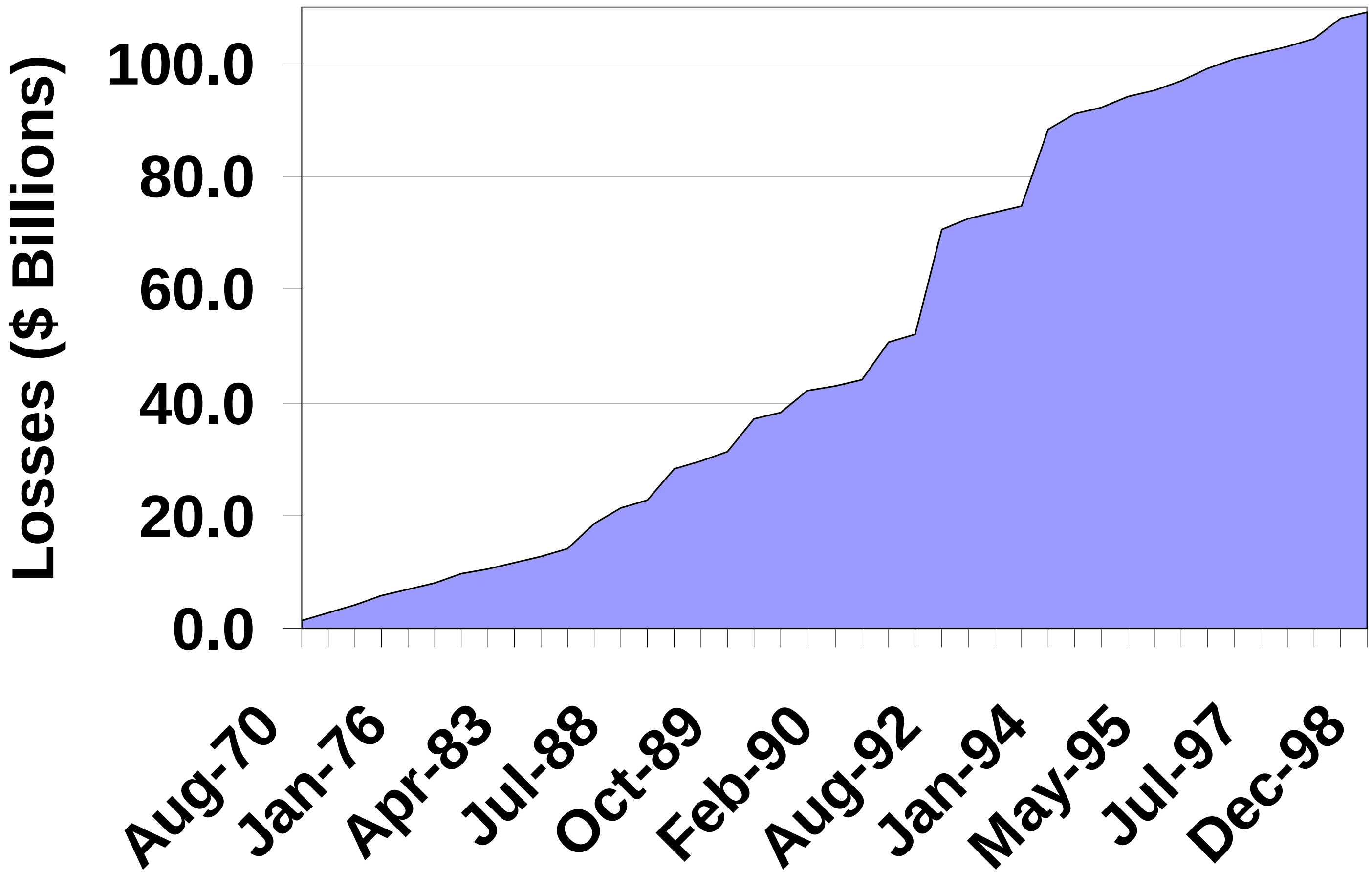
Number of CAT Losses: 1970-98



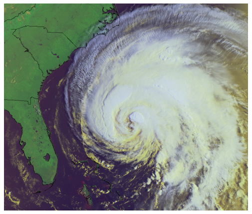


Cost of Top 40 CAT Losses: 1970-1998 (Cumulative)





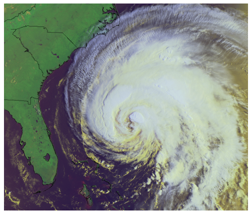
Top 10 CAT Losses: 1970-98



Date	Loss ($ billions)	Event	Location
Aug-92	18.60	Hurricane Andrew	US
Jan-94	13.76	Northridge Earthquake	US
Sep-91	6.65	Typhoon Mireille	Japan
Jan-90	5.73	Hurricane Daria	Europe
Sep-89	5.52	Hurricane Hugo	US
Oct-87	4.30	Autumn Storm	Europe
Feb-90	3.98	Hurricane Vivian	Europe
Aug-98	3.53	Hurricane Georges	US, Carib.
Jul-88	2.76	Oil Rig Explosion	UK
Jan-95	2.65	Kobe Earthquake	Japan

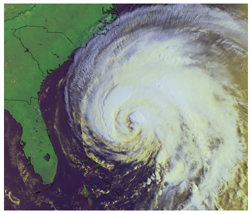
Source: Swiss Re.

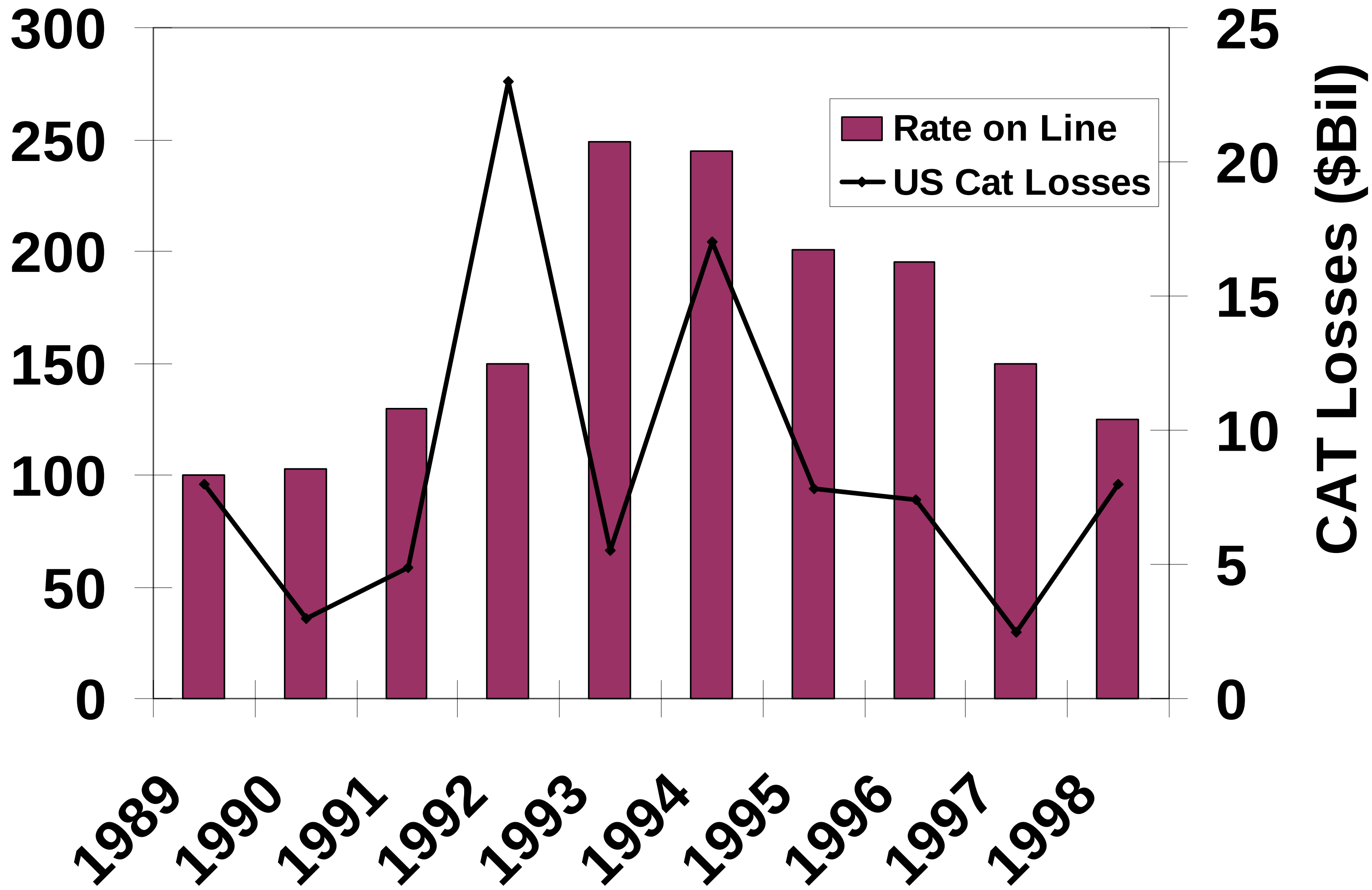
Projected Catastrophes



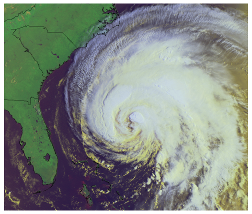
- $75 billion Florida hurricane
- $21 billion Northeast hurricane
- $72 billion California earthquake
- $100 billion New Madrid earthquake

Reinsurance Market:
Rates on Line & CAT Losses

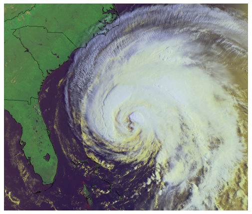




Failure of Diversification: Types of Events



- High-Frequency, Low-Severity
  - Auto collision
  - Non-CAT homeowners losses
- Low-Frequency, High-Severity
  - Property catastrophes
  - *Failure of Law of Large Numbers*

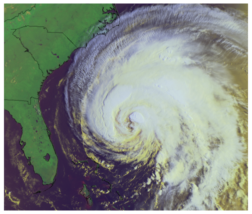


Why Time-Diversification Fails

"Holding large amounts of capital to finance infrequent events is not possible in practice."

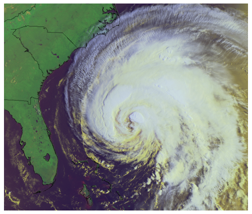
- Holding capital is costly due to agency costs and other market imperfections
- "Underutilized" capital attracts raiders
- Tax and accounting rules discourage holding "excess" capital

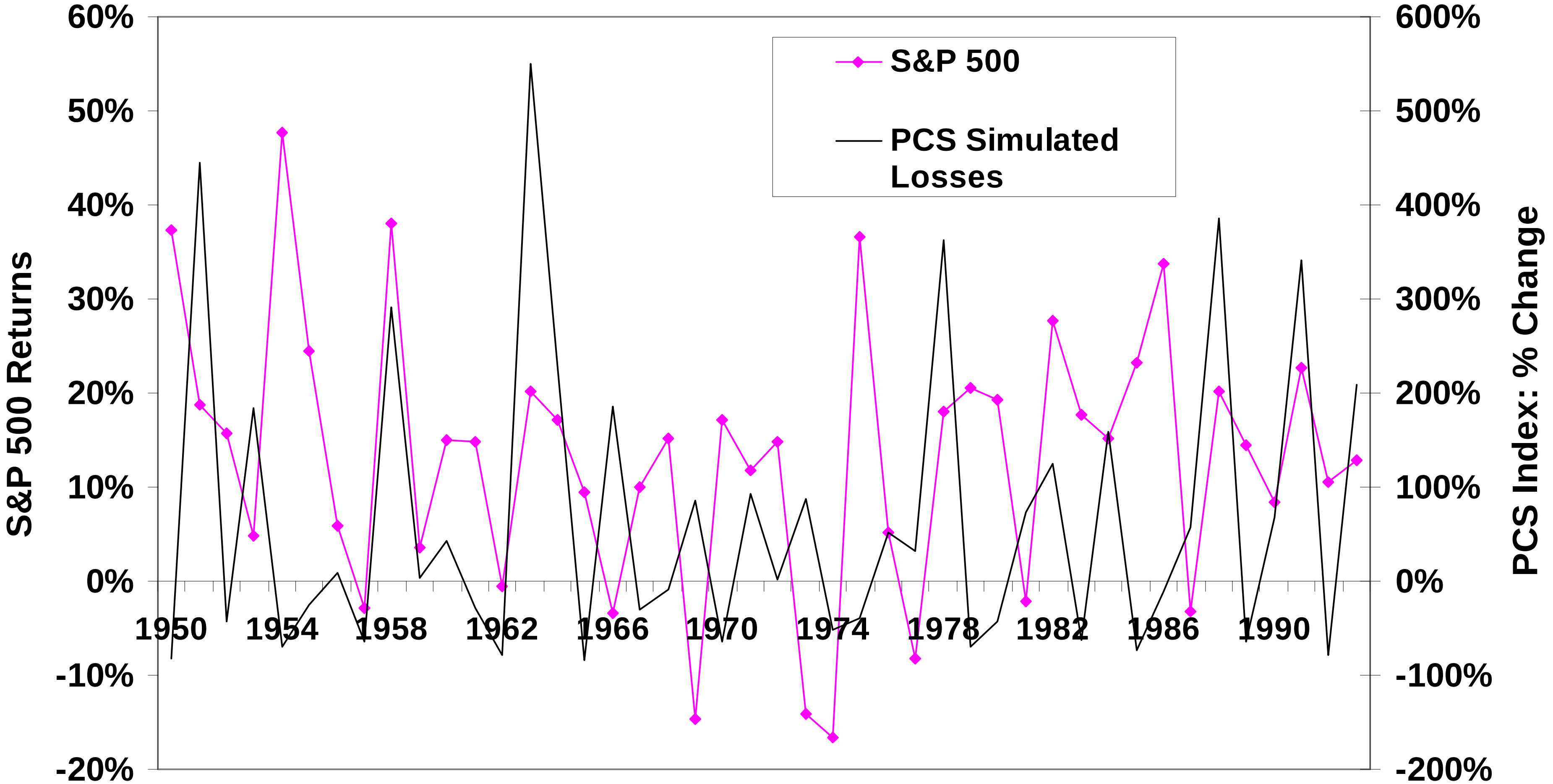
Why Securitization Is the Solution



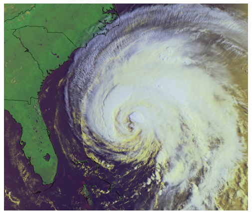
- US Bonds & Stocks – $25 trillion
  $75 billion < 0.5%
- CATs uncorrelated with other events that move markets (zero-beta securities)
- Markets reveal information -- reduce reinsurance price/quantity cycles

CAT Securities: "Zero-Beta" Assets



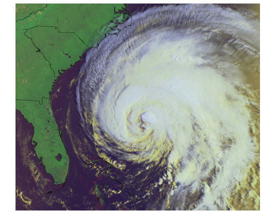


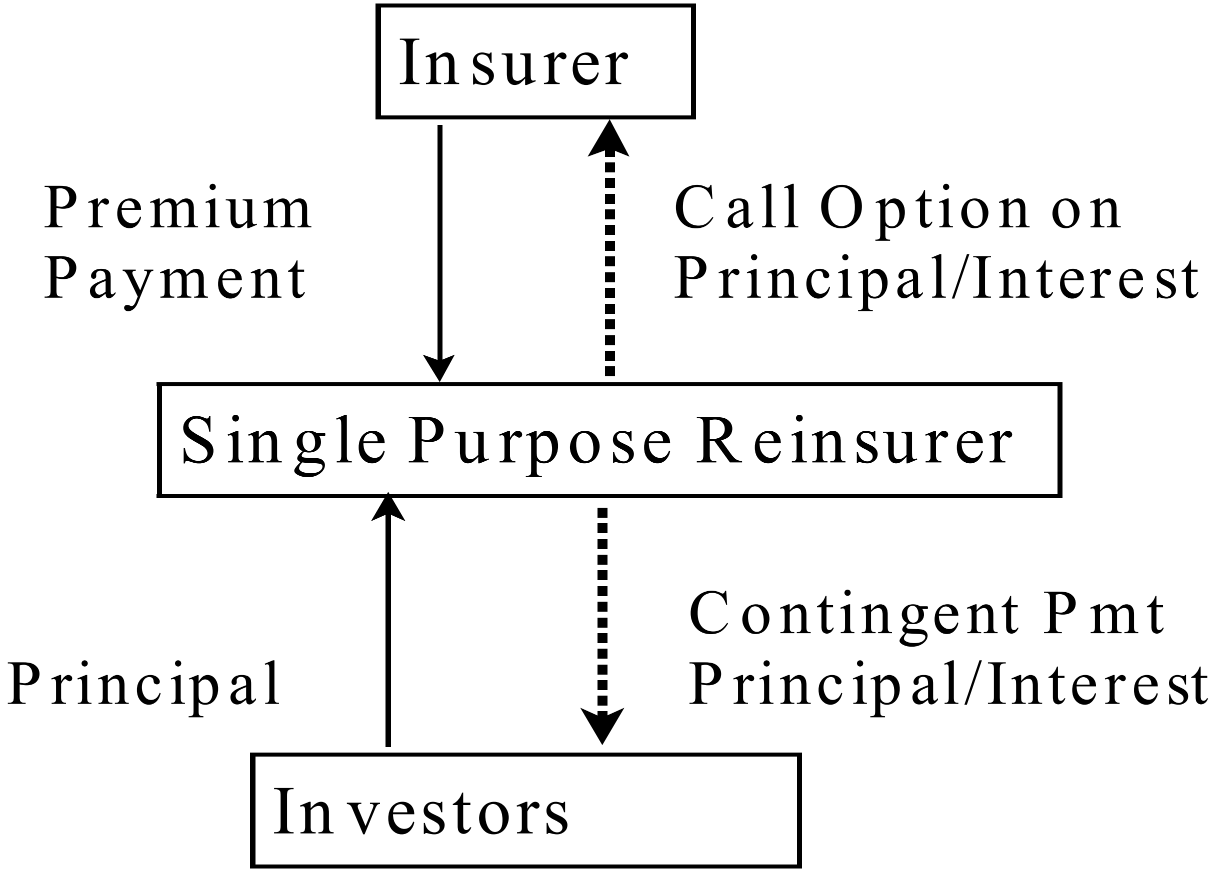
CAT Loss Securities



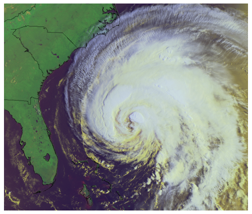
- CBOT CAT Option Spreads
- CAT Bonds
- CAT E-Puts
- Federal Excess of Loss (XOL) Reinsurance

CAT Bonds



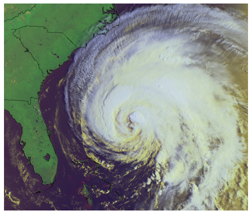

Insurer
Premium
Payment
Call Option on
Principal/Interest
Single Purpose Reinsurer
Principal
Contingent Pmt
Principal/Interest
Investors

The Case for a Federal Role



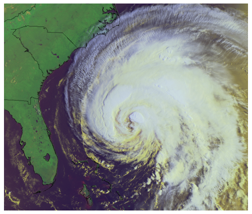
- Catastrophe risks violate independence requirement of an insurable risk
  - Cross sectional vs. inter-temporal diversification
- Constraints on private market solutions
  - Limits on insurer capitalization
  - Tax limitations
  - Accounting limitations
  - Vulnerability to raiders
  - Prohibitive post-loss cost of capital
- Unstable reinsurance markets
- Inadequate capital markets solutions

The Case for a Federal Role II



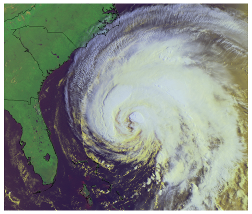
- Private insurers have difficulty in diversifying large losses across time
  - Once in 100 year event difficult to fund in advance
  - Information asymmetries and other market imperfections raise the cost of capital following a large event (even if the insurer remains solvent)
- Government is the borrower of last resort
  - Can borrow at the risk-free rate
  - Inter-generational financing of large events may be desirable
- Contracts could be priced to break-even or make a profit in expected values ("Crowding Out")

The Case Against a Federal Role



- Government contracts might slow the growth of private market CAT securitization
- Mis-pricing could unfairly penalize taxpayers
- The program might be difficult to kill once an adequate private market develops

CAT Loss Contract Payoff Function



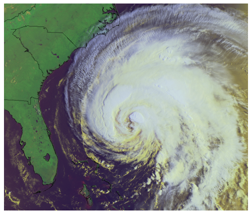
- Option spreads are the dominant contractual form
  - CBOT options
  - CAT bonds
  - XOL reinsurance

- The payoff function

$$P = Max[0,\delta(L - C)] - Max[0,\delta(L - T)]$$

  - C = lower strike
  - T = upper strike
  - δ = coinsurance proportion

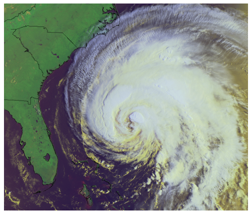
Defining the Underlying (L)



The contracts could pay off based on:

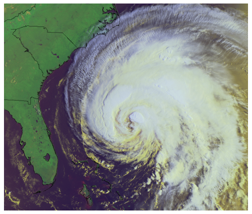
- The insurer’s own losses (XOL reinsurance, CAT bonds)
- An industry loss index (CBOT options, CAT bonds)
  - National
  - Statewide
  - Sub-state
- A “parametric” index (CAT bonds)
  - Richter scale reading
  - Saffir-Simpson severity class

Contract Details: Federal XOL Contracts



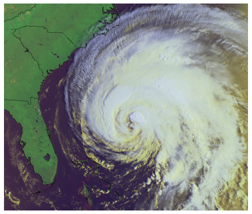
- Underlying (L) = Industry-wide property cat losses
  - As reported by independent statistical agent
- Coverage period - 1 calendar year
  - Loss development period - 18 months
  - Single event policies
    - Renewal provision
  - Sold annually
- Authorized purchasers
  - Insurance companies
  - Reinsurers
  - State pools

Contract Details II: Federal XOL Contracts



- Types of contracts and qualifying lines of business
  - Hurricane contract
    - Homeowners, wind policies, commercial multi-peril, fire, allied, farmowners, commercial inland marine
  - Earthquake/volcanic activity contract
    - Earthquake shake policies, commercial multi-peril, commercial inland marine
- Trigger to be set above current market capacity, e.g., $25 to $50 billion spreads

Hedging with Federal XOL Catastrophe Contracts



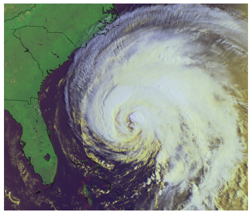
- Loss ratio w/o XOL contracts

$$R = \frac{L_{NA}}{P_A} + \frac{L_{CA}}{P_A}$$

- Loss ratio with N XOL contracts

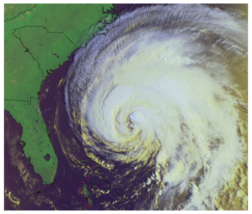
$$R = \frac{L_{NA}}{P_A} + \frac{L_{CA}}{P_A} - \frac{N}{P_A} \times \left[\frac{Max(L_{CI} - C, 0)}{1000} - \frac{Max(L_{CI} - T, 0)}{1000} \right]$$

Hedging With Federal XOL Options: Hedging Objectives



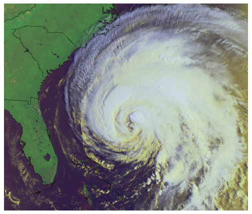
- Cap the loss ratio
- Reduce the variance of the loss ratio

Approaches To CAT Risk Modeling



- Engineering/actuarial simulation modeling – AIR, RMS
- Statistical modeling using realized CAT losses

Pricing Model:
The Loss Distribution Function



$$F(L) = \sum_{N=0}^{\infty} p(N)q(L > T \mid N)S(L \mid L > T)$$

$$= S(L \mid L > T)\sum_{N=0}^{\infty} p(N)q(L > T \mid N)$$

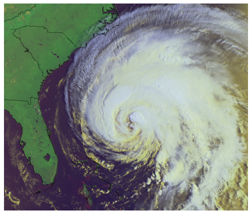
F(L) = distribution of CAT losses

p(N) = probability of N CATs occur during year

q(L>T|N) = probability that one CAT is > T, given N CATs

S(L|L>T) = distribution of CAT loss severity conditional on L>T

Contracts Covering a Single Event: Frequency Distribution



let $P_< = \text{Prob}(L<T)$

$P_> = 1 - P_<$,

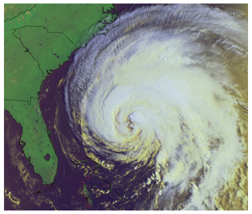
$$q(L > T \mid N) = P_> + P_< P_> + P_<^2 P_> + \ldots + P_<^{N-1} P_>$$

$$= P_> \frac{1 - P_<^N}{1 - P_<} = 1 - P_<^N$$

Taking the expectation over N yields and assuming Poisson arrival rate λ, yields

$$p^* = 1 - e^{-\lambda P_>}$$

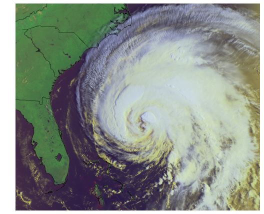
Contracts Covering a Single Event: Severity Distribution



Pareto $$S(L) = \alpha d^{\alpha} L^{-(1+\alpha)}$$

Lognormal $$S(L) = \frac{1}{L\sigma\sqrt{2\pi}} e^{-\left(\frac{\ln(L)-\mu}{\sigma}\right)^2}$$

Contracts Covering a Single Event: Severity Distribution



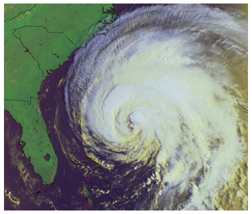
Burr12

$$S(L) = \frac{|a|qL^{a-1}}{b^a\left[1+\left(\frac{L}{b}\right)^a\right]^{q+1}}$$

GB2

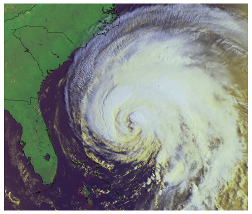
$$S(L) = \frac{|a|qL^{ap-1}}{b^{ap}B(p,q)\left[1+\left(\frac{L}{b}\right)^a\right]^{p+q}}$$

Loss Estimates - Historical Data



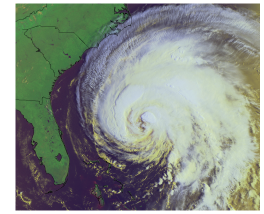
- Database
  - Compiled by Property Claims Service (PCS)
  - Covers all insured CAT losses since 1949
  - CAT = single event losses > $5M
  - Catastrophes included
    - » Hurricanes
    - » Tornadoes
    - » Windstorms
    - » Hail
    - » Fire and Explosions
    - » Riots
    - » Brush fires
    - » Floods

Adjusting Historical Data



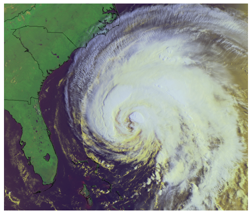
- Need to adjust for both
  - Changes in exposure levels
  - Price levels
- Adjustment method 1 - PA
  - Exposure - State Population Index
  - Price Levels - State Construction Cost Index
- Adjustment method 2 - VA
  - Exposure and price levels
    - » U.S. Census of Housing, Series HC80-1-A

Property Catastrophe Loss Statistics: Since 1949



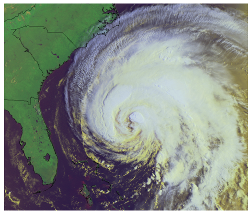
Type of Catastrophe	Number	Mean	Standard Deviation	Skewness	Minimum	Maximum
Earthquake	14	$1,079.9M	$ 3,313.6M	3.6	$ 11.9M	$12,500.0M
Brush Fire	27	228.4M	434.8M	4.4	3.8M	2,296.6M
Flood	14	73.1M	117.5M	2.2	7.0M	356.5M
Hail	53	82.1M	90.2M	2.1	8.0M	443.3M
Hurricanes	57	1,222.7M	2,763.0M	4.8	5.3M	18,391.0M
Ice	1	20.6M	-	-	20.6M	20.6M
Snow	11	102.9M	194.8M	3.1	7.2M	677.6M
Tornado	21	74.6M	116.1M	3.7	3.2M	546.7M
Tropical Storm	8	73.9M	58.9M	1.8	20.0M	204.9M
Volcanic Eruption	1	69.9M	-	-	69.9M	69.9M
Wind	864	96.0M	429.8M	23.5	2.8M	11,746.3M
All Other	66	109.0M	191.9M	3.3	3.8M	983.1M
Total	1137	167.0M	849.1M	14.8	2.8M	18,391.0M

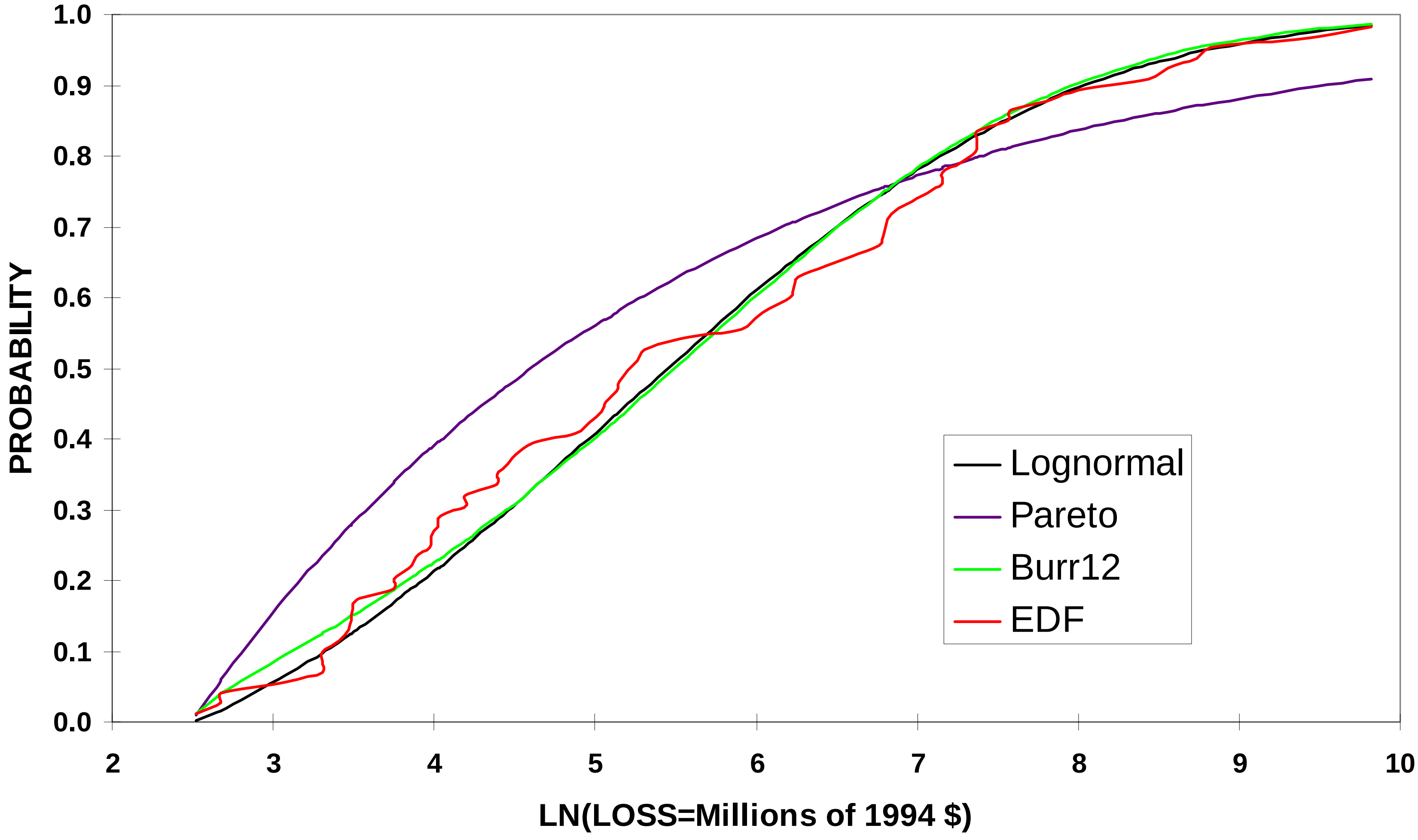
Estimating Severity Distributions: Hurricanes and Earthquakes



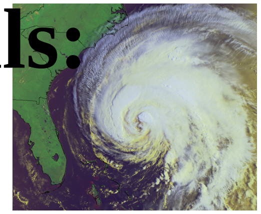
Distribution	Parameter	PCS-VA	PCS-PA
Lognormal	μ	5.40	4.59
	σ	2.06	2.17
	-LOG(L)	471.67	426.96
Pareto	α	0.33	0.34
	d	12.04	6.85
	-LOG(L)	430.04	470.04
Burr 12	a	0.66	0.80
	b	874.30	95.78
	q	1.99	1.00
	-LOG(L)	502.54	461.54
GB2	a	0.15	0.08
	b	2.91E+08	0.00
	p	10.97	121.91
	q	88.98	50.20
	-LOG(L)	501.44	460.48
Frequency		2.20	2.20

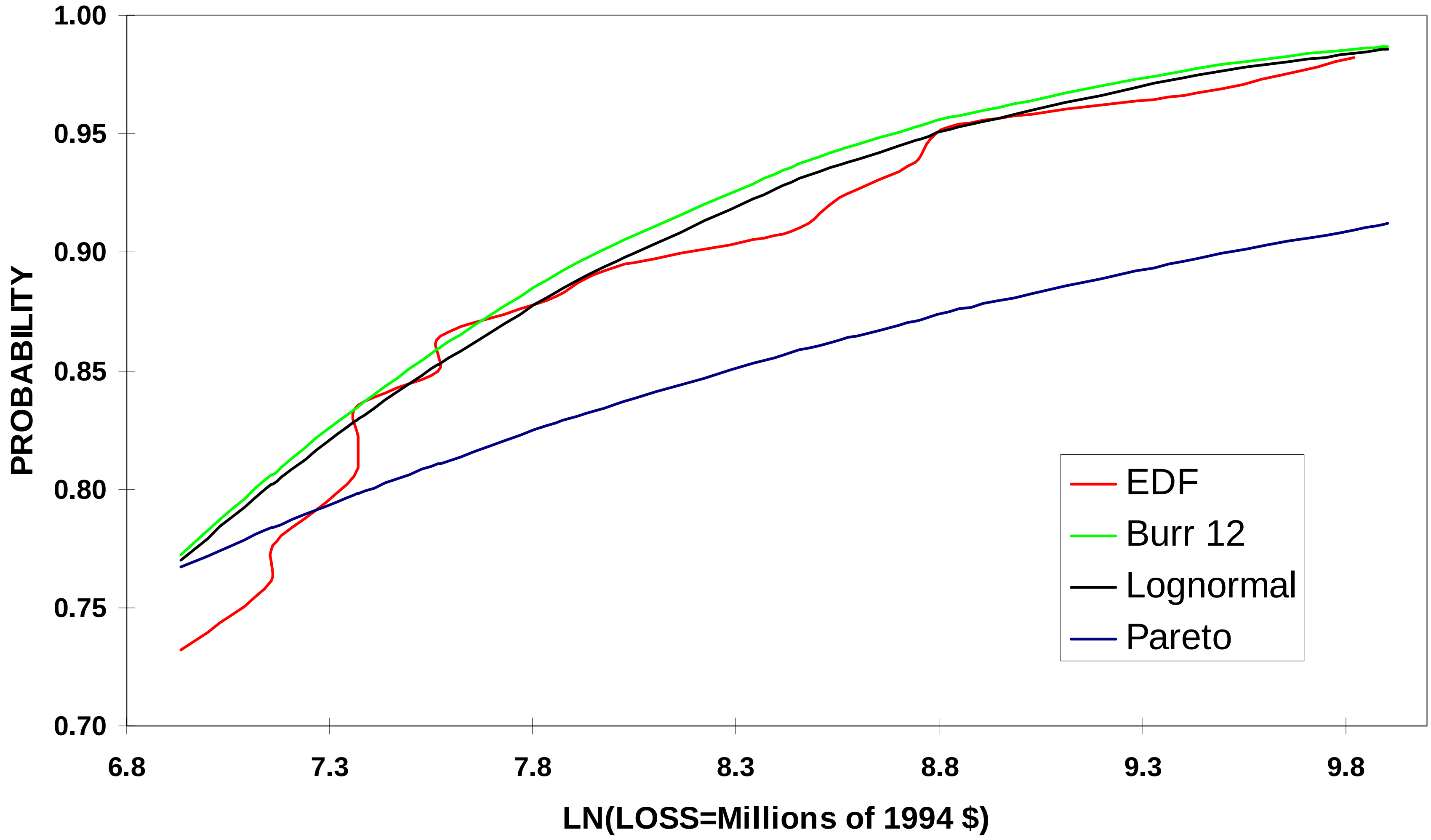
Severity of Loss Distribution Functions: PCS-VA Hurricanes and Earthquakes



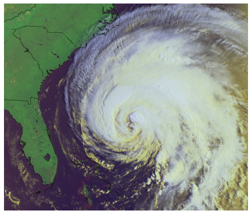


Severity of Loss Distribution Function Tails: PCS-VA Hurricanes and Earthquakes



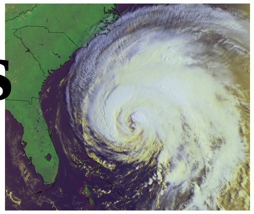


Expected Loss for the $25-$50B Layer: PCS Historical Data



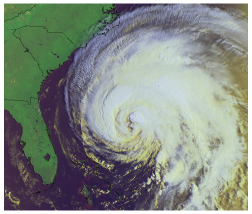
Losses Inflated By Housing Values:	Lognormal	Pareto	Burr 12	GB2
E(L;$25B,$50B,$12.04M)	$ 170.2M	$ 1,805.8M	$ 162.4M	$ 112.0M
PROB[L>$25|EVENT OCCURS] = P>	1.10%	8.18%	1.00%	0.79%
PROB[L>$25] = p* (Poisson param = 2.2)	0.024	0.165	0.022	0.017
E(L;$25B,$50B,$12.04M|L>$25B)	$ 15,518.1M	$ 22,073.6M	$ 16,194.7M	$ 14,179.1M
Total E(L): $25-50B Layer	$ 370.0M	$ 3,635.7M	$ 353.3M	$ 244.2M
Losses Inflated By Population:				
E(L;$25B,$50B,$6.85M)	$ 81.0M	$ 1,319.5M	$ 211.0M	$ 97.1M
PROB[L>$25|EVENT OCCURS] = P>	0.53%	6.01%	1.13%	0.61%
PROB[L>$25] = p* (Poisson param = 2.2)	0.012	0.124	0.025	0.013
E(L;$25B,$50B,$6.85M|L>$25B)	$ 15,286.1M	$ 21,950.1M	$ 18,617.9M	$ 15,839.4M
Total E(L): $25-50B Layer	$ 177.2M	$ 2,719.1M	$ 458.5M	$ 212.1M

Summary Statistics: PCS Reported Losses Vs. RMS Simulated Losses



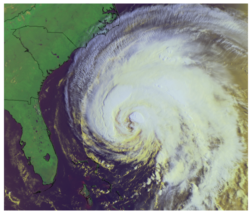
	Number	Mean	Standard Deviation	Minimum	Maximum
PCS Severity of Losses 1949-1994, Losses > 12.04M	67	$1,284.0M	$2,943.0M	$12.4M	$18,391.0M
RMS Severity of Losses All Losses	95182	$736.5M	$3,790.5M	$5.0M	$107,546.3M
RMS Severity of Losses Losses > $12.04M	66138	$1,048.0M	$4,493.5M	$12.1M	$107,546.3M
PCS Frequency of Losses 1949-1994, Losses > 12.04M	67	1.54	1.31	0	6
RMS Frequency of Losses All Losses	95182	9.52	3.06	0	23
RMS Frequency of Losses Losses > $12.04M	66138	6.67	2.56	0	19

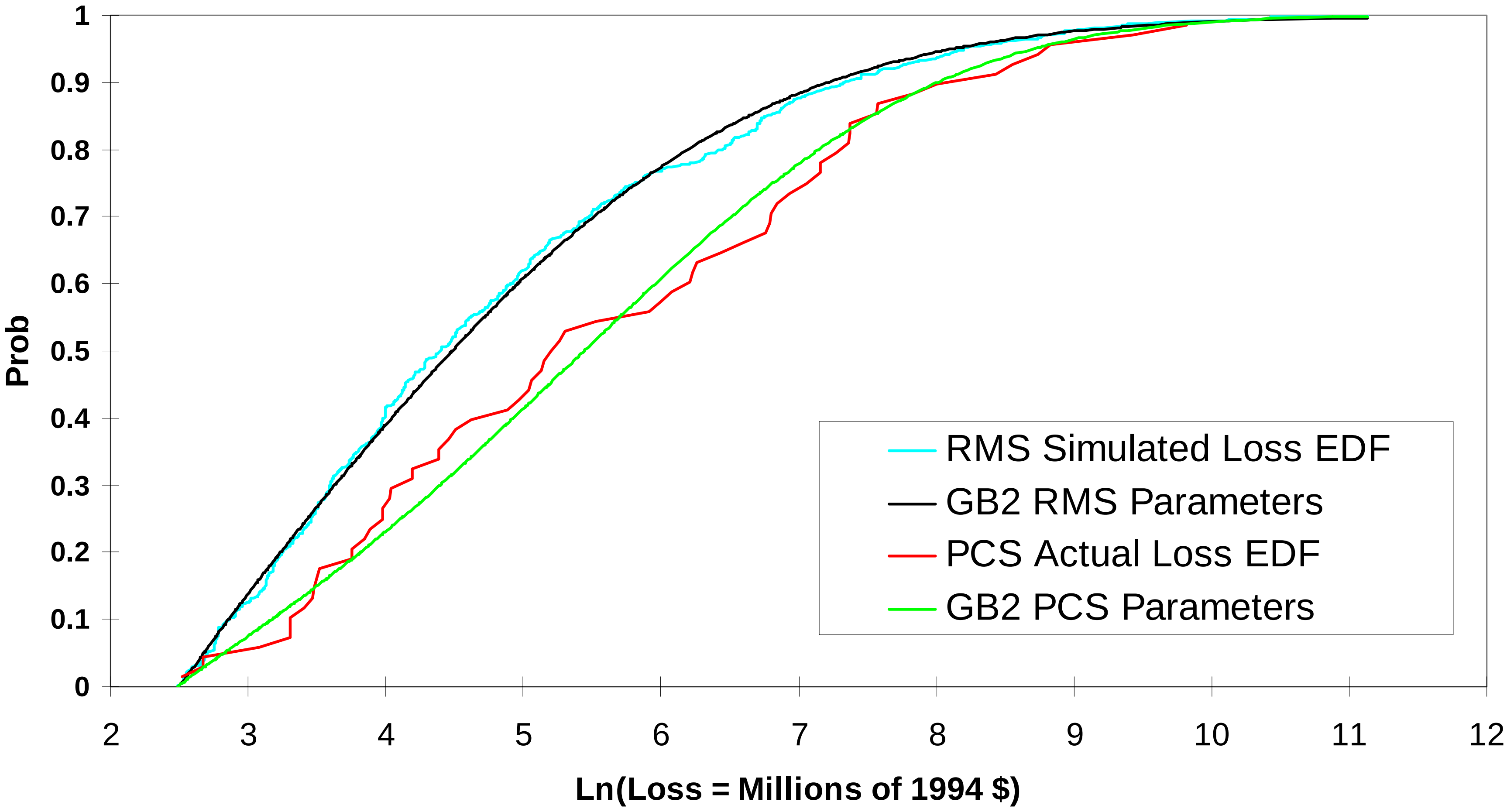
Estimating Severity Distributions: PCS Losses vs. RMS Simulated Losses



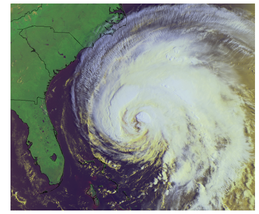
Distribution	Parameter	PCS-VA	PCS-PA	RMS - US
Lognormal	μ	5.40	4.59	4.40
	σ	2.06	2.17	2.20
	-LOG(L)	471.67	426.96	6108.24
Pareto	α	0.33	0.34	0.43
	d	12.04	6.85	12.04
	-LOG(L)	430.04	470.04	6653.26
Burr 12	a	0.66	0.80	0.91
	b	874.30	95.78	44.60
	q	1.99	1.00	0.74
	-LOG(L)	502.54	461.54	6609.18
GB2	a	0.15	0.08	0.40
	b	2.91E+08	0.00	23.51
	p	10.97	121.91	3.82
	q	88.98	50.20	2.49
	-LOG(L)	501.44	460.48	6604.77
Frequency		2.20	2.20	6.60

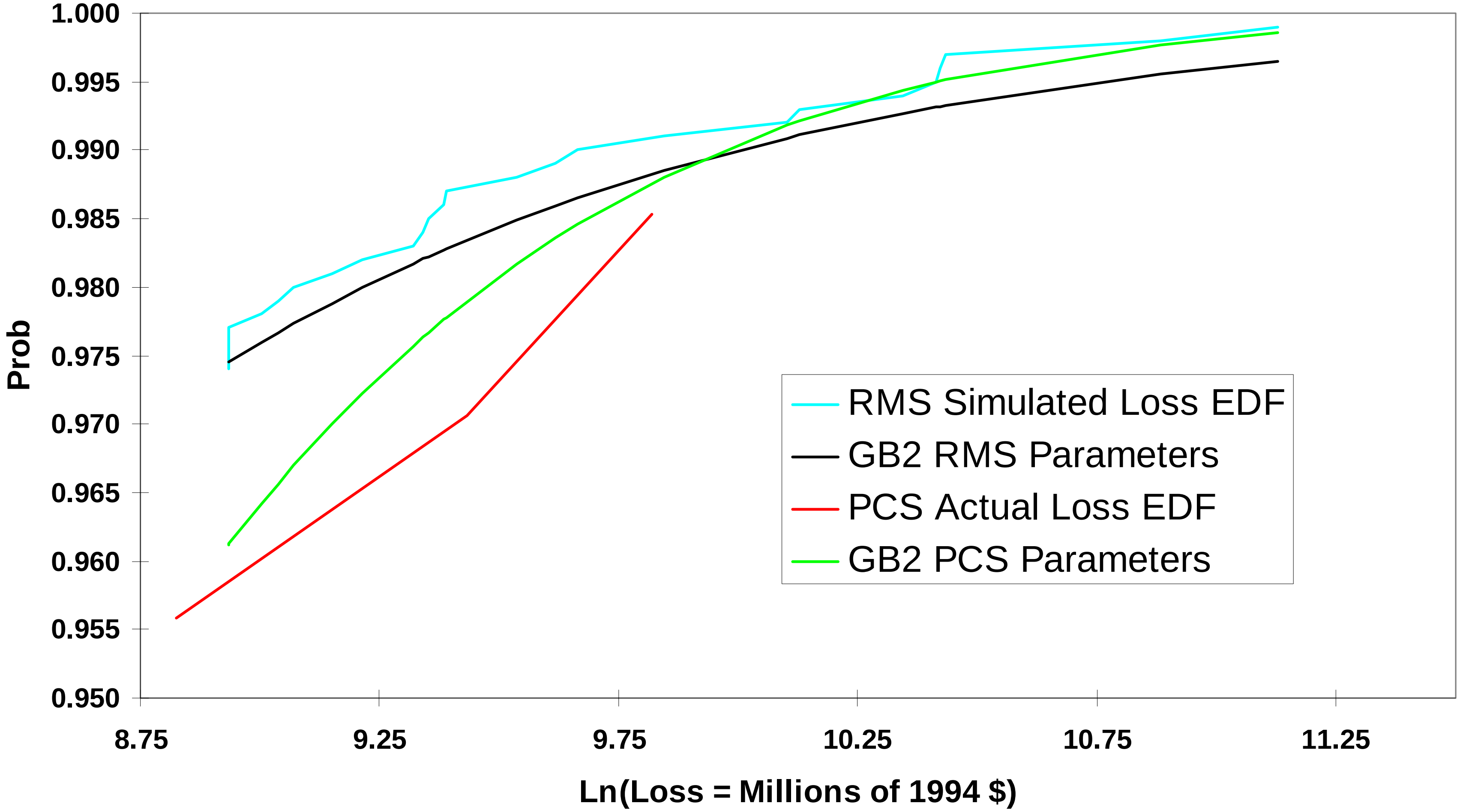
Fitting Severity Distributions: PCS-VA Losses Vs. RMS Simulated Losses



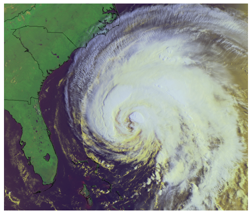


Fitting Severity Distributions Tails: PCS-VA Losses Vs. RMS Simulated Losses



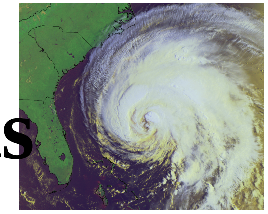


Total Expected Loss for $25-$50B Layers: PCS Losses Vs. RMS Simulated Losses



Losses Inflated By Housing Values:	Empirical	Lognormal	Pareto	Burr 12	GB2
E(L;$25B,$50B,$12.04M)		$ 170.2M	$ 1,805.8M	$ 162.4M	$ 112.0M
PROB[L>$25|EVENT OCCURS] = P>		1.10%	8.18%	1.00%	0.79%
PROB[L>$25] = p* (Poisson param = 2.2)		0.024	0.165	0.022	0.017
E(L;$25B,$50B,$12.04M|L>$25B)		$15.52B	$22.07B	$16.19B	$14.18B
Total E(L): $25-50B Layer		$ 370.0M	$ 3,635.7M	$ 353.3M	$ 244.2M
Losses Simulated by RMS					
E(L;$25B,$50B,$6.85M)	$ 82.0M	$ 69.7M	$ 792.3M	$ 279.2M	$ 159.1M
PROB[L>$25|EVENT OCCURS] = P>	0.70%	0.46%	3.73%	1.43%	0.89%
PROB[L>$25] = p* (Poisson param = 6.7)	0.045	0.030	0.218	0.090	0.057
E(L;$25B,$50B,$6.85M|L>$25B)	$11.71B	$15.27B	$21.25B	$19.48B	$17.85B
Total E(L): $25-50B Layer	$ 528.8M	$ 453.4M	$ 4,635.5M	$ 1,758.2M	$ 1,019.7M

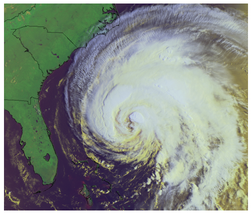
Price Estimates of Federal XOL Contracts



Region	Historical Frequency	Severity Distribution Assumption Lognormal	Pareto	Burr12	GB2
PCS - VA	2.2	$ 370.0M	$ 3,635.7M	$ 353.3M	$ 244.2M
PCS - PA	2.2	$ 177.2M	$ 2,719.1M	$ 458.5M	$ 212.1M
RMS - US	2.2	$ 152.6M	$ 1,673.5M	$ 604.6M	$ 346.6M
RMS - CA	0.217	$ 87.0M	$ 500.6M	$ 80.7M	$ 56.9M
RMS - FL	0.378	$ 4.7M	$ 102.3M	$ 53.5M	$ 69.0M
PCS - SE	0.844	$ 219.3M	$ 1,331.0M	$ 103.0M	$ 70.9M
RMS - SE	0.844	$ 206.8M	$ 1,526.2M	$ 249.6M	$ 187.7M

	RMS Frequency	Lognormal	Pareto	Burr12	GB2
PCS - VA	6.7	$ 1,083.7M	$ 9,209.2M	$ 1,037.1M	$ 720.1M
PCS - PA	6.7	$ 525.5M	$ 7,188.6M	$ 1,341.9M	$ 627.7M
RMS - US	6.7	$ 453.4M	$ 4,635.5M	$ 1,758.2M	$ 1,019.7M
RMS - CA	3.6	$ 44.6M	$ 950.5M	$ 502.2M	$ 645.1M
RMS - FL	0.83	$ 331.6M	$ 1,861.3M	$ 307.9M	$ 217.3M
PCS - SE	1.35	$ 349.4M	$ 2,090.0M	$ 164.5M	$ 113.2M
RMS - SE	1.35	$ 330.3M	$ 2,395.2M	$ 398.4M	$ 299.9M

Average Prices and Rates on Line: Federal XOL Contracts



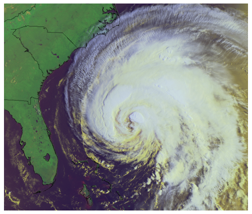
AVERAGE PRICE

	Severity Distribution Assumption			
Region	Lognormal	Pareto	Burr12	GB2
PCS - VA	$ 726.8M	$ 6,422.5M	$ 695.2M	$ 482.1M
PCS - PA	$ 351.3M	$ 4,953.8M	$ 900.2M	$ 419.9M
RMS - US	$ 303.0M	$ 3,154.5M	$ 1,181.4M	$ 683.1M
RMS - CA	$ 65.8M	$ 725.6M	$ 291.5M	$ 351.0M
RMS - FL	$ 168.2M	$ 981.8M	$ 180.7M	$ 143.1M
PCS - SE	$ 284.4M	$ 1,710.5M	$ 133.8M	$ 92.0M
RMS - SE	$ 268.6M	$ 1,960.7M	$ 324.0M	$ 243.8M

AVERAGE RATE ON LINE

Region	Lognormal	Pareto	Burr12	GB2
PCS - VA	2.91%	25.69%	2.78%	1.93%
PCS - PA	1.41%	19.82%	3.60%	1.68%
RMS - US	1.21%	12.62%	4.73%	2.73%
RMS - CA	0.26%	2.90%	1.17%	1.40%
RMS - FL	0.67%	3.93%	0.72%	0.57%
PCS - SE	1.14%	6.84%	0.54%	0.37%
RMS - SE	1.07%	7.84%	1.30%	0.98%

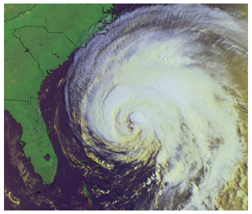
Risk Loadings: Problem and Solutions



- Problem: Market incompleteness
  – difficult to hedge jump risk

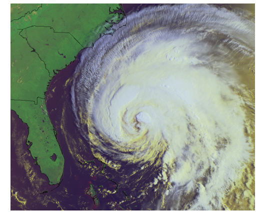
- Solutions
  - Asset pricing model with unsystematic jump risk (Merton 1976)
  - Option pricing with assumption about investor preferences (e.g., Chang 1995)

Is CAT Risk Really Zero-Beta?



- CATs to date are zero beta *but*
  - We have not observed a $100 billion event
  - Could cause a solvency crisis in insurance markets
  - Could be spillovers to other parts of the economy, e.g., Federal or private borrowing could raise interest rates, etc.

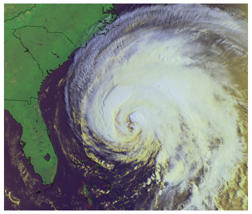
Prices: Selected CAT Bond Issues



Date	Transaction Sponsor	Spread Premium	Prob of 1st $ of Loss	E[L | L >0]	Expected Loss	Prem to E[Loss]	Risk
Mar-00	SCOR	14.00%	5.47%	59.23%	3.24%	4.32	EQ, Wind
Mar-00	Lehman Re	4.50%	1.13%	64.60%	0.73%	6.16	EQ, Wind
Nov-99	American Re	5.40%	0.78%	80.77%	0.63%	8.57	EQ, HC
Nov-99	Gerling	4.50%	1.00%	75.00%	0.75%	6.00	EQ
Jun-99	USAA	3.66%	0.76%	57.89%	0.44%	8.32	HC
Jul-99	Sorema	4.50%	0.84%	53.57%	0.45%	10.00	EQ, HC
Jul-98	Yasuda	3.70%	1.00%	94.00%	0.94%	3.94	HC
Mar-99	Kemper	4.50%	0.62%	96.77%	0.60%	7.50	EQ
May-99	Oriental Land	3.10%	0.64%	66.04%	0.42%	7.35	EQ
Feb-99	St. Paul/ F&G Re	8.25%	5.25%	54.10%	2.84%	2.90	Agg CAT
Dec-98	Centre Solutions	4.17%	1.20%	64.17%	0.77%	5.42	HQ
Dec-98	Allianz	8.22%	6.40%	56.41%	3.61%	2.28	Wind,Hail
Aug-98	X.L./MidOcean Re	5.90%	1.50%	70.00%	1.05%	5.62	Mult CAT
Jul-98	St. Paul/ F&G Re	4.44%	1.21%	42.98%	0.52%	8.54	Agg CAT
Jun-98	USAA	4.16%	0.87%	65.52%	0.57%	7.30	HC
Mar-98	Centre Solutions	3.67%	1.53%	54.25%	0.83%	4.42	HC
Dec-97	Tokio Marine & Fire	2.09%	1.02%	34.71%	0.35%	5.90	EQ
Jul-97	USAA	5.76%	1.00%	62.00%	0.62%	9.29	HC
Aug-97	Swiss Re	2.55%	1.00%	45.60%	0.46%	5.59	EQ
Aug-97	Swiss Re	2.80%	1.00%	46.00%	0.46%	6.09	EQ
Aug-97	Swiss Re	4.75%	1.00%	76.00%	0.76%	6.25	EQ
Aug-97	Swiss Re	6.25%	2.40%	100.00%	2.40%	2.60	EQ

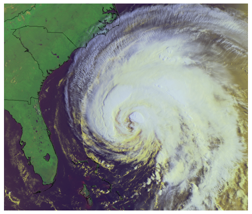
Source: Goldman Sachs & Co. **Premium/E[Loss] Average = 9.00; Median = 6.77.**

Why Are CAT Bond Spreads So High?



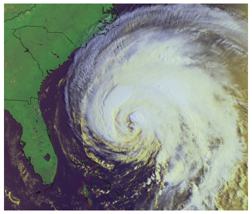
- Lack of liquidity – few issues/limited secondary market
- Investor unfamiliarity with CAT securities
- Parameter uncertainty

Conclusions



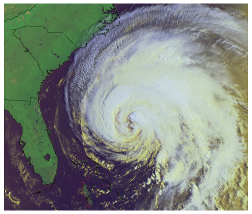
- CAT securities can be priced using statistical modeling and/or engineering/actuarial simulation
- Prices remain high due to illiquidity, investor unfamiliarity, and parameter uncertainty
- Significant potential for development of world-wide market

Insurance Linked Securities: The Future – I



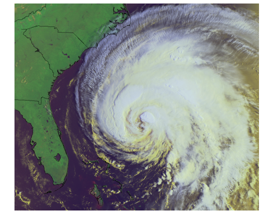
- Extension to other types of insurance
  - Liability insurance
  - Health insurance
  - Life insurance and annuities
  - Automobile insurance

Insurance Linked Securities: The Future – II



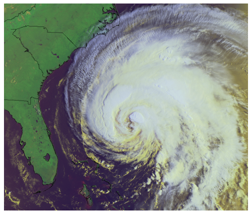
- Increasing geographical diversification
  - US states and regions
  - Asian countries and regions
  - European countries and regions
  - Australia
- Added liquidity will undercut the reinsurance price cycle & stabilize markets

Insurance Linked Securities: The Future – III



- Reinsurers
  - Perform underwriting function
  - Manage basis risk
  - Bear less risk directly
- Convergence of reinsurance & investment banking
- Continued role for OTC contracts

Insurance Linked Securities: The Future – IV



- Moving towards a public market
  - Increasing standardization
  - Better indices
  - Reducing regulatory barriers
  - Educating insurers and investors
- "Corporate" CAT derivatives – industrial firms bypass insurers & go direct to capital markets