Optimal Annuitization with Background Risk and Equity Exposure During Retirement*

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Abstract

We analyze annuity demand in a realistic life-cycle model in which we optimize over consumption and asset allocation. We incorporate background risk and incomplete annuity menus as possible drivers of deviations from full annuitization. Retirees face longevity risk, capital market risk, inflation risk, and background risk. We model annuitization as a one-time decision at retirement. Contrary to what is often suggested in the literature, we find that in these settings full annuitization remains close to optimal, irrespective of whether real or only nominal annuities are available. Under all circumstances we find optimal annuitization levels above 95% of initial wealth. On the one hand annuitization is attractive due to the additional wealth created by the mortality credit, on the other hand annuities are irreversible and the annuity menu is incomplete. We show that the additional wealth effect dominates, optimally individuals annuitize almost their entire wealth at retirement to capture the mortality credit. Whenever liquidity or equity exposure is desired, individuals save sizeable amounts out of their annuity income to smooth shocks due to background or inflation risk and/or to get equity exposure. We can identify this result, because we do not assume a priori that consumption equals annuity income in retirement and solve a dynamic programming problem for consumption and savings. Similarly, adding variable annuities to the menu does not increase welfare significantly, since individuals can save in order to get the desired equity exposure. Furthermore we find that for individuals who do not face (real) background risk, it is optimal to annuitize substantially less to receive the equity premium. Hence if both possible motives to annuitize less are considered jointly, they generally interact in such a way that full annuitization is optimal.

Keywords: Asset allocation, retirement, optimal life-cycle portfolio choice, annuity, savings
JEL classification: D14, D91, G11, G23

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1 Introduction

In this paper we model optimal decumulation of retirement wealth. Prior research has shown that in simple stylized settings full annuitization of available wealth upon retirement is optimal for individuals who only face uncertainty about their time of death. Yaari (1965) shows that risk averse agents with intertemporally separable utility who are only exposed to longevity risk, and with no desire to leave a bequest, find it optimal to hold their entire wealth in annuities if these are actuarially fair. This argument is extended by Davidoff, Brown, and Diamond (2005) to cases with more risk factors and more general utility functions. Full annuitization is optimal in these models since the annuities generate a mortality credit that cannot be captured otherwise.

In the literature the policy recommendation that all pension wealth should be annuitized has been challenged. These papers are partly motivated by the observation that very few individuals voluntarily purchase annuity products when they reach the retirement age (Bütler and Teppa (2007) and Mitchell, Poterba, Warshawsky, and Brown (1999)). This empirical fact is often referred to as the annuity puzzle. In this paper we focus on two of the main factors that have been put forward to challenge the claim that full annuitization is optimal. The first factor emphasizes that annuities are irreversible due to adverse selection. This implies that annuities cannot be sold if liquidity is needed because of unforeseen shocks, for instance in health costs or breakdown of a durable consumption good. In addition, people face borrowing constraints, hence such background risk generates a reduced demand for annuities (Turra and Mitchell (2004), Pang and Warshawsky (2008), and Sinclair and Smetters (2004)). The second factor in our analysis is that annuity menus are typically incomplete. In many cases only nominal annuities are available rather than annuities which hedge inflation risk or which give exposure to equity markets. Such incomplete annuity menus may also reduce annuity demand (Milevsky and Young (2007a), Horneff, Maurer, Mitchell, and Dus (2008), and Koijen, Nijman, and Werker (2008b))

We analyze a comprehensive stochastic life-cycle model from the retirement phase onwards. An individual optimally allocates a fraction of wealth to an annuity at age 65. Every period an agent decides how much to consume, how much to save, and how to allocate his wealth between stocks and a riskless bond. The model includes the most important risks a retiree faces, namely longevity risk, background risk, inflation risk, and capital market risk. Recently developed numerical methods are used to solve the model.

We find that almost full annuitization is optimal irrespective of whether real or only nominal annuities are available. Neither background risk nor the lack of equity exposure has a sizeable effect on optimal annuitization levels. Individuals allocate about 95% of their wealth to real annuities if these are available. If background risk hits them the liquid wealth is used as a buffer and consumption is temporarily reduced to rebuild the buffer. Furthermore we find that for individuals who only have access to nominal annuities full annuitization remains optimal. During retirement they accumulate a sizeable amount of wealth. The median savings account is at its maximum (in real terms) at about age 84 and amounts to approximately 25% of initial wealth. Saving during retirement is driven by four factors: (1) redistribution of consumption to later periods when the real value of the nominal annuity income is low. Furthermore people save to hedge against (2) inflation risk and (3) background risk. Finally, wealth accumulation allows people to benefit from the (4) equity premium. These four effects cannot be disentangled in the many papers that assume that consumption and annuity income coincide in retirement or are based on simple draw-down rules (see for instance Horneff, Maurer, Mitchell, and Dus (2006b)). We disentangle these four reasons and find that it is optimal to redistribute a large sum of annuity income to later periods, the anticipatory motive to save. Furthermore the second reason, inflation risk, induces a large amount of precautionary savings, it increases the amount accumulated in the savings account by 50%. Expenses due to background risk are a substantial but less important reason for saving than inflation risk (savings impact of 20%). The final reason, to gain equity exposure, does not increase the amount of savings significantly. In contrast to previous research we find that adding variable annuities to the menu does not increase welfare significantly. We find this different result because we do not assume that consumption equals the annuity income, hence individuals can save out of the annuity income and invest in equity.

Furthermore we examine both possible motives to reduce annuity demand, namely incomplete annuity markets and background risk, separately. We find that in a model without inflation risk, which acts as a background risk, equity is indeed a reason not to annuitize fully, namely 85% annuitization is optimal. Hence strikingly, the two potential reasons to annuitize less interact in such a manner that when considered separately, annuity demand is lowered, but analyzed jointly, full annuitization remains optimal. This seems counterintuitive, but is due to the fact that background and inflation risk reduce the demand for equity exposure. Individuals who already face inflation risk have such a high overall risk level that they do not want additional risk in the form of equity exposure. The benefits of less annuitization, more equity exposure, are thus outweighed by the additional wealth created via annuitization, since the demand for equity is low. For this reason the effect that the lack of equity exposure has on optimal annuitizion levels is reduced by (real) background risk.

Our study is closely related to that of Pang and Warshawsky (2008). The main differences are that they restrict the analysis to real annuities and that additional annuities can be bought every year. They find that early in retirement it is optimal to annuitize nothing of your wealth and that from age seventy onwards the optimal annuitization fraction increases with age. Full annuitization is only reached for people in their early eighties. In contrast to their results, we find that full annuitization is optimal at retirement. The difference in results is due to their model setup, namely that additional annuities can be bought every year. Pang and Warshawsky (2008) state that annuities represent a specific asset class with its own unique risk and return profile. They model the annuitization decision essentially as a portfolio allocation decision between bonds, equity, and annuities. Since the mortality credit increases with age, an annuity bought at a later age earns a higher return than an annuity bought at age 65. In that case individuals find it optimal to first invest in equity to receive the risk premium, but eventually annuities crowd out equity. Horneff, Maurer, Mitchell, and Dus (2008) and Horneff, Maurer, and Stamos (2006a) also find that the optimal annuitization level increases with age. In contrast to these studies we find that (almost) full annuitization at retirement is optimal. The difference between our study and those mentioned above is that we assume that annuitization can only take place at retirement. We make this assumption for various reasons. First of all in several countries the decision whether to annuitize your pension account or take a lump sum is, due to the tax legislation, to take place at retirement. Furthermore mandatory annuitzation of a fraction of wealth at younger ages reduces adverse selection costs that are generated when the annuity date can be chosen. These adverse selection costs are typically ignored in the papers referred to above. A third reason for our assumption of a single conversion opportunity at retirement is that in reality people make financial decisions very infrequently rather than annually. Furthermore Agarwal, Driscoll, Gabaix, and Laibson (2007) show that the capability of individuals to make financial decisions declines dramatically at higher ages, hence it seems optimal to make these decisions at younger ages when a person is still able to do so.

The assumption of a single conversion opportunity implies that, contrary to the recommendation of the papers referred to, (almost) full annuitization at age 65 is optimal. The benefit of receiving the mortality credit and getting longevity risk insurance outweighs the initial loss of liquidity and equity exposure. In a similar setting Horneff, Maurer, Mitchell, and Dus (2006b) find that the optimal annuitization level is approximately 70%, however they do not optimize dynamically over either the equity/bond portfolio or consumption, as we do.

In our model we treat the magnitude of background risk as independent of age, which seems realistic for most European countries. A number of papers have analyzed annuity demand from a US perspective where health expenses are in general only partially covered by insurance policies. Sinclair and Smetters (2004) find that exogenous health shocks decrease the demand for life annuities since they simultaneously raise the demand for liquid assets and shorten the life expectancy. In contrast Turra and Mitchell (2004) model annuitization as a one-time decision at retirement and examine the effect of heterogeneity in health status among agents on the decision to annuitize. They find that differences in health and anticipated health costs can help explain why many individuals annuitize only partly. As a robustness check we modeled background risk explicitly as out-of-pocket medical expenses via estimates by De Nardi, French, and Jones (2008). The health costs increase sharply with age and there is a negative relation between health costs and survival probabilities. We find that (almost) full annuitization remains optimal if we model health costs as the source of background risk.

In this paper we ignore a number of other potential drivers of annuity demand. These include the presence of loads in annuity prices (see for instance Mitchell, Poterba, Warshawsky, and Brown (1999)), bequest motives (Brown (2001) and Inkmann, Lopes, and Michaelides (2008)), private information on health status (Turra and Mitchell (2004)), high pre-annuitized wealth levels (Dushi and Webb (2004)), and family composition (Brown and Poterba (2000) and Kotlikoff and Spivak (1981)). These extensions could be considered in subsequent work.

The remainder of the paper is organized as follows. In Section 2 we describe the individual's preferences, the setup of the financial market, the benchmark parameters, and the numerical method to solve the dynamic programming problem. Section 3 contains detailed simulation results for the benchmark case. Robustness checks are subsequently performed in section 4. Section 5 concludes.

2 The retirement phase life-cycle model

2.1 Individual's preferences and constraints

We consider a life-cycle investor during retirement with age $t \in 1, ..., T$, where $t = 1$ is the retirement age and T is the maximum age possible. The individual's preferences are presented by a time-separable, constant relative risk aversion utility function and the individual derives utility from real consumption, C_t. More formally, the objective of the retiree is to maximize the following function

$$V = E_1 \left[\sum_{t=1}^{T} \beta^{t-1} \left(\left(\prod_{s=1}^{t} p_s \right) \frac{C_t^{1-\gamma}}{1-\gamma} \right) \right], \tag{1}$$

where β is the time preference discount factor, γ denotes the level of risk aversion, and C_t is the real amount of wealth consumed at the beginning of period t. The probability of surviving to age t, conditional on having lived to period $t-1$ is indicated by p_t. We define the nominal consumption as $\overline{C_t} = C_t \Pi_t$, where Π_t is the price index at time t.

The individual invests a fraction w_t in equity, which yields a gross nominal return of R_{t+1}. The remainder of the wealth is invested in a riskless bond and the return on this bond is denoted by R_t^f. The intertemporal budget constraint of the individual is, in nominal terms, equal to

$$W_{t+1} = (W_t + Y_t - B_t - \overline{C_t})(1 + R_t^f + (R_{t+1} - R_t^f) w_t), \tag{2}$$

where W_t is the amount of financial wealth at time t, Y_t is the annual nominal annuity income, and the expenses due to background risk are indicated by B_t. The timing of decisions is as follows First the individual receives his annuity income and incurs expenses due to background risk. After this exogenous shock he decides how much to consume and subsequently invests the remaining wealth. In case the annuity income plus wealth at the beginning of the period is insufficient to pay the expenses and consume, the individual receives a subsistence consumption level. In subsequent periods that person first needs to pay of his debt before he can consume more than the subsistence level. The decision frequency is annually.

The individual faces a number of constraints on the consumption and investment decisions. First, we assume that the retiree faces borrowing and short-sales constraints

$$w_t \geq 0 \text{ and } \iota' w_t \leq 1. \tag{3}$$

Second, we impose that the investor is liquidity constrained

$$\overline{C_t} \leq W_t, \tag{4}$$

which implies that the individual cannot borrow against future annuity income to increase consumption today.

2.2 Financial market

The asset menu of an investor consists of a riskless one-year nominal bond and a risky stock. The return on the stock is normally distributed with an annual mean nominal return μ_R and a standard deviation σ_R. The interest rate dynamics are described by an Ornstein-Uhlenbeck process

$$dr_t = -a(r_t - \mu_r)dt + \sigma_r dW_t, \tag{5}$$

where r_t is the instantaneous short rate and a indicates the mean reversion coefficient. μ_r is the long run mean of the instantaneous short rate and σ_r denotes the instantaneous standard deviation of the short interest rate. The yield on a risk-free bond with maturity h is a function of the instantaneous short rate in the following manner:

$$R_t^{f(h)} = -\frac{1}{h}\log(A(h)) + \frac{1}{h}B(h)r_t, \tag{6}$$

where $A(h)$ and $B(h)$ are scalars and h is the maturity of the bond.

In our market, inflation is modeled as follows. For the instantaneous *expected* inflation rate we assume

$$d\pi_t = -\alpha(\pi_t - \mu_\pi)dt + \sigma_\pi dZ_t, \tag{7}$$

where α is the mean reversion parameter, μ_π is long run expected inflation, σ_π is the standard deviation of the expected inflation, and dZ_t are the innovations. Subsequently the price index Π follows from

$$\Pi_{t+dt} = \Pi_t \exp(\pi_{t+dt} + \sigma_\Pi dB_t), \tag{8}$$

where dB_t are the innovations to the price index. We assume there is a positive relation between the expected inflation and the instantaneous short interest rate, that is the correlation coefficient between Z_t and B_t is positive.

We consider single-premium immediate life-contingent annuities with real or nominal payouts. Consequently, the annuity income is given by

$$Y = PR_0 A^{-1}, \tag{9}$$

where PR_0 is the premium and A is the annuity factor. The single premium is equal to the present value of expected benefits paid to the annuitant and we assume an actuarially fair annuity. The annuity factor, A, is equal to

$$A = \sum_{t=1}^{T} \left(\left(\prod_{s=1}^{t} p_s \right) \exp(-tR_0^{(t)} \right), \tag{10}$$

where $R_0^{(t)}$ is the time zero yield on a zero coupon bond maturing at time t. The interest rate term structure that is applied is either nominal or real depending on the type of annuity.

The annuity factor for a variable annuity payout is similar to equation (10), but $R_0^{(t)}$ is equal to the assumed interest rate (AIR), which is fixed. The annual annuity income depends on the return of the portfolio backing the annuity, R_t^A, and is equal to

$$Y_t = PR_0 A^{-1} \prod_{t=1}^{T} \left(\frac{1 + R_t^A}{1 + AIR} \right). \tag{11}$$

The AIR determines whether, in expectation, the annuity payout stream increases or decreases over time. The annuity income is constant over time in case the AIR is equal to the return of the underlying portfolio, R_t^A. If the AIR is below R_t^A, then the nominal income stream is upwards sloping over time.

We postulate that the expenses due to background risk are lognormally distributed with an annual mean μ_B and a standard deviation σ_B. Furthermore we assume that the expenses do not exhibit autocorrelation.

2.3 Benchmark parameters

In the previous paragraphs we presented the specification of the life-cycle model. In this section we set the parameter values for the benchmark case. In accordance with Pang and Warshawsky

(2008) and Yogo (2008) we set β, the time preference discount factor equal to 0.96. The risk aversion coefficient γ is assumed equal to 5 for ease of comparison, since this is equivalent to Pang and Warshawsky (2008) and near the parameter choice of Yogo (2008) and Ameriks, Caplin, Laufer, and Van Nieuwerburgh (2008). Initial wealth is such that, if the individual would annuitize fully in real annuities, the (real) income for the rest of the lifespan equals unity. We call this real annuity income if 100% is invested in a real annuity the Full Real Annuity Income (FRAI). The mean expenses due to background risk are 10% of the FRAI, with a standard deviation of 7%. Furthermore we choose a subsistence consumption level of about 15% of the FRAI.[1]

The equity return is normally distributed with a mean annual nominal return of 8% and an annual standard deviation of 20%, which is in accordance with historical stock performance. The mean instantaneous short rate is set equal to 4%, the standard deviation to 1%, and the mean reversion parameter to 0.15. The correlation between the instantaneous short rate with the expected inflation is 0.4. The parameters on the inflation dynamics are taken from Koijen, Nijman, and Werker (2008a). They find a mean inflation of 3.48%, the standard deviation of the instantaneous inflation rate is equal to 1.38%, the standard deviation of the price index equals 1.3%, and the mean reversion coefficient equals 0.165. The assumed interest rate is equal to 4%, which is similar to Horneff, Maurer, Mitchell, and Stamos (2008) and Koijen, Nijman, and Werker (2008b).[2] The portfolio linked to variable annuity consists 100% of equity. Furthermore we will perform robustness checks to assess whether the results hold for different values for the individual preference parameters and financial market parameters. Time ranges from $t = 1$ to time T, which corresponds to age 65 and 100 respectively. The number of simulated paths N is equal to 1000. The survival probabilities are the current male survival probabilities in the US and are obtained from the Human Mortality Database.[3] We assume a certain death at age 100.

[1]If an individual has invested his entire wealth in a real annuity, then in less than 0.01% of the cases the individual receives the subsistence consumption level.

[2]The US National Association of Insurance Commisionairs requires that the AIR may not be higher than 5%. Furthermore Horneff, Maurer, Mitchell, and Stamos (2008) remark that 4% is commonly used in the US insurance industry.

[3]We refer for further information to the website, www.mortality.org.

2.4 Numerical method for solving the life-cycle problem

Due to the richness and complexity of the model it cannot be solved analytically hence we employ numerical techniques instead. We use the method proposed by Brandt, Goyal, Santa-Clara, and Stroud (2005) and Carroll (2006) with several extensions added by Koijen, Nijman, and Werker (2008a). Brandt, Goyal, Santa-Clara, and Stroud (2005) adopt a simulation-based method which can deal with many exogenous state variables. In our case $X_t = (R_t^f, \pi_t)$ is the relevant exogenous state variable. Wealth acts as an endogenous state variable. For this reason, following Carroll (2006), we specify a grid for wealth *after* (annuity) income, expenses due to background risk, and consumption. As a result, it is not required to do numerical rootfinding to find the optimal consumption decision.

The optimization problem is solved via dynamic programming and we proceed backwards to find the optimal investment and consumption strategy. In the last period the individual consumes all wealth available. The value function at time T equals:

$$J_T(W_T, R_T^f, \pi_T) = \frac{W_T^{1-\gamma}}{1-\gamma}. \tag{12}$$

The value function satisfies the Bellman equation at all other points in time,

$$V_t(W_t, R_t^f, \pi_t) = \max_{w_t, C_t} \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \beta p_{t+1} E_t(V_{t+1}(W_{t+1}, R_{t+1}^f, \pi_{t+1})) \right). \tag{13}$$

In each period we find the optimal asset weights by setting the first order condition equal to zero

$$E_t(C_{t+1}^{*-\gamma}(R_{t+1} - R_t^f)/\Pi_{t+1}) = 0, \tag{14}$$

where C_{t+1}^* denotes the optimal real consumption level. Because we solve the optimization problem via backwards recursion we know C_{t+1}^* at time $t+1$. Furthermore we simulate the exogenous state variables for N trajectories and T time periods hence we can calculate the realizations of the Euler conditions, $C_{t+1}^{*-\gamma}(R_{t+1} - R_t^f)/\Pi_{t+1}$. We regress these realizations on a polynomial expansion in the state variables to obtain an approximation of the conditional expectation of the Euler condition

$$E\left(C_{t+1}^{*-\gamma}(R_{t+1} - R_t^f)/\Pi_{t+1}\right) \simeq \tilde{X}_p' \theta_h. \tag{15}$$

In addition we employ a further extension introduced in Koijen, Nijman, and Werker (2008a). They found that the regression coefficients θ_h are smooth functions of the asset weights and consequently we approximate the regression coefficients θ_h by projecting them further on polynomial expansion in the asset weights:

$$\theta_h' \simeq g(w)\psi. \tag{16}$$

The Euler condition must be set to zero to find the optimal asset weights

$$\tilde{X}_p'\psi g(w)' = 0. \tag{17}$$

The procedure to determine the optimal consumption strategy is similar to the optimal asset weights. The Euler condition for optimal consumption is determined via regressing the realizations of marginal utility on the state variables. In this manner the optimal consumption for every trajectory, time period, and wealth grid point is determined.

3 Results for the benchmark case

As shown by Davidoff, Brown, and Diamond (2005) full annuitization is optimal if the annuity market is complete. This is however not the case if no annuity is available which offers equity exposure. Figure 1 presents the certainty equivalent consumption for various levels of annuitization, conditional on optimal consumption and asset allocation strategies. The dashed line shows that for the case with background risk and real annuities, annuitization of about 96% of total wealth is optimal. The welfare gain over no annuitization is substantial: An increase in annual certainty equivalent consumption from 50% of the FRAI to 100% of the FRAI.[4] If no annuities are available, welfare is thus reduced by about 50%. The magnitude of the welfare gains are comparable to the findings in Davidoff, Brown, and Diamond (2005) and Mitchell, Poterba, Warshawsky, and Brown (1999). For many individuals part of their wealth will be annuitized for institutional reasons, for example in the form of social benefit payments or Defined Benefit pensions. The results show that an increase in the level of annuitization from say 50% to 100% also brings about a very substantial

[4]In section 2.3 we stated that, for ease of comparison, we set initial wealth such that, if the individual would annuitize fully in real annuities, the (real) income for the rest of the lifespan equals unity. We call this real annuity income if 100% is invested in a real annuity, the Full Real Annuity Income (FRAI).

welfare gain which is in line with Mitchell, Poterba, Warshawsky, and Brown (1999). Full annuitization, as compared to the optimal level of about 96%, generates a negligible welfare loss. This implies that the fact that the annuity market is incomplete does not have a material impact on the optimal annuitization level, given that we allow dynamic saving strategies.

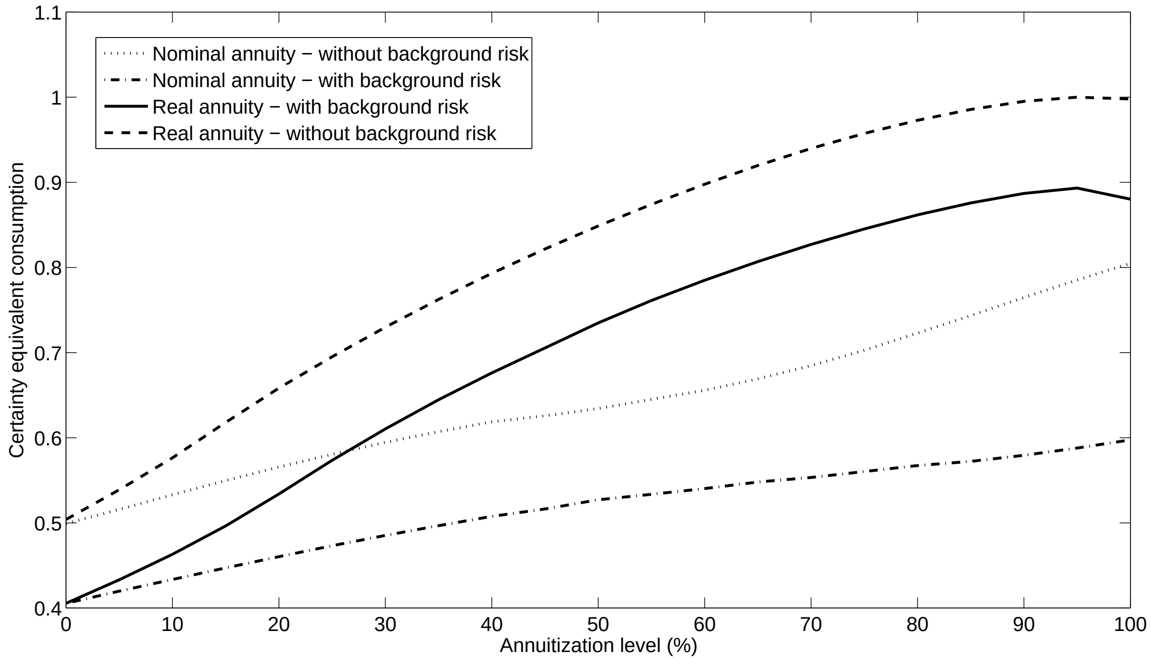


Figure 1: Optimal annuitization levels
The figure displays the certainty equivalent consumption for the life-cycle model with and without background risk and nominal or real annuity income. Equity is included in the model. The optimal annuitization strategy is the level that generates the highest certainty equivalent consumption.

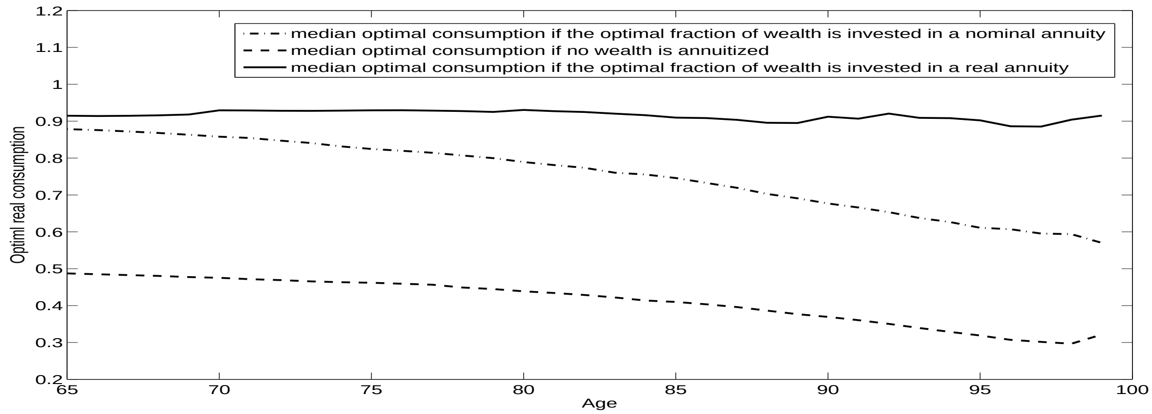
In addition, the results on the optimal annuity demand are hardly affected by the presence of background risk, the solid line in Figure 1 shows that full annuitzation is still close to optimal. Obviously, background risk reduces the attainable utility levels, but the curves are still essentially increasing: more annuitization leads to more utility. Later we will see that the main difference with the case without background risk is that the agent accumulates wealth out of annuity income to cover shocks in background risk and plans consumption to rebuild these buffers when needed. Pang and Warshawsky (2008) find that in a life-cycle model with health costs as background risk, annuity demand increases due to background risk. The reason for this contrasting result is that they do not model annuitization as a one-time decision that needs to be made at retirement age, but optimize annually over the equity-bond-annuity portfolio. In effect, the annuitization decision is modeled as a portfolio allocation decision. Health costs are an additional risk factor which drives households to shift demand from risky to riskless assets, namely from equity to bonds and annuities. Then as a consequence of the superiority of annuities over bonds, annuity demand increases due to health costs. In sum we find that it is optimal to annuitize fully. The benefits of insurance against longevity risk and the mortality credit outweigh the reduction in liquidity and less ability to get equity exposure.

The third and fourth curve in Figure 1 refer to the case where only nominal annuities are available and thus inflation risk cannot be hedged via annuities. Full annuitzation remains optimal and generates substantial welfare gains over fractional annuitization or the pure use of draw-down strategies. Later we will see that the agent will rebuild liquidity and exposure to equity markets through capital accumulation after retirement. Note that this capital accumulation after retirement could not be identified by earlier studies which equated consumption after retirement to the cash flows generated by the annuity portfolio.

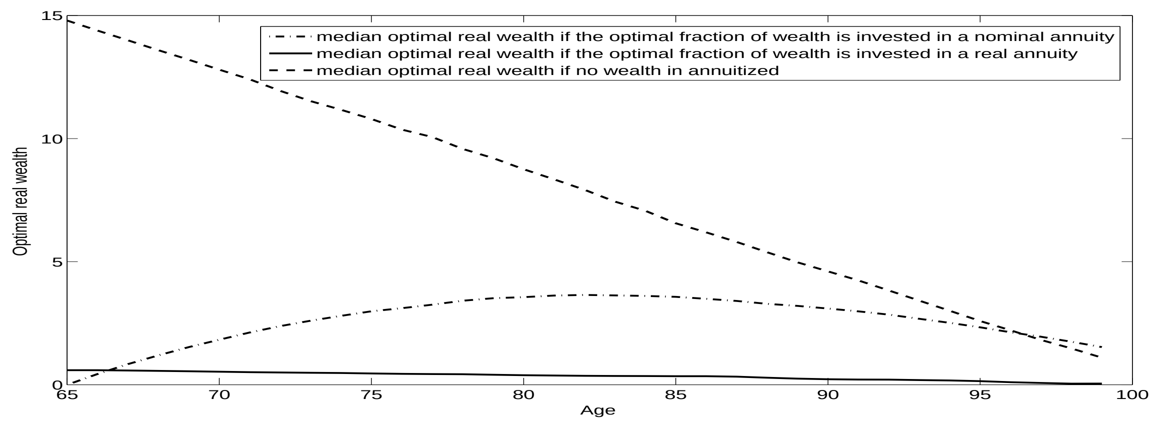
The optimal consumption, wealth trajectory, and asset allocation rules are illustrated in Figures 2 to 5. In Figure 2 we present the median consumption, wealth, and asset allocation for three cases. The three cases that are considered are (1) no annuitization, (2) the optimal (100%) level of nominal annuities, and (3) the optimal (96%) level of real annuities. Figure 3 presents optimal median wealth trajectories for a number of alternative model settings. In Figures 4 and 5 we illustrate how the consumption level depends on the wealth level and how consumption is affected by shocks in wealth.

Figure 2a shows that in case (1) and case (2) the optimal consumption path is decreasing over time. This reflects the fact that if the longevity risk in the real consumption level is not hedged, agents do not plan much consumption at ages where the probability is high that one will have passed away. If real annuities are used, inflation risk can be hedged and the planned consumption path is approximately flat (in real terms) because of the fact that the time preference parameter and interest rates approximately coincide.

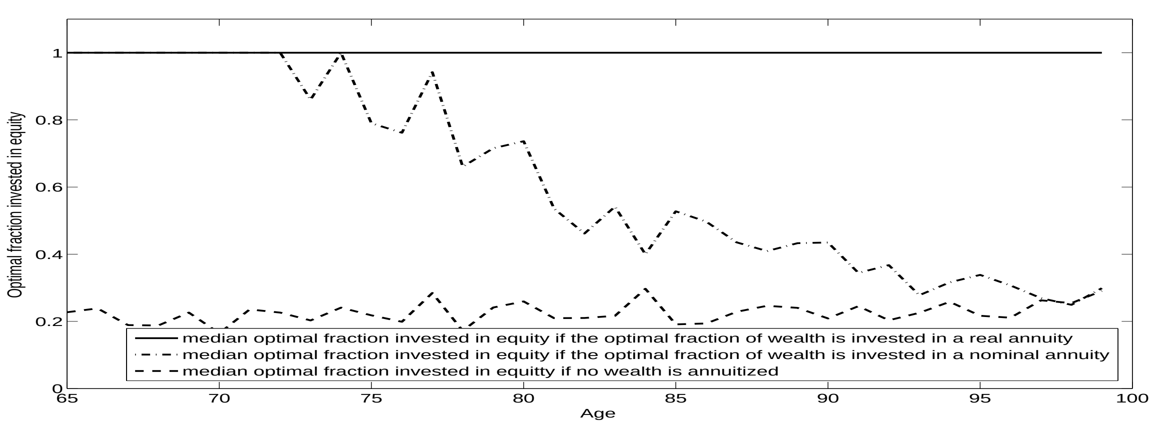
Figure 2b shows that wealth is slowly decumulated if real annuities are used. The level of liquid wealth is sufficient to cover for unexpected shocks in background risk. The median wealth trajectory is very different if nominal annuities are used to cover longevity risk. In that case the



(a)



(b)



(c)

Figure 2: Optimal real consumption, optimal real wealth and optimal asset allocation
Panel (a) displays the optimal real consumption for the optimal real annuitization level, optimal nominal annuitization level and without annuities. Panel (b) displays the optimal real wealth for the optimal real annuitization level, optimal nominal optimization level and without annuities. Panel (c) presents the optimal fraction invested in the risky asset for the optimal real annuitization level, optimal nominal optimization level and without annuities. Expenses due to background risk are included in the model.

individual saves substantially out of the nominal annuity income and a median real wealth of 3.5 times the FRAI is attained at the age of 80. This liquid capital is needed to have sufficient *real* consumption if the agent happens to get very old. This is in accordance with Love and Perozek (2007), they also find that background risk increases the optimal amount of liquid assets.

In panel C of Figure 2 we see that the optimal fraction invested in the risky asset if a person has annuitized nothing is about 20% and is fixed over time. Instead the optimal fraction is 100% if an individual has invested optimally in a real annuity. We see that the optimal fraction depends negatively on the fraction of liquid wealth compared to total wealth (liquid wealth plus discounted value of annuity income). This result is in line with Cocco, Gomes, and Maenhout (2005).

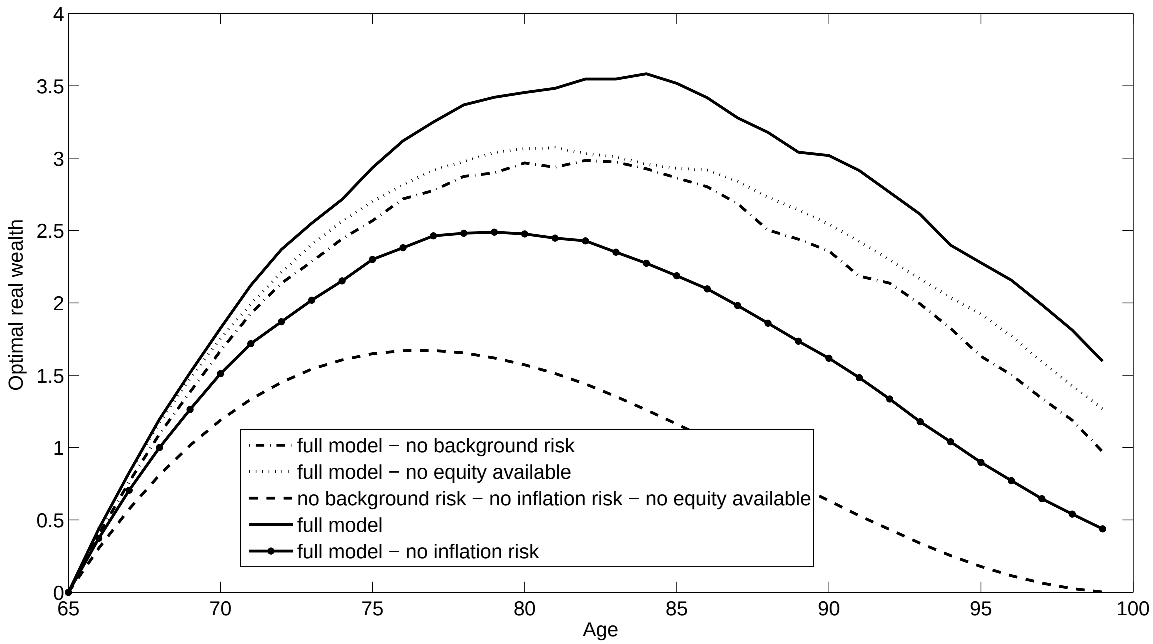
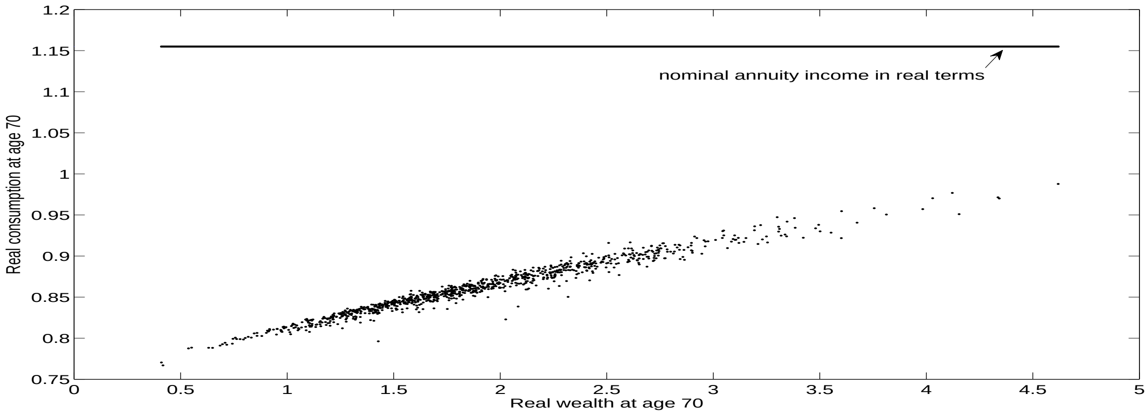


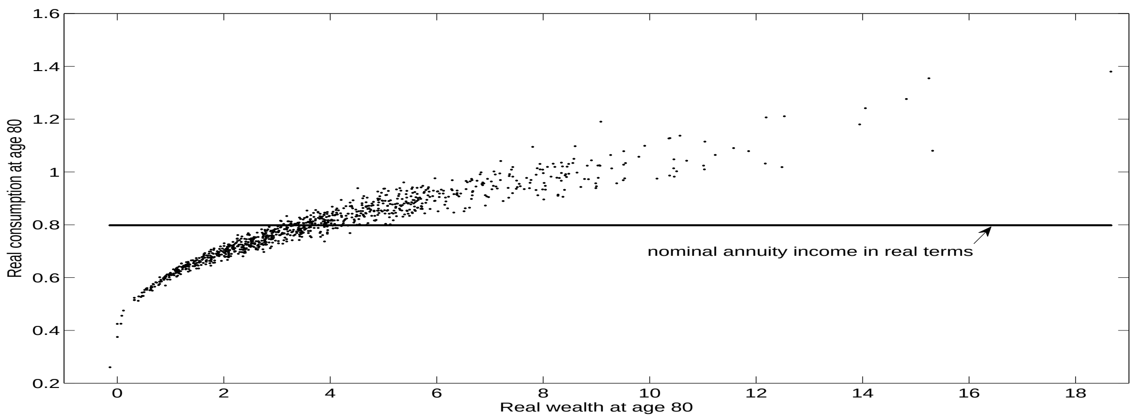
Figure 3: Optimal real wealth trajectories
This figure shows the optimal real wealth trajectories for five variations of the parameter values. The wealth trajectories are for the case where 100% is invested in a nominal annuity. In the model setup where inflation risk is excluded, the inflation level is fixed at 3.48%.

Figure 3 analyzes in more detail the most striking result in Figure 2, the capital accumulation in case of nominal annuities. Individuals save out of nominal annuity income for four different reasons. A first reason is real consumption smoothing, because even deterministic inflation erodes the real consumption that can be obtained from the nominal annuity income. A second reason relates to inflation risk. Inflation risk generates precautionary saving as inflation risk can be seen in this setting as a (partly) unhedgeable background risk. The third reason is precautionary saving to hedge for the background risk in our model. The fourth and final motivation to accumulate capital is exposure to equity risk to capture its risk premium. Figure 3 presents the optimal median wealth path for five different specifications of the model to disentangle the different reasons for capital accumulation. If only deterministic inflation is incorporated (i.e. background risk, inflation risk, and equity risk are excluded from the model) the maximum amount of wealth accumulated is about 1.7 times the FRAI. In expectation an individual accumulates an amount equal to approximatively 11% of his initial wealth at age 65 to redistribute income to years where the nominal income in real terms is low. This maximum savings is reached at age 75. If all three risk factors are included, then the optimal median wealth level is substantially larger and reached at later age. It is evident that hence the *level* of inflation explains only a small part of the results. The median savings is reduced from approximately 3.5 times the FRAI if all risk factors are included to 3 times the full real annuity income if the equity risk is ignored. Similarly if background risk is taken out, the amount of savings is slightly lower than 3 times the FRAI. Similarly Palumbo (1999) finds that uncertain medical expenses increases the amount of precautionary savings. The main driver of the accumulation of capital is inflation *risk* as the optimal maximum savings amount decreases with some 40% if that risk factor is taken out. The level of precautionary savings is enhanced by the persistency in inflation. In sum, an individual could also simply annuitize less to keep wealth liquid and extract wealth from the savings account to insure against inflation shocks. However, we find that instead it is optimal to annuitize fully to receive the mortality credit and subsequently save out of the annuity income.

To illustrate the savings behavior and the impact of background shocks on consumption and capital accumulation we added Figures 4 and 5. Figure 4 covers the case of real annuities and Figure 5 that of nominal annuities. Panel A of Figure 4 shows the optimal consumption level for varying wealth levels for a 70-year old. The real saving is 0.33 times the FRAI (nominal annuity income in real terms, 1.15, minus real consumption, 0.82) a year for wealth level 1 and 0.23 a year for a wealth level of 3 times the FRAI. A two standard deviations shock in terms of background risk corresponds to a wealth reduction of 0.14 times the FRAI. Linear interpolation suggests that after a background risk shock of that magnitude, consumption is reduced by (only) 0.007 times the FRAI. Background risk will therefore only have a small impact on subsequent consumption in the



(a)



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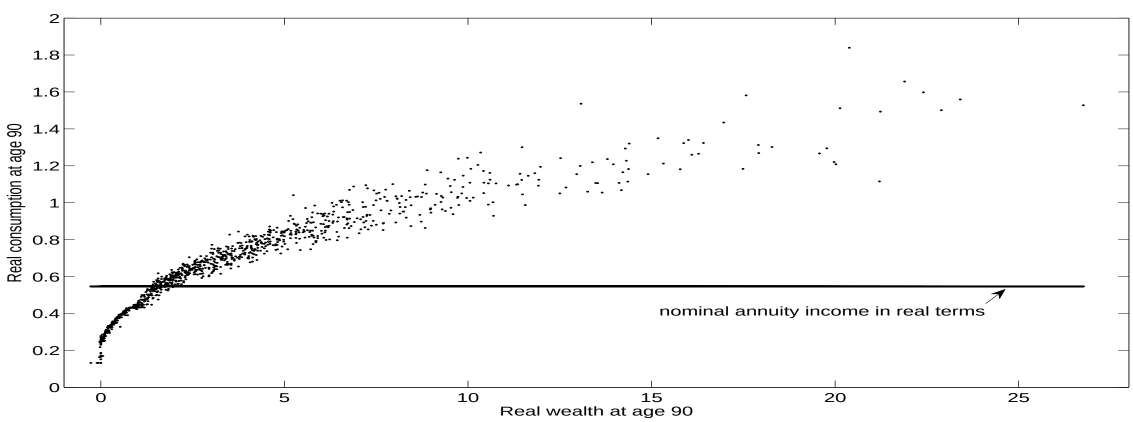
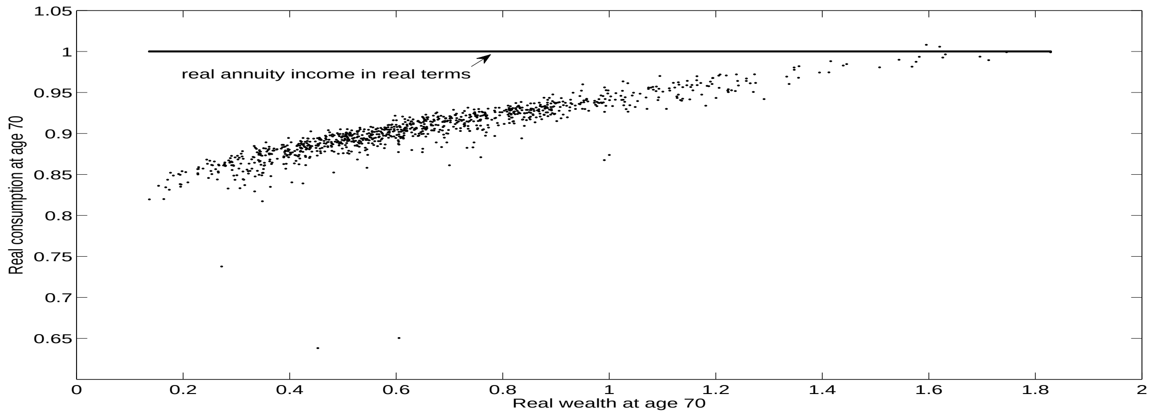


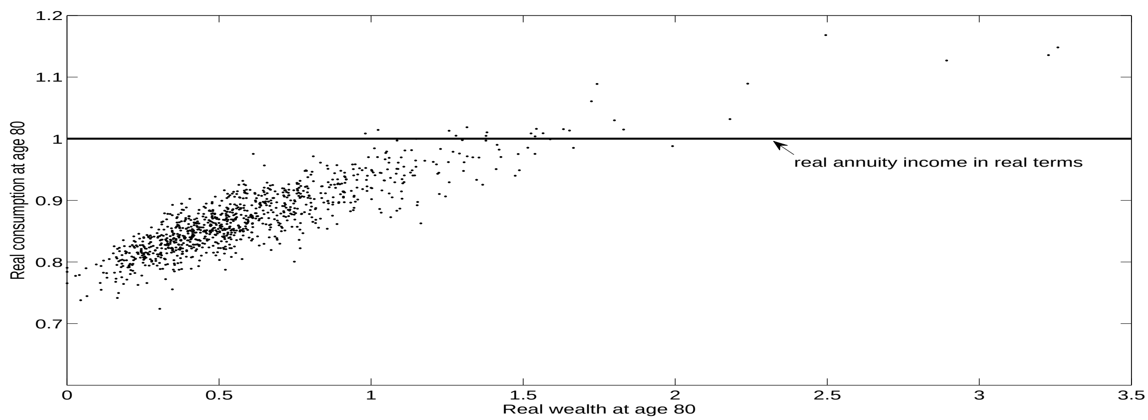
(c)

Figure 4: Optimal consumption for varying wealth levels for a person with an optimal nominal annuity income

The above panel displays the optimal real consumption for a 70 year old for several real wealth levels. In case the optimal consumption is below the line that displays the annuity income, then it is optimal to save for the corresponding wealth level. The middle panel shows the optimal real consumption levels per real wealth level for a 80 year old and the lower panel for 90 year old. The parameters are that of the benchmark set up and the optimal amount is invested in a nominal annuity (100%).



(a)



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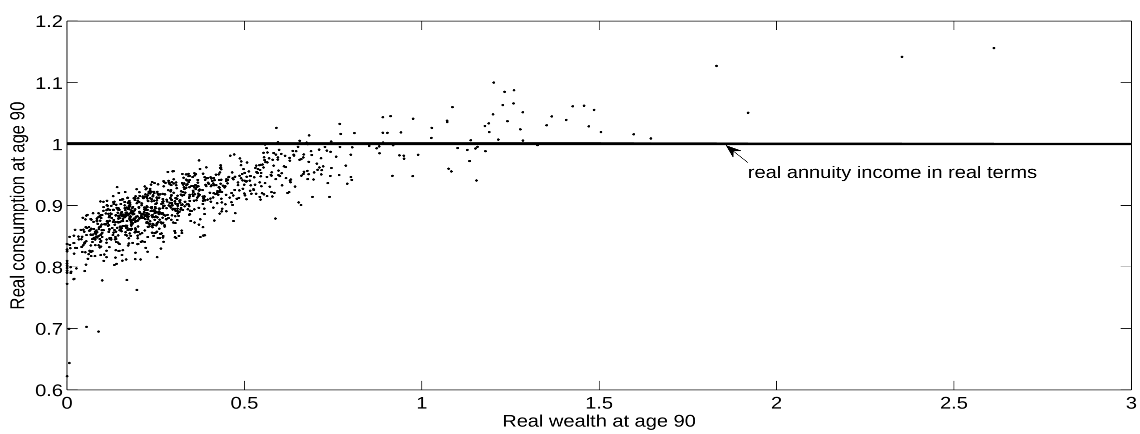

(c)

Figure 5: Optimal consumption for varying wealth levels for a person with an optimal real annuity income

The above panel displays the optimal real consumption for a 70 year old for several real wealth levels. In case the optimal consumption is below the line that displays the annuity income, then it is optimal to save for the corresponding wealth level. The middle panel shows the optimal real consumption levels per real wealth level for a 80 year old and the lower panel for 90 year old. The parameters are that of the benchmark set up and the optimal amount is invested in a real annuity (96%).

nominal case where the buffers are large. However, if an individual receives a real annuity income he has less savings, hence he reacts more after a large background shock. In Figure 5a we see that an individual with a wealth of 0.5 times the FRAI saves 0.11 and if he has a wealth level of 1.5 times the FRAI he saves nothing. Linear interpolation suggests that savings are increased by 0.016 times the FRAI after a two standard deviation shock in background risk, which is almost twice as high compared to the nominal annuity case. As a side effect, the figures illustrate the saving behavior of those with low wealth. In the case of nominal annuities, a 90-year old with wealth less than 2 times the FRAI should still save to hedge against background risk and inflation risk.

To be able to separate the effect that either background risk, inflation risk, or a lack of equity exposure has on optimal annuitization demand, we present several specifications of the model in Table 1. Recall that if all risks are considered jointly, almost full annuitization is optimal. However we see in Table 1 that annuitizing 84% is optimal if background risk and inflation risk are excluded from the model. Interestingly, if individuals do not face (real) background risk, a lack of equity exposure is indeed a reason to annuitize substantially less. So even though background risk and equity exposure are theoretically both reasons to decrease annuity levels, they interact in such a manner that when considered jointly, full annuitization remains optimal. The explanation for this is that background and inflation risk reduce the demand for equity, because an individual who is exposed to (real) background risk already faces a high overall risk level. In that case the benefits from reducing annuitization, namely more equity exposure, are not that high and outweighed by the benefits of annuitization, namely receiving the mortality credit. However, in the case that an individual is only exposed to longevity risk and not background risk, then he does prefer to annuitize less to increase his equity exposure. Horneff, Maurer, Mitchell, and Dus (2008) and Babbel and Merril (2007) find that equity exposure is a reason to annuitize less, however they do not incorporate background risk. We find similar results if we exclude background risk, but in a model which incorporates background and inflation risk, a lack of equity exposure no longer induces lower annuity demand.

Finally we examine whether adding variable annuities to the menu increases welfare sizeably. In Table 2 we display the welfare gains from allocating the optimal amount to a variable and a real annuity, compared to only a real annuity. We see in Table 2 that the welfare gains are a maximum of 1.5%, hence adding a variable annuity to the menu does not increase welfare significantly. The combined optimal annuity portfolio for an individual who faces background risk is only 10% in a variable annuity and the remaining wealth in a real annuity. The reason is that individuals can save out of their annuity income to get equity exposure and real annuities provide a much better hedge against inflation risk than equity-linked annuities. Koijen, Nijman, and Werker (2008b) find an optimal allocation of 40% to real annuities, however they do not include equity in the asset menu. Horneff, Maurer, Mitchell, and Stamos (2008) find a welfare gain of 6% at age 80 and 30% at age 40 of investing in variable annuities instead of nominal annuities. The reason for this contrasting result is that inflation is excluded in that paper and the asset allocation of the portfolio linked to the variable annuity can vary over time.

3.1 Draw-down strategy for consumption

We report the welfare gains from following an optimal consumption pattern compared to a simple rule of thumb consumption strategy. In the simple strategy for consumption that we consider the annuity income is fully consumed and the current wealth divided by the expected remaining life time of the individual is consumed as well. This draw-down strategy is the one that is preferred in Horneff, Maurer, Mitchell, and Dus (2006b). This strategy never accumulates wealth which we know from the previous sections to be important if individuals receive a nominal annuity income. Moreover this draw-down strategy can result in rather low consumption levels if agents live longer than expected. As expected the welfare impact of the use of suboptimal consumption strategies can be quite substantial, which are reported in Table 3. When 100% is invested in annuities the welfare loss ranges from 0% to 9.5%. The welfare loss is about zero for the case without back-

Table 1: Optimal annuitization levels (in %) for varying specifications of the model
The parameters are equal to the benchmark case and optimal allocation to real annuities is calculated. If inflation risk is excluded from the model (a nominal annuity is in effect equal to a real annuity), then the optimal level is similar for a real and a nominal annuity.

	inflation risk included		inflation risk excluded	
	background risk included	background risk excluded	background risk included	background risk excluded
Equity included	96	97	83	84
Equity excluded	97	99	96	100

Table 2: Welfare gains (in %) of investing the optimal amount in a combination of variable and real annuities compared to only real annuities
The assumed interest rate (AIR) is either 4% or 2%. The rest of the parameters are as in the benchmark case.

	AIR 4%	AIR 2%
background risk included		
welfare gain	1.2	1.1
optimal real/variable annuity	90/10	90/10
background risk excluded		
welfare gain	1.5	1.3
optimal real/variable annuity	85/15	85/15

ground risk and 100% allocated to real annuities, because the optimal strategy is then to consume approximately the entire annuity income. Thus the optimal consumption strategy does not differ sizeably from the simple rule of thumb. The importance of optimal consumption strategies is more important for nominal annuities.

Table 3: Welfare loss (in %) of following a draw-down rule for consumption compared to the optimal consumption strategy

% in annuities	without background risk		with background risk	
	nominal	real	nominal	real
0%	43.3	44.3	38.3	41.0
20%	36.4	13.6	41.3	45.6
40%	21.8	2.0	40.8	21.7
60%	8.1	1.3	35.6	6.5
80%	3.4	1.5	24.3	3.4
100%	3.5	0	9.5	7.3

4 Alternative individual characteristics and financial market parameters

The evidence in the previous section suggests that background risk and an incomplete annuity menu have only a small effect on optimal annuitization levels. Instead of annuitizing only partially to insure against background risk and inflation risk, it is found to be optimal to allocate almost all your wealth to an annuity and save out of the annuity income, where needed. In this section we show that these results are robust to alternative assumptions in individual characteristics and financial market parameters. We present results for two benchmark cases: An individual who can freely invest in a real annuity and someone who can freely invest in a nominal annuity. In all cases the other assumptions, including those on background risk are as before, unless explicitly stated otherwise. We investigate the robustness of the results for changes in the risk aversion coefficient, the equity premium, the size of the background risk, and the inclusion of a load factor. The results on optimal annuitization levels are displayed in Table 4.

Table 4: Robustness tests

The table reports the optimal annuitization levels (in %) for several alterations of the parameters in the model. For every robustness check one parameter is changed and the rest stays the same as in the benchmark model.

Parameter setup	Optimal level real annuities	Optimal level nominal annuities
Benchmark parameters	96	100
Mean gross equity return 10% instead of 8%	93	100
Subsistence consumption level 0.3 instead of 0.15	95	100
Mean expenses due to background risk 0.2 instead of 0.1	91	100
Expense factor 7.3% instead of 0%	94	100
Risk aversion coefficient 2 instead of 5	92	100

As a first robustness check we increase the size of the equity premium to obtain an expected stock return of 10% rather than 8%. Not surprisingly this implies a reduction in annuity demand, but the numerical effect is small. The optimal demand for real annuities reduces from 96% to 93%. For the nominal annuity case, full optimization always remains optimal. As a subsequent test we doubled the subsistence consumption level to examine whether this alters the optimal level, Table 4 shows that this is hardly the case.

As another check for robustness the mean (real) expenses due to background risk have been increased from 10% to 20% of the full real annuity income. Moreover the standard deviation was doubled as well. The optimal level allocated to a real annuity decreases from 96% to 91%. Again the direction of the effect is as expected and the numerical differences are small.

In addition we consider the effect on optimal annuitization of including a load factor on the annuity income. The load factor was set at 7.3% in line with Mitchell, Poterba, Warshawsky, and Brown (1999). The optimal annuitization level falls by only 2%. Naturally the welfare loss of the load is large, 8.5%.[5] Finally a less risk averse individual ($\gamma = 2$) invests 92% of his initial wealth

[5]This result is not presented in the paper. The percentage welfare loss is larger than the load, because the amount

in real annuities. Thus the change in the optimal annuitization level is quantitatively small and the previous results are also robust for an alternative risk preference.

5 Conclusion

In this paper we analyze whether optimal annuity demand is strongly affected by either an (initial) loss of equity exposure or background risk and inflation risk. We solve a realistic life-cycle model and optimize dynamically over the consumption level and asset allocation, as well as the annuitization level at age 65. If no variable annuities are available and borrowing constraints are imposed, then in order to get equity exposure it can potentially be optimal to annuitize only a part of your wealth. However we find that (almost) full annuitization remains optimal, irrespective of whether nominal or real annuities are available. In addition, we examine another possible reason for individuals to annuitize less, namely background risk and inflation risk (if only nominal annuities are available). In case of nominal annuities, the agent will save considerably out of the annuity income during retirement to gain equity exposure and hedge against background risk and inflation risk. If an individual receives a real annuity income instead of nominal, he saves only a small amount as a buffer against (real) background risk. Furthermore if a person does not face background risk and inflation risk, annuity demand does decrease substantially due to a lack of equity exposure. Hence both possible explanations for lower annuitization, a lack of equity exposure and (real) background risk, relate in such a way that if both are included, full inflation is optimal. The explanation is that an individual who already faces inflation risk and background risk, does not want to have a high equity risk. The amount of risk exposure is already so high, that it is not optimal to add more risk and receive the equity premium. Finally we show that adding variable annuities to the menu does not increase welfare sizeably, since individuals can save to get the adequate equity exposure.

This paper can be extended in several ways. We ignored bequest motives and heterogeneity in survival rates (for instance between education levels). Moreover, the timing of the annuitization could be modeled as an endogenous decision to analyze whether in that case background risk and an incomplete annuity market alter the optimal timing of annuitization. Milevsky and Young

of income after paying the expenses due to background risk falls by a larger percentage than the load. The income available for consumption does not scale down by the load percentage, due to the expenses for background risk.

(2007b) find that in a one-time annuitization setting which is similar to this paper, the value of delaying annuitization can be large. Furthermore the impact of health costs on the background risk could be modeled explicitly. In some countries, including the US, many agents face large unforeseen health expenses. Health costs are persistent and increase with age. Moreover high health costs and life expectancy are negatively related. The persistency in health costs could induce higher precautionary savings, but on the other hand, if the survival probability decreases sharply after high health costs, this would decrease the need for liquid wealth. De Nardi, French, and Jones (2008) model health costs in this manner and find that for the US setting medical expenses have a large effect on the savings behavior of the elderly. For many other countries this factor will be far less dominant. Finally we have restricted our analysis to immediate annuities, however the annuity menu may also include deferred annuities.

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