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Traverses through the Landscape of Reflective Thinking: Teachers' Actions in the Context of Lesson Study

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Abstract

The notion of reflective practice has its antecedent in the works of Dewey (1933) and Schön (1983) and is key to a teacher's professional growth. In-service professional development models (e.g., lesson study, action research) attend to and embed this vital construct in their program design. In this paper, I feature a lesson study–based professional development program that aimed to foster teachers' content and pedagogical knowledge as well as their reflective thinking. I document the learning experiences of a group of elementary mathematics teachers and describe their journey toward becoming reflective practitioners. The insights that the teachers gained not only helped them traverse the landscape of reflective thinking but also enabled them to become more aware of the significance of using student thinking as a vehicle to inform and enhance their classroom practice.

Keywords: lesson study, reflective thinking, process reflection, MSP grant

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> *It was important to listen to other teachers' approaches to their teaching; we learned to give more of the authority and direction of learning to the students and not rely solely upon ourselves to pass on knowledge … we realized that it is important and necessary to have reflection time as a teacher and as a student.*

The above quotation offers a snapshot of the shared reflective experiences of a group of elementary mathematics teachers on a lesson study project set against the backdrop of a larger professional development program. The quote highlights the teachers' acknowledgment of the significance of engaging in reflective practice that is grounded in an understanding of students' mathematical thinking. I have launched this discussion with this excerpt from the teachers' written reflections both to kindle readers' inquisitive minds and to address questions that might arise, such as: What is the significance of this quote? Who are these teachers? What are the specifics of the lesson study project? In what ways did teachers' participation in this project guide and influence their actions and reflections? This paper addresses such questions by tracing and describing the teachers' journey toward becoming reflective practitioners.

Under the auspices of an Ohio Mathematics and Science Partnership (MSP) grant, the Miami University Partnership for Enhancing the Teaching of Mathematics (MUPET-Math) project was launched in 2007 to nurture mathematics pedagogy in partnership schools in southwest Ohio.

The overall goals of the MUPET-Math program were to provide participant teachers with an opportunity to: (a) enhance their mathematical content knowledge, (b) understand, respect, and use student thinking to guide their instruction, (c) engage in reflective thinking to more thoroughly support student learning, and (d) become members of an emerging and caring community of learners (Keiser & Gloeckner, 2009). To these ends, the teachers participated in focused projects that included thinker-doer activities (Hart, Schultz, & Najee-ullah, 2004), student interviews (Schifter & Fosnot, 1993), and lesson study (Yoshida, 1999).

The MUPET-Math program encompassed three cohorts of teachers from three school districts. Each cohort participated in two phases of professional development: an initial 2-week summer workshop followed by a year-long lesson study project. In this paper, I describe one cohort's reflective learning experiences related to the lesson study project, using the lens of reflective thinking in particular to unravel those aspects of learning that fostered the teachers' engagement in the four stages of process reflection outlined by Ricks (2011).

Theoretical Background

Lesson Study: Origin and Progression

Lesson study, a continuous form of professional development, has its roots in the Japanese education system. Japanese teachers engage in sustained collaboration with their peers, both within and outside their schools and school districts, to enhance their classroom teaching and consequently to provide better learning experiences for their students. Perry and Lewis (2009) describe lesson study as a cyclic process in which teachers collaborate to:

> formulate goals for student learning and long-term development; collaboratively plan a "research lesson" designed to bring to life these goals; conduct the lesson in a classroom, with one team member teaching and others gathering evidence on student learning and development; reflect on and discuss the evidence gathered during the lesson, using it to improve the lesson, the unit, and instruction more generally. (p. 366)

The following tenets underlie the heart of the lesson study process: (a) teacher-led peer-to-peer collaboration, (b) a sustained focus on student learning, and (c) teacher learning grounded in classroom practice (Yoshida, 1999). Three major directions have been proposed for the effective study and analysis of the lesson study process, namely: "expansion of the descriptive knowledge base, explication of the innovation mechanism, and testing of design-based improvements" (Lewis, Perry, & Murata, 2006, p. 3).

Lesson study documents teacher practices along multiple dimensions that include: (a) in-service and pre-service teacher education programs (e.g., Presmeg & Barret, 2009; Parks, 2007; Fernandez, 2005), (b) diverse classrooms (e.g., Taylor, Anderson, Meyer, Wagner, & West, 2005), (c) multiple grade levels (e.g. Wanko, Harper, Edwards, Johnson, & de Saint-Rat, 2010), and (d) various content strands (e.g., Ricks, 2011; Becker, Ghenciu, Horak, & Schroeder, 2008).

Researchers have argued that more theoretical and empirical research is needed to understand the "specific processes that make a lesson study work – choosing tasks that reveal student thinking, designing tools that support effective data collection by teachers, crafting discussion protocols that keep the focus on student learning" (Lewis, Perry, & Murata, 2006, p. 9). This report adds to the realm of lesson study research in that it documents and interprets practitioners' lesson study participatory and learning experiences using the theoretical lens of process reflection (Ricks, 2011).

Lesson Study: The Role of Students' Mathematical Thinking

Teachers' knowledge about their students' mathematical thinking and understanding plays a significant role in their lesson planning and instruction (Ambrose & Vincent, 2003). To better analyze and utilize students' mathematical thinking in their teaching, teachers need to attend to their students' thinking in a purposeful manner. The goal is to understand students' mathematical thinking and to use it as a lens to: (a) further explore mathematical ideas, and (b) inform and enhance their teaching (D'Ambrosio, 2004).

To use students' mathematical thinking in their teaching, teachers must first know what listening entails. In particular, they must know how to question students, what to listen for, and how to identify students' understanding based on their verbal and written responses. One of the strengths of the lesson study model is that it places student learning at the heart of professional development activity. During the initial phase of a lesson study project, teachers identify and describe the overall goals of the project in terms of student learning. In the subsequent phases (i.e., research lesson development and enactment), teachers maintain focus on student learning by: (a) developing lessons that build on students' prior knowledge, and (b) enacting lessons both to make student learning visible and to gain insights about students' thinking (Murata, 2011).

Lesson Study: The Role of Reflective Thinking

Lesson study calls for teacher reflection upon multiple aspects of their teaching, such as lesson planning, task development, lesson implementation, and student learning. The notion of reflective practice grounds teachers' reflective thoughts and actions in the context of their everyday teaching and is central to their professional growth (Van Manen, 1977). Teachers who reflect critically on their classroom teaching practices develop a broadened view of mathematics teaching and learning and are better able to facilitate a nurturing learning environment for their students (Cavanagh & Prescott, 2010).

Reflective practice has its antecedent in the works of Dewey (1933) and Schön (1983).
Dewey posited reflection as an interconnected multi-stage process and proposed open-mindedness, responsibility, and wholeheartedness as the three essential traits of reflective

individuals. Schön (1983) elaborated on Dewey's principles and argued that there are two types of reflection: *reflection in action* and *reflection on action*. He later proposed a third type, *reflection for action*, which is the anticipated outcome of reflection in and on action and refers to a practitioner's reflections that lead to future action.

Both Dewey's and Schön's models of reflection offer a venue for practitioners to engage in a cyclic four-step process: (a) encountering the problem situation, (b) identifying the specific problem(s), (c) positing possible solutions, and (d) problem solving by testing and collecting data to enhance their classroom instruction (Ricks, 2011). Such models serve as useful tools for viewing and analyzing teachers' reflective actions shaped by their participation and learning experiences in the different stages of a lesson study project.

Methodology

I adopted a three-tiered teaching experiment methodology (Lesh & Kelly, 2000) because it allowed exploration of researcher (Tier 1), teacher (Tier 2), and student (Tier 3) learning in the context of classroom teaching. In the current research project, teachers were pursuing research goals centered on student learning and then reflecting upon and learning from their experiences. In this paper, the focus is on Tier 2. In particular, the goal was to describe how teachers learned about: (a) the lesson study and its processes, (b) their students' mathematical thinking, and (c) their own reflective processes. My descriptions are rooted in an understanding and analysis of teachers' reflections specific to the lesson study experience.

Participants

The larger MSP project involved three cohorts of teachers from three school districts. Within each cohort, the teachers split into different groups based on the grade levels they taught. Since the MSP project was funded by a state grant, the participating teachers were paid a stipend, which was awarded in the form of teaching materials. Furthermore, the grant money paid for substitute teachers when participating teachers attended the professional development sessions. The focus of this report is on a group of seven fifth and sixth grade teachers. Each teacher had been teaching at their grade level for at least 3 years prior to their participation in this project and had been using the *Investigations in Number, Data, and Space* for grades K–5 for their mathematics curricula.

Prior to their engagement in the year-long lesson study project, the teachers had participated in a 2-week summer workshop during which they were introduced to the essential tenets of lesson study through lesson study videos and related literature. The group was introduced to the four phases of a lesson study (Murata, 2011) and to the notion of a research lesson (Perry & Lewis, 2009). Teachers were encouraged to identify specific goals for their year-long lesson study project. In particular, they were asked to: (a) identify a content strand (e.g., number sense),

(b) select a specific topic within that content strand (e.g., division of fractions), and (c) develop lessons that aligned with their lesson study goals.

In planning for the lessons, teachers consulted a variety of curricular materials that included the district curriculum map, the *Investigations* textbook, and the state and national standards. To develop preliminary drafts of the lessons (henceforth known as research lessons), they used a research lesson template (Lesson Study Group, 2001).

Data Sources

To describe teacher learning, it was necessary to situate the teachers' tasks within the overall context of the lesson study as well as the specific context of their classroom teaching. The main purpose was to gain insights into the teachers' thoughts and actions as inspired and informed by their work specific to a research lesson. To this end, multiple data sources (i.e., observations, field notes, physical artifacts, and reflections) were employed and analyzed to understand and interpret the teachers' actions.

Observations. Direct observation and participant observation methods (Yin, 1984) were used to gain insight into teachers' actions related to the research lesson planning and implementation. During the planning and debriefing sessions, I was a participant-observer as I was also the lesson study project facilitator. As I listened to the teachers' conversations, I frequently intervened to provide suggestions, seek clarifications, and answer questions related to the research lesson. During the research lesson implementation session itself, one of the participating teachers implemented the lesson while the other members of the research team observed various aspects of the lesson.

Field notes. In my field notes, I included descriptions of teachers' actions, drawing upon my observations related to lesson planning and implementation. In particular, I detailed teachers' lesson planning strategies, their struggles and successes, their dilemmas and "aha" moments. In a similar vein, the participating teachers themselves wrote field notes based on their own observations of the research lesson implementation. Analysis of teacher-documented field notes played a major role in triggering and fostering teacher reflection as well as lesson revision.

Artifacts. The lesson study group maintained a 'mini-portfolio' that included drafts of research lessons, mathematical tasks, teachers' solutions to mathematical tasks, a list of curricular resources, and student worksheets. Research lesson drafts were used to document teachers' work during the different stages of the lesson study. Teacher-developed mathematical tasks and their responses to these tasks offered insights into the teachers' understanding of the mathematics involved. Analysis of student worksheets helped the research team gain insights into the students' mathematical understandings.

Reflections. In my e-journal, I reflected on my research lesson participatory experiences, noted my thoughts and feelings, traced my emerging insights and intuitions, and recorded my failures and successes. Similarly, teachers maintained a journal in which they, too, reflected on their lesson planning and lesson enactment experiences.

Table 1 provides an overview of the data collected and analyzed for this study.

Table 1

Summary of Data Sources

Data Type	Contributor	Events
Observations	Researcher	Lesson planning, enactment, and debriefing
	Teachers	Lesson enactment
Field notes	Researcher	Lesson planning, enactment, and debriefing
	Teachers	Lesson enactment
Artifacts	Teachers	Initial and revised lesson plans, mathematical tasks, and student handouts
	Students	Solution/solution processes specific to the mathematical tasks
Reflections	Researcher	Lesson study participation experiences
	Teachers	Research lesson, its enactment, and lesson study participation experiences

Procedures

In Japan, teachers are immersed in a lesson study culture and have a thorough understanding of the dynamics of this model. Use of this established and successful professional development model in a non-Japanese culture, however, necessitated certain adaptations and variations. We made slight alternations to the lesson study model to better suit the goals of the MUPET-Math program and to fit the needs of our teachers.

The first alteration concerned the role of the facilitator. In the current study, the teachers simultaneously engaged in grasping the principles of lesson study and enacting the research lessons. To facilitate this process, I (in my role as facilitator) maintained a more active role in this project. Having a facilitator as part of the lesson study research team is somewhat of a departure from the traditional Japanese *Lesson Study*, but it has become the norm in the North American form of lesson study (Murata, 2011).

During the initial planning and the post-lesson enactment reflection sessions, I intervened to scaffold discussions using appropriate questioning strategies: questions designed both to foster thoughtful, purposeful discussions centered on the research lesson and to promote meaningful reflections. Such questions addressed a variety of aspects including the goals of the lesson, the choice of the topic, and the choice of tasks.

The second alteration concerned the development and implementation of the research lessons themselves. In the traditional Japanese lesson study model, teachers: (a) develop a research lesson, (b) enact the research lesson and gather data on student learning, and (c) use the data both to reflect on the lesson and to make revisions to it (Murata, 2011). This cycle is repeated as many times as needed to develop a repertoire of knowledge regarding any mathematical topic. In our case, participating teachers expressed concerns about teaching a single lesson (designed at the beginning of the academic year) three times over the course of a year. Moreover, external pressure came into play in the form of standardized tests as well as the need to adhere to the district curriculum guide. Hence, we decided to focus on a different mathematical topic for each cycle of the lesson study.

During the school year, the research group met several times to plan for, implement, and reflect on the research lessons. For each research lesson, the group met three times, once each for the initial planning session, the lesson implementation session, and the debriefing (reflection and revision) session. The group developed three research lessons: one on number sense, one on division word problems, and one on data analysis.

The focus of this report is the research lesson on division word problems. The research lesson was taught to a group of 22 Grade 6 students (hitherto referred as the class). An intervention specialist was present in the room to assist five students who required special accommodation. In line with lesson study principles, the research lesson was taught by one teacher in her class while the research team observed the lesson in action.

Each member of the research team played a unique role: One teacher implemented the lesson; another teacher focused on the research lesson implementation, paying attention to aspects such as task implementation and teacher and student questions; and the remaining five teachers sat with student groups, one at each table, to observe and listen to the students' mathematical conversations.

My role was to observe both the teachers and the students in action. During scheduled class time, the classroom teacher taught the research lesson while the rest of the group members enacted their assigned roles. At the end of the research lesson implementation, the members of the team regrouped to reflect on their research lesson enactment experiences.

Typically, lesson study reports are organized along the four phases of the lesson study process: lesson planning, implementation, debriefing, and revising and re-teaching. This report deviates from that norm in that I use the four stages of the process-reflection framework (Ricks, 2011) to describe the teachers' lesson study participatory experiences. The lesson study group began their project with the following overarching goal: to gain a greater and deeper understanding of their students' mathematical thinking. The research lessons and related tasks were designed in accord with this central goal.

Process Reflection Model

This paper uses Ricks's (2011) process reflection model to discuss teachers' thoughts and actions. Ricks proposed the process reflection framework to interpret practitioners' reflective actions as a cyclic progression of ideas. He distinguishes between two types of reflection: *incident reflection* and *process reflection*. *Incident reflection* refers to a practitioner's mental thoughts and/or activities that emerge and fade away in the moment. Although this form of reflection requires significant mental activity, Ricks (2011) terms this a passive form of reflection because there are no explicit methods that connect the "processed past with the anticipated future action" (p. 252). *Process reflection*, on the other hand, is an active form of reflection and allows for series of incident reflective thoughts to be linked together into "cohesive mental continuums as ideas develop through action" (Ricks, 2011, p. 252).

Ricks's process reflection model consists of four stages: *experiential event*, *idea suspension and problem creation*, *idea formation*, and *testing*. The *experiential event* refers to a triggering experience that sows the initial seed of doubt or concern in the practitioner's mind. Although the individual may not instantly address the concern or offer an impromptu explanation to discard doubt, this event stimulates his or her subsequent reflective processes. The second phase, *idea suspension and problem creation*, refers to the processes that lead to the identification of a problem situation. Instead of extemporaneously reacting to the trigger event, the practitioner suspends reaction to allow for a more in-depth analysis of the initial experience.

During the *idea formation* stage, the practitioner is able to think more deeply about the problem at hand. Processes such as reading relevant literature, researching problem-related information, and reflecting on past experiences allow the practitioner to consider possible solutions to the problem, draft a plan of approach, and explore venues for implementation. The plan is enacted during the fourth stage, *idea testing*. The insights gleaned from this stage of the process reflection cycle might just be the new beginnings that launch the next iteration.

Findings

Experiential Event

Concurrently with the lesson study project, the teachers were engaged in content exploration using thinker-doer activities. During one of the content exploration sessions (prior to the launch of the second research lesson), the teachers worked on contextual word problems in the division of whole numbers. In addition to sharing their solutions and solution processes for specific tasks, they also reflected on their pedagogical experiences related to this topic. Here is a summary of their reflections (my field notes):

> Many students could do the division procedurally. However, the added context in a word problem posed [an] additional challenge. In contrast to plain problems, the

> contextual problems do not tell the students what operation to use. Students must gather this information from the problem. Moreover, the plain problems seek just the numerical answer as opposed to a contextual problem where students must interpret the remainder in the context of the problem. Many students end up using an incorrect operation to solve the problem and/or are unable to interpret the solution in the context of the problem.

These reflections, though experiential and thoughtful, happened in the moment, and the group did not make specific plans to follow up on this issue. At the time, the teachers were concentrating more on deepening their own mathematical understandings and were less concerned about their plans for the research lesson. However, their discussions on this topic sowed the initial seed for the research lesson that they would develop much later. In fact, we could view this as an example of incident reflection that continued to take a deeper hold as the team progressed through their lesson study work.

When the teachers met for their initial research lesson planning session, the class was working on the number theory and fractions unit. The teachers decided to develop a research lesson that would align with the topics in this unit. One of the group members recalled the thinker-doer discussion related to division word problems and suggested this as a topic for the research lesson. The rest of the members immediately picked up on this idea as they all had previously expressed concern about student understanding of this concept. Their early discussion on this topic was the "singular event for which the resultant reflective processes could trace all roots" (Ricks, 2011, p. 255). The group had taken its first step along the process reflection model continuum.

Idea Suspension and Problem Creation

The group was now ready to follow up on their incident reflections. However, they were cautious in their approach. Rather than offering spontaneous quick-fix solutions, the group deliberated and drafted a plan of approach. In this plan, they identified the preliminary steps that had to be completed prior to their research lesson exploration. These steps included: (a) consulting the curriculum map to situate the research lesson, (b) developing specific goals for the research lesson, (c) identifying the students' prior knowledge as well as their difficulties with the topic, and (d) researching curricular materials such as textbooks, research literature, and each other's catalogue of activities. The group suppressed their "spontaneous ideas about the event … [to allow for] … more detailed mental scrutiny, leading to the recognition of problematic characterizations" (Ricks, 2011, p. 253). Teachers temporarily suspended crafting the research lesson to set the stage for problem creation, coming up with the following problem statement:

> The focus of the research lesson will be on word problems on division. The lesson will support student collaboration and require them to communicate their thinking.

Problem identification is one of the key phases of a lesson study project (Yoshida, 1999). Teachers' incident reflections related to their students' difficulties with word problems enabled them to complete this key phase, and furthermore, they were able to make an explicit connection to their initial triggering event, enabling them to progress further along the process reflection continuum.

Idea Formation

The group was now ready to brainstorm ideas and structure a research lesson centered on the problem. During this idea formation phase, the teachers mulled over the problem statement in an attempt to identify specific goals for their research lesson. In addition, they proposed and discussed activities and tasks for inclusion in the research lesson, and the best ideas were "further developed and refined in preparation for acting" (Ricks, 2011, p. 253).

Although all the teachers had taught their own lessons on the chosen topic, this was the first time they engaged in peer-to-peer collaboration to develop a joint lesson. Their individual pedagogical experiences, rather than being a detriment to this process, helped them to fine-tune their lesson goals, and they subsequently identified several key goals for their research lesson (group artifact).

1. Elicit students' understanding of remainders in the fractional and the decimal form.
2. Enable students to use their prior knowledge of division, fractions, and decimals to explore connections between the three.
3. Enable students to understand the connection between the fractional form and the decimal form of a remainder.
4. Help students understand and interpret the remainder (in any form) in the context of the given problem.
5. Foster student collaboration and enable students to communicate their thinking.

Next, the teachers brainstormed ideas for a research lesson that would address these goals, consulting the district curriculum map and the state standards to establish specific connections to the goals. The group named the research lesson "What Remains?". This lesson was designed to enable students to: (a) explore problem situations that involved division with remainders; (b) use their knowledge of division, fractions, and decimals to explore connections between the three;

(c) collaborate in groups to make sense of problem situations that involved division with remainders; and (d) write number sentences to depict a story problem and then solve their problem by dividing (with and without using a calculator).

The lesson comprised three tasks: (a) a launch activity, (b) a follow-up activity, and (c) an exit assessment. The launch activity was intended to enable students to draw upon and apply their prior knowledge of division to solve a contextual problem and to encourage them to interpret the solution in context. The follow-up activity was proposed to facilitate small-group discussions and to foster an in-depth exploration of the concept of the remainder. The exit activity was designed to allow the teachers to assess and gain insight into their students' understanding of the topic.

The group's next assignment was to identify suitable task(s) for the launch activity. The classroom teacher (Andy, pseudonym) suggested using the students' own classroom setting as the context for this activity. The other group members accepted this suggestion and continued their deliberations. Their discussions covered a range of topics that included task identification and selection, task presentation, time allocation, student worksheets, questioning strategies, and anticipated student responses. This information was entered into the lesson planning template.

To showcase the teachers' plans for the launch activity, I present the relevant portions from their template in Table 2.

Once the teachers had completed their launch activity discussions, they moved on to the follow-up activity. Their goal here was to develop an activity that would enable students to build on their launch activity explorations and to explore in depth the notion of the remainder. One of the teachers suggested the following task:

> Five people picked up shells at the beach. They found 171 shells. They want to share them equally. How many shells will each person get?

A detailed discussion ensued. The group liked the context; however, they were not sure if this task in itself was sufficient to address the goals for the activity. This problem did not give students an opportunity to focus on the remainder, let alone require them to discuss fractional and/or decimal remainders. One of the teachers proposed that they develop several tasks related to the number sentence 171 ÷ 5 and pose a different question for each task.

The group welcomed this idea and began developing tasks. Rather than presenting the entire class with a single problem, the teachers decided to assign each student group a different task and then invite the groups to share their solution and solution processes. This act, they hoped, would encourage the rest of the class to consider the possibility of different solutions and solution processes for the problem, even though each student group's initial number sentences were identical.

Table 2

Launch Activity (Duration: 15–20 minutes)

Plan of Approach/Teacher Actions	Tasks	Anticipated Student Response(s)
Before class: Arrange 5 tables and 4 chairs around each table. Make copies of student worksheets. Arrange teaching materials (transparencies, markers, worksheets, seating chart, etc.). Before posing Task 1: Count the number of students and note this number on the board. Instruct students to line up at the board. Ask students to find a seat around a table. Seat those students who are not able to find a seat on chairs next to the board. (Note: Later these students will be assigned to work with other groups)	Task 1: Can anyone tell me what kind of math problem was represented in this activity?	Task 1: Student responses will vary: Addition: Add the number of students at each table. Repeated addition: Group by fives and then add the remainder. Multiplication: Multiply the number of students at each table by the number of tables. Repeated subtraction: You have 21 students and then take away 5 at a time; do this repeatedly. Division: Divide the number of students by the number of chairs at each table. Divide the number of students by the number of tables.
After posing Task 1: Record student answers to Task 1 on the board or overhead.	Task 2: Could you give me a number sentence to illustrate the math in this problem? (If someone responds with a division number sentence, focus on this student's response and steer other students' attention to this idea.)	Task 2: Answers will vary. $5 + 5 + 5 + 5 + 1 = 21$ $5 * 4 = 20$; $20 + 1 = 21$ $20 – 5 = 15$; $15 – 5 = 10$… $21 \div 5 = 4$ r 1
After posing Task 2: State the following: "Okay, these are all correct ways to find the answer to this particular problem. What is the quickest/fastest way to find the answer? Follow up with this question: Why do you think so? After posing Task 3: Write the responses on the board or overhead.	Task 3: What is $21 \div 5$? Follow-up questions: What do the numbers 21, 5, 4 represent here? Some of you have a quotient 4 with remainder of 1. However, the group with the calculator has 4.2 as the quotient. Now what do the remainders 1 and 0.2 mean to you? How would you write 0.2 as a fraction? How do you write the remainder 1 as a fraction? How many tables/stations would be needed to seat every student?	Task 3: Answers will vary: 4 r 1 4 and 1/5 (fractional form) 4.20 (decimal form) Follow-up questions: 21 (number of students in the class) 5 (number of tables/stations) 4 (number of students at each table/station)

The follow-up tasks and the teachers' plans for task enactment are presented in Table 3.

Table 3

Intermediate Activity (Duration: 30–40 minutes)

Plan of Approach/Teacher Actions	Tasks	Anticipated Student Response(s)
Before starting this activity: Give each group a marker plus one problem from the *Division Situations* worksheet printed at the top of the transparency. Each table will work with a different problem from the worksheet. Group work: Give students about 10 minutes to solve the problem (display timer). Stress that students will need to show how they reached their solution. They will also need to identify the dividend, divisor, quotient, and remainder in their number sentence. Whole class discussion: Invite one representative to share his/her group's findings with the whole class. Make notes on the board, as needed. Pose follow-up question(s) and direct the students' attention to the number sentence represented in each question and to the different forms of answers. Note: In particular, identify tasks where the remainder matters and what form of the remainder matters vs. situations where remainder does not matter.	Q1: Five people picked up shells at the beach. They found 171 shells. They want to share them equally. How many shells will each person get? Q2: Five friends earned $171 with a car wash. They want to share the money equally. How much will each person get? Q3: A group of 171 children are going to the zoo. They will travel in cars that hold 5 passengers each. How many cars will they need? Q4: There are 171 sixth graders. They need to form 5 teams for a field day. How many fifth graders should be on each team? Q5: There are 171 sixth graders who will attend the Adams Open House. The gym's bleachers hold 5 students per row. How many rows will Cleveland take up? Follow-up questions: If each of you were working with the same number sentences, how is it that you each came up with a correct answer that is so different from the other 4? Which tasks required you to focus on the remainder? In what ways did you work with the remainder?	Responses may vary slightly: Q1: Students will divide 171 by 5 and then drop off the 1 remaining shell to conclude that each person should get 34 shells. Q2: Students will divide 171 by 5 and then decide that each person gets $34.20 or use the fractional form of remainder (34 1/5) or just have 34 R1. Q3: Students will divide 171 by 5 to get 34 with a remainder of 1 and decide that they would need 35 cars to fit the extra student. Q4: Students will divide 171 by 5 and will come up with the answer 34 with the remainder of 1 and conclude that they will need 34 students per team. Q5: Students will divide 171 by 5 and get the answer of 34 with a remainder of 1 and conclude that they need 35 rows to sit on the bleachers as a group. Follow-up questions: Students might point to the structuring of different problems where the denominator can be ignored, where it cannot be ignored, or where it makes sense to express it as a decimal quantity.

As the final component of their research lesson, the teachers developed a two-task exit assignment designed to gather student data and to gain insights into the students' understanding of division tasks. To this end, the teachers included specific instructions to encourage students to explain their thinking. The task, as worded for the assignment, is presented here (Exit slip: teacher artifact):

> For the following problem, write a number sentence; label your number sentence; and show how you solved your problem.
> Exit Task 1: There are 160 packages. To deliver most of the packages, it will take 3 small planes. Each plane will take the same number of packages. How many packages will each plane take?
> Exit Task 2: There are 160 packages. If each plane can only carry 3 packages, how many planes will be needed to deliver all of the packages?

This culminating activity marked the end of the research lesson as well as the conclusion of the teachers' discussion related to the planning of the research lesson. The teachers had collaborated and developed a joint lesson in an attempt to address the problem situation. They were excited and were now ready for their lesson enactment session.

Idea Testing

Lesson implementation marked the fourth phase in the group's journey on the process reflection continuum. Furthermore, it denoted the "experimentation of the formulated possible solutions" of the problem event (Ricks, 2011, p. 253). On the day of implementation, the lesson study group assembled in the Grade 6 classroom ahead of the scheduled class time. During class time, the classroom teacher (Andy) taught the research lesson while the rest of the group members enacted their assigned roles. Preliminary observations are noted here (teacher field notes):

> For the most part, Andy followed the lesson plan. She finished the launch activity within the stipulated time. However, she ran overtime during the intermediate activity (10 minutes). The exit slip activity ran overtime as well. There were specific instances where student-generated responses were completely different from the teacher-anticipated responses. Such situations required Andy to make instant decisions and digress from the planned lesson. This was a necessary deviation as Andy used a student-generated response (not anticipated or identified in the lesson plan) to generate a class discussion. Students were engaged, and [they] collaborated within their small groups to solve the tasks. Andy encouraged students to explain their thinking.

At the end of the research lesson implementation, Andy handed over teaching responsibilities to the substitute teacher, and the lesson study group dispersed for a break. The end of this lesson enactment session indicated the culmination of this cycle of the group's process reflection processes (see Table 4).

Table 4

Cycle 1 of the Process Reflection Framework

Process Reflection Phases	Lesson Study Group's Actions
Experiential event	Reflecting on past teaching experiences, the group identified a topic for discussion.
Idea suspension and problem creation	The group identified specific difficulties related to the topic, then developed and presented a problem.
Idea formation	The group developed the research lesson, identifying research lesson goals and mapping detailed plans for the lesson and its implementation.
Idea testing	The lesson was enacted in a live classroom. Student data was gathered to gain insights into their understanding.

Discussion

In a lesson study project, the research lesson implementation marks the end of the third phase of the project (Yoshida, 1999), while in a process reflection model, the research lesson implementation concludes a cycle of teachers' reflective processes. However, the end of the implementation phase also signals a new beginning. In light of their lesson enactment experiences, the teachers delved into what Ricks (2011) termed "deeper realms of reflective thought, if new puzzling observations arise during the testing, beginning a new reflective cycle" (p. 254).

As mandated by the tenets of the lesson study (Murata, 2009; Yoshida, 1999) and reflective thinking models (Dewey, 1933; Schon, 1983; Ricks, 2011), the teachers reflected upon multiple dimensions such as lesson planning, teacher actions, and student learning. The first iteration of the research lesson (i.e., experiential event) triggered and guided the next phase of the process reflection cycle. Teachers continued to engage in meaningful discussions to identify venues for revision and to enhance their research lesson. Teachers' in situ reflections on the lesson enactment in turn triggered and guided the next phase of the process reflection cycle. Here in my field notes, I cite some of these reflections:

> The lesson was too long. Some students barely had time to work on their exit slip tasks. Anticipated student responses were completely different from what the students [actually] said. The tasks on the exit slips posed problems for some students. They had trouble deciding what to do with the "extra package" in the

> exit slip tasks. Students were confused and asked questions such as: “What to do with the extra package? What is in the package? Could we split it between three planes?”

During the problem creation (i.e., identifying and posing a problem) and idea formation (i.e., revising the research lesson) phases of process reflection, the group continued to act upon their reflective thoughts. Drawing upon their research lesson experiences from Cycle 1, teachers now acted in a more “thoughtful manner as [their] actions [were] reasoned and purposeful, rather than hasty” (Ricks, 2011, p. 253). Based on their classroom observations and a preliminary analysis of student work, teachers identified this key issue for action: *Analysis of student work indicates that they have trouble understanding and differentiating between situations that need rounding up (of the remainder) and situations that do not. We need to address this issue*. Two broad themes thus emerged from the teachers’ reflections on the lesson enactment: reflections on the research lesson and reflections on students’ thinking.

During the research lesson debriefing session, teachers attended to (but did not limit their discussion to) the following aspects: (a) discussing if and how the lesson addressed the goals of the research lesson as well as those of the lesson study; (b) discussing if and how each of the activities lessons (launch, exploratory, exit) addressed the research lesson goals; and (c) assessing the difficulty level of the chosen tasks. Teachers noted that the research lesson certainly helped them probe their students’ thinking about word problems related to remainders and that the lesson fostered student collaboration and communication. However, in observing the students in action, they realized that some students had difficulty interpreting and understanding the remainder in context. Teachers concluded that the lesson was not helpful in addressing Research Goal 4 and that they needed to attend to this issue.

The discussion on the research lesson naturally paved the way for the teachers to reflect on their observations and analysis of their students’ thinking and understanding. Two key reflections were shared: (a) surprise about the students’ responses to the launch and introductory activities, and (b) concern about the students’ responses to Exit Task 1. By placing the emphasis on student learning, teachers began to identify “the gap between their students’ state of learning and understanding and the teachers’ aspirations for their students” (Yoshida, 2003).

The teachers concluded that the launch tasks did not pose a significant challenge to the students. They were surprised, however, that the students’ responses to the launch activity tasks were not ones that the group had anticipated. To model the mathematical situation (refer to launch activity, Task 1), the students started proposing fraction-related responses such as 21/20. This ‘solution’ took several teachers off-guard. The classroom teacher (Andy) immediately pointed out that the class had recently begun discussing fraction-related topics and offered this as a plausible explanation for the students’ fraction-related responses. The group concluded that the student response of 21/20 could be construed as a comparison of the number of students to the number of chairs.

They also realized that their *anticipated* student responses might in fact stem from their own past teaching experiences and thus might not always match the *actual* student responses. The introductory activity was deemed a success: The tasks challenged and engaged the students, fostered small group collaboration, and required students to communicate their thinking. Above all, it helped the teachers to develop a repertoire of knowledge regarding their students' understanding specific to these tasks.

Many teachers expressed concern about student responses to the tasks on the exit slips. Initially, teachers merely classified student responses as correct or incorrect. A more purposeful and careful analysis generated an intense discussion centered on the following types of responses:

- Two planes will carry the same amount of packages but the third plane will carry extra
- Each plane will carry 53.333 packages
- Each plan will carry 53 and 1/3 of the packages
- Each plane will carry 54 packages

The group hypothesized about student difficulties in distinguishing between situations that required rounding up and those that did not. One of the teachers noted that some students had not realized that there could not be 54 packages in each plane because this total (54*3=162) would exceed the available number (160) of packages. As a plausible explanation, another teacher stated that these students might have compared this situation to the situation in the second task where an extra plane (plane number 54) was required to accommodate the extra package (package number 160). A third teacher pointed out that the students had not realized that there weren't any extra packages in this situation and added that the students would have realized this mistake if they had checked their answer against the given facts. The teachers did realize, however, that the limited data (i.e., the sample student work) and related analyses were insufficient to validate these hypotheses. In my research notes, I observed that in their discussion, they noted:

> We realize that students had difficulty interpreting the problems and making connections to their answer and the given context. We have student work and their explanations. However, we would like to further probe students' thinking on this topic.

Teachers spent a significant amount of time critiquing the exit activity, in particular questioning the choice of tasks, wording of the tasks, and the placement of the tasks. They came up with a number of questions that needed attention as well as proposing countermeasures to address these issues (teacher field notes):

- How can we gain a better understanding of students' thinking? (Use the same tasks again as a launch or intermediate activity so we can listen to their explanations.)
- In what ways can the exit tasks be refined? (Revise the wording; allow extra time for completion; use the exit task activity it as a group task.)
- What probing questions will enable students to make connections to the math content and the given context? (What does 53.333 mean in this context? Does it make sense to have 0.333 package? Does it make sense to round down or round up? When would you round down or round up?)

As the discussion continued, the teachers proposed several revisions to their research lesson that included: (a) using the launch activity as needed, based on the student grade level; (b) using the exit task as a follow-up activity both to direct students' attention to the difference in the two situations and to provide teachers with more data that would enable them to better understand their students' thinking; (c) using another division task (in a follow-up activity), this time using the context of monetary units; (d) using student work to develop tasks (e.g., using actual student responses to the exit slip tasks to launch and generate a discussion on the topic); and (e) using exit slip activity as needed .

By the end of this phase (i.e., idea formation) of the process reflection cycle, the teachers had significantly revised their original lesson plan and had "primed the mind for better acquisition of detailed observation during the testing of the idea" (Ricks, 2011, p. 253). However, at this point, they could not engage in the final phase (i.e., idea testing) of the reflection cycle. As mentioned earlier, this was one of the alterations we had to make to our project to abide by the district curriculum schedule. Nevertheless, the revised lesson was in place and ready for future implementation.

Conclusions and Recommendations

The theoretical constructs of lesson study, reflective thinking, and children's mathematical thinking were used to parse out facets of teacher learning in a group of fifth and sixth grade teachers who participated in a lesson study project set against a larger professional development program. To do so, I used Ricks's (2011) process reflection model to describe the teachers' journey toward becoming reflective practitioners. Use of this model helped both to maintain the focus on teacher learning and to uncover those aspects of teacher learning that enabled them to engage in process reflection. The four phases of the process reflection model served as useful tools to uncover, understand, analyze, and describe teachers' reflective thoughts and actions as they traveled through the different phases of lesson planning and enactment.

The lesson study context provided an ideal venue for teachers to engage in peer-peer collaboration and critical reflection. During the lesson planning and debriefing sessions, participants reflected on multiple aspects of the lesson study, including the research lesson, mathematical tasks and activities, teacher actions, and student learning. In addition, they developed a repertoire of knowledge consisting of research lessons, lesson enactment plans, written student work, and documentation of their analysis and reflections on the research lesson and student work.

As the teachers navigated through each phase of the lesson study cycle, they paid more attention to students' thinking in ways that supported their own learning and reflection, as evidenced here (teacher artifact):

> The peer-peer interactions were such a benefit to us individually because we all took something from the experience and changed the way we planned lessons for our classrooms for this school year, keeping in consideration how we want our students to interact and discuss math in our classrooms.

However, our findings should be treated with caution; participation in a single professional development project will not and cannot completely reform a teacher's practice. This process requires engaging teachers continuously in best practices in a purposeful manner (Lewis, Perry, & Murata, 2006). Participation in a lesson study process entails a serious investment of time and resources. The grant from the MSP project helped our teachers take time away from their teaching on a regular basis to engage in professional development.

Some persistent questions remain: *How can we support teacher participation in such projects in the absence of external funding? How can we help teachers sustain their enthusiasm from this project and carry it forward?* Such questions do not have simplistic answers, and further research is required to address them.

In the traditional Japanese professional development model, lesson study is entirely teacher-led and fosters teacher learning, and as a consequence, greatly impacts student learning. By design, in the North American versions of lesson study, the research team includes a facilitator. In this context, the following questions should be explored: *(a) whether and in what ways a facilitator (e.g., mathematics teacher educator) impacts peer-peer teacher collaboration in a lesson study project, and (b) whether and in what ways a facilitator enhances teachers' engagement in process reflection.* There is more to ponder on this topic, and a focus on both Tier 1 (researcher learning) and Tier 2 (teacher learning) as outlined by the teaching experiment methodology is needed to address such questions.

As a mathematics teacher educator, I believe that there is much more to be learned about the impact of lesson study on all constituents: teacher educators, teachers, and students. This belief will guide and fuel my future research efforts. I conclude by quoting this excerpt from our teachers' final reflections:

> It was a positive and beneficial experience to have the opportunity to observemath teaching in other people's classrooms and in multiple buildings. We were able to benefit from seeing how other people set up their classrooms to maximize the learning environments for our own students. We rarely get the opportunity to observe other teachers teach. We know that it is helpful to witness people who are using best practices, and without the exposure to these observations, how can we grow as teachers?

These thoughts convey a positive message. For teachers, the best takeaway message is the realization that engagement in teacher-led, peer-peer collaborations and purposeful reflections is crucial to their professional growth as teachers of mathematics.

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