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Pre-service Teachers' Mathematical Understanding: Searching for Differences Based on School Curriculum Background

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Abstract

Elementary teachers' understanding of mathematics as required for effective classroom teaching is an area of intense research interest. Our research examined the differences in mathematical understanding between pre-service teachers who had been exposed to a reformed mathematics curriculum for up to 8 years during their elementary and secondary education (n = 77) and those who had not had such exposure in school (n = 51). Examination of responses related to integer subtraction and fraction division questions on a paper-and-pencil survey revealed little distinction between the two participant groups. Participants in both groups did not provide any conceptual justification or model to support the operations they were able to perform and resorted to simply restating rules by way of explanation. This inability to think conceptually has implications for in- and pre-service teacher education.

Keywords: mathematics education, pre-service education, teacher mathematics knowledge, curriculum change

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Ontario made the move to institute government-mandated revisions of school mathematics curricula following the initial publication of *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 1989). The province released a fundamentally changed elementary school mathematics curriculum in June 1997, followed by a further revised version in 2005 (Ministry of Education and Training, 1997; Ontario Ministry of Education, 2005). The release of the initial curriculum was accompanied by little professional development for classroom teachers, however, even though both the teaching and learning principles – as well as the mathematics content itself – were quite distinct from previous curricula.

With the unveiling of its new elementary school curriculum in 1997, our province mandated that all elementary grade level teachers were to begin using it in September of that year. This meant that a student starting Grade 7 in September 1997, for example, would have experienced six previous grades of mathematics using a different curriculum before changing to the new one. This new curriculum (Ministry of Education and Training, 1997) contained Grade 7 'expectations' requiring students to "demonstrate an understanding of operations with fractions using manipulatives” and to “represent the addition and subtraction of integers using concrete materials, drawings, and symbols” (p. 24), as well as Grade 8 expectations that asked students to “demonstrate an understanding of operations with fractions” (p. 26). Also included were a series of ‘process expectations’ drawn largely from the initial NCTM *Standards* document (1989).

The process expectations included in the revised curriculum strands describe the intended student learning processes: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating. For example, the problem-solving process expectation states that students “will be expected to describe what they are doing in mathematics and explain why they are doing it” (Ministry of Education and Training, 1997, p. 3). Furthermore, the problem-solving process expectation specifies that “knowledge of mathematical language, structure, and operations will help students to reason, to justify their conclusion, and to express ideas clearly” (p. 5).

Although process expectations were embedded within the content strands for each of the grade levels in the 1997 document, they are more clearly defined and separated from the other expectations in the 2005 revision. These learning processes, influenced by social constructivist notions, place significant demands on teachers, in particular on those who were most likely to have been educated in what might be perceived as a traditional paradigm (McNeal & Simon, 2000). The mathematical processes described in the Ontario Mathematics Curriculum (Ministry of Education and Training, 1997; Ontario Ministry of Education, 2005) describe classroom learning that goes far beyond the rote learning of procedures.

Both expectations, implying those related to content, and process expectations were further fleshed out in the 2005 iteration of the curriculum document, which is the one currently in use. In this document, Grade 8 content expectations include having the student “solve problems involving operations with integers, using a variety of tools (e.g., two-coloured counters, virtual manipulatives, number lines)” and “represent the multiplication and division of fractions, using a variety of tools and strategies (e.g., use an area model to represent $^1/_4$ multiplied by $^1/_3$)” (Ministry of Education, 2005, p. 111). This is the curriculum document that pre-service teachers graduating from teacher education programs since 2005 would be expected to use in Ontario elementary classrooms.

In this paper, we compare mathematical knowledge of pre-service teachers based upon their prior education in either the new or old curriculum. Participants were drawn from a related 5-year, cross-sectional study exploring mathematical knowledge (Kajander, 2010). We hypothesized in our larger study that, given the emphasis on conceptual reasoning implicit in the process expectations, pre-service teachers who had been primarily educated through the new curriculum would outperform those who had not been in terms of modeling, explaining, and justifying ideas. However, data from our survey that asked pre-service teachers to model and/or explain basic elementary procedures after performing them, revealed little overall difference between the old- and new-curriculum mean scores.

A goal of our current study was to more deeply examine the initial written survey responsesof pre-service teachers to search for more subtle differences that might not be apparent from looking only at the overall survey means. We still wondered if differences might exist in teacher understanding based on the curriculum they themselves had experienced in school. Accordingly, we wanted to search for evidence of conceptual understanding among teachers who had been taught via the reformed curriculum compared with those who had been educated under the older curriculum.

Although we acknowledge that procedural and conceptual learning and understanding are interrelated in a complex manner, for the purposes of our current study, we refer to “a mastery of basic skills” (Ontario Ministry of Education, 2005, p. 3) and “operational skills” (p. 4) as *procedural knowledge,* and to an understanding of “structures, operations, and processes” (p. 3) and the “interrelated concepts that form a structure for learning mathematics in a coherent way” (p. 4) as *conceptual knowledge*.

Our current paper focuses on the data from 2009, the beginning of the fifth year of our larger study, which was gathered when participants entered the one-year teacher certification program. At this stage, just over half of our pre-service teacher cohort had experienced the revised Ontario curriculum (Ministry of Education and Training, 1997) for up to 4 years of their elementary school experience, as well as during high school. (The high school curriculum was changed on a one-year-at-a-time basis beginning with the first cohort of students finishing Grade 8 under the new curriculum.)

As part of our larger study, all participants completed a paper-and-pencil survey instrument, the “Perceptions of Mathematics Survey” (Kajander, 2007), during the first meeting of their mathematics methods class. This instrument, whose questions were strongly drawn from the original Ma (1999) interviews, explored both a procedural and conceptual understanding of elementary mathematics concepts. However, we were mostly interested in the conceptual understanding aspects as potentially related to the knowledge of mathematics as needed by teachers (Ball, Hill, & Bass, 2005).

Overall mean scores reported for 2005 to 2008 (Kajander, 2010) suggested that although procedural computational skills were reasonably adequate even at the beginning of each year of the study, conceptual understanding (as evidenced by participants’ written ability to explain, justify, or model the procedural calculations they had just performed) of those same participants was extremely weak among the entering pre-service teachers. We thus wanted (a) to explore whether discernible differences in the conceptual understanding of pre-service teachers, as based on their own school curriculum experience, was observable at this stage, and (b) to further examine participants’ understanding of concepts.

Theoretical Background

Our research is grounded in a social constructivist theoretical framework, which proposes that understanding is built through a combination of the actions of the individual and the social interactions the individual undertakes, and that these two facets cannot be separated (Palincsar, 1998). Another underpinning belief of the social constructivist theorists is that classroom discussions can provide a place for developing deep conceptual understandings (Palincsar, 1998). Although the basic tenets of social constructivism are in themselves a basis for mathematics education, the nature of the subject matter calls for an expansion on them (Ball & Bass, 2000; Lesh & Doerr, 2003). In particular, we are interested in pre-service teachers' knowledge of "why the methods and algorithms work" (Kahan, Cooper, & Bethea, 2003, p. 226), which Kahan et al. argue is an important aspect of the mathematical content knowledge required by teachers.

Previous research has suggested that elementary teachers, often subject generalists, have typically not been specially prepared in mathematics and thus generally enter teacher preparation programs with a conceptually weak mathematical understanding (Kajander, 2010; Karp, 2010). As well, Ball, Thames, and Phelps (2008), Silverman and Thompson (2008), and others argue that a specialized body of knowledge particular to teaching – initially termed "pedagogical content knowledge" by Shulman (1986, p. 9) and more recently referred to as "mathematical knowledge for teaching" (Ball et al., 2008, p. 389; Silverman & Thompson, 2008, p. 499) – is needed in teaching, and that such knowledge relies heavily on strong conceptual underpinnings.

Although "mathematical knowledge for teaching" does include ideas pertaining in general to classroom teaching, such as knowledge of both content and students, there are other aspects of this knowledge that have been identified as 'purely' mathematical. Ball et al. (2008) claim that "deciding whether a method or procedure would work in general … determining the validity of a mathematical argument, or selecting a mathematically appropriate representation … requires mathematical knowledge and skill, not knowledge of students or teaching" (p. 398). We feel that the development of this particular aspect of teachers' mathematical knowledge is an area that should be supported during pre-service teachers' education.

We posited that teachers' specialized knowledge must be grounded in a deep conceptual understanding of the mathematics content, as this understanding is needed, for example, to support the enactment of mathematical processes such as problem solving and representation in the classroom. Not only has such deep teacher understanding been cited as critically important for student success (Ball et al., 2005; Heck, Banilower, Weiss, & Rosenberg, 2008), a recent large-scale study reported measureable differences in student learning based on knowledge of mathematics that is specialized for teachers (Baumert et al., 2010).

In 2007, Year 3 of our 5-year study, we began to see pre-service teachers who had experienced the new curriculum in school starting in Grade 7. However, these early participants would have had teachers who were teaching from the new curriculum for the first time, and since these participants had not experienced the previous years of this new curriculum for Grades 1 to 6, it is unlikely that full implementation of the new curriculum had been experienced by the pre-service teachers during those initial years.

By Year 5 of our study, however, many teachers would have been in their third year of teaching from the new document (Ministry of Education and Training, 1997), and students would have experienced the new curriculum as of Grade 5. Hence, we chose to focus our current study on the Year 5 pre-service teacher sample because we felt that it was important to study a group other than the initial cohort.

Methodology

Participants

We collected data from a cohort of 140 pre-service teachers enrolled in a mathematics methods course as part of a one-year teacher certification program. At our mid-sized institution, the ‘junior-intermediate’ cohort (for teaching Grades 4 to 10) receives a total of 36 course-hours in mathematics curriculum and instruction, which we refer to as the methods course, which may or may not be preceded by any post-secondary mathematical coursework.

Of the 140 original participants, 12 were excluded from the current study because of incomplete survey responses or other factors, such as two who did not go to school in Ontario for all of their schooling. Consequently, there were 51 pre-service teachers who had received 11 to 13 of years of mathematics under the old curriculum only, and 77 who had received up to 8 years under the new curriculum (with Grades 1 to 4 being old curriculum).

Neither gender nor age data were collected, but the cohort (as might be expected) had a somewhat larger number of female participants, and although there was a handful of mature students, most had come to the teacher certification program almost directly following their undergraduate degree.

Data Sources

The main data sources informing this discussion are responses from a written survey given to pre-service teachers on the first day of the methods course. We selected two questions from the “Perceptions of Mathematics Survey” (POM) (Kajander, 2007) completed at the beginning of the first class in the methods course. Then we categorized and analyzed response types and searched for overall observations as well as differences possibly attributable to the participants’ earlier school curriculum experience.

The original 2007 POM survey contained five questions, each with one subpart related to procedural skill as well as one subpart related to conceptual understanding; see Kajander (2007) for the full survey. We chose two specific questions for our current study: Question 2, pertaining to subtraction of integers [5 – (–3)], and Question 3, pertaining to division of fractions (1¾ ÷ ½). We chose these questions for two reasons: first, we thought that beginning our current study with numeracy-related items seemed a good starting point, and second, these two items in particular had the lowest overall mean scores of any of the survey items in our original study.

For both questions, participants were asked to complete both “part a),” which asked participants to “Answer the question showing your steps as needed,” and “part b),” which asked them to “Explain why the method you used in ‘a)’ works, using words, diagrams, models and examples as appropriate. If possible, do the question another way.”

“Part a)” was scored as procedural knowledge, and “part b)” was scored as conceptual knowledge (see Kajander, 2010). Only the responses to “part b)” of each question, as used for the scoring of conceptual knowledge in the original analysis, were used for the current analysis and discussion. We felt that these two survey items, 2b) and 3b), that were chosen aligned quite strongly with the new curriculum expectations as previously described, and hence were of interest for that reason as well as for the reasons just mentioned.

Based on the work of Kajander and Lovric (2005), a two-point scale was used in the original survey scoring for each question part; hence, the total possible combined score for the fractions and the integers conceptual questions 2b) and 3b) was four points. To explore the responses to the items more fully, we also developed a set of qualitative codes for the two items as part of our current study.

Data Analysis

To create the codes to describe the data, we examined examples of individual survey responses chosen semi-randomly. Every 14th survey was chosen, for a total of 10 selected surveys. This sample was then used to create the initial set of potential codes (shown in Table 1). These initial codes were then independently re-tested on the sample by each researcher.

After the re-testing and subsequent discussion between the two researchers, two further codes were developed to fully encompass all the responses found in the sample. The two new codes were (a) “mixture,” which represented a response with some correct and some incorrect or irrelevant elements, and (b) “two expl,” which represented a response with two or more correct conceptual explanations. Table 1 contains the initial set of codes plus these two additional codes. The 14-survey sample was then returned to the dataset (in the original order).

Both researchers, working independently, then tested the codes (Table 1) further by applying them to the two chosen questions (2 and 3) on the first 15 of the 140 surveys. Any discrepancies (about 10%, i.e., three questions out of the total of 30 questions were initially scored somewhat differently by the two researchers) were resolved via discussion and, in one case, clarification of the code description. At this point, the 15 surveys were returned to the dataset, and the entire sample was re-coded by the first author (AK) for the analysis.

In the scoring for the current study, the one single best descriptor from the list in Table 1 was chosen to represent each response. This was in contrast to the scoring method used for the previous analysis of this data in which a numeric score (0, 1, or 2) was used to score each response because for this study, we wanted to probe and describe the data more fully and broadly by grouping the responses via the response types shown in Table 1.

Table 1

Codes Used in Data Analysis

	Code	Description of Response	Example/Details of Response	Supporting Literature
1	blank	Blank or ?	Either a "?" or left completely blank	IEA TIMSS, 1997
2	inc/other	Incorrect rule or other irrelevant comment only	"because of BEDMAS"	Ball, 1988a, b
3	mixture*	Mixture of correct and incorrect rule, rule stated only	"find a common denominator and then invert and multiply"	Ball, 1988a, b; Conference Board of the Mathematical Sciences, 2001
4	rule	Procedural calculation redone from "a)" and/or correct rule stated only	"because two negatives make a positive"	Ball, 1988a, b; Kajander & Mason, 2007; Karp, 2010
5	alt proc	Alternate correct procedure, different from the one provided in "a)"	For example, converting fractions division to decimals and solving	Ball et al., 2008
6	part expl	Partly correct attempt at model or conceptual explanation	"dividing by a fraction is like finding out how many you have of it"	Tirosh, 2000
7	one expl	One of [the following:] a correct model or a conceptual explanation	For example, a chips, number line, or other model of integer subtraction, or a conceptual explanation such as "dividing by one half is counting how many one halves are in 1¾. There are three complete halves and half of the next one so that's 3½"	Ball et al., 2008
8	two expl*	More than one correct model and/or explanation	More than one of the previous response type provided	Ball, et al., 2008; Small, 2009

Note: Responses were scored in the current analysis by choosing the one single best descriptor from this table for each separate question response. No numeric degree of scoring was used for the chosen descriptor.

* Codes that were developed after the first round of testing.

The first code used for analyzing the participants' written responses was either a blank response or the participant simply writing "?" in the answer spot; this code is supported by the IEA Third International Mathematics and Science Study (TIMSS, 1997), in which items where blank answers were given by students were specifically noted in its scoring guide.

The next three codes dealt with the idea that "knowing math had always meant to be able to produce the answer the teacher wanted through memorizing formulas" (Ball, 1988b, p. 14); in some cases, these memorized formulas may not be recalled in their entirety, so the second code was worded as “stating an incorrect rule or making another irrelevant comment,” and the third code was worded as “giving a mixture of correct and incorrect rules but still only stating them.”

As Ball (1988b) discovered through examining teacher responses to mathematics questions, procedures may be remembered incorrectly or incompletely. Pre-service teachers might be trying to demonstrate their understanding of the procedural steps through rules created by their former teachers to help them learn the content, but these rules were either forgotten or “mis-remembered” (Conference Board of the Mathematical Sciences, 2001, para. 45).

The fourth code, “procedural calculation redone or correct rule restated only,” connected to descriptions in the literature of teachers ‘explaining’ a concept by simply demonstrating or stating a procedural method or rule (Holm & Kajander, 2011; Kajander & Mason, 2007; Karp, 2010). As Ball (1988a) noted in her study of pre-service teachers, many of the participants “equated remembering with knowing” (p. 21).

These first four codes were used to describe responses that did not align with the definition of the specialized knowledge needed for teaching, namely, the need for a teacher to “show what the steps of the procedure mean and why they make sense” (Ball et al., 2008, p. 398), whereas the remaining codes were meant to tease out different levels of a more conceptual and specialized knowledge.

The fifth code was “alternate correct procedure that was different from the one provided in the survey question part a” (“part a” of each survey question simply asked participants to do the question to find the answer). The need for this code is supported by one of the aspects of “mathematics for teaching” described by Ball et al. (2008), which states that teachers must be able to examine non-standard student solutions and “figure out what students have done, whether the thinking is mathematically correct for the problem, and whether the approach would work in general” (p. 397); hence, we considered it noteworthy if participants could themselves provide an alternate method.

The sixth code reflected “a partly correct attempt at a model or conceptual explanation,” as described by Tirosh (2000), and the seventh code applied to participants showing “one of [the following:] a correct model or a conceptual explanation for the question.” These two codes align with the definition of “mathematical knowledge for teaching” that refers specifically to “knowledge of content and teaching,” in that teachers require the ability to provide conceptual models for their students (Ball et al., 2008, p. 401).

Finally, we felt that based on the definitions of mathematics needed for teaching, pre-service teachers would ideally need to show multiple models or conceptual explanations for each question. As such, we created a final code that identified participants who created “more than one correct model and/or conceptual explanation.” As Small (2009) noted, students who truly understand a mathematics concept would be able to use it in multiple ways, connect it to other concepts, and understand more complicated questions. Teachers would in turn need an understanding of multiple models to be able to support their students because “knowing content is also crucial to being inventive in creating worthwhile opportunities for learning that take learners' experiences, interests, and needs into account" (Ball & Bass, 2000, p. 86). Our codes were intended to differentiate between different responses to allow us to potentially unpack more subtle differences, specifically those differences between individuals with different curriculum experiences.

In the original numeric scoring of the items, a score of zero was given for "part b)" responses in which the participant did not give any model, justification, or conceptual explanation. The first four codes in Table 1 all refer to responses that scored zero in the original item scoring of conceptual knowledge (i.e., a blank, an irrelevant comment or incorrect rule, a partly correct rule, or simply stating a correct rule such as "because two negatives make a positive").

An attempted or incomplete model or explanation scored one point, and a correct and reasonably complete model, justification, or conceptual explanation (code short form: "one expl") was originally scored as a full two points for the survey item. Adequate knowledge of "mathematical knowledge for teaching," on the other hand, would ideally require knowledge of more than one correct explanation or model; hence, the codes in Table 1 identify this response with the final code "two expl."

To support our analysis, we also provide descriptive statistics to summarize the scores that the complete cohort of incoming pre-service teachers, as well as the groups used in the current study, had originally received on these sub-items on the earlier POM survey. (See Kajander, 2010, for a description of the survey scoring process).

Findings

Descriptive statistics suggest that the responses to the two selected paper-and-pencil survey items indicated a serious weakness in the pre-service teachers' conceptual understanding. The overall mean score of the Year 5 cohort on these two sub-items was 0.10 out of 4 points, or 2.5% (see Table 2).

Table 2

Combined Mean for Year 5 of All Participants

	N	*M*	*SEM*	*SD*
CK score	140[a]	.100	.029	.346

Note: Overall combined mean for all Year 5 participants on the two survey question responses, out of 4 possible points. SEM = standard error of the mean, SD = standard deviation. For more on survey, see Kajander (2007).

[a] 12 of these 140 participants were not included in the more detailed analysis because they either did not respond to the question about previous curriculum experience or their response was unclear, or, in two cases, because some of their schooling took place outside of Ontario.

When separated by group (those who had experienced some of the new curriculum compared to those who had not, with those who left the question blank or gave unclear responses removed from the dataset), the mean for old-curriculum participants was 0.137 (3.4%) compared with the new-curriculum participant mean of 0.091 (2.3%) (see Table 3). Independent t-tests suggested this difference was not significant, $t(126) = .711$, $p = .478$.

Table 3

Means by Group

	N	*M*	*SEM*	*SD*
Old curriculum (pre-1997)	51	.137	.063	.448
New curriculum	77	.091	.033	.289

Note: Overall combined mean for the two groups of participants on the two survey question responses, out of a possible 4 points. SEM = standard error of the mean, SD = standard deviation. For more on survey, see Kajander (2007).

Individual responses, coded according to the codes in Table 1, are provided in Figures 1 and 2. The results show that few pre-service teachers responding to the two items demonstrated an understanding of any conceptual basis for numeric methods used for subtracting a negative integer or for dividing two fractions, regardless of their past curriculum experience.

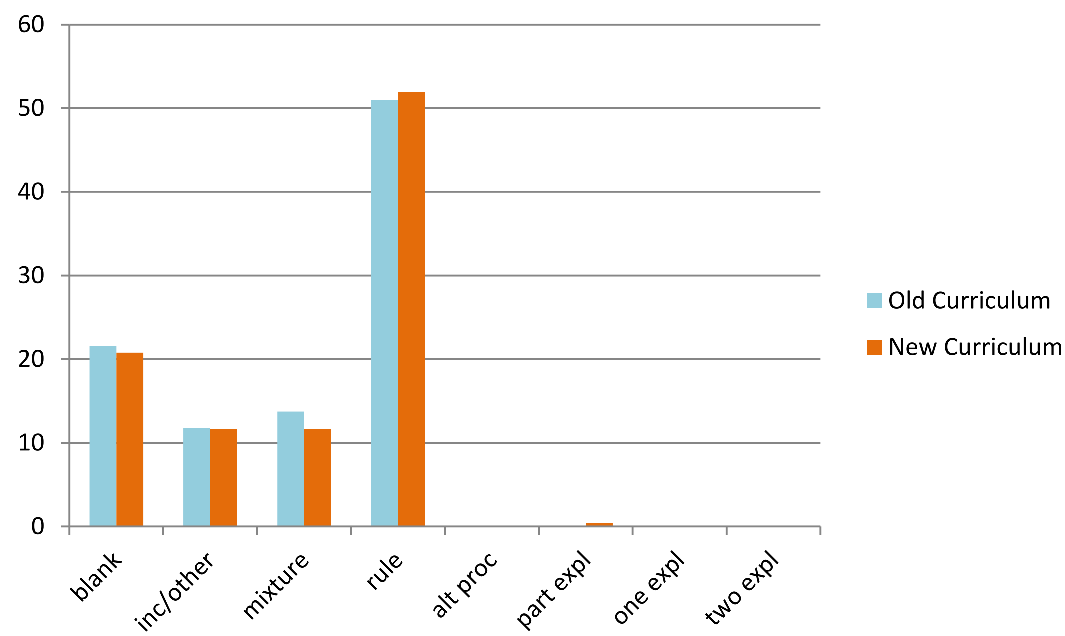

Figure 1. Responses to integers question only: Old- vs. new-curriculum participants. Numbers are shown as the percentage of participants from each group giving each response, using codes from Table 1.

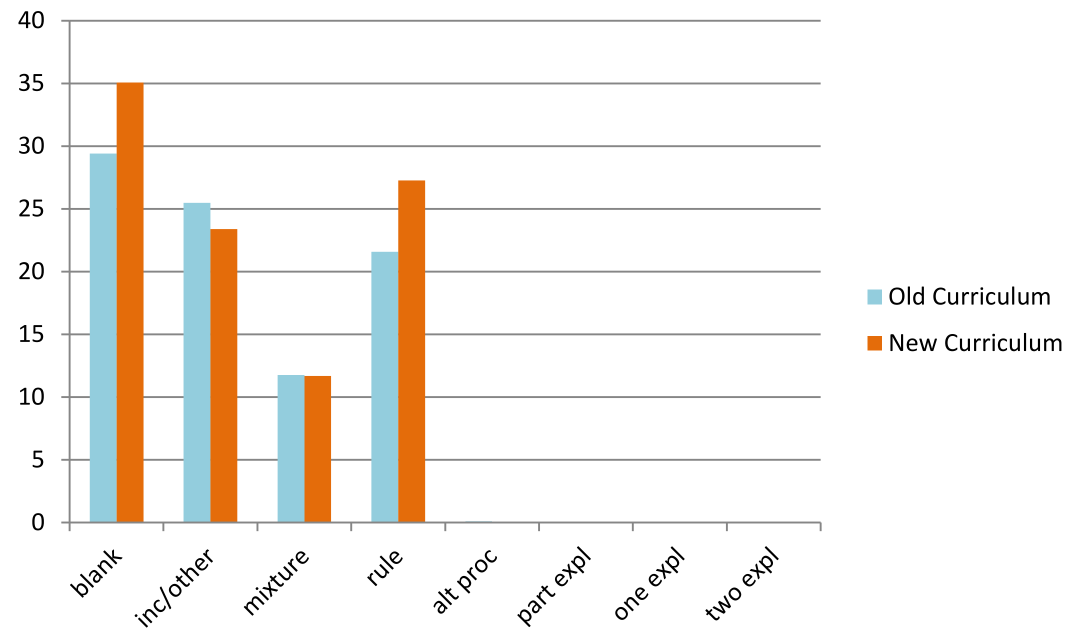

Figure 2. Responses to fractions question only: Old- vs. new-curriculum responses. Numbers are shown as the percentage of participants in each group giving each response, using the codes shown in Table 1.

Integers

Figure 1 shows that more than 95% of the responses fell into the first four code categories, namely: (a) a blank response, (b) an irrelevant comment or something unclear or incorrect (e,g., "reverse all the signs"), (c) a mixture of correct and incorrect rule-based responses, or (d) a rule simply stated as the 'explanation' ("because two negatives make a positive"). More than 20% of both old- and new-curriculum participants left this question completely blank. Just over 50% of participants in both groups simply stated a rule along the lines of "two negatives make a positive" in an attempt to explain the integer questions. Just over 10% of both groups fell into the incorrect/other irrelevant comment category, examples of which were "use BEDMAS" or "reverse signs on all numbers." Overall, we were not able to identify any striking differences between the two curriculum groups via our analysis.

Fractions

We found a similar result with the fractions question, where more than 95% of responses fell into the first four code groupings; once again, the analysis did not reveal any major differences in response types between the two curriculum groups (see Figure 2). For the old-curriculum group, almost 30% of all responses were left blank, whereas around 35% of the new-curriculum participants left the question blank. More than 20% of the responses from both groups to the "Explain" prompt stated a procedural rule – generally "invert and multiply" – but without explanation or elaboration.

Most of the remaining responses from both groups for both questions fell into the two code categories in between (i.e., “inc/other” and “mixture”). About 25% of the responses were in the “incorrect/other irrelevant comment” category; an example of a response in this category would be “invert the first fraction.” The “mixture” category, which involved a mixture of correct and incorrect rule-based responses, received around 10% of the responses in both groups.

Only one participant, who happened to be in the old-curriculum group, supplied a model or conceptual explanation of any sort. The idea that dividing fractions draws upon the ‘measurement’ concept of counting amounts or groups of the second quantity (e.g., “1 ¾ ÷ ½ is like counting how many portions of size ½ there are in 1¾”) is an important conceptual idea that was almost never mentioned by participants.

Discussion

We wanted to explore whether differences based on past curriculum experience would be evident, given the emphasis on conceptual reasoning in the new curriculum. Our initial quantitative analysis, which formed the initial stage of the research reported here, suggested that differences were not observable quantitatively. To further explore these data, we performed a more nuanced analysis, but the results from our initial work in 2009 to 2010 remained unchanged and validated.

Initial overall mean scores in our paper-and-pencil survey had suggested no significant improvement in conceptual understanding on the part of the new-curriculum participants compared to those who had studied under the pre-reform curriculum.

Unpacking and further categorizing individual responses, as we did in this current analysis, again revealed little difference between the two groups in terms of type of response: in both cases, virtually no participants were able (or, one might suggest, were not motivated to do so, or perhaps were simply not interested) to provide a conceptual basis or explanation of any sort for the calculations they were performing. Furthermore, there were many instances when participants simply stated a procedural rule as their ‘explanation’ for a numeric method.

The early indications suggested by our data, when seen through the perspective of the longer-term results of school curriculum change, have not yet revealed evidence of an improved conceptual understanding of elementary concepts, at least in those students who choose to become pre-service teachers; this is of course assuming we are interpreting the new curriculum to imply that students should be able to explain and model the conceptual basis of the procedures they are using, as was argued earlier in this paper.

Hence, whether by choice, motivation, or lack of preparation, new-curriculum participants did not appear to be any more conceptually oriented than their old-curriculum counterparts. We also found evidence of this result during pre-service classes, in particular those classes in which participants working together in small groups were asked to model and explain mathematical ideas. Our informal classroom observations thus support our current survey-based results, in that participants across the board generally found explaining and modeling mathematical ideas very challenging, especially at first.

Although the challenges of developing a conceptual understanding have been reported before (Ball et al., 2005), we continue to be concerned, as we are faced yet again with the extent of this on-going problem in our own province, especially given the literature that cites teacher knowledge as very important to student learning (e.g., Karp, 2010). Even with our more nuanced analysis, we were unable to show any evidence of deeper understanding within our cohort of pre-service teachers than had existed before the curriculum reform.

Alternately, our data might suggest a different interpretation, which relates to our efforts to better understand “mathematics for teaching” (Ball et al., 2008). Could it be argued that the understanding required to complete our written survey questions – that is, an ability to explain basic fractions and integers operations using models or conceptual reasoning – is *specialized* content knowledge (Ball et al., 2008) and hence not something addressed in most elementary classrooms?

If so, then our data highlight both the specialized nature of this content (which seemed to be unknown to the majority of the entry-level pre-service teachers) and more generally the challenge this content poses for new teachers. Such an interpretation also underscores the need for extra time in teacher preparation to focus on these ideas, which has been argued elsewhere as critically important (Kajander, 2010).

In our considered opinion, the curriculum expectations in the province’s document, which is supposed to govern the mathematics content and processes addressed in Ontario schools, do clearly state a need for students to both “understand” as well as “demonstrate operations” using “concrete materials” (Ministry of Education and Training, 1997), as described earlier in this paper. Although students may not need to provide multiple explanations or models in school math – doing so might arguably fit into the realm of “mathematics for teaching” or specialized knowledge, according to Ball et al. (2008) – we feel the emphasis on at least some conceptual understanding was indeed described in the 1997 curriculum document and even more clearly detailed in the 2005 revision.

However, given that our data continue to suggest that these objectives have not been met, at least as evidenced by our current pre-service teacher cohort, the categorization of this knowledge becomes a moot point. Indeed, the pre-service teachers in our study are in real need of opportunities to develop their content-based understanding before they enter their own classrooms.

We argue that the development of some initial conceptual understanding – at the very least to the level described in the reformed curricula – is an important starting point for pedagogical discussions in teacher preparation. For example, discussions about what a problem-based lesson might look like in elementary school do not make much sense for participants for whom the instruction “explain” typically means stating a rule.

Hence, we believe that mathematical support for pre-service teachers should begin during their undergraduate studies, be further supported and contextually deepened during their teacher education, and be continued during professional development as in-service teachers. Such support working in parallel at the in-service level to improve classroom learning also has the potential to effect change in future pre-service teacher cohorts, which might in turn provide an enhanced ‘starting point’ for new teachers.

In summary, even when we employed more fine-grained and qualitative methods, we were unable to demonstrate any evidence of a deepened and more conceptually based understanding in those new pre-service teachers who had experienced several years of the reformed curriculum document (which may or may not have been effectively or fully implemented as a guide to their learning).

It must further be remembered that in our current study (2009), the very earliest a study participant could have begun studying under the new curriculum was Grade 5. In the late 1990s when the new curriculum was first unveiled, not only were teachers faced with the task of ‘catching students up’ with concepts in the new strands and processes that were supposed to have been addressed in the earlier years of the new curriculum (which students changing curricula in Grade 7 and 8 would not have experienced beforehand), they were also teaching the new curriculum with, at most, a few years’ experience and may have been doing so with minimal professional development and support (Kajander & Mason, 2007).

Although the context of our current study was regional, we believe the outcomes are not unique to our location. It must also be remembered that our results may apply only to that group of school graduates who chose to go into elementary teacher education at the post-secondary level and would not necessarily apply more broadly to all high school graduates. Hence, making any judgements about the effect of the new curriculum on students’ mathematical conceptual development in general cannot be done from our data.

Nevertheless, the data suggest that we still have a long way to go in terms of curriculum implementation, at least in terms of the effect of the curriculum on those students who choose to enter teacher preparation, particularly in relation to those curriculum expectations involving representations, explanations, and demonstrations of understanding that are found in both the 1997 and 2005 versions of the provincial elementary mathematics curriculum (Ministry of Education and Training, 1997; Ontario Ministry of Education, 2005). Given other research that cites teachers’ knowledge of mathematics as a critical component of effective teaching (e.g., Baumert et al., 2010; Wong & Lai, 2006), our data underscore the need to provide appropriate and extensive mathematical experiences for pre-service teachers.

Conclusions and Recommendations

Our study suggests that elementary mathematics teacher education continues to be significantly challenged by teachers’ mathematical views and content-related understandings (or lack thereof), despite their exposure to a new and arguably more progressive curriculum for much of their own elementary and secondary education.

Regardless of curriculum background, over 95% of the pre-service teachers entering teacher preparation at our university did not demonstrate, whether because of aptitude or choice, the ability to provide explanations, models, or justifications for the computational rules they were using for the fractions and integers calculations they performed. Furthermore, we were unable to find any evidence in our sample of pre-service teachers of improved conceptual understanding of basic fraction and integer operations by those participants who had studied under the new (1997) curriculum during their upper elementary school years.

Data from our other work do suggest to us that pre-service teachers are, in fact, keenly interested in deepening their conceptual understanding (Holm & Kajander, 2011) and typically get very excited by conceptual ideas, models, and explanations along with their connection to previously known procedural methods. Fractions in particular seems to be a content area that provides many aha! moments as participants develop the understanding to connect contexts and new models with existing procedures. We have found evidence that strong gains are possible in even a relatively short time and that most participants are highly motivated to learn these ideas when they are in an environment focused on concepts and models (Kajander, 2010).

In particular, we agree with Karp's (2010) concern that prospective teachers' "ability to listen to their students and to react to what their students said [is] largely limited by their own conception of mathematical knowledge and conception of mathematics" (p. 135). As part of our ongoing quest to ultimately improve student learning outcomes in mathematics, our study suggests that opportunities to simultaneously deepen conceptual mathematical understanding still need to be aggressively sought in teacher preparation. Although this need has been argued elsewhere (e.g. Sullivan, 2011), our study contributes to the discussion by further illustrating and underscoring the problem. Since teachers are in effect the 'front-line' workers in the curriculum delivery process, teachers have the potential to significantly influence the degree to which a given curriculum document is implemented as intended.

Furthermore, we feel that data such as ours need to be taken into account when further revisions of the school curriculum take place in Ontario. Appropriate learning experiences for teachers must be based on accurate information about their current levels of understanding, and our study contributes to this body of knowledge. It appears that much work must be done in teacher-development programs before the kinds of problem-based and conceptual-learning experiences described in the curriculum-reform documents have any hope of being realized in the classroom.

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