Rigidity and Circuits on the Cylinder

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A framework (G, p) is continuously flexible if there exists a continuous function $x : [0, 1] \to C^{|V|}$ such that the following hold:

1. $x(0) = p$,
2. $(G, x(t))$ is equivalent to (G, p) for all $t \in [0, 1]$,
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Definition

A framework (G, p) is (continuously) rigid on C if it is not continuously flexible and is minimally rigid if $(G \setminus e, p)$ is flexible for any edge $e \in E(G)$.

Infinitesimal Rigidity

Definition

The cylinder rigidity matrix $R_C(G, p)$ is the $(|E| + |V|) \times 3|V|$ matrix where the usual 3-dimensional rigidity matrix is augmented by $|V|$ extra rows, one for each vertex. The entries in the row for vertex i are zero except in the column triple corresponding to i where the entry is $(x_i, y_i, 0)$.

$$\left[\begin{array}{cccc} p_i - p_j & \dots & p_j - p_i & \dots \\ \hline (x_i, y_i, 0) & \dots & & \\ & \ddots & & \\ & \dots & (x_j, y_j, 0) & \dots \end{array}\right]$$

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Definition

(G,p) is infinitesimally rigid on C if $R_C(G,p)$ has maximal rank. Moreover (G,p) is minimally infinitesimally rigid on C if $R_C(G,p)$ has maximal rank and linearly independent rows.

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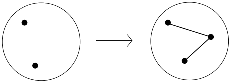
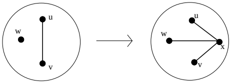
Proposition

For generic p, (G, p) is infinitesimally rigid on C if and only if (G, p) is continuously rigid on C.

Henneberg-Laman





Recall for the plane:

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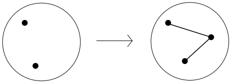

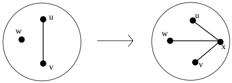

Recall for the plane:

Theorem: Maxwell, Henneberg, Laman, Lovasz and Yemini

For generic p and a graph $G = (V, E)$, the following are equivalent

1. (G, p) is minimally rigid (in $\mathbb{R}^2$),
2. $|E| = 2|V| - 3$ and for every subgraph X of G with at least one edge, $|E(X)| \leq 2|V(X)| - 3$,
3. G can be derived from K_2 by Henneberg 1 and 2 moves,
4. for any non-edge e (including a second copy of an existing edge) $G + e$ is the edge disjoint union of two spanning trees.

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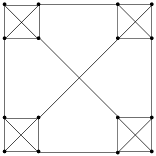

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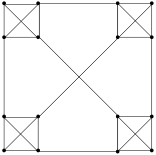

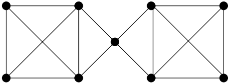

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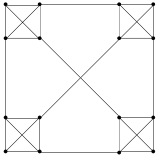

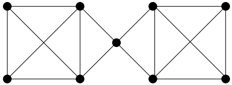

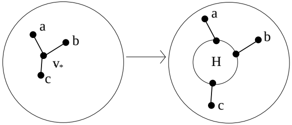

Let $H \subset G$ be a proper subgraph with $|E(H)| = 2|V(H)| - 2$. Write G/H for the (possibly multi)graph formed by contracting H to a point v_*. G/H is called a contraction of G by H and conversely, G is an extension of G/H by H.

Theorem. Tutte, Nash-Williams 1964, N., Owen and Power 2010

For a simple graph $G = (V, E)$ with $|V| \geq 4$ and generic p the following are equivalent:

1. (G, p) is minimally infinitesimally rigid on C,
2. G is derivable from K_4 by the Henneberg 1, Henneberg 2 and graph extension moves,
3. $|E| = 2|V| - 2$ and every subgraph $G' = (V', E')$ satisfies $|E'| \leq 2|V'| - 2$,
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- Aside: the theorem extends to a union of concentric cylinders.
- Half turn ($\mathbb{Z}_2$) symmetry 'predicts' flexibility in minimally rigid graphs...

Back to the Plane

$G = (V, E)$ is a $\mathcal{R}_2$-circuit if and only if G satisfies $|E| = 2|V| - 2$ and all proper subgraphs $G' = V', E')$ (with at least one edge) satisfy $|E'| \leq 2|V'| - 3$.

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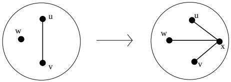
Theorem - Berg and Jordan 2003

Let G be a 3-connected $\mathcal{R}_2$-circuit with $|V| \geq 5$. Then some inverse Henneberg 2 move is possible on G.



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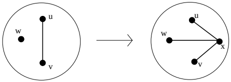

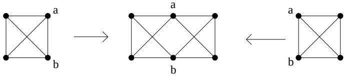

Theorem - Berg and Jordan 2003

G is a $\mathcal{R}_2$-circuit if and only if G can be formed from disjoint copies of K_4 by Henneberg 2 moves and sums.

Circuits

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Definition

A simple graph $G = (V, E)$ is a circuit (in $\mathcal{R}_C$) if and only if $|E| = 2|V| - 1$ and all proper subgraphs $G' = (V', E')$ satisfy $|E'| \leq 2|V'| - 2$.

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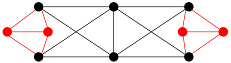
- In the corresponding multigraph matroid every circuit has an admissible vertex.
- In $\mathcal{R}_C$ not every circuit has an admissible vertex.

Examples



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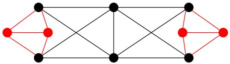

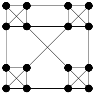

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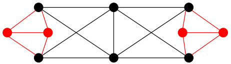

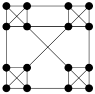

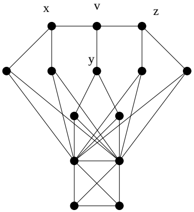

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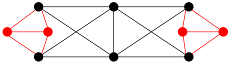

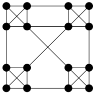

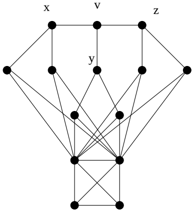

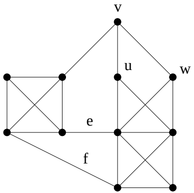

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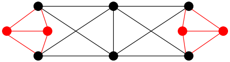

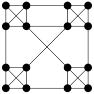

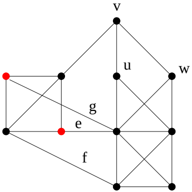

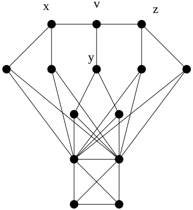

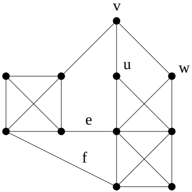

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Proposition

Let $G=(V,E)$ be a 3-connected circuit with no non-trivial 3-edge-cutsets and $|V|\geq 6$. Then some vertex is admissible.

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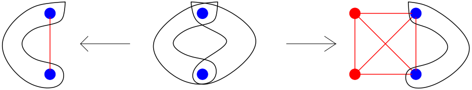

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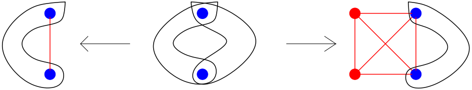
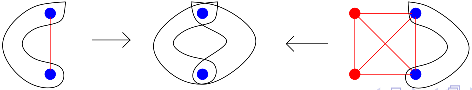


Let G_1, G_2 be circuits such that G_1 contains an edge $a_1 b_1$ and G_2 contains a two vertex cut a_2, b_2 within $K_4(a_2, b_2, c_2, d_2)$. A 1-sum operation takes G_1 and G_2 and forms $G_1 \oplus_1 G_2$ by removing $a_1 b_1$, c_2, d_2 and $a_2 b_2$ and superimposing a_1, b_1 onto a_2, b_2 and calling the resulting vertices a, b.



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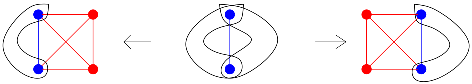

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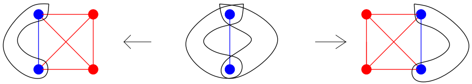
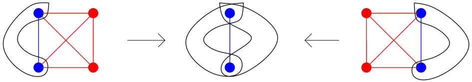


Let G_1, G_2 be circuits such that G_i contains a two vertex cut a_i, b_i within $K_4(a_i, b_i, c_i, d_i)$. A 2-sum operation takes G_1 and G_2 and forms $G_1 \oplus_2 G_2$ by removing c_i, d_i and superimposing a_1, b_1 onto a_2, b_2 and calling the resulting vertices a, b and keeping only one copy of the edge ab.



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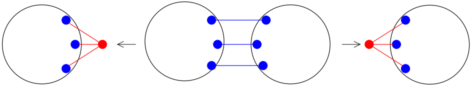

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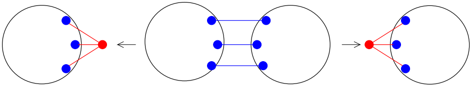

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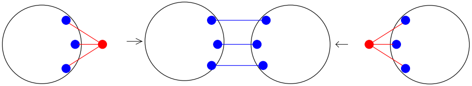
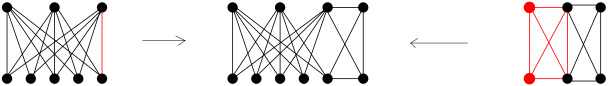


Examples



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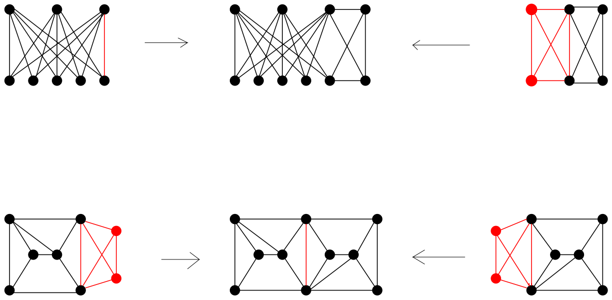

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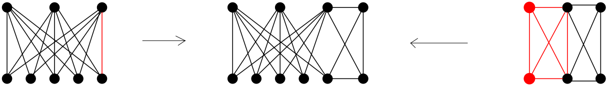

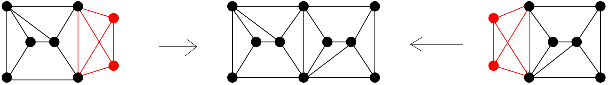

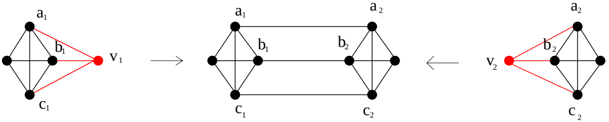

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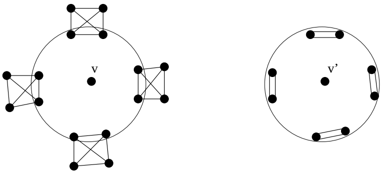

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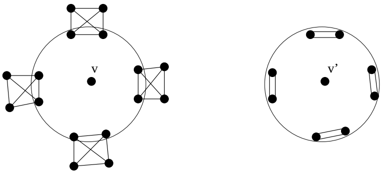

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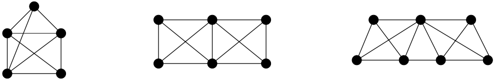

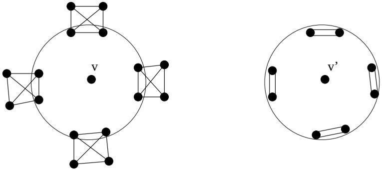

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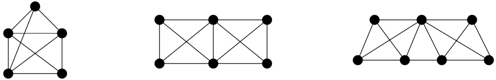

Theorem

G is a circuit if and only if G can be formed from disjoint copies of $K_5 \setminus e$, $K_4 \sqcup K_4$ and/or $K_4 \veebar K_4$ by Henneberg 2 moves, 1-sums, 2-sums and 3-sums.

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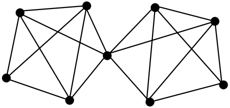

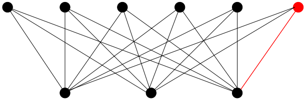

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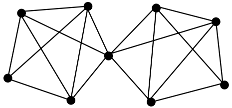

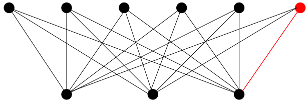

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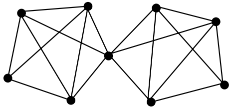

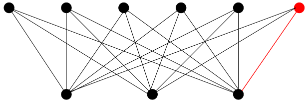

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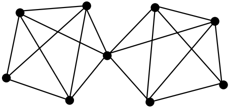

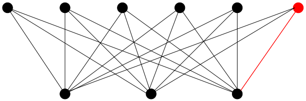

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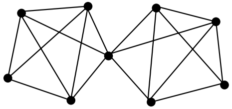

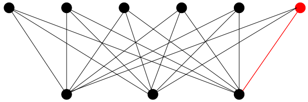

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  - it remains to prove these operations preserve rigidity on a cone/torus: Henneberg 1 and Henneberg 2 work on 'any' surface, other moves to work out.