Topics in Rigidity Theory from the Aspect of Assur Graphs

Offer Shai

- Rigid structures are compositions of cycles of atoms (Assur Graphs)

- The main definition of Assur Graphs.

- Characterization of Assur Graphs through the form of the rigidity matrix

-Assur Graphs are "fully sensitive" related to loads and velocities.

- Why body bars are important in engineering?

- Pinned or floating Assur Graphs

- Geometric constraint problem = Analysis of Linkages (with Reich, by *DASSAULT*)

- Relation between Rigidity matrix and Jocobian matrix (with Muller, Germany)

- The **singularity** of Assur Graphs

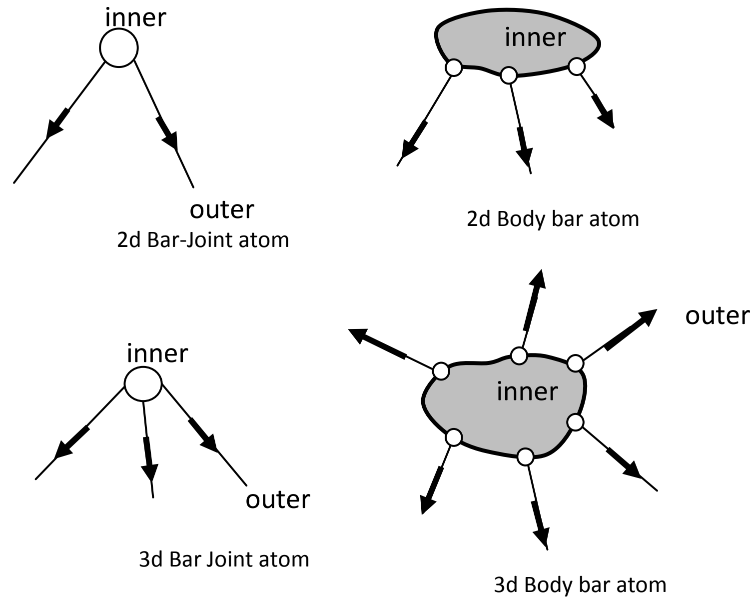
  - Deployable/foldable Tensegrity Assur Graph

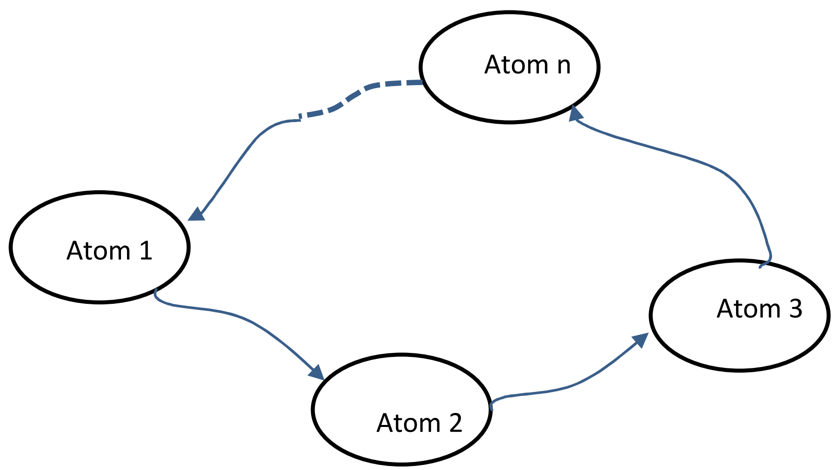
  - Towards soft/rigid robots (with Ayali, zoologist)

  - From singular Assur Graphs into floating mechanisms

Rigid structures = compositions of cycles of Atoms.

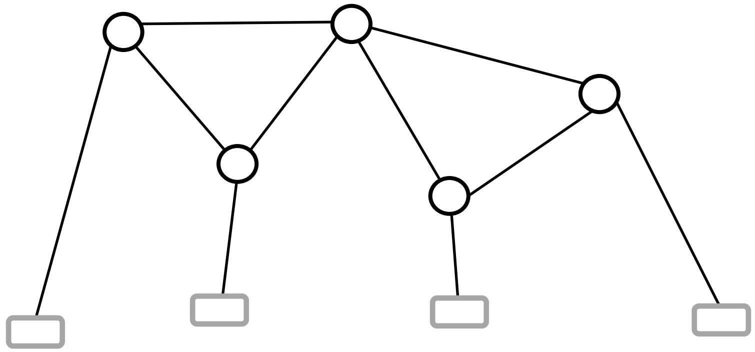
The Atoms of Assur Graphs





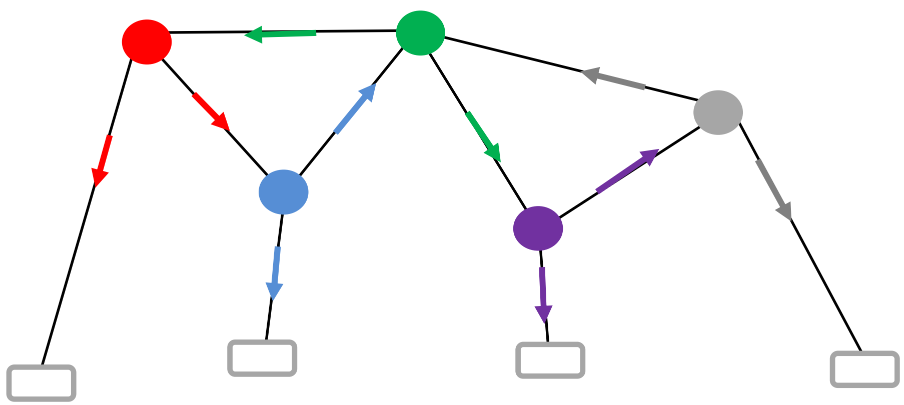
Assur Graph =cycles of Atoms= strongly connected

Example of 2d Atoms of Bar&Joint Assur Graph



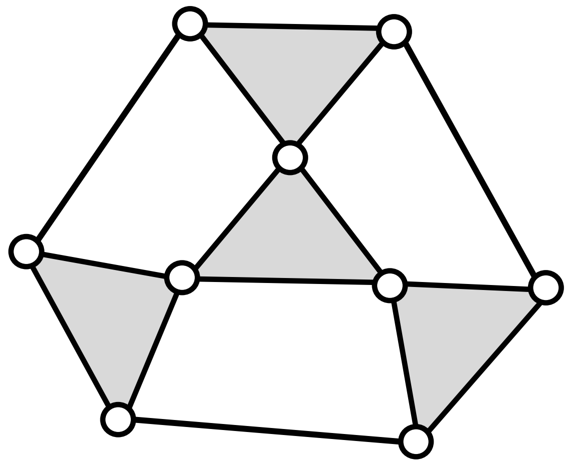
2d Bar-Joint

Assur Graph



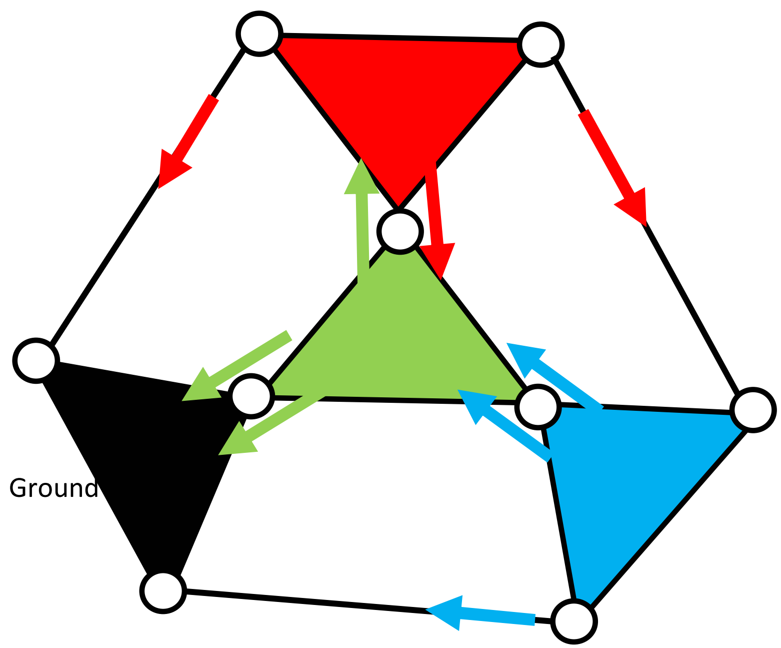
2d cycle of atoms

Example of 2d Atoms of body bar Assur Graphs



2d Body-bar

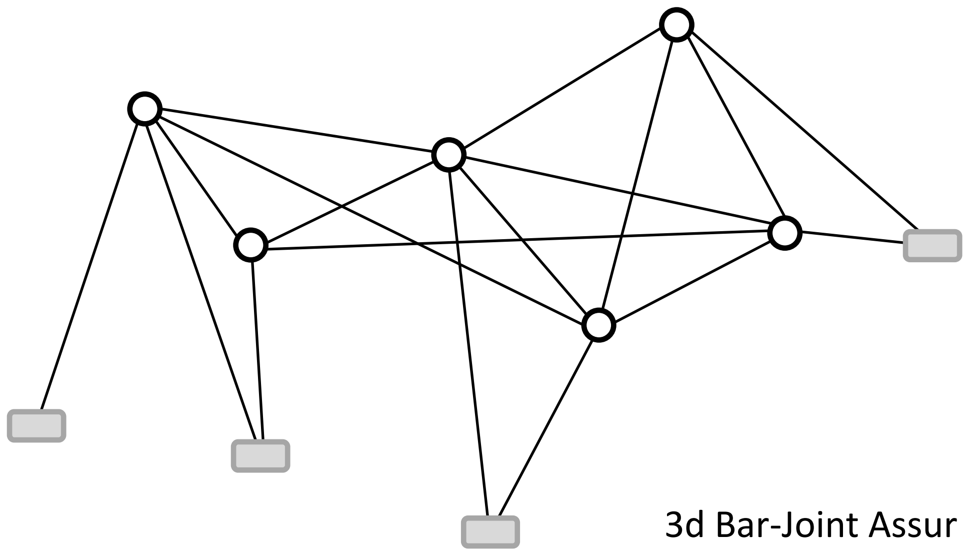
Assur -Graph



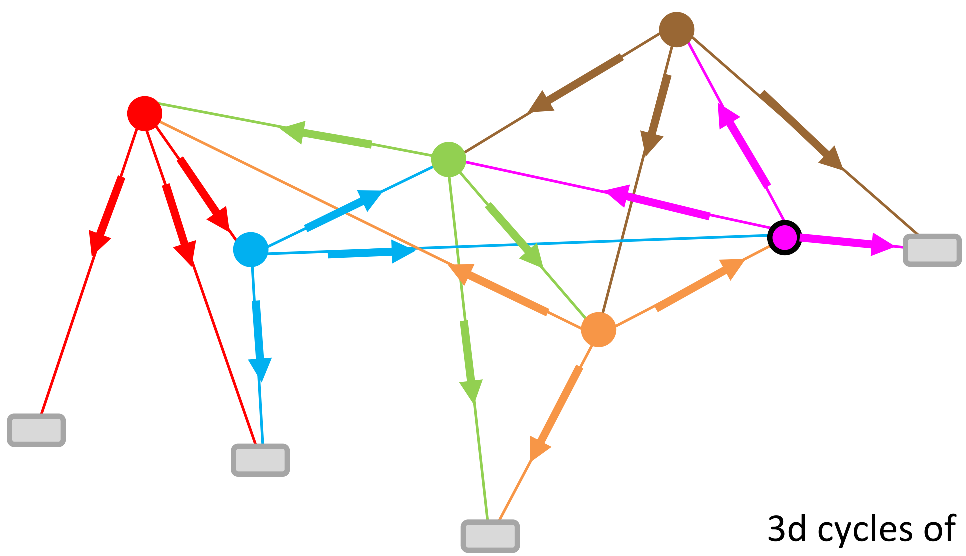
2d cycle of

atoms

Example of 3dAtoms of Bar&Joint Assur Graphs



3d Bar-Joint Assur Graph



3d cycles of atoms

The main definition of Assur Graphs.

There are more equivalent definitions, as appears in :

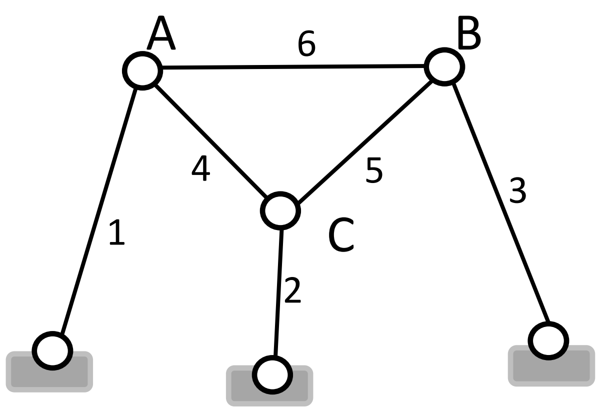
Servatius B., Shai O. and Whiteley W., "Combinatorial Characterization of the Assur Graphs from Engineering", *European Journal of Combinatorics*, Vol. 31, No. 4, May, pp. 1091-1104, 2010

Shai O., Sljoka A. and Whiteley W., "Directed Graphs, Decompositions, and Spatial Rigidity", submitted to *Discrete Applied Mathematics*, 2010.

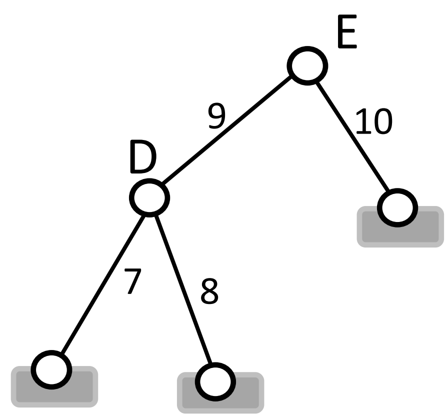
Sljoka A., Shai O and Whiteley W., “Checking mobility and decomposition of linkages via Pebble Game algorithm”, ASME Design Engineering Technical conferences, August 28-31, 2011, Washington, USA.

The main definition of Assur Graphs:

G is a bar&joint Assur Graph IFF G is a **pinned isostatic graph** and it does not contain any **proper pinned isostatic graph**.



Asssur Graph



Not Asssur Graph

Characterization of Assur Graphs through the form of the rigidity matrix.

The decomposition of the rigidity matrix into lower-block-triangulation is important in kinematics/statics analysis.

It can be derived from the matrix the influenced zone of the vertex (external force/load).
The influenced zone of an edge – velocity.

Decomposing into Assur Graphs from the aspect of rigidity matrix = lower-block-triangular form.

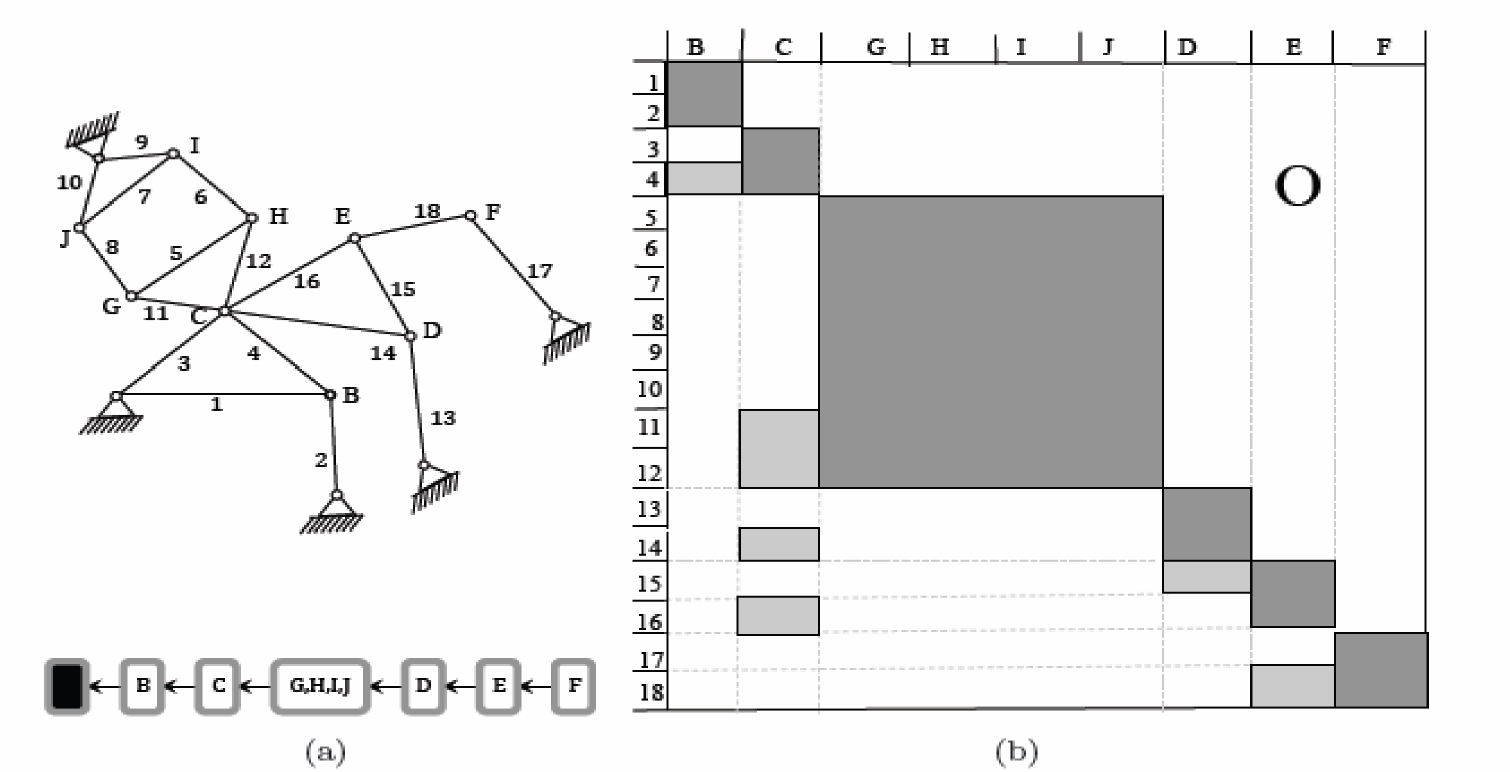
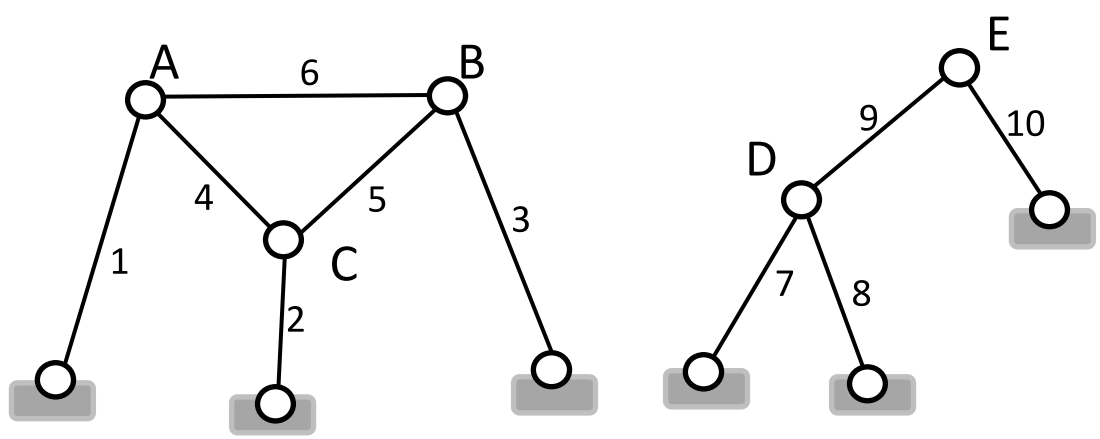


Figure 7: The pinned 2-isostatic framework in (a), with the selected linear order extending the partial order of Figure 2, generates the block triangular matrix in (b).

Assur Graphs are “fully sensitive”

Let G be a 2d pinned isostatic graph. G is an Assur Graph IFF applying an external force on ANY vertex results in forces in ALL the edges.



Asssur Graph

Not Asssur Graph

In 3d a new phenomenon appears
(as expected)

Not all the edges are active in supporting external forces (there are passive edges).

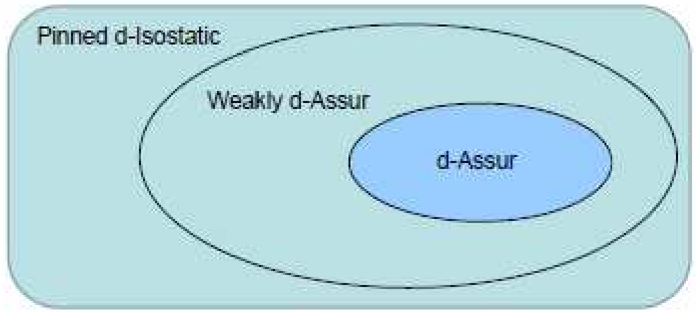
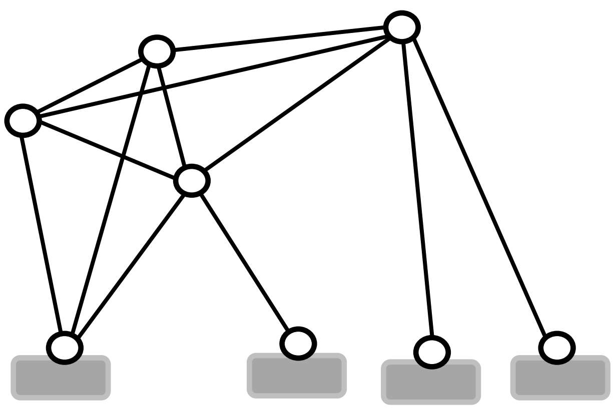
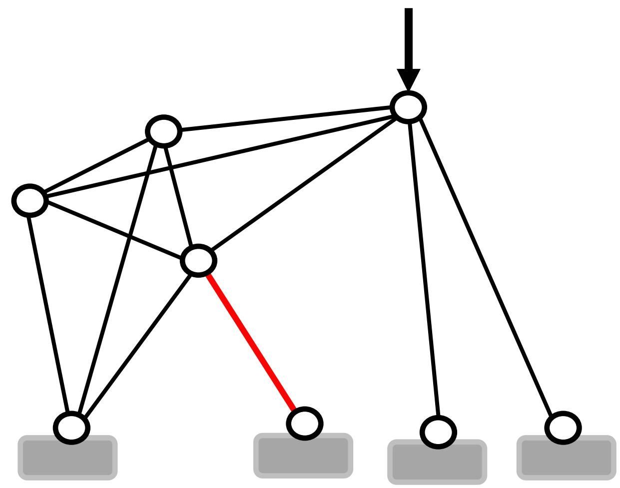


Figure 15: d-Assur graphs are a special case of Weakly d-Assur graphs.



Weakly 3-Assur graph

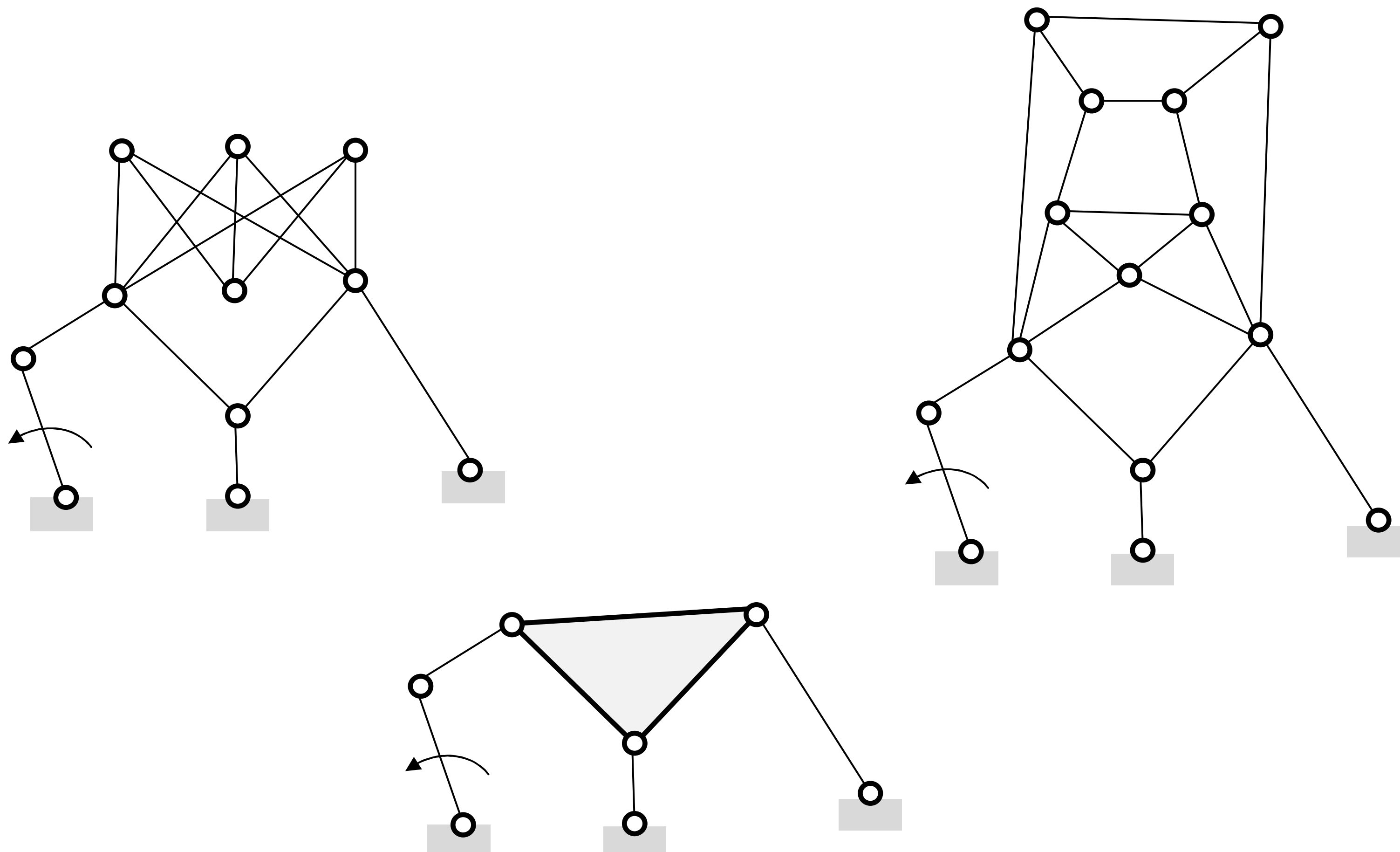


Passive edge = zero force edge

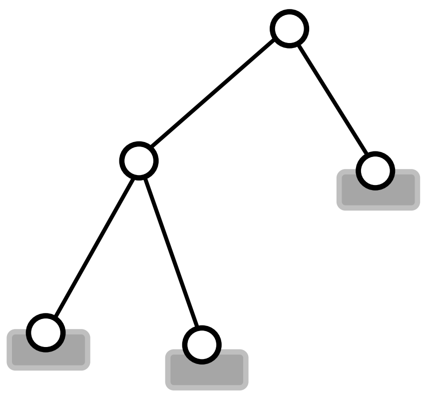
Weakly 3-Assur graph

Why body bars are important in engineering?

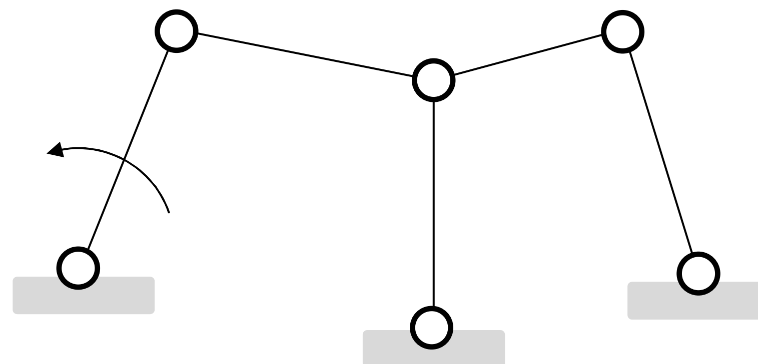
From the aspect of kinematics, all these three linkages are the same!



But, we need both: bar and joint and body-bar.



Structural mechanics



In linkages, sometimes we have to use bar and joints.

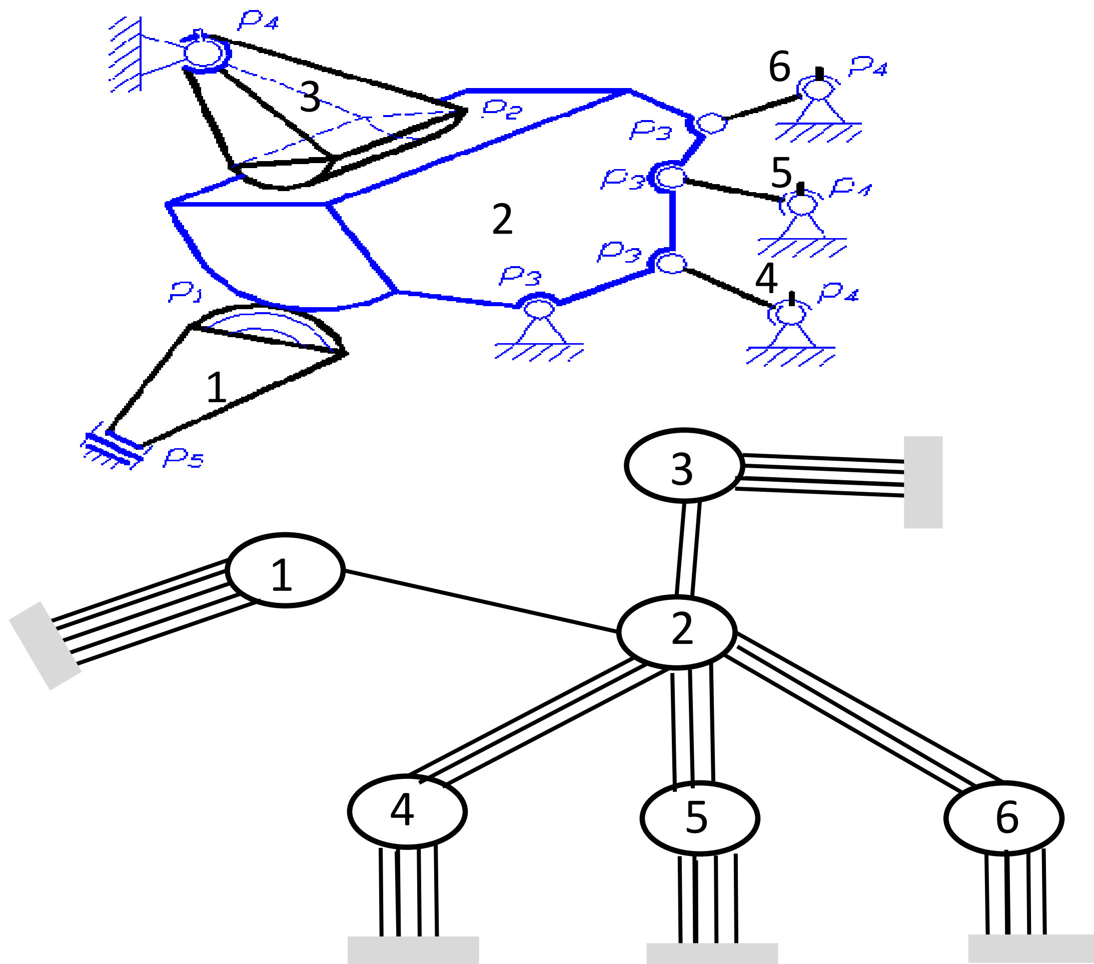
Body bar is a proper graph for 3d Mechanical systems

$$F = 6*N - 5*p_5 - 4*p_4 - 3*p_3 - 2*p_2 - 1*p_1$$

p_5	p_4	p_3	p_2	p_1
hinges	Restricted spherical	spherical	revolute	Gear pair

$n=6;\ p_5=1;\ p_4=4;\ p_3=4;\ p_2=1;\ p_1=1.$

$W=6\cdot 6-5\cdot 1-4\cdot 4-3\cdot 4-2\cdot 1-1=0$

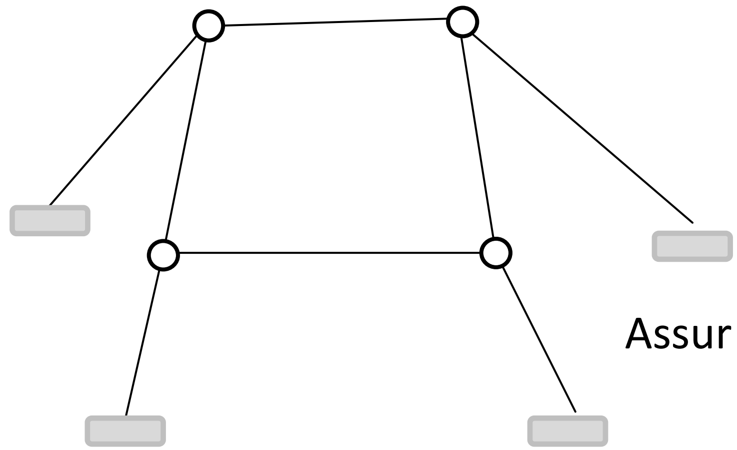


Pinned or floating Assur Graphs?

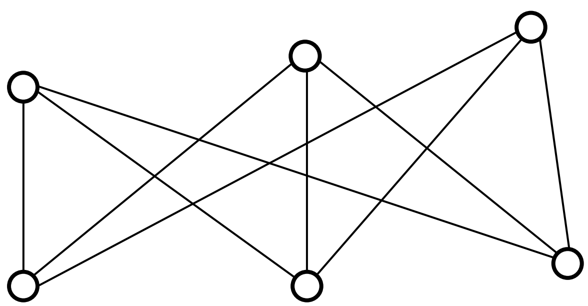
Isostatic Assur Graphs?

The boundaries of Assur Grpahs

Bar and joint Assur Graphs are pinned graphs.



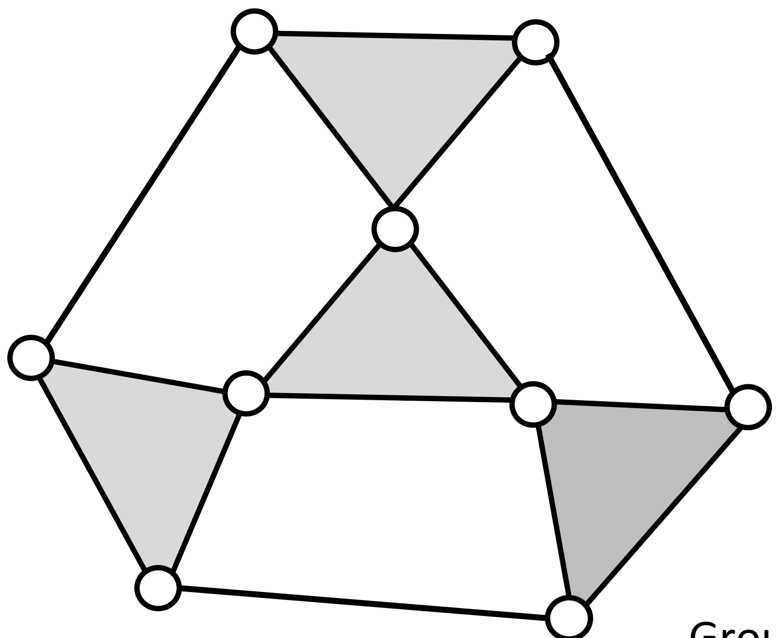
Assur Graph – Tetrad



This can be thought of as floating Assur Graph – grounding any edge results in a Tetrad.

What about body-bar Assur Graphs? Pinned or floating?

Body bar Assur Graphs are floating , unpinned graphs.

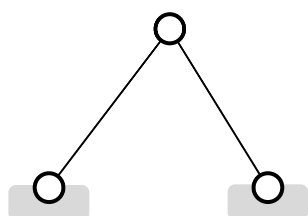


Body bar Assur Graph

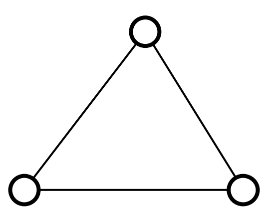
Grounding any body/bar results in a pinned body-bar Assur Graph.

Strange, the origin graph of ALL body bar Assur Graphs and bar-joint Assur Graphs is …

The origin graph of both types of Assur Graphs is the same/similar .



The origin graph of bar-joint Assur Graph:
the 2d dyad.



The origin graph of body-bar Assur Graph:
the triangle, floating 2d dyad.

Geometric constraint problem

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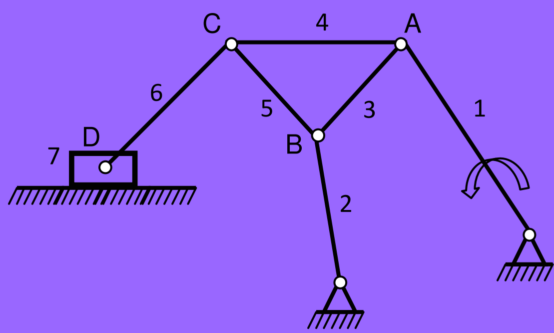
Analysis of Linkages

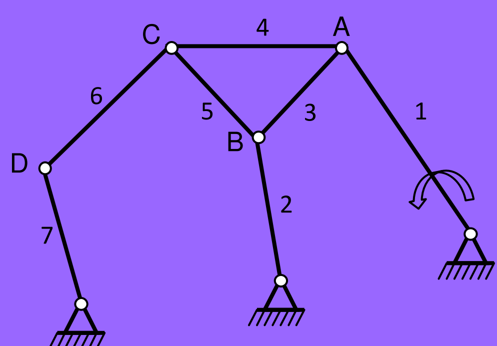
The idea is to use the same software for both.

Joint work with Prof. Y. Reich, Tel-Aviv University.
Introduced to *DASSAULT* SYSTEMES, January 2011, Paris.

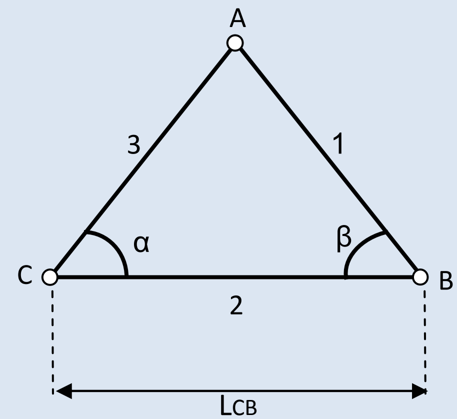
Decomposition – separate the system (mechanism, geometric constraint) into minimal inseparable components (Assur Graphs- AGs).



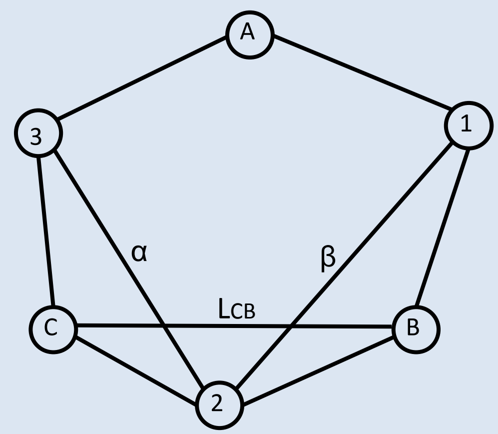
The mechanism



The structural scheme



The geometric constrains

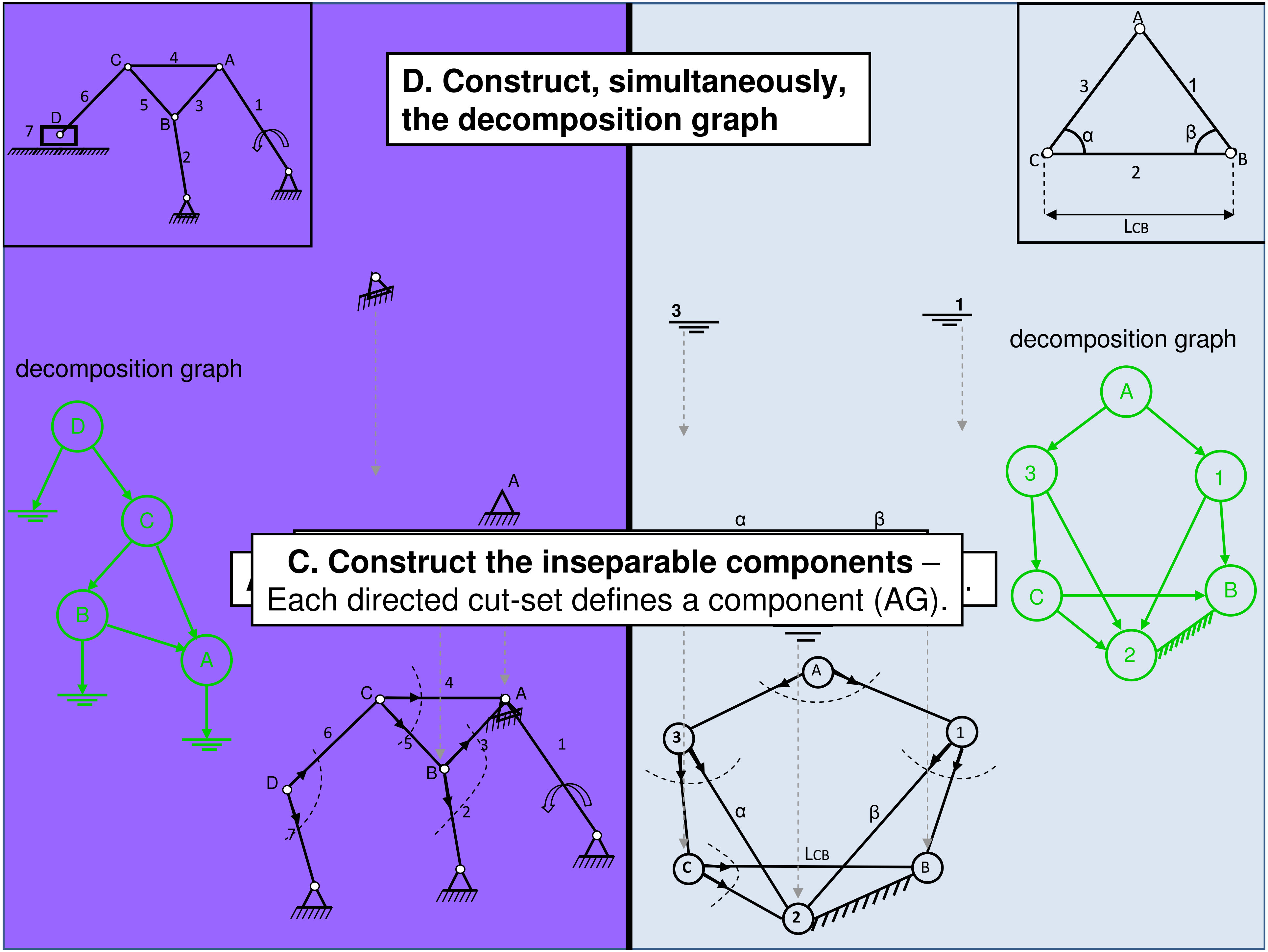


The geometric constrains graph

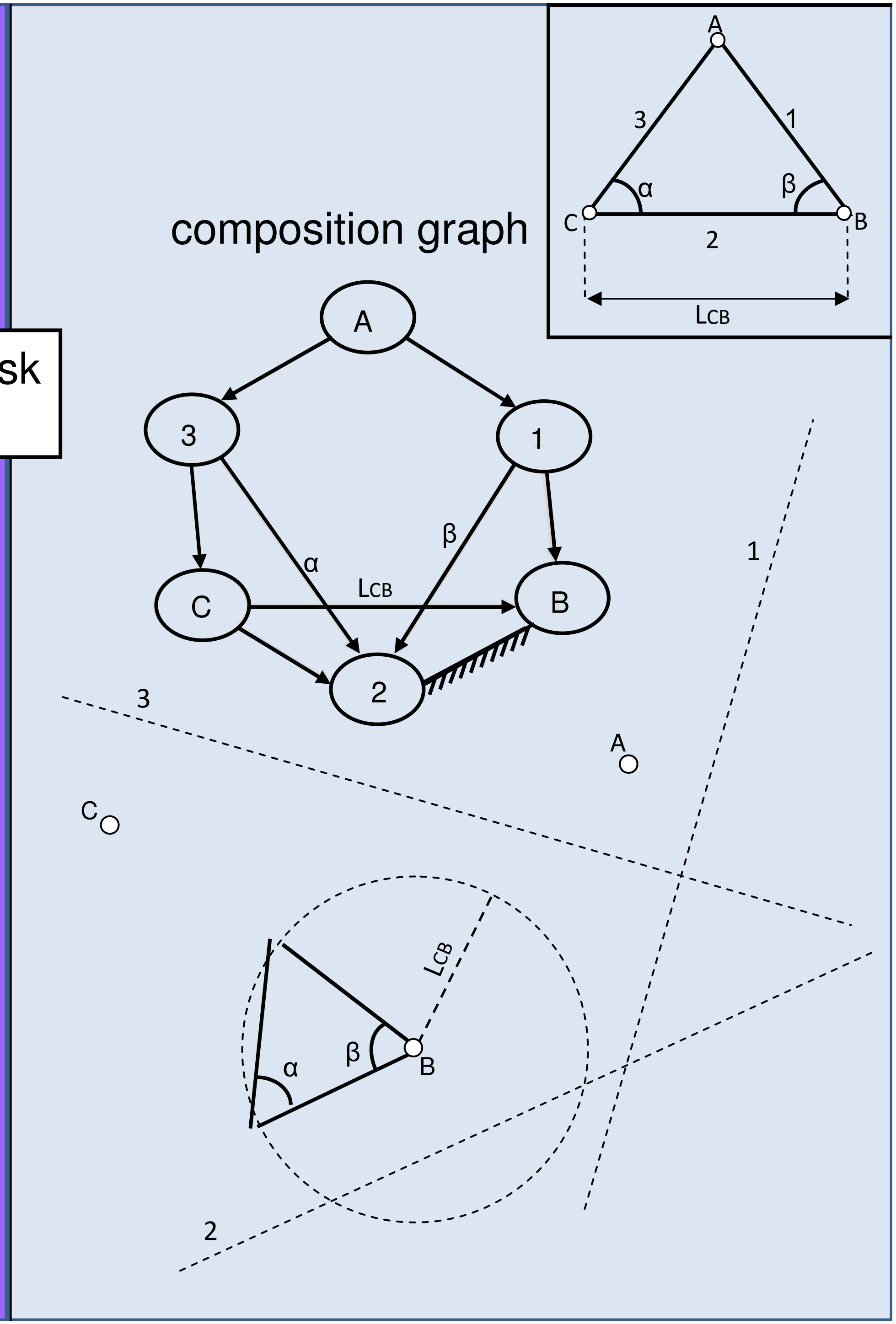
D. Construct, simultaneously, the decomposition graph

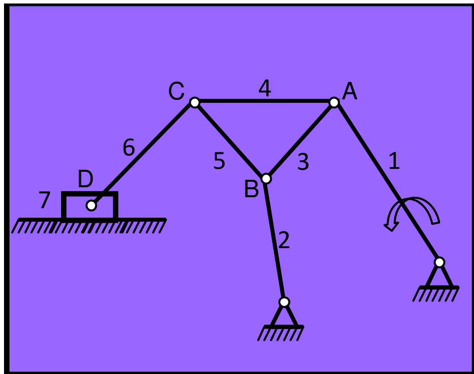
C. Construct the inseparable components –

Each directed cut-set defines a component (AG).

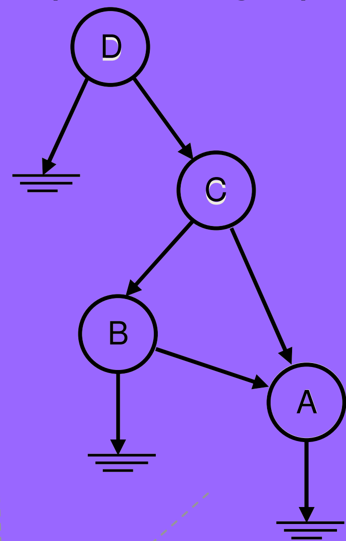


C. Continue till you have completed the task
Constructing the geometric object.



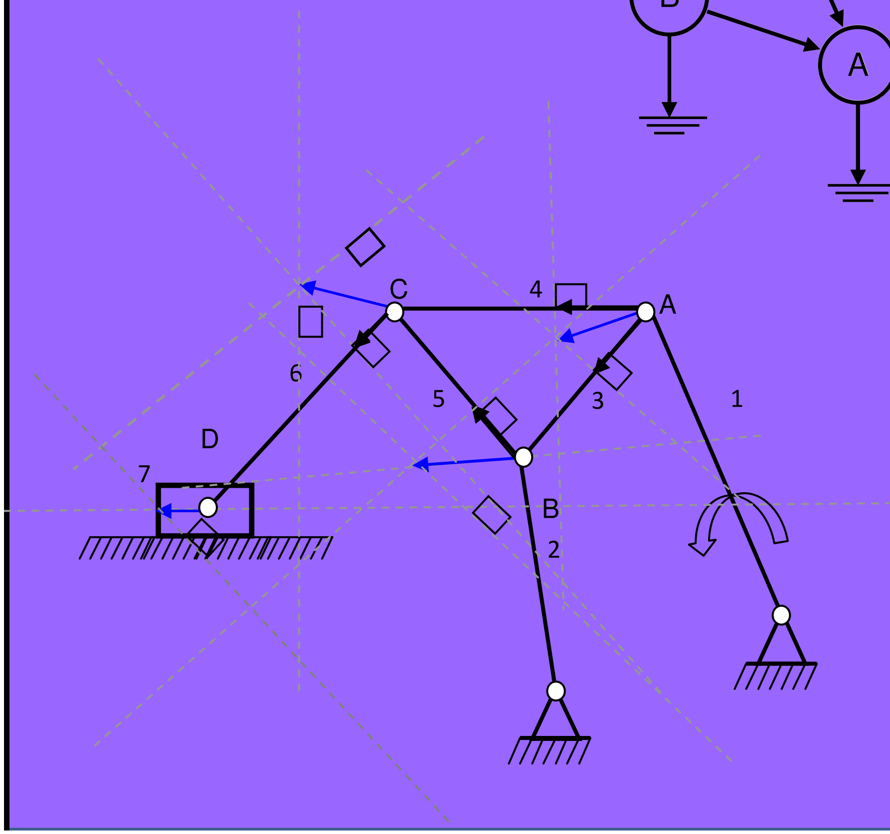


composition graph

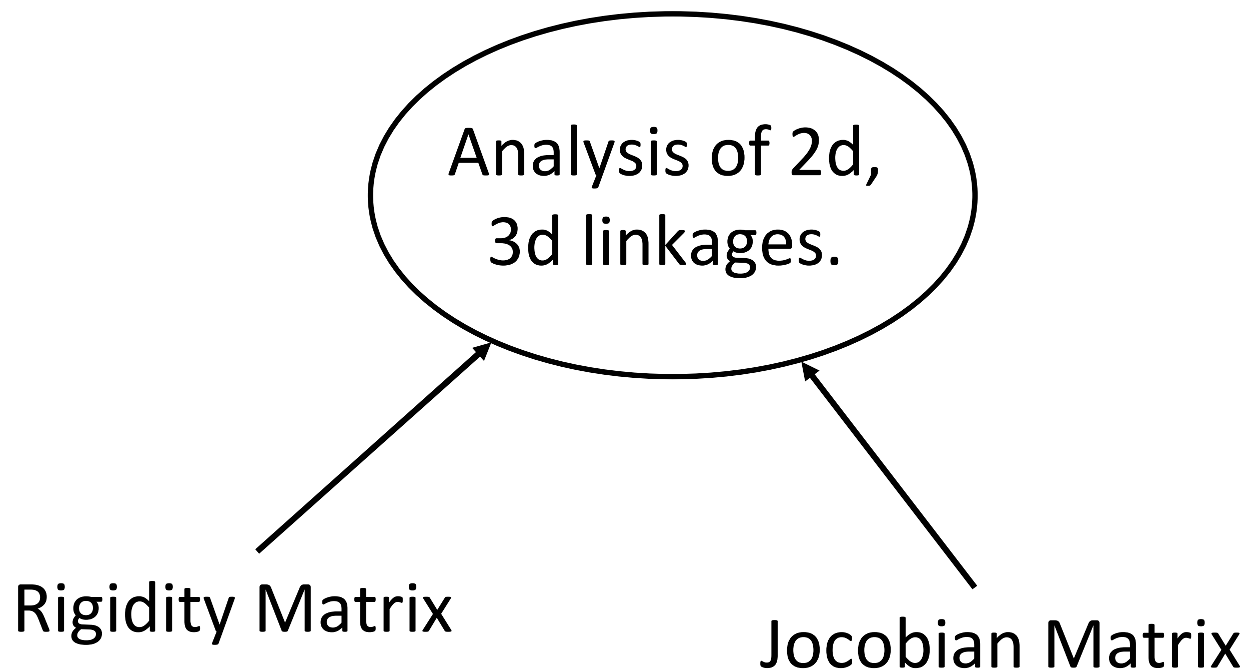


C. Continue till you have completed the task. Analyzing the mechanism.

graph) and analyze/solve them.



Relation between Rigidity matrix and Jocobian matrix (with Muller, Germany)



- The idea is also to “ **fertilize**” between the two methods.
- We want to establish **common terminology** between rigidity community and mechanical engineering.

-The responses for the presentation of the paper were very good.

Sljoka A., Shai O and Whiteley W., “Checking mobility and decomposition of linkages via Pebble Game algorithm”, **ASME Design Engineering Technical conferences**, August 28-31, 2011, Washington, USA.

- Preliminary results indicate on problems with rigidity matrix to analysis linkages with “**floating sliders**”. We succeeded to overcome the problem.

The **singularity** of Assur Graphs

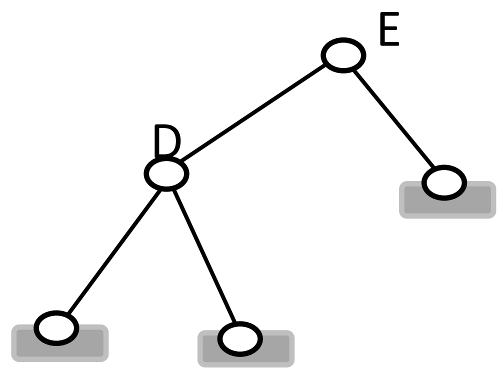
and

its applications

Servatius B., Shai O. and Whiteley W., "Geometric Properties of Assur Graphs", European Journal of Combinatorics, Vol. 31, No. 4, May, pp. 1105-1120, 2010.

Assur Graph Singulariy Theorem (Servatius et al., 2010):

G is an Assur Graph IFF it has a configuration in which there is a **unique self-stress** on **all** the edges and **all** the inner vertices have **1dof** an infinitesimal motion .



This topology does not have such configuration.

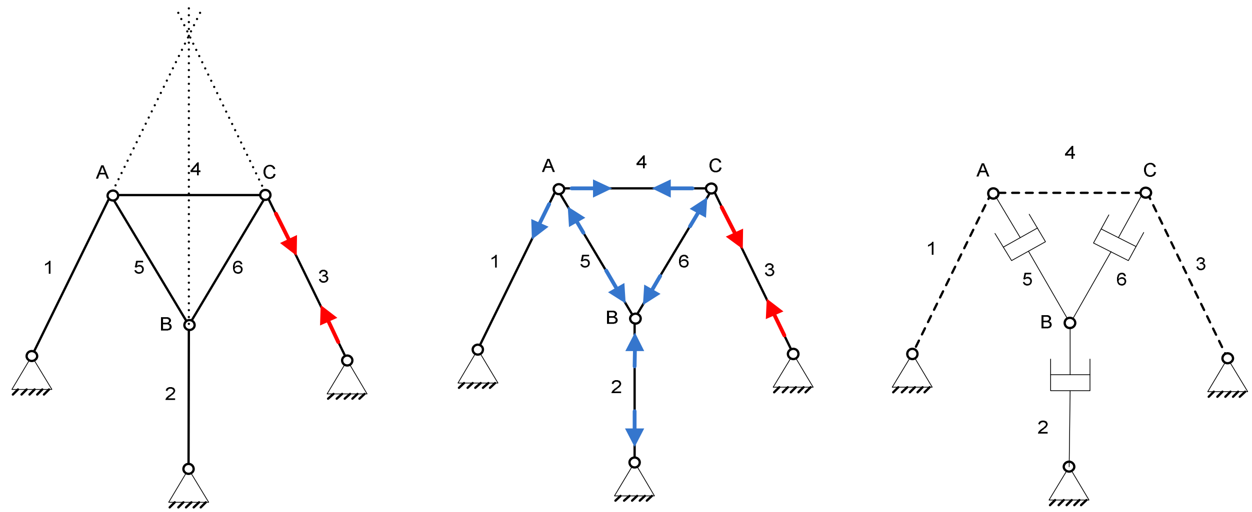
Deployable/foldable Tensegrity Assur Graph

-The robot can sustain external forces/loads only when it is at the **singular** position.

- The control is very easy (control only one element).

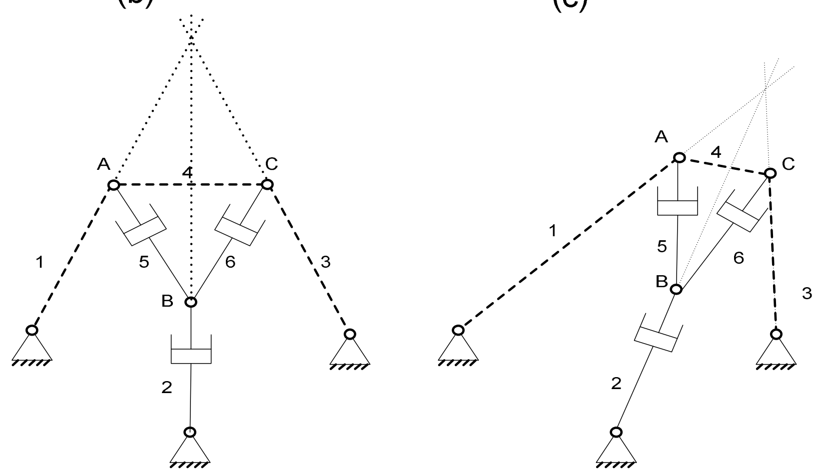
-Usually, the tensegrity structures are indeterminate (over braced) and the control is the main problem.

Combining the Assur triad with a tensegrity structure



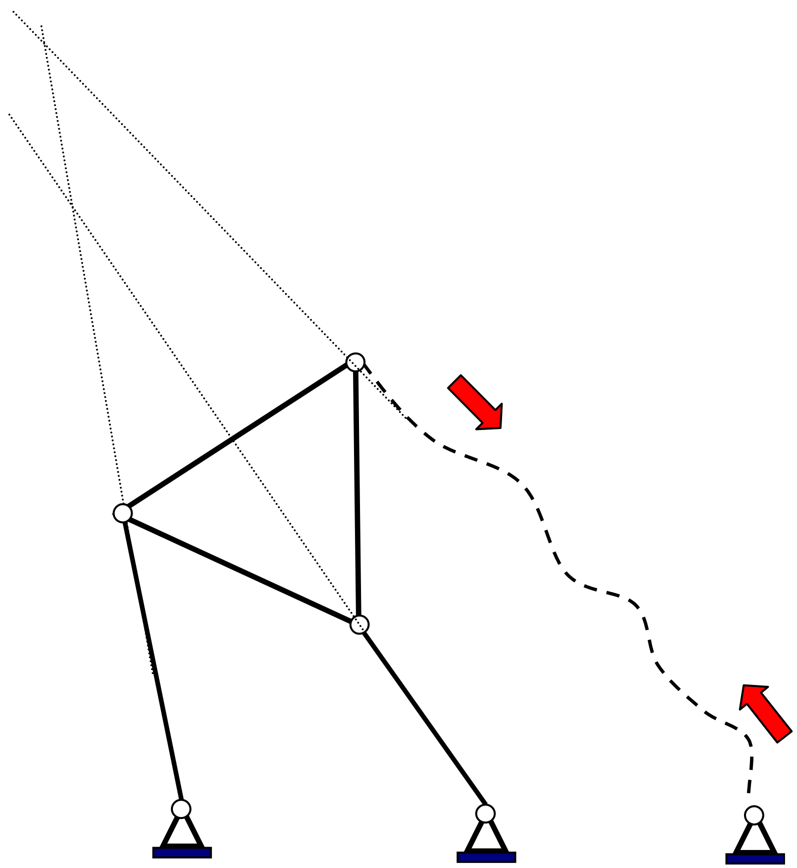
(a) (b) (c)

Changing the singular point in the triad

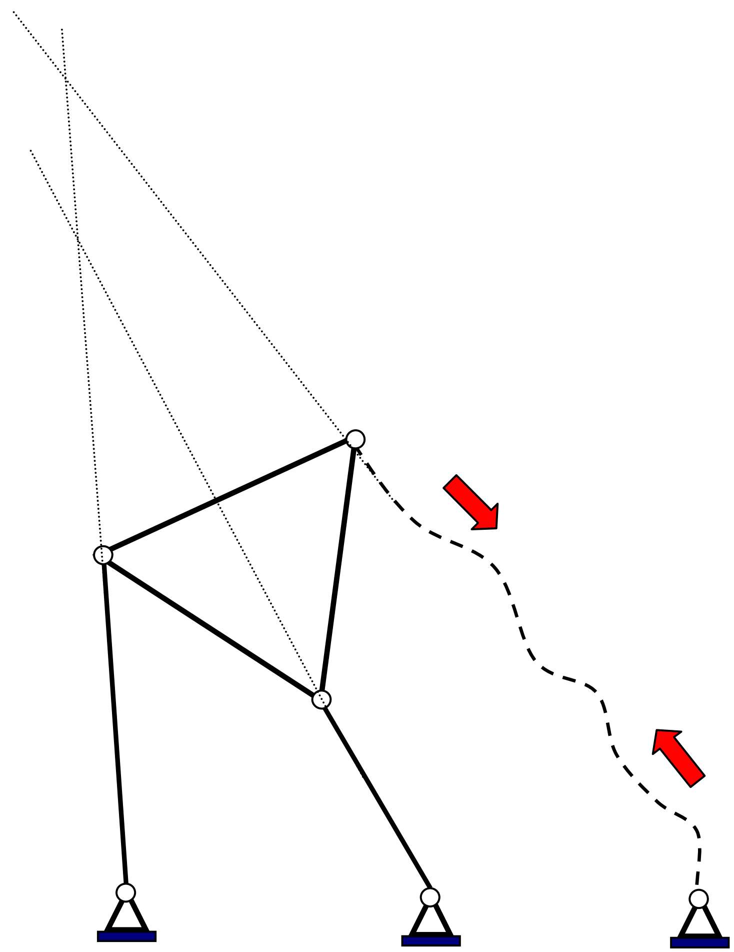


(a) (b)

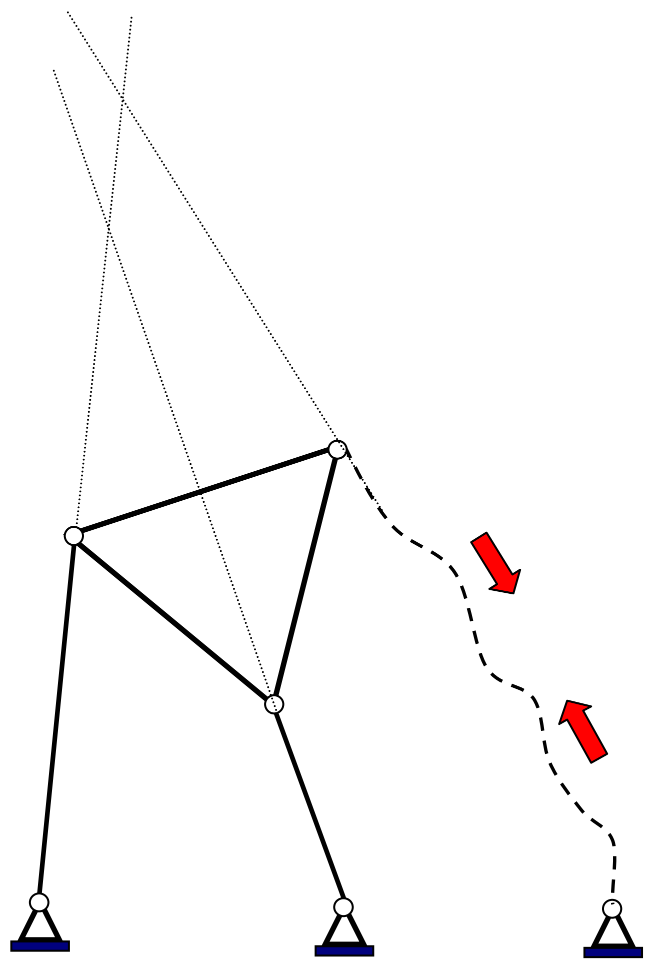
Transforming a soft (loose) structure into Rigid Structure



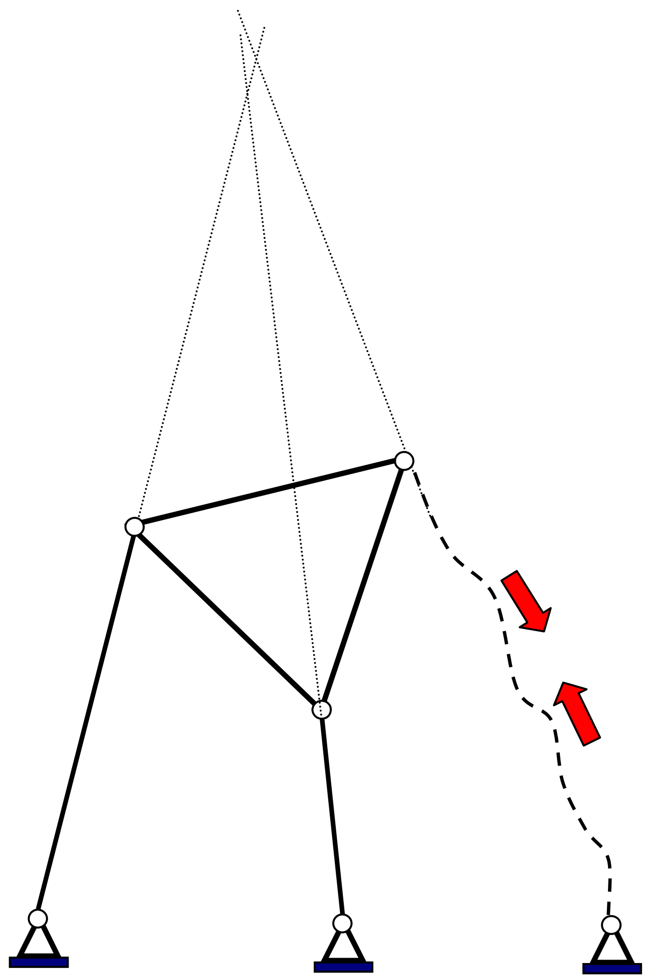
Shortening the length of one of the cables



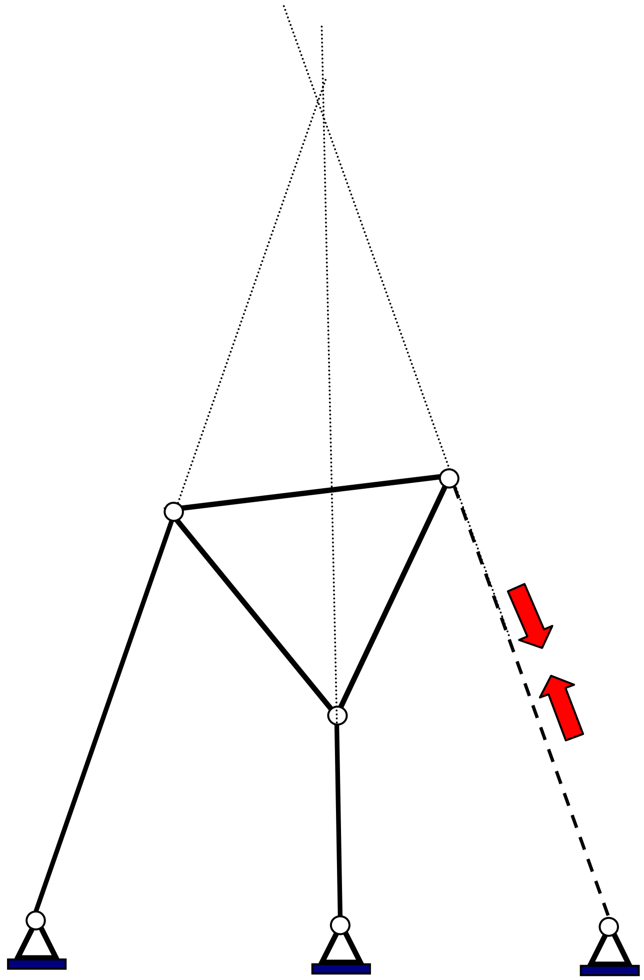
Shortening the length of one of the cables



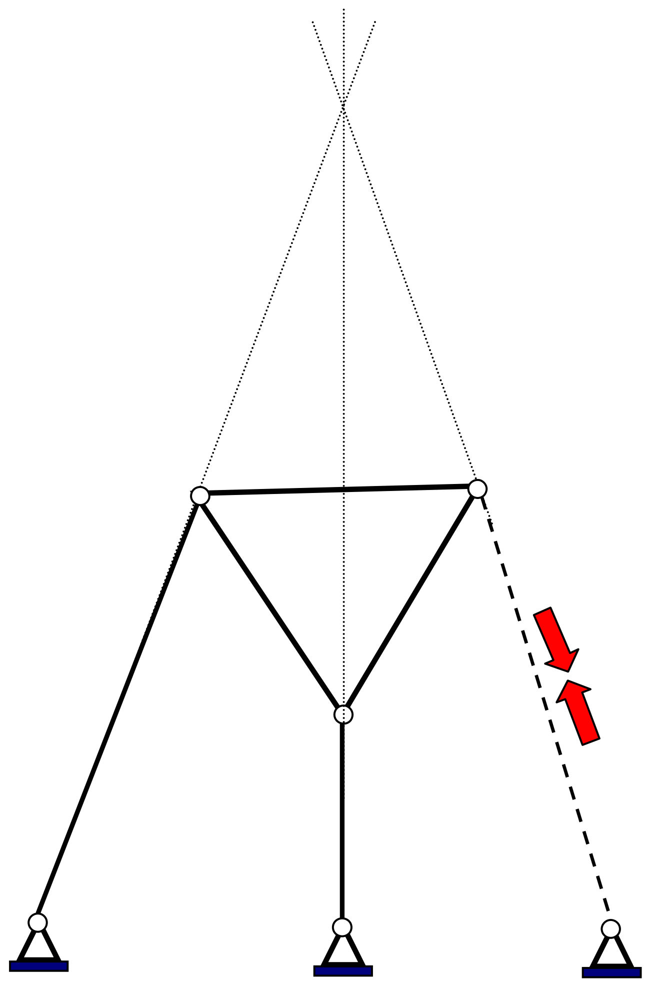
Almost Rigid Structure



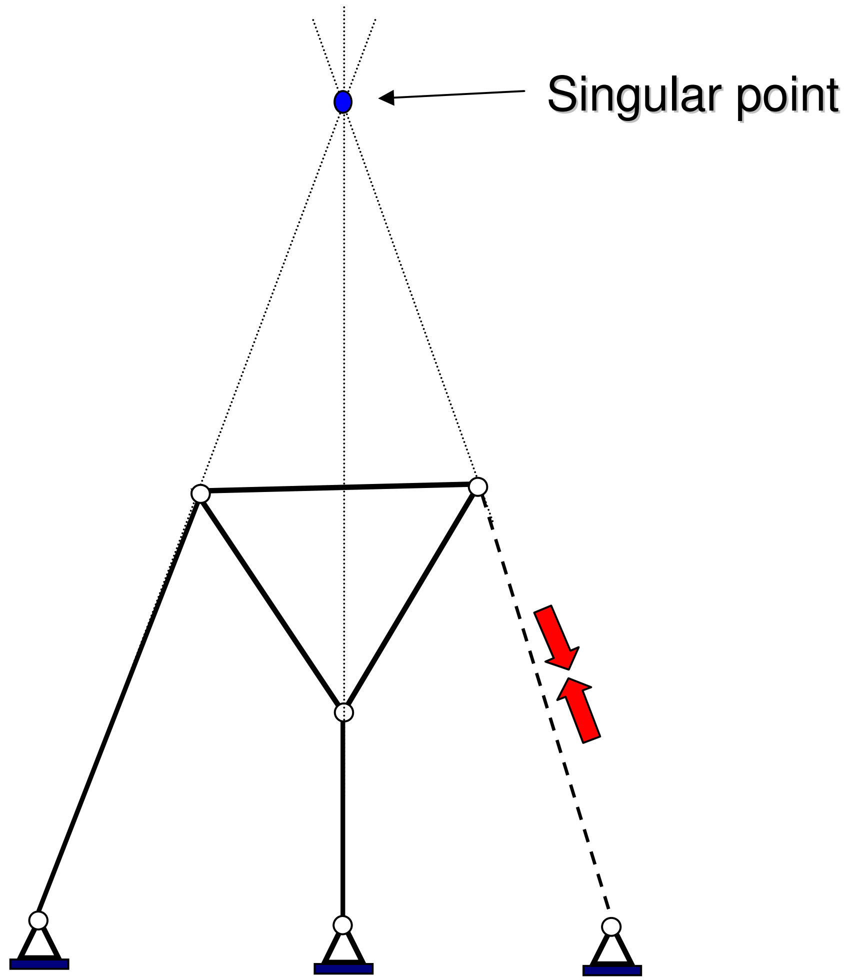
Almost Rigid Structure



At the Singular Position



The structure is Rigid



The Deploayble/foldable Tensegrity Assur Graph robot that was built in our lab.

Fig. 4a: The two floor prototype robot (left) and a model of the three floor prototype (right)

From Soft to Rigid Structure

Theorem: it is enough to change the location of only ***one*** element so that the Assur Graph is at the **singular position**.

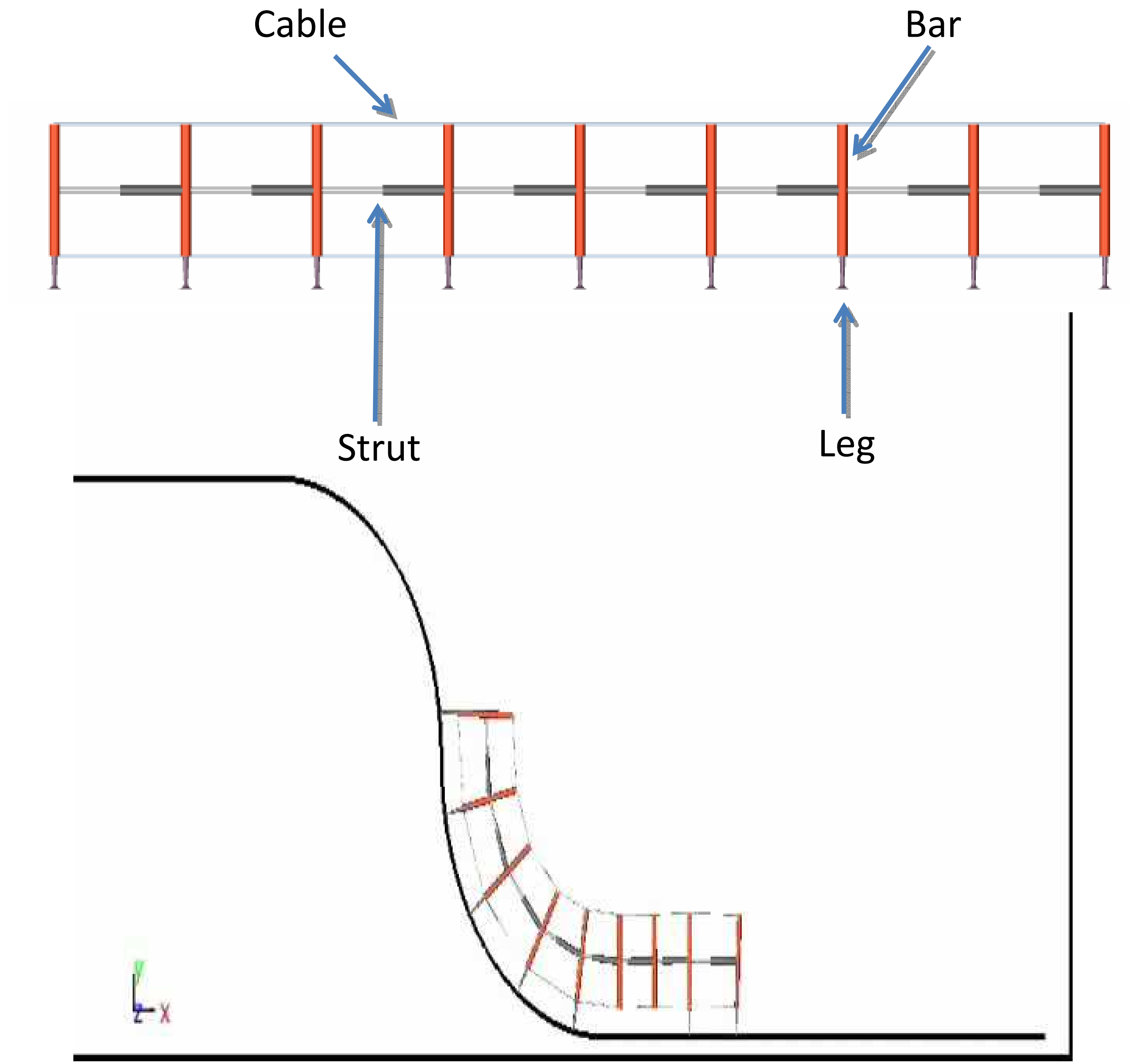
In case the structure is loose (soft) it is enough to **shorten** the length of only **one** cable so that the Assur Graph is being at the singular position.

Simulation of the caterpillar and other soft/rigid animals through Assur Tesnegrity Graphs.

Joint work with the zoologist Prof. Ayali and M.Sc student - Omer

Towards soft/rigid robots.

It is very important that the animal robot is “fully sensitive”.


Cable
Bar
Strut
Leg
y
x
z

From singular Assur Graphs into floating mechanisms.

This is done in three steps:

1. **Characterize the singularity of the given Assur Graph** – apply a combinatorial method that use face force (projection of polyhedron).
2. **Transform the infinitesimal motion into finite motion** using sliders and other methods (not systematic, yet).
3. **Replication** – resulting with floating overbraced framework.

Characterization of the singularity of the Tetrad

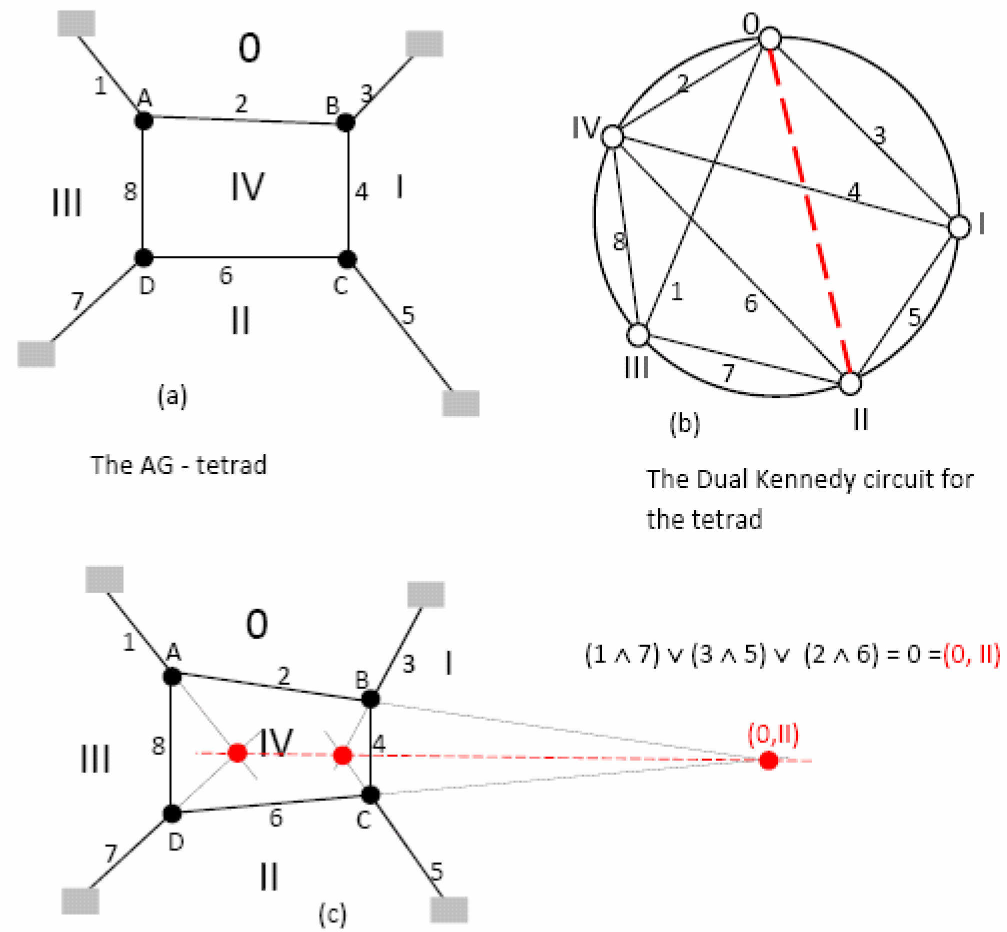
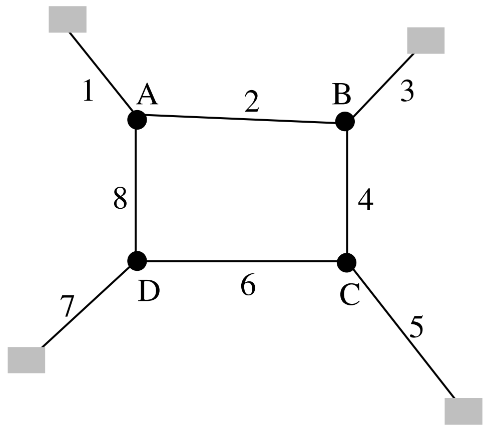
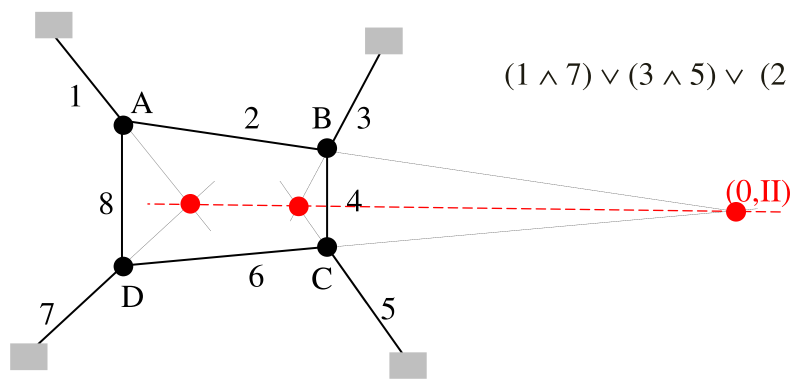


Figure 4. The characterization of the singular position of the tetrad – the three red points should be collinear.

The singular position of the Tetrad



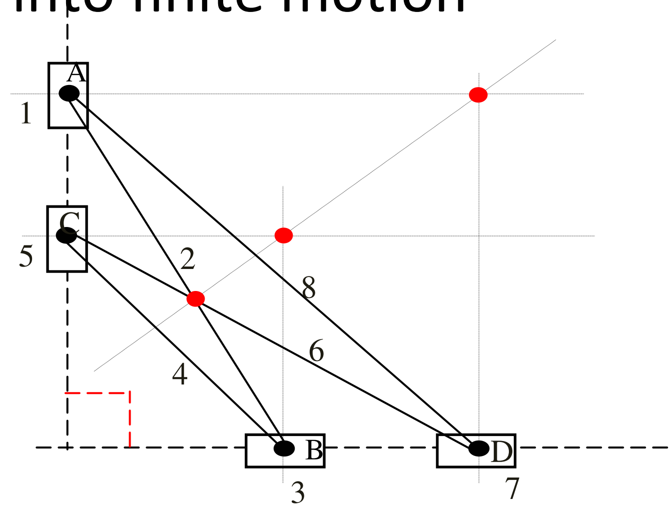


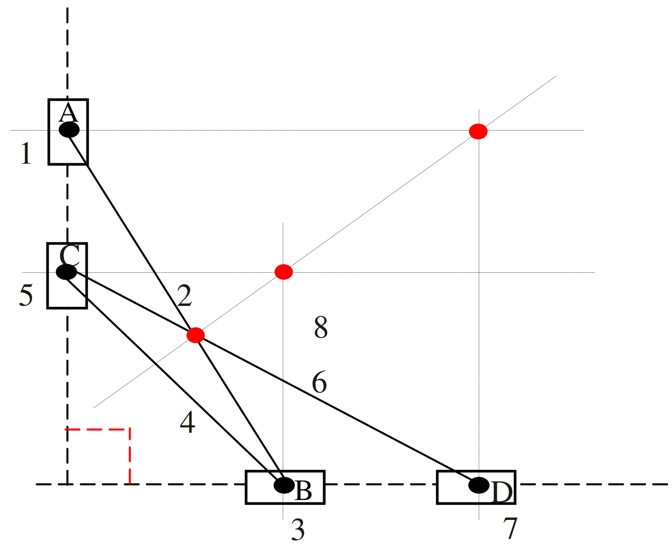
$(1 \wedge 7) \vee (3 \wedge 5) \vee (2 \wedge 6) = 0$

$= (0, \mathrm{II})$

Characterization done by
equimomental lines and face force

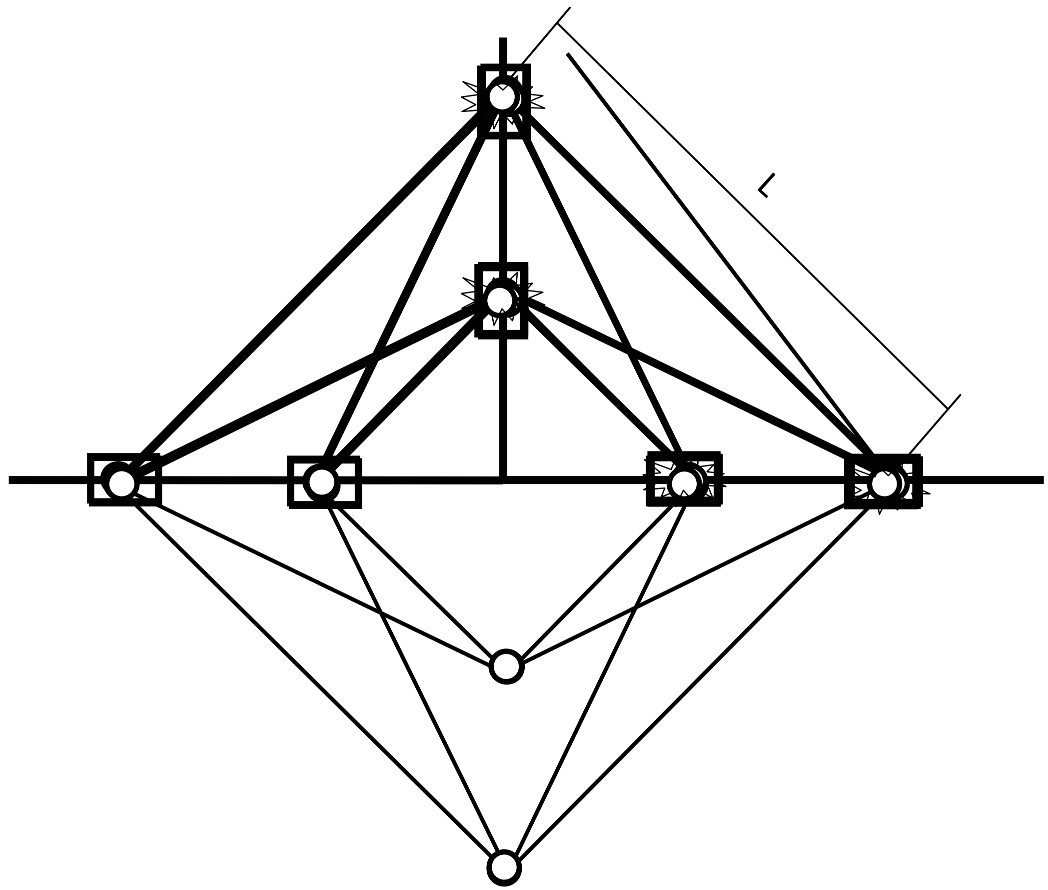
2. Transforming the infinitesimal motion of the Tetrad into finite motion



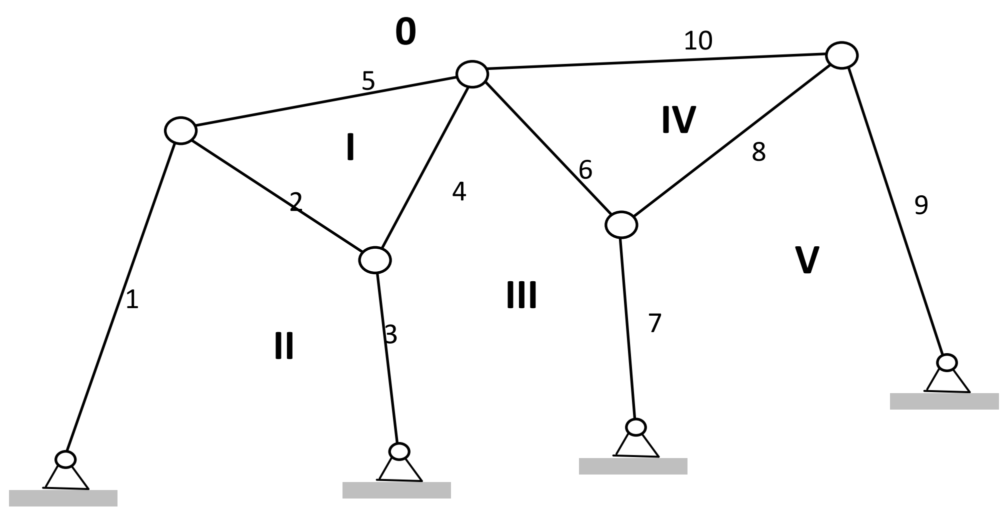


$$Y_A^2 + X_D^2 = \left(L_2^2 - X_B^2\right) + \left(L_6^2 - \left(L_4^2 - X_B^2\right)\right) = const$$

The replication step.



Last example – The double Triad Assur Graph



1. Singularity characterization of the Double Triad.

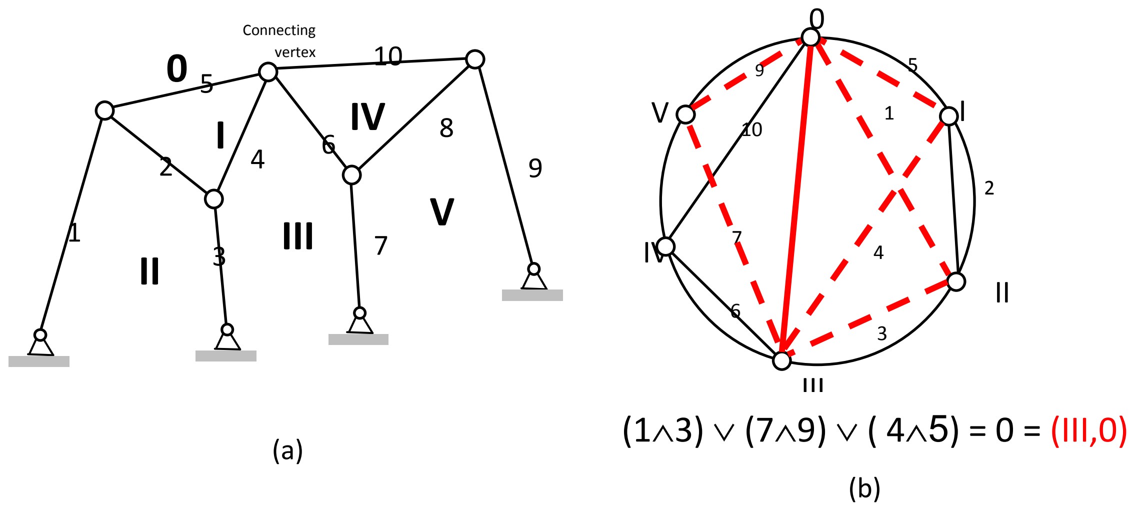


Figure 7. Characterization of the singular configuration of double triad through dual Kennedy circuit in statics.

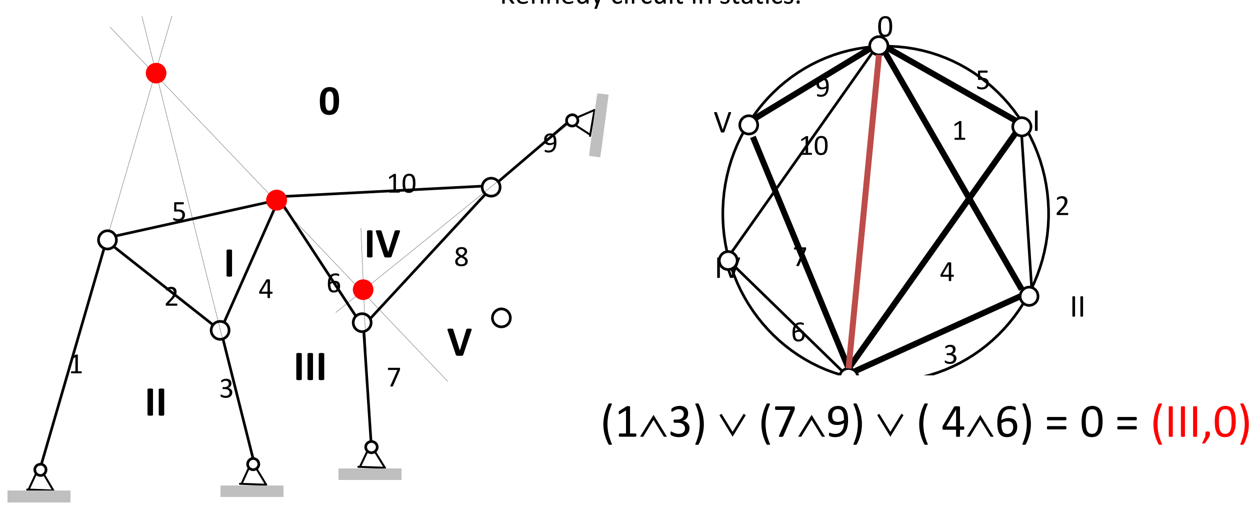
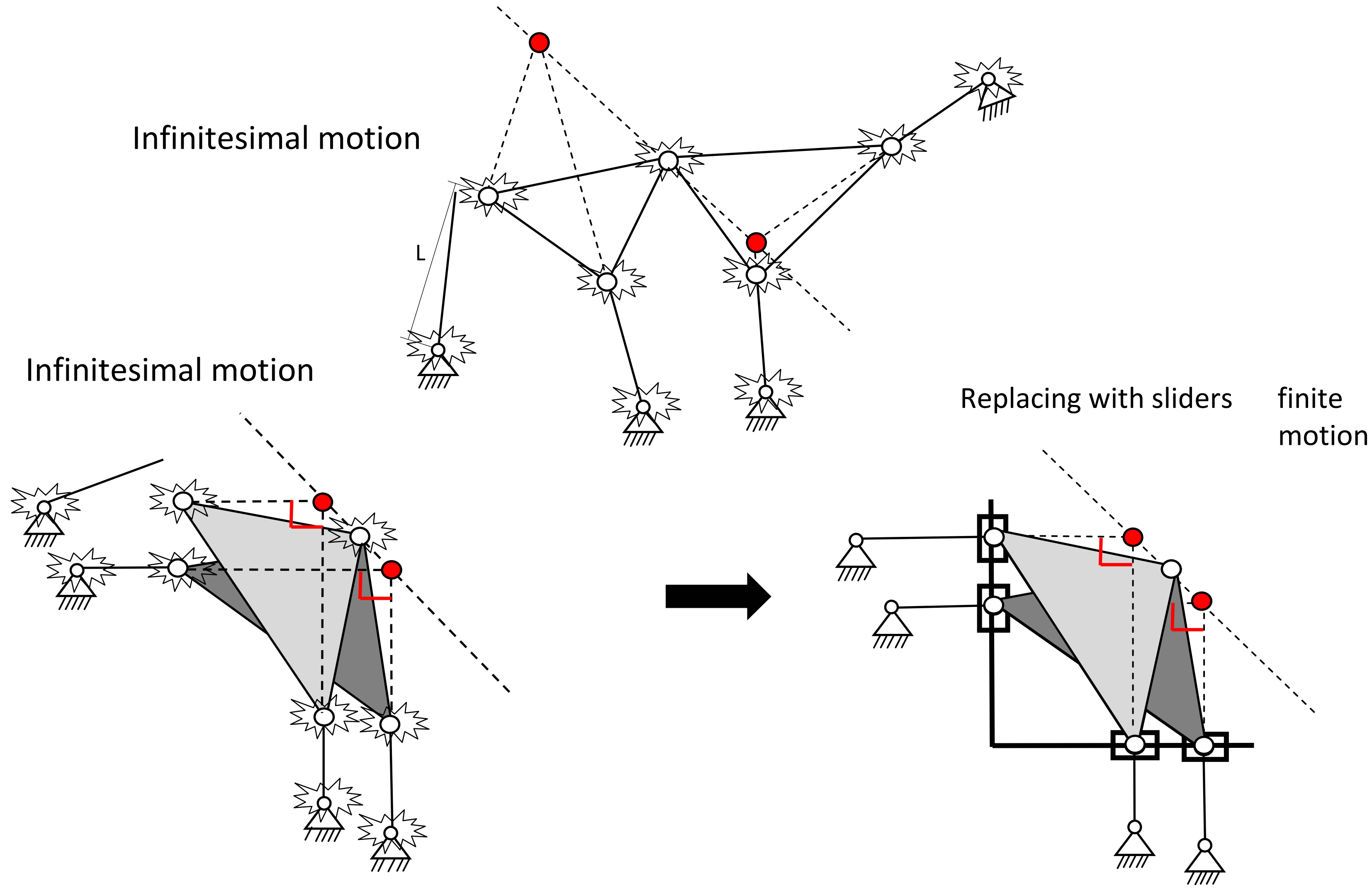
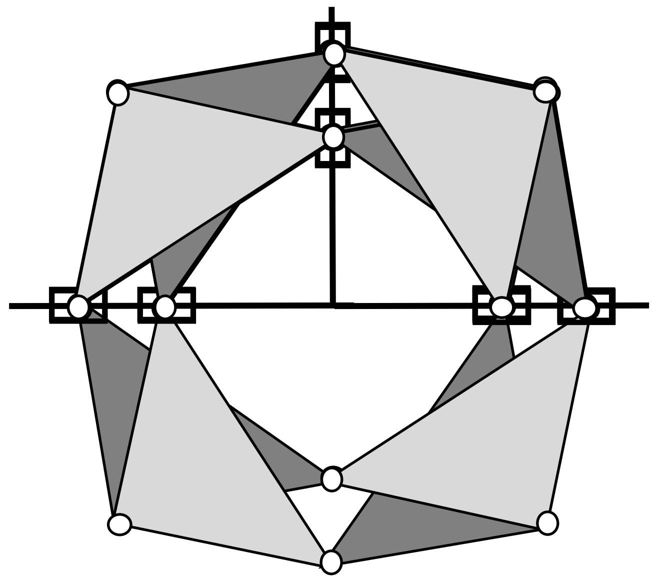


Figure 8. The double triad at the singular configuration having an infinitesimal motion.

2. Transforming the infinitesimal motion of the Double-triad into a finite motion



3. The replication step



Thank you !