

Geodesic Voting for biomedical image segmentation

Laurent D. COHEN

Directeur de Recherche CNRS
CEREMADE, UMR CNRS 7534 Université Paris-9 Dauphine
Place du Maréchal de Lattre de Tassigny 75016 Paris, France

Cohen@ceremade.dauphine.fr

<http://www.ceremade.dauphine.fr/~cohen>

Some joint works with Y. Rouchdy, and J. Mille.

Toronto, May 2012



dépasser les frontières

CEREMADE

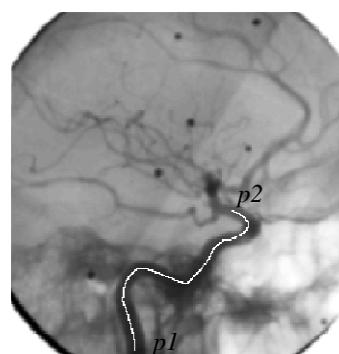
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Paths of minimal energy



Looking for a path along which a feature Potential $P(x,y)$ is minimal

$$E(C) = \int_0^L P(C(s))ds$$

example: a vessel
dark structure
 P = gray level

Input : Start point $p1=(x1,y1)$

End point $p2=(x2,y2)$

Image

Output: Minimal Path

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Overview

- Minimal Paths, Fast Marching and Front Propagation
- Geodesic Density for tree structures
 - Using Geodesic voting with Radius energy
 - Using Geodesic voting as a prior shape
 - Using Geodesic voting as initial deformable tree

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Minimal Paths: Eikonal Equation

$$E(C) = \int_0^L P(C(s))ds$$

Potential $P > 0$ takes lower values near interesting features :
on contours, dark structures, ...

STEP 1 : search for the surface of minimal action U of $p1$ as the minimal energy integrated along a path between start point $p1$ and any point p in the image

Start point $C(0)=p1$

$$U_{p1}(p) = \inf_{C(0)=p1; C(L)=p} E(C) = \inf_{C(0)=p1; C(L)=p} \int_0^L P(C(s))ds$$

STEP 2: Back-propagation from the end point $p2$ to the start point $p1$:

Simple Gradient Descent along U_{p1}

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Minimal Paths: Eikonal Equation

STEP 1 : minimal action U of $p1$ as the minimal energy integrated along a path between start point $p1$ and any point p in the image

Start point $C(0)=p1$;

$$U_{p1}(p) = \inf_{C(0)=p1; C(L)=p} E(C) = \inf_{C(0)=p1; C(L)=p} \int_0^L P(C(s)) ds$$

$$\|\nabla U_{p1}(x)\| = P(x) \text{ and } U_{p1}(p1) = 0$$

Example $P=1$, U Euclidean distance to $p1$
in general, U weighted geodesic distance to $p1$

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Minimal Paths: Eikonal Equation

$$E(C) = \int_0^L P(C(s)) ds$$

STEP 2: Back-propagation from the end point $p2$ to the start point $p1$:

Simple Gradient Descent along U_{p1}

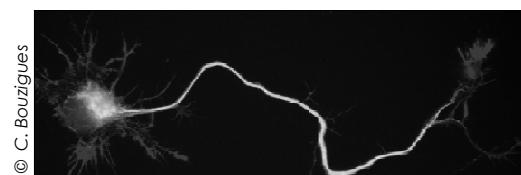
$$\frac{dC}{ds}(s) = -\nabla U_{p1}(C(s)) \text{ with } C(0) = p2.$$

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Minimal Path between $p1$ and $p2$

image I

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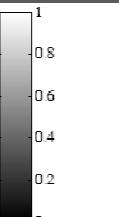
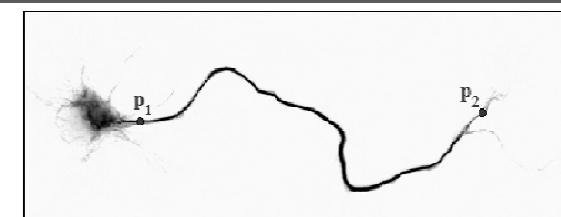
potential $\mathcal{P} : \Omega \rightarrow \mathbb{R}^{+*}$
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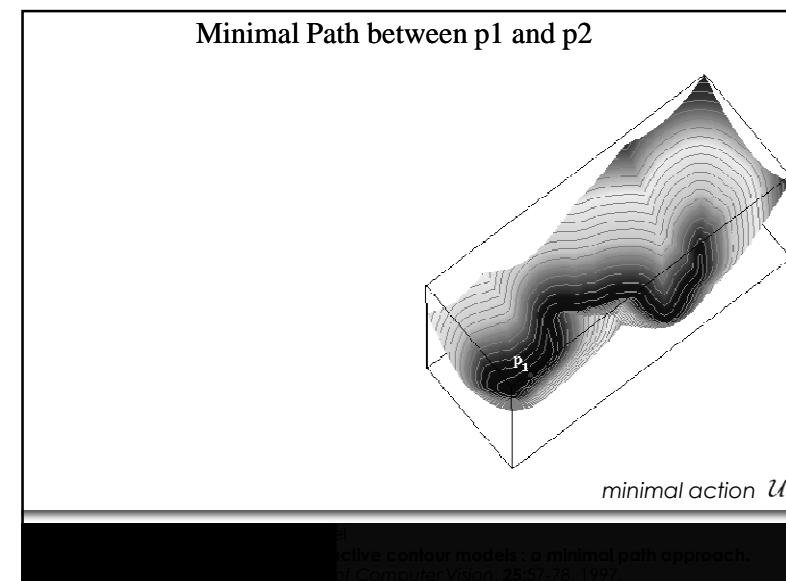
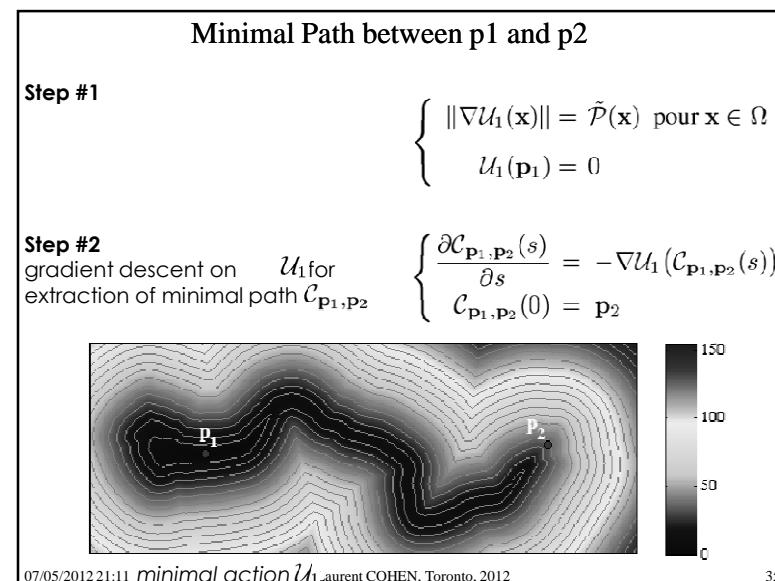
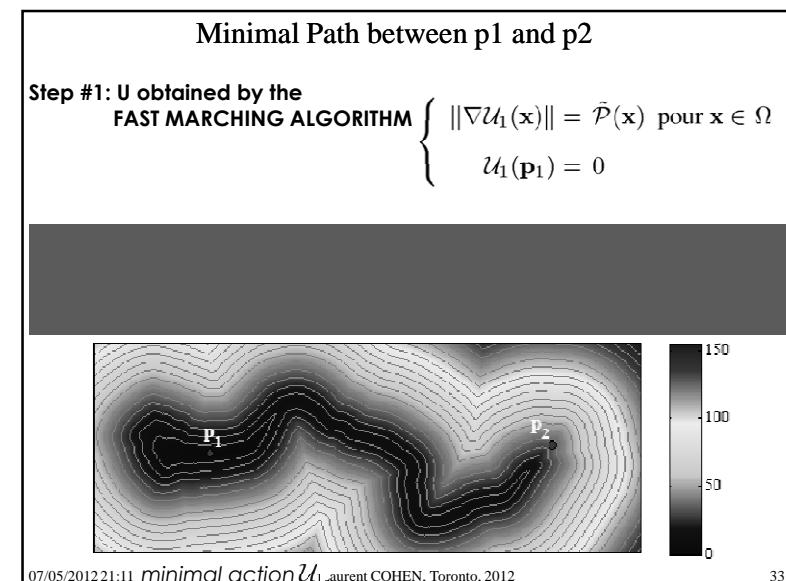
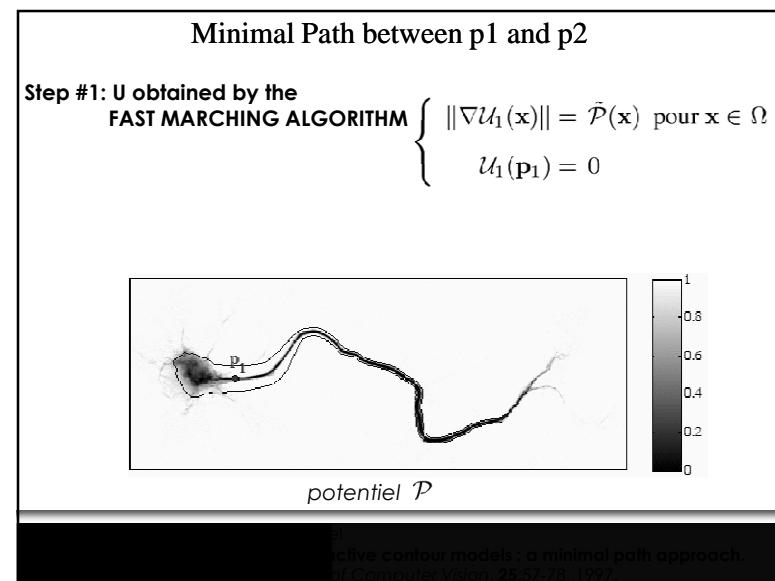
Minimal Path between $p1$ and $p2$

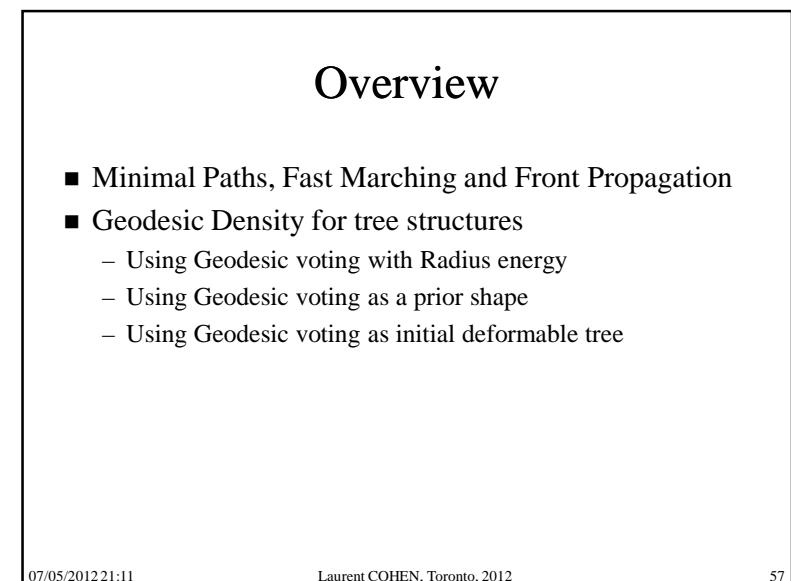
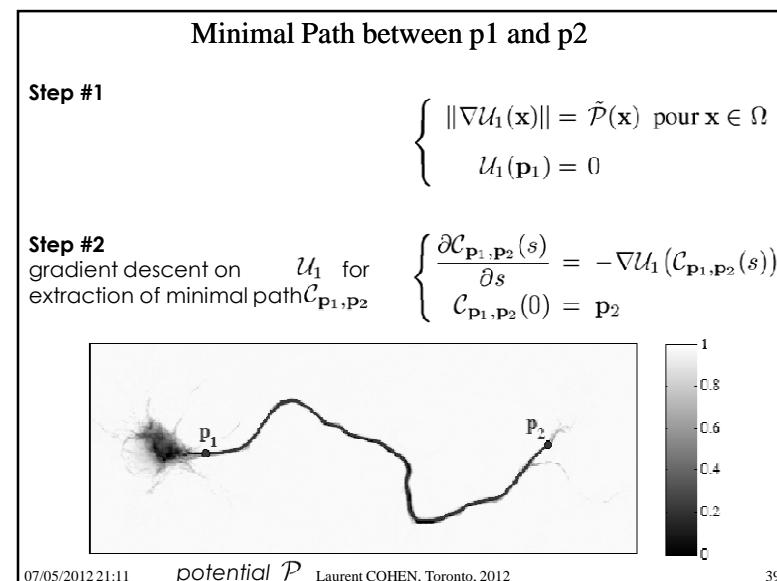
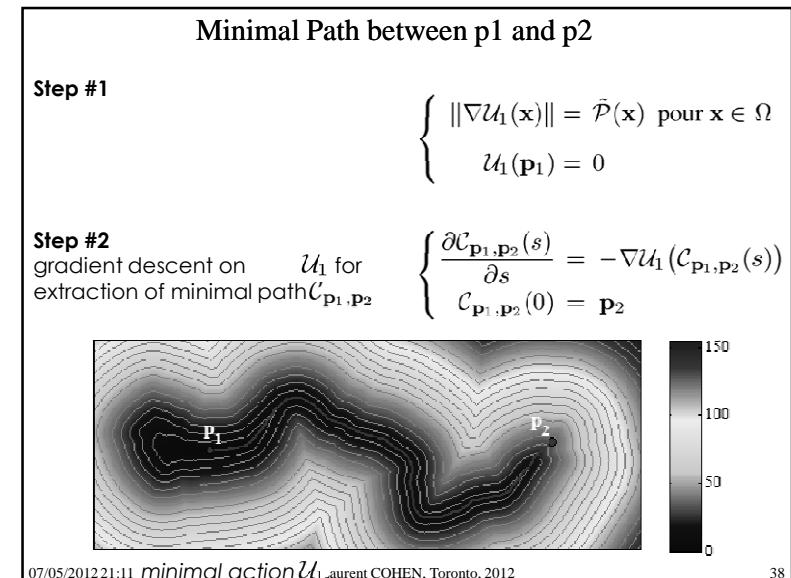
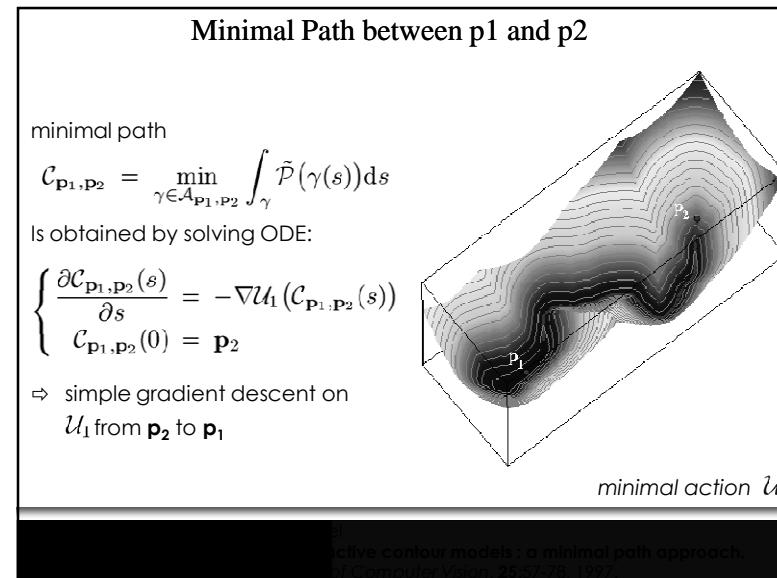
Step #1

$$\begin{cases} \|\nabla \mathcal{U}_1(\mathbf{x})\| = \tilde{\mathcal{P}}(\mathbf{x}) \text{ pour } \mathbf{x} \in \Omega \\ \mathcal{U}_1(p_1) = 0 \end{cases}$$

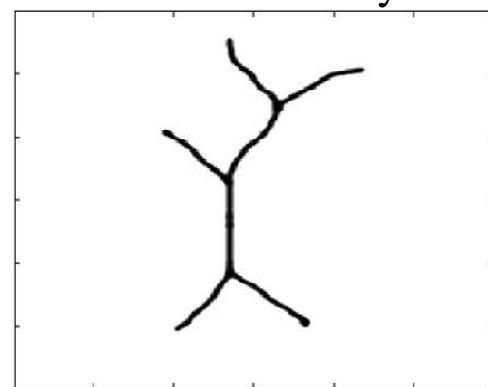
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Geodesic Density



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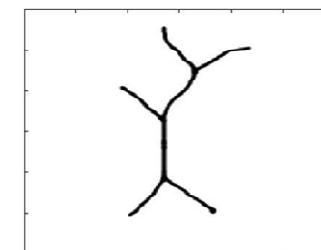
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Geodesic Density

One could give a root point and the set of endpoints for each branch.

Goal: user gives only one root point.

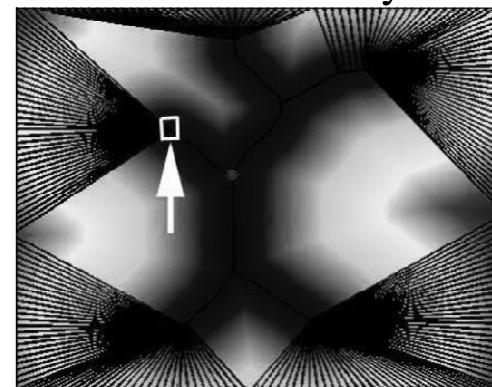


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Geodesic Density

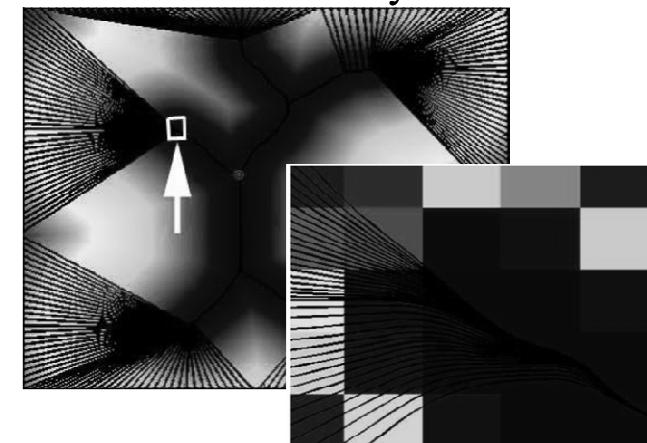


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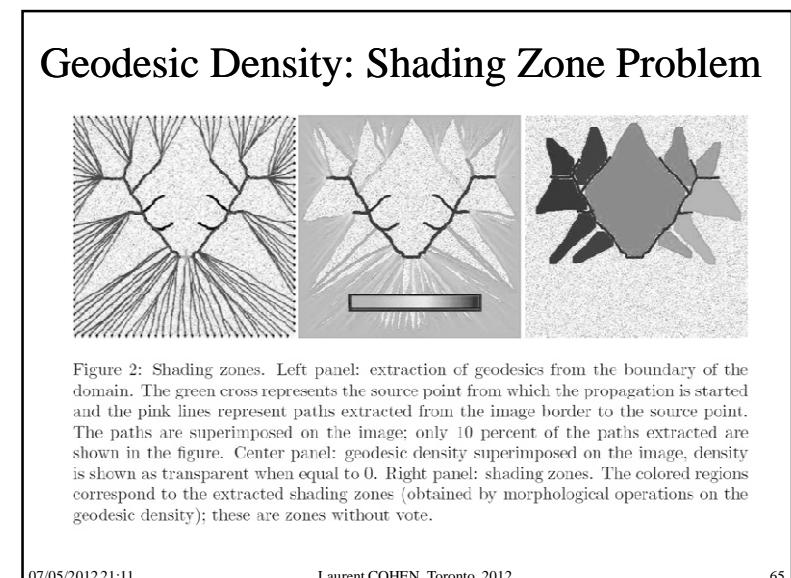
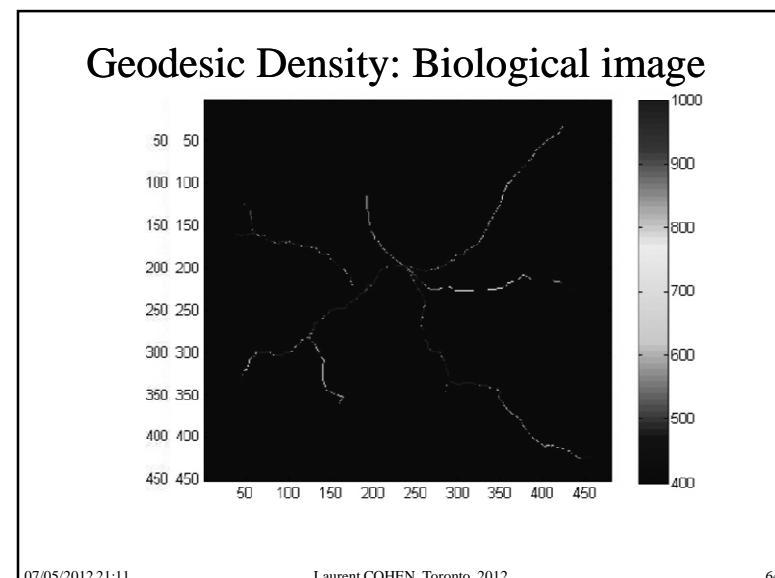
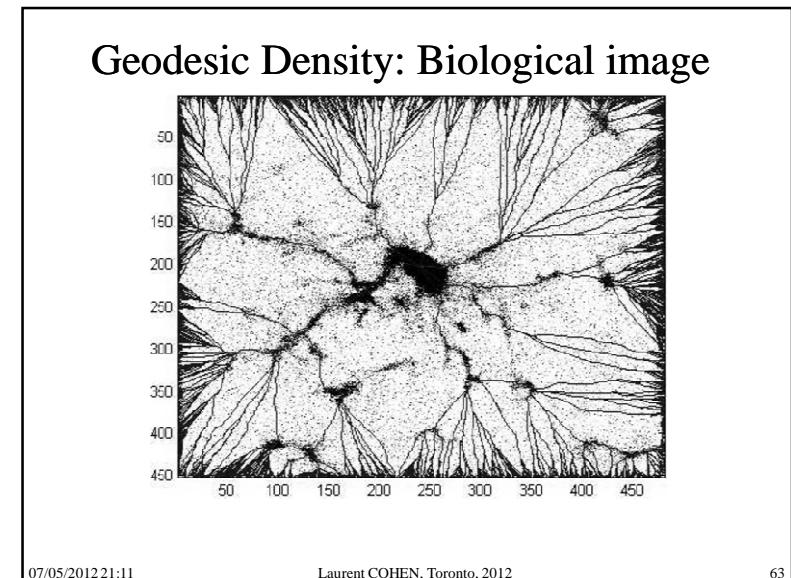
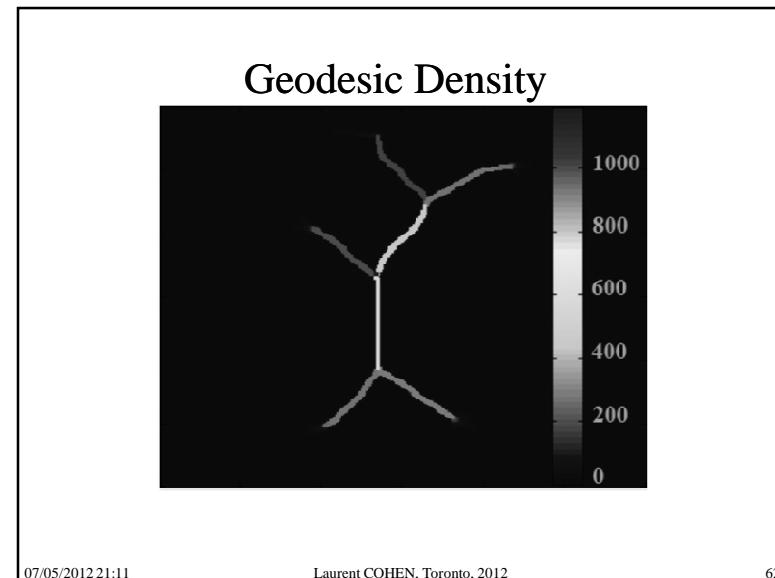
Geodesic Density



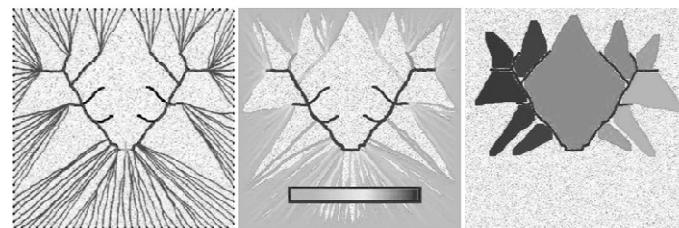
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Geodesic Density: Shading Zone Problem



Different solutions proposed:

- Second step in each shading zone
- Multiple source points
- Transport Equation
- Adaptive Voting: End points adaptively scattered

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Geodesic Density: Shading Zone Problem

Multiple source points

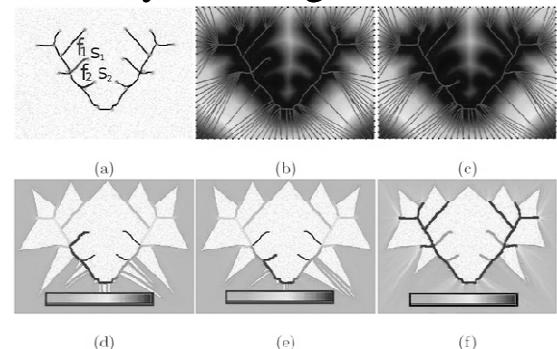


Figure 3: Voting by multi-propagation. (a) The green circles correspond to Harris points used to run multi-propagation superimposed to the image. (b) and (c): the pink lines represent paths extracted from the image border to the source points S_1 and S_2 , the paths are superimposed on the geodesic distance map U . Only 10 percent of the paths extracted are shown in the figures; (d) and (e) are respectively the voting score maps associated respectively to the geodesics map (b) and (c). (f) corresponds to the global score map computed by multi-propagation.

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Geodesic Density: Transport Equation

$$u_t + \operatorname{div}(vu) = 0, \quad (t, x) \in]0, T[\times \Omega, \quad (8)$$

where $v = -\nabla U$ denotes the velocity field computed from the distance map U . Due to the conservation of the information transported by the equation (8) toward the source point, we can define a geodesic density as the integral of the solution of the transport equation (8) in the time T

$$\mu(x) = \int_0^T u(t, x) dt.$$

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Geodesic Density: Transport Equation

$$u_t + \operatorname{div}(vu) = 0, \quad (t, x) \in]0, T[\times \Omega, \quad (8)$$

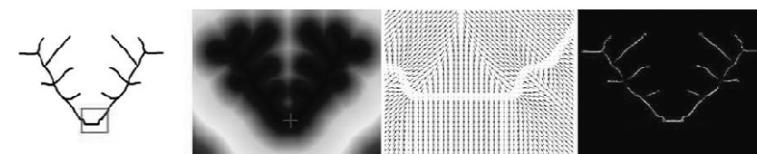


Figure 6: Voting by transport equation. First panel: synthetic image representing a tree structure. Second panel: distance map, the source point is indicated by the red cross. Third panel: zoom on the velocity field shown in the region indicated by the red square in the first panel. Fourth panel: geodesic density computed by the relation (10).

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Geodesic Density: Shading Zone Problem

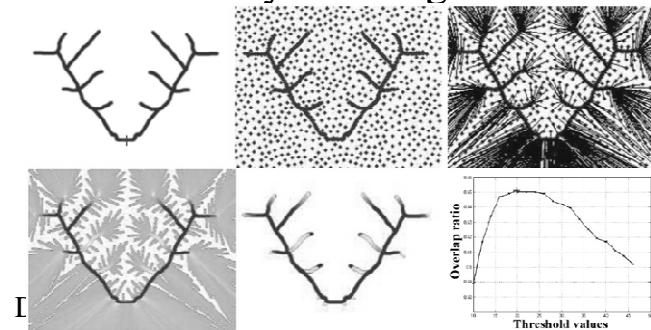


Figure 4: Adaptive voting. First row: the left panel shows the synthetic tree, the red cross represents the root of the tree; the center panel shows the farthest points; the right panel shows in blue the geodesics extracted from the farthest points to the root. Second row: the left panel shows the geodesic density; the center panel shows the geodesic density after thresholding; the right panel plots the effect of the variation of the threshold on the overlap ratio, the red square represents the value

$$Th = \frac{\max(\text{geodesic density})}{100}$$

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Geodesic Density

Not sensitive to the source point location

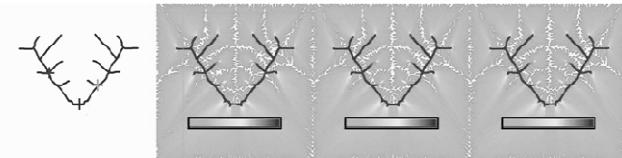


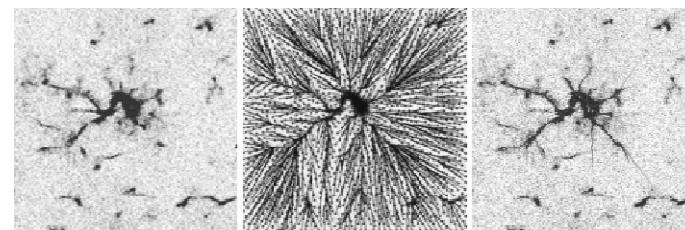
Figure 5: Illustration of the effect of the localization of the source point on the geodesic voting density. From left to right: red, green, and blue crosses indicate the localization of the source points; geodesic density generated with the source point indicated by the red cross; geodesic density generated by the source point indicated by the blue cross; geodesic density generated by the source point indicated by the green cross.

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Geodesic Density: Real Biological image

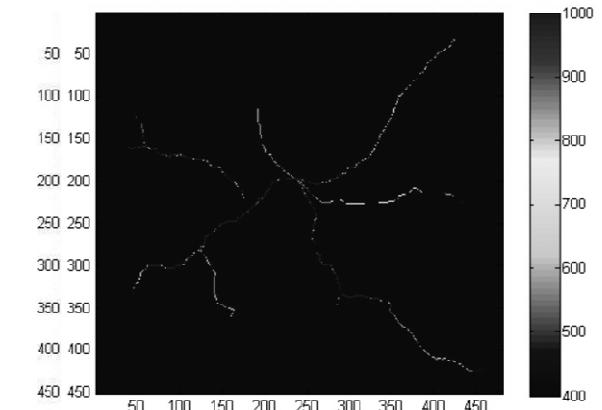


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Geodesic Density: Biological image

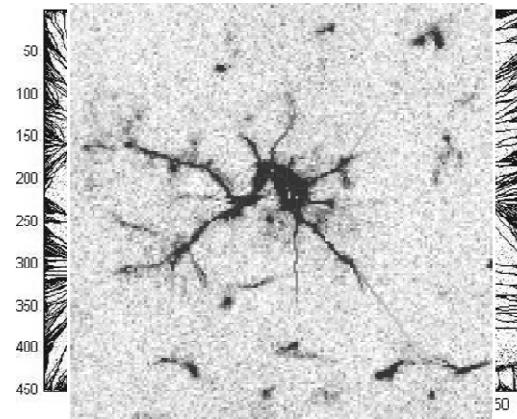


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Geodesic Density: Biological image

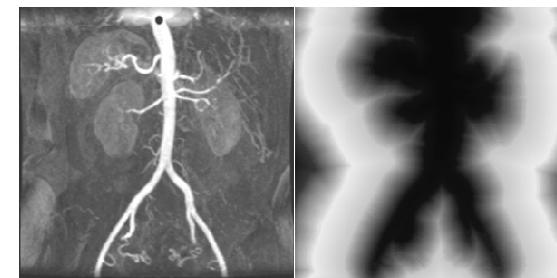


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Geodesic Density: Real example

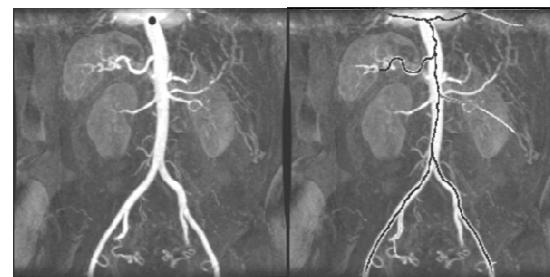


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Geodesic Density: from the image boundary



Boundary : 4*256 end points

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Geodesic Density: all points



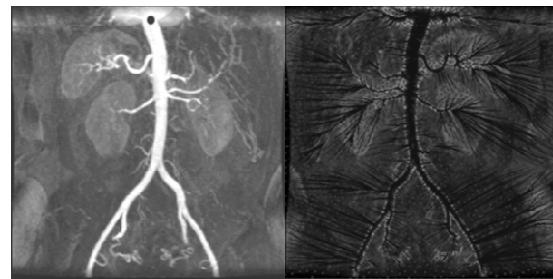
From all points : 256*256 end points

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Geodesic Density: adaptive voting



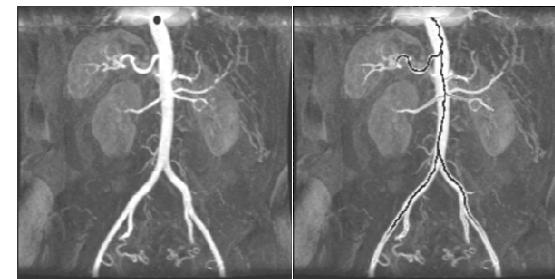
Adaptive voting : 1000 end points

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Geodesic Density: adaptive voting



Adaptive voting : 1000 end points

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Geodesic Voting and centerline

(with Y. Rouchdy, ISBI'11)

voting using Space + radius distance

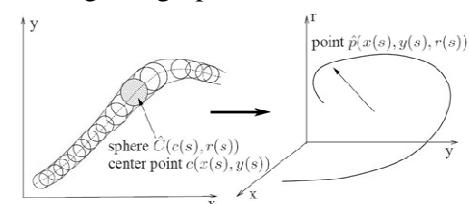


Figure 1. A tubular surface is presented as the envelope of a family of spheres with continuously changing center points and radii.

$$\tilde{P}(x, r) = \omega + \frac{\lambda_1}{r^\beta} (m(x, r) - m_0)^2 + \frac{\lambda_2}{r^\beta} (\sigma^2(x, r) - \sigma_0^2)^2$$

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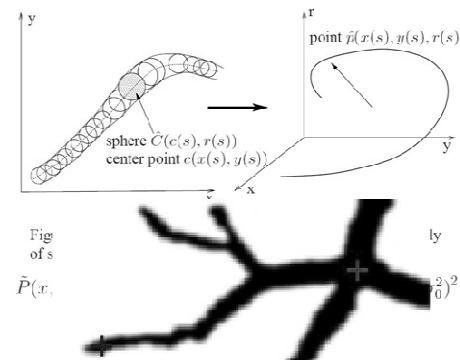
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Geodesic Voting and centerline

(with Y. Rouchdy, ISBI'11)

voting using Space + radius distance



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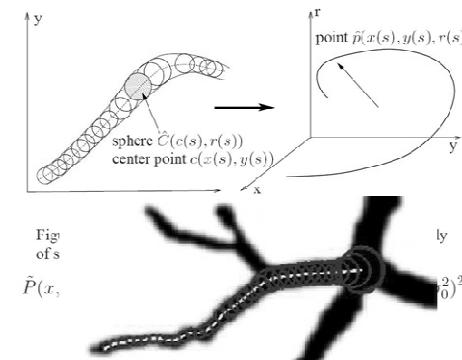
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Geodesic Voting and centerline

(with Y. Rouchdy, ISBI'11)

voting using Space + radius distance



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3D Minimal Path for tubular shapes in 2D

2D in space , 1D for radius of vessel

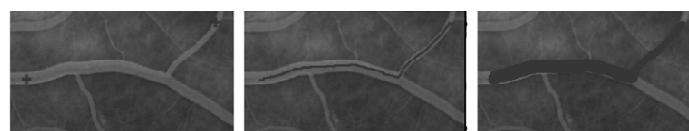


Fig. 2. Vessel segmentation for an angiogram 2D projection image based on the proposed method

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Geodesic Voting and centerline

voting using Space + radius distance
Different ways of using the voting results.

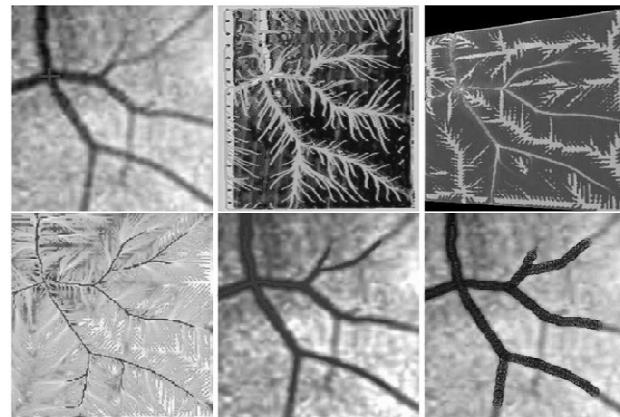
$$\tilde{\mu}_s(x) = \sum_{r=0}^{r_{max}} \mu(x, r), \quad \tilde{\mu}_m \doteq (x) = \max_{r \in [0, r_{max}]} \mu(x, r)$$

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Geodesic Voting and centerline



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Geodesic Voting and centerline

voting using Space + radius distance

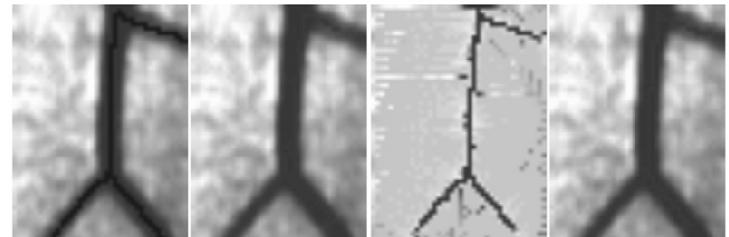


Figure 8: Comparison of the original voting method and our approach. From left to right: in blue the manual segmentation of the centerlines of the tree; results obtained by the original voting method (overlap ratio $O = 0.41$); the geodesic density $\bar{\mu}_m$ obtained by our approach; the density $\bar{\mu}_m$ after thresholding (overlap ratio $O = 0.75$).

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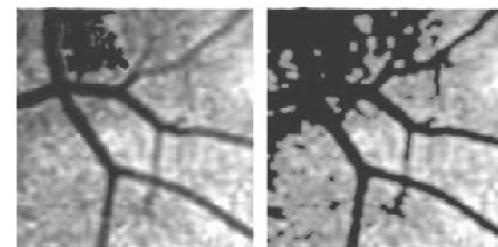
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Geodesic Voting and Segmentation with Prior

Level set method either edge based or region based



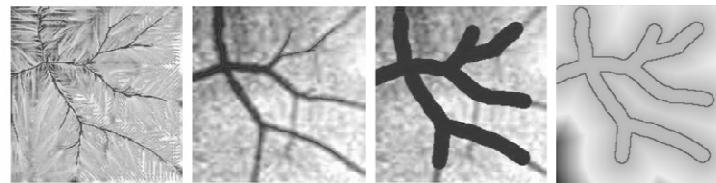
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Geodesic Voting and Segmentation with Prior

Level set initialized with result of voting $\tilde{\phi}$



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Geodesic Voting and Segmentation with Prior

(with Y. Rouchdy, SSVM'11)

Chan Vese Energy including shape Prior given by $\tilde{\phi}$

$$\mathcal{V}(\phi, c_1, c_2) = \int_{\Omega} \left(\lambda_1(u_0 - c_1)^2 H_{\epsilon}(\phi) + \lambda_2(u_0 - c_2)^2 (1 - H_{\epsilon}(\phi)) + \mu \delta_{\epsilon}(\phi) |\nabla \phi| + \nu H_{\epsilon}(\phi) \right) dx,$$

$$E_b(\phi, c_1, c_2) = \mathcal{V}(\phi, c_1, c_2) + \gamma \int_{\Omega} \frac{(\phi - \tilde{\phi})^2}{2\sigma^2} \delta_{\epsilon}(\phi) dx,$$

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Geodesic Voting and Segmentation with Prior

(with Y. Rouchdy, SSVM'11)

Chan Vese Energy including shape Prior given by $\tilde{\phi}$

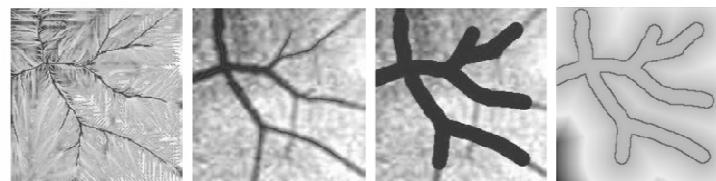


Figure 10: Geodesic voting segmentation of vessels from a 2D retinal image. The left panel shows in red the voting tree on the image; the second panel shows the voting tree obtained by thresholding the geodesic density; the third panel shows in red the voting tree after morphological dilatation; the right panel shows the segmentation result obtained with the geodesic voting method presented in Section 4.2.2 (GVP).

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Geodesic Voting and Segmentation with Prior

(with Y. Rouchdy, SSVM'11)

Chan Vese Energy including shape Prior given by $\tilde{\phi}$

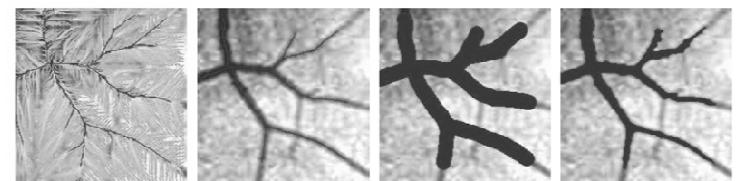


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Geodesic Voting and Segmentation with Prior

3D extension

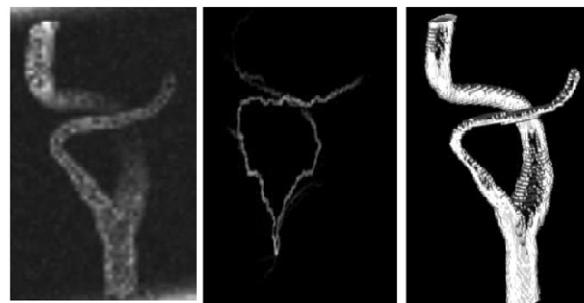


Figure 16: Lumen segmentation from simulated 3D data. The left panel shows the original image, the center panel shows the geodesic density, the right panel shows the segmentation result obtained with our approach.

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Geodesic Voting and Segmentation

3D extension

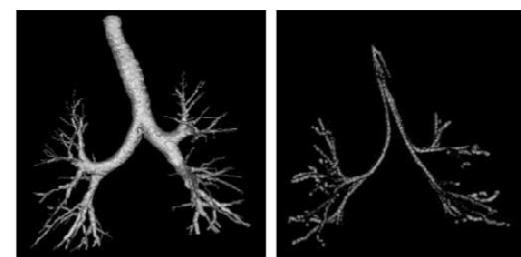


Figure 18: Segmentation of the airway tree with the geodesic voting method. The left panel shows the surface rendering of the manual segmentation. The right panel shows the rendering of the voting tree after dilation of 1 mm.

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Geodesic Voting and Deformable Tree

(with Julien Mille, MMBIA'09, ISBI'10)

initial image with root point



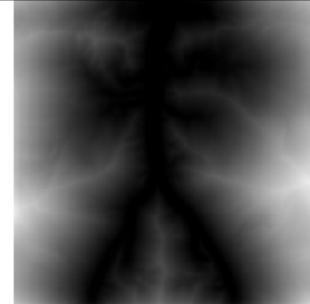
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Geodesic Voting and Deformable Tree

minimal action map from root point



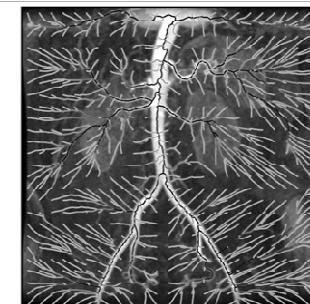
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Geodesic Voting and Deformable Tree

Initial tree from voting score



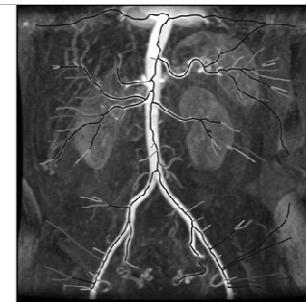
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Geodesic Voting and Deformable Tree

Removing unsignificant segments by thresholding the geodesic voting

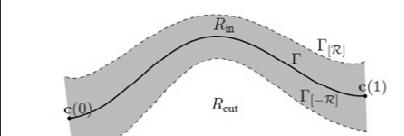


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$$\Gamma_{[R]}(u) = \Gamma(u) + R(u)\mathbf{n}(u) \quad \text{et} \quad \Gamma_{[-R]}(u) = \Gamma(u) - R(u)\mathbf{n}(u)$$

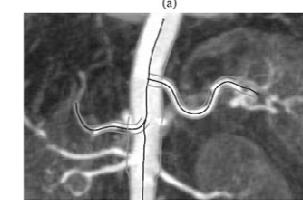


Figure 2. Deformable band defined by medial curve and thickness
(a), representation of the tree by discontinuous bands (b)

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$$E_{\text{smooth}}[\Gamma, \mathcal{R}] = \int_{\Omega} \left\| \frac{d\Gamma}{du} \right\| + \left(\frac{d\mathcal{R}}{du} \right)^2 du$$

$$E_{\text{region}}[\Gamma, \mathcal{R}] = \iint_{R_{in}} g_{in}(\mathbf{x}) d\mathbf{x} + \iint_{R_{out}} g_{out}(\mathbf{x}) d\mathbf{x}$$

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Intermediate steps of tree evolution.



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Geodesic Voting and Deformable Tree

final step of tree evolution.



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Geodesic Voting and Deformable Tree

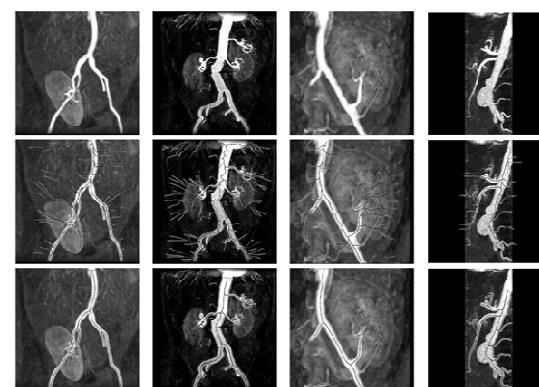


Figure 5. Original images (top row), trees after thresholding on voting score (middle row), final trees with boundary curve (bottom row)

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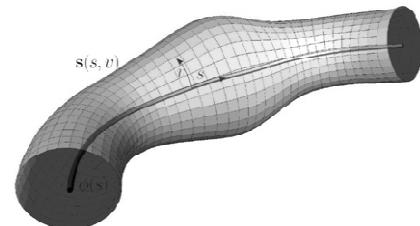
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Geodesic Voting and Deformable Tree

Energy minimizing Deformable tube

$$\mathbf{s}(s, v) = \phi(s) + \mathcal{R}(s, v)(\cos v\mathbf{N} + \sin v\mathbf{B})$$



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Geodesic Voting and Deformable Tree

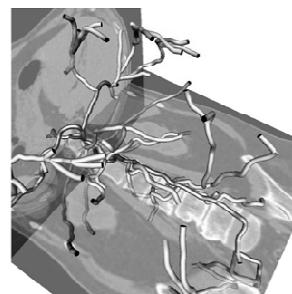
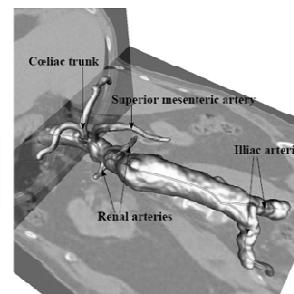


Figure 5. Tree after thresholding on voting score (left)



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Conclusion

- Minimally interactive tools for vessels and vascular tree segmentation (tubular branching structures)
- User provides only one initial point
- Fast and efficient propagation algorithm
- Voting approach as a powerful tool to find the structure, which can be completed with other approach.

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Thank you !

Cohen@ceremade.dauphine.fr

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www.ceremade.dauphine.fr/~cohen

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Geodesic Voting for biomedical image segmentation

Laurent D. COHEN

Directeur de Recherche CNRS
CEREMADE, UMR CNRS 7534 Université Paris-9 Dauphine
Place du Maréchal de Lattre de Tassigny 75016 Paris, France

Cohen@ceremade.dauphine.fr

<http://www.ceremade.dauphine.fr/~cohen>

Some joint works with Y. Rouchdy, and J. Mille.

Toronto, May 2012



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