Workshop on Microlocal Methods in Medical Imaging

A New Adaptive S Transform with Applications in Studying Brain Functions

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Outline

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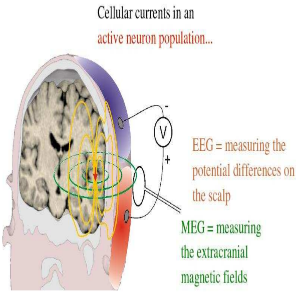
Studying brain functions

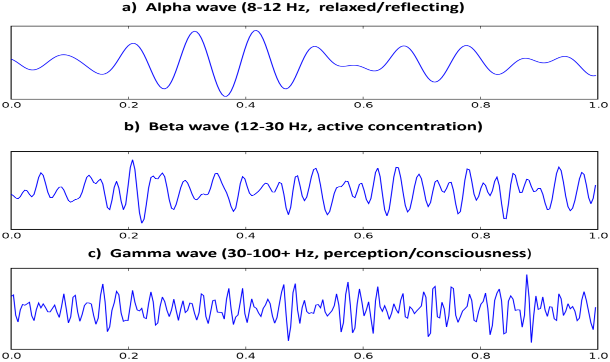
- Understand the brain's control mechanisms
- Guide treatment of mental disorders and other neurological diseases

- Brain signals: Electroencephalography (EEG) and Magnetoencephalography (MEG)



Adapted from Lauri Parkkonen, HBM course 2006

Brain rhythms



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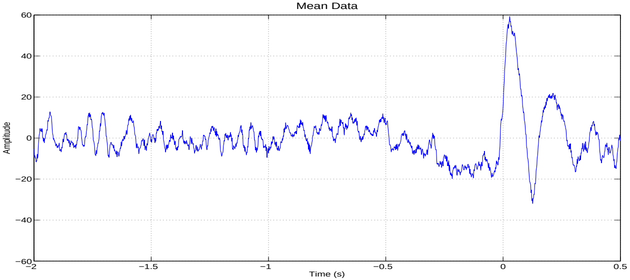
- Spectrum analysis becomes a popular approach in analyzing brain signals.

Challenges in brain research

- Non-stationarity
- A huge range of variations of the signal structure
- Complication of interactions among different brain regions



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Challenges in brain research

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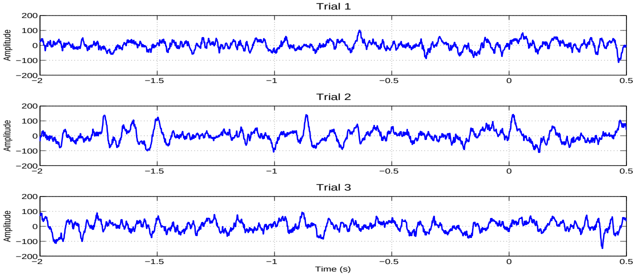

Challenges in brain research

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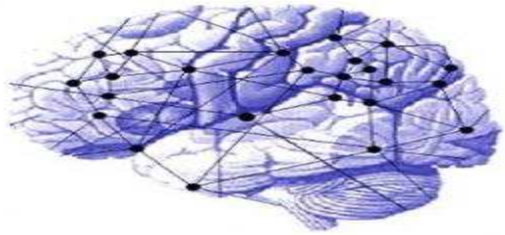

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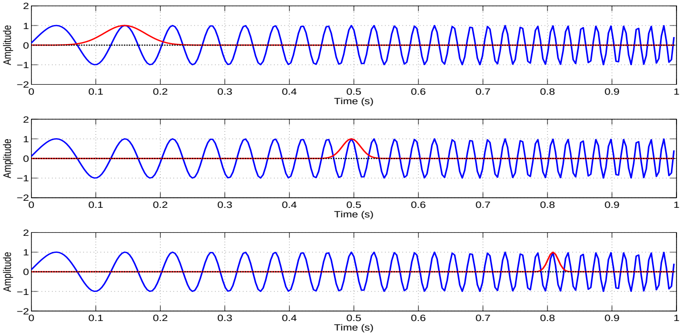
The Stockwell transform

- A moving window Fourier transform whose window function is scaled by $\frac{1}{|f|}$.



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A generalized Stockwell transform (GST)

- We propose a generalized Stockwell transform with the window function scaled by $\sigma(f) = \frac{p}{|f|^q}$. (Pinnegar and Mansinha, 2003: $\sigma(f) = \frac{p}{|f|}$; Sejdić, et al., 2008: $\sigma(f) = \frac{1}{|f|^q}$)

Definition: The Generalized Stockwell Transform

Let $\psi \in L^2(\mathcal{R})$. The generalized Stockwell transform of $x \in L^2(\mathcal{R})$ is defined as

$$GST_x^{(p,q)}(t,f) = \frac{|f|^q}{p} \int_{-\infty}^{\infty} x(\tau) \overline{\psi\left(\frac{|f|^q(\tau - t)}{p}\right)} e^{-j2\pi\tau f} d\tau, \quad f \neq 0,$$

and

$$GST_x^{(p,q)}(t,0) = \int_{-\infty}^{\infty} x(\tau) d\tau,$$

where $p > 0$ and $q \geqslant 0$.

Main mathematical properties

A Frequency Domain Equivalence

$$\mathrm{GST}_x^{(p,q)}(t,f) = \int_{-\infty}^{\infty} X(\alpha)\overline{\Psi\left(\frac{p(\alpha-f)}{|f|^q}\right)} e^{j2\pi(\alpha-f)t}\,d\alpha, \quad f \neq 0.$$

Connection to the Fourier Spectrum

$$\int_{-\infty}^{\infty} \mathrm{GST}_x^{(p,q)}(t,f)\,dt = X(f).$$

An Inversion Formula

$$x(t) = \mathcal{F}^{-1}\left\{\int_{-\infty}^{\infty} \mathrm{GST}_x^{(p,q)}(t,f)\,dt\right\}.$$

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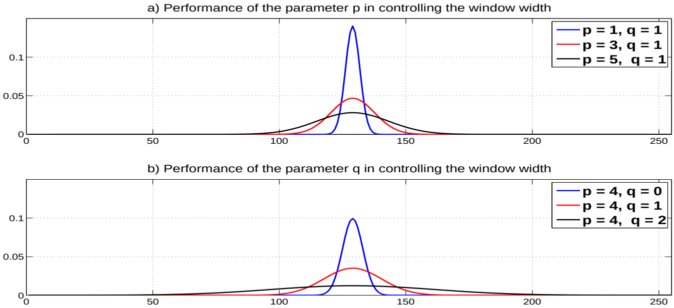
Effects of the parameters p and q ($\sigma(f) = \frac{p}{|f|^q}$)

- The width of the window function in the GST increases as the value of p or q increases, and q has a stronger influence.



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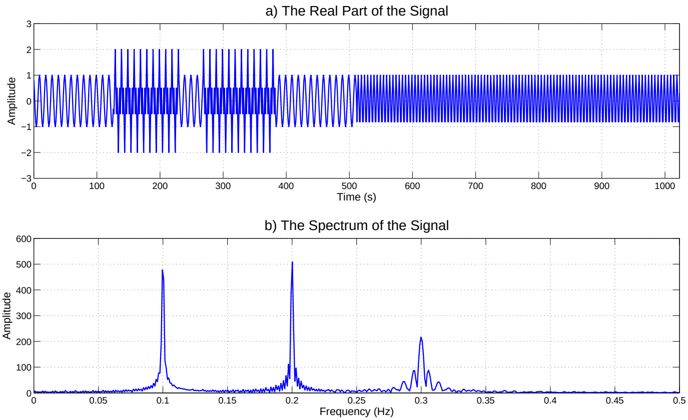
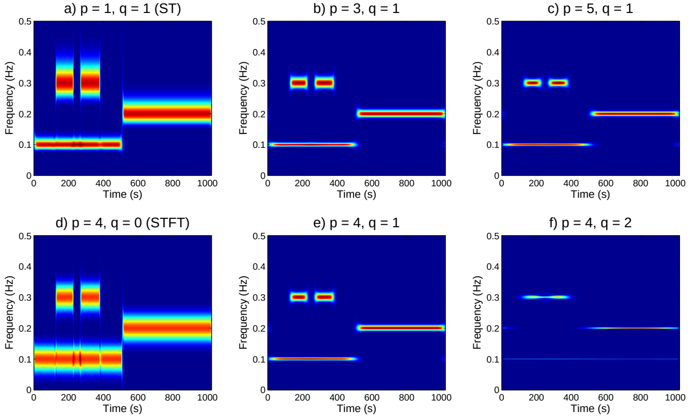
A simulated signal



An illustration ($\sigma(f) = \frac{p}{|f|^q}$)



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Question

- Given a specific signal, how to automatically identify optimal parameters such that the corresponding GST provides better resolution to reveal the signal characteristics?

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Evaluate the GST performance by measuring the energy concentration

- A good performed time-frequency representation (TFR) is expected to have the signal energy only concentrated at the involved frequencies.
- Energy concentration measures have been widely used to evaluate the performance of the TFR.
- The Stanković's measure (2001) with an order k:

$$CM_k(TFR) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |TFR_x(t,f)|^{\frac{1}{k}}\, dtdf,$$

where $TFR_x(t,f)$ is an energy distribution, *i.e.*, $\int\int_{-\infty}^{\infty} TFR_x(t,f)dtdf = \| x \|_2^2$.

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The energy distribution based on the $\mathrm{GST}^{(p,1)}$

Theorem: Resolution of the Identity Formula of the $\mathrm{GST}^{(p,1)}$

Let $\psi \in L^2(\mathcal{R})$ be such that

$$C_\psi^p = \int_0^\infty \frac{|\Psi(f-p)|^2}{f} df - \int_{-\infty}^0 \frac{|\Psi(p-f)|^2}{f} df < \infty, \tag{1}$$

then for any signal $x \in L^2(\mathcal{R})$,

$$\frac{1}{C_\psi^p} \int\int_{-\infty}^{\infty} \left|\mathrm{GST}_x^{(p,1)}(t,f)\right|^2 \frac{dtdf}{|f|} = \| x \|_2^2 .$$

- The representation $\frac{1}{C_\psi^p}\left|\mathrm{GST}_x^{(p,1)}(t,f)\right|^2$ offers an energy distribution in the time-frequency domain with the measure $\frac{dtdf}{|f|}$.
- If $q \neq 1$, the GST with an arbitrary window function generally does not satisfy the resolution of the identity formula.

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A practical example of the GST-based energy distribution

Theorem: A Modified Gaussian Window

For

$$\Psi_g^p(f) = \frac{1}{K^p}\left((1+e^{-8\pi^2p^2})W_g(f) - e^{-2\pi^2p^2}W_g(f+p) - e^{-2\pi^2p^2}W_g(f-p)\right)$$

where $W_g(f) = e^{-2\pi^2 f^2}$ and $K^p = (1-e^{-4\pi^2p^2})^2$, we then have

$$C_\psi^p = \int_0^\infty \frac{|\Psi(f-p)|^2}{f}df - \int_{-\infty}^0 \frac{|\Psi(p-f)|^2}{f}df < \infty.$$

- The GST with the modified Gaussian window function provides a valid energy distribution.

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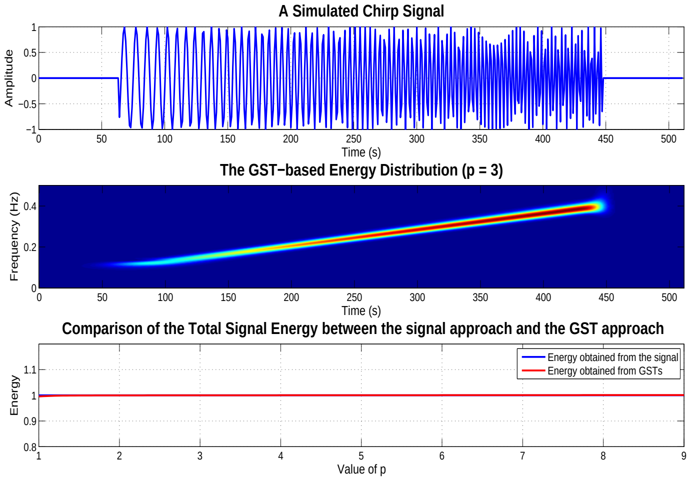
An illustration of the GST-based energy distribution



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Procedure to determine the adaptive Stockwell transform

Step 1. For each p from a given set, calculate the C_ψ^p and the GST $GST_x^{(p,1)}(t, f)$ of the signal x.

Step 2. Compute the concentration measure
$CM_2(GST(p)) = \int\int \frac{\left|GST_x^{(p,1)}(t,f)\right|}{\sqrt{C_\psi^p |f|}} dtdf.$

Step 3. Determine the optimal value of p through minimizing the measure $CM_2(GST(p))$.

Step 4. The adaptive Stockwell transform associated with the signal x is given by $AST_x(t, f) = GST_x^{(p_{opt},1)}(t, f).$

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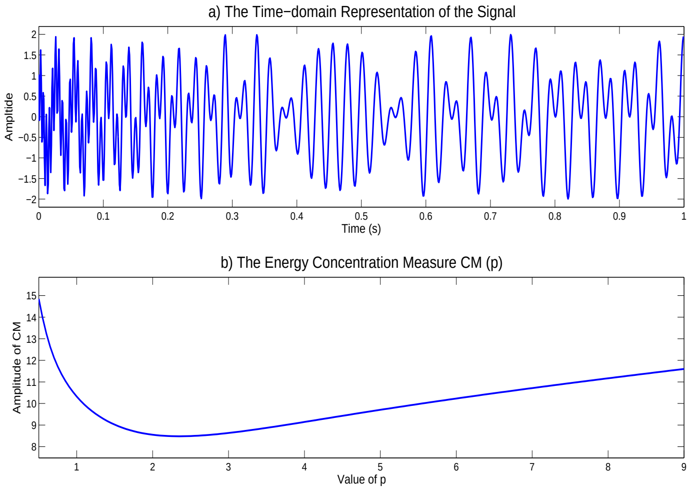
A simulation example



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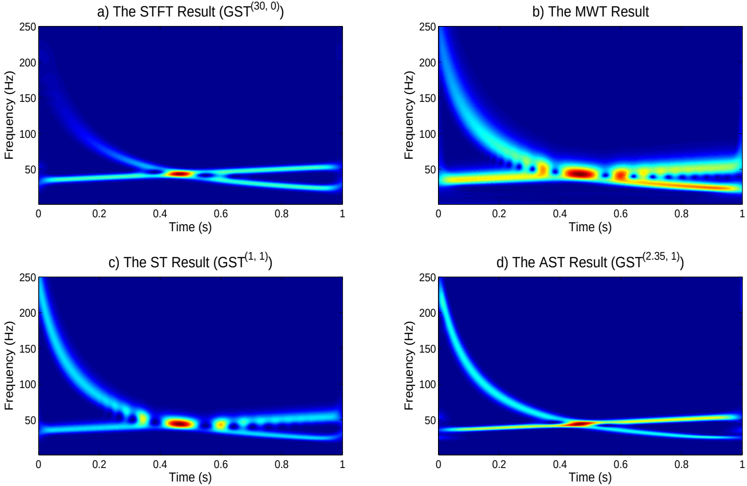
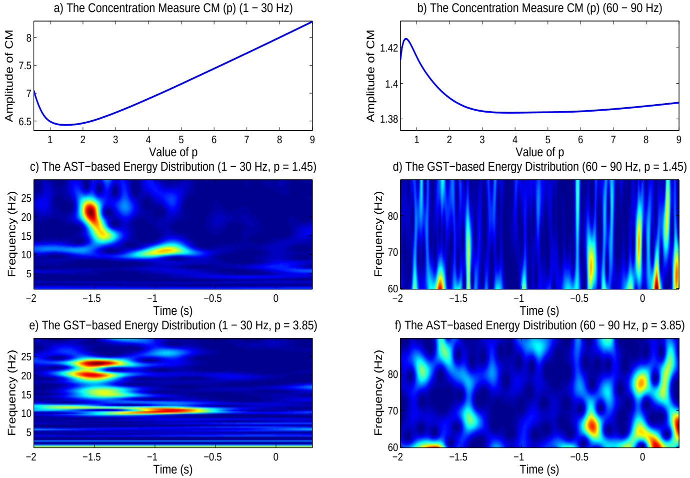


A MEG example



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GST-based measures

1. The GST-based power spectrum: reveals the time-varying spectral characteristics of a single time series.
2. The GST-based coherence function: describes the dynamics of linear interrelations between a pair of time series.
3. The GST-based phase-locking statistic: measures the phase synchronization between a pair of time series.

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- Developed by M.B. Priestley, which presents a theoretical time-frequency domain measure of locally stationary time series.
- An approximation form

$$ES_{xx}(t, f) = \mathcal{F}_{\tau \to f}\{\Gamma_{xx}(t, \tau) \otimes_t g(t, \tau)\},$$

where $\Gamma_{xx}(t, \tau)$ is the autocorrelation function given by

$$\Gamma_{xx}(t, \tau) \quad = \quad E\left\{\left(x^*(t - \frac{1}{2}\tau)x(t + \frac{1}{2}\tau)\right)\right\}.$$

and $g(t, \tau)$ is a two-dimensional localization function.

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The power spectrum based on the Cohen's class distributions (CCDs)

- The Cohen's class distribution function was introduced by L. Cohen (1966), which is able to represent all bilinear TFRs.
- The CCD-based power spectrum:

$$\begin{aligned} TS_{xx}^{(Cohen)}(t,f) &= E\{CCD_x(t,f)\} \\ &= \mathcal{F}_{\tau\to f}\{\Gamma_{xx}(t,\tau)\otimes_t \Phi(t,\tau)\}. \end{aligned}$$

  where $\Phi(t,\tau)$ is the time-lag kernel function.
- $\Phi(t,\tau)$ performs as the localized window function, which implies that any CCD-based power spectrum can be interpreted as an estimated evolutionary spectrum.

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The time-varying power spectrum based on the GST

Definition: The GST-based Power Spectrum

$$TS_{xx}^{(GST)}(t, f) = E\left\{GST_x^{(p,q)}(t, f) \cdot \overline{GST_x^{(p,q)}(t, f)}\right\}.$$

- The GST-based power spectrum presents a time-frequency domain measure of time series power.
- By choosing $q = 1$ and $p = p_{opt}$, this definition leads to the AST-based power spectrum.

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The interpretation of the GST-based power spectrum

Theorem: The Extended Cohen's Class of the GST

$$TS_{xx}^{(GST)}(t, f) \quad = \quad \mathcal{F}_{\tau \to f}\left\{\Gamma_{xx}(t, \tau) \otimes_t \tilde{\Phi}^{(p,q)}(t, \tau; f)\right\}$$

with the frequency-dependent time-lag kernel function

$$\tilde{\Phi}^{(p,q)}(t, \tau; f) = \frac{f^{2q}}{p^2}\psi\left(\frac{|f|^q\left(-t - \frac{1}{2}\tau\right)}{p}\right)\overline{\psi\left(\frac{|f|^q\left(-t + \frac{1}{2}\tau\right)}{p}\right)}.$$

- The GST-based power spectrum presents an advanced estimation of the evolutionary spectrum with a frequency-dependent localization function.

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The coherence function

- A frequency domain measure of the interrelation between a pair of stationary time series.
- The cross-correlation function:

$$\Gamma_{xy}(t,\tau) = E\left\{\left(y^*(t-\frac{1}{2}\tau)x(t+\frac{1}{2}\tau)\right)\right\}.$$

- The coherence function:

$$C_{xy}(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)\cdot S_{yy}(f)}, \quad \text{if} \;\; S_{xx}(f)\cdot S_{yy}(f) \neq 0.$$

where $S_{xy}(f) = \mathcal{F}_{\tau\to f}\{\Gamma_{xy}(\tau)\}$.

The GST-based coherence function

Definition: The GST-based Coherence

For two time series $x(t)$ and $y(t)$, the time-varying coherence based on the generalized Stockwell transform is defined as

$$TC_{xy}^{(GST)}(t,f) = \frac{|TS_{xy}^{(GST)}(t,f)|^2}{TS_{xx}^{(GST)}(t,f) \cdot TS_{yy}^{(GST)}(t,f)},$$
$$\text{if} \;\; TS_{xx}^{(GST)}(t,f) \cdot TS_{yy}^{(GST)}(t,f) \neq 0.$$

Note that $0 \leqslant TC_{xy}^{(GST)}(t,f) \leqslant 1$.

- The GST-based coherence provides a time-frequency domain measure of the interrelation characteristic, which can be interpreted as an advanced estimation of the evolutionary coherence.

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Phase synchronization

- The coherence function measures the linear interrelation between two time series, *i.e.*,

$$y(t) = \int_{-\infty}^{\infty} H(\tau)x(t-\tau)d\tau,$$

  which does not separate the effects of amplitude and phase in the interaction.

- The phase synchronization evaluates the nonlinear interactions based on the synchronization of the instantaneous phase, *i.e.*,

$$E\{\phi_x(t) - \phi_y(t)\} = \text{const.}$$

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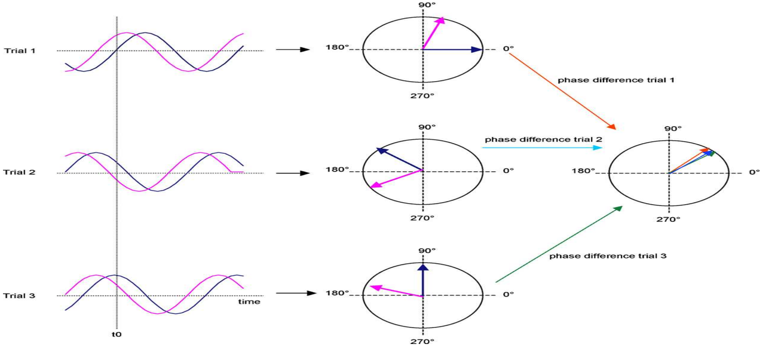
The phase-locking statistic (PLS)

- Proposed by Lachaux, et al. (1999) as

$$PLS_{xy}(t) = \left| E\{e^{i(\phi_x(t)-\phi_y(t))}\} \right|,$$

where $\phi_x(t)$ and $\phi_y(t)$ are instantaneous phases of the analytic signals.



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The instantaneous phase of the GST

Theorem: An Explicit Approximation Formula of the GST

Let the signal $x(t) = a(t)e^{j\phi(t)}$, with $\psi(t) = \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}$, we have

$$GST_x^{(p,q)}(t,f) \approx A^{(GST)}(t,f)e^{j\phi^{(GST)}(t,f)},$$

where

$$\phi^{(GST)}(t,f) \approx \phi(t) - j2\pi tf.$$

- The GST holds the absolutely referenced phase information.
- An approximation formula of the ST with a general window function can be found in (Guo, Molahajloo and Wong, 2010).

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The GST-based phase-locking statistic (PLS)

Definition: The GST-based Phase-locking Statistic

For two time series $x(t)$ and $y(t)$, the phase-locking statistic based on the generalized Stockwell transform is defined as

$$\mathrm{PLS}_{xy}^{(GST)}(t,f) = \left| E\left\{ \frac{GST_x^{(p,q)}(t,f)\cdot \overline{GST_y^{(p,q)}(t,f)}}{|GST_x^{(p,q)}(t,f)|\cdot|GST_y^{(p,q)}(t,f)|} \right\} \right|$$

- $$\mathrm{PLS}_{xy}^{(GST)}(t,f) \approx \left| E\left\{ e^{i[\phi_x(t)-\phi_y(t)]} \right\} \right| .$$

  Phases $\phi_x(t)$ and $\phi_y(t)$ here are the phases of the filtered signals around the considered frequency f.
- The GST-based PLS provides the phase synchronization measure across a wide range of frequencies.

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The Multi-source Interference Task (MSIT)

- Designed to study normal human cognition and psychiatric pathophysiology (Bush, et al., 2003).
- A behavioral experiment involving tasks at multiple levels of difficulty.
- There are two types of task trials in the MSIT: control trials and interference trials.

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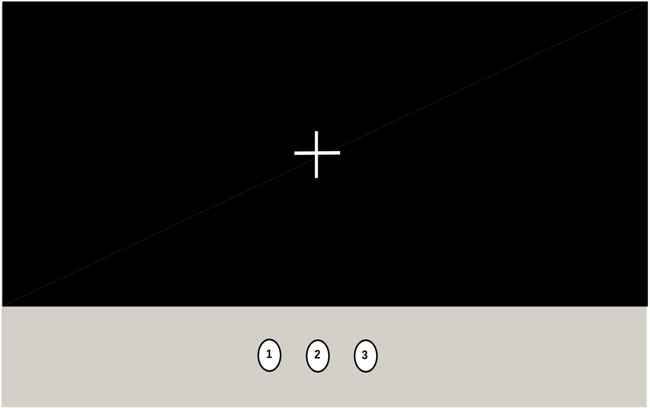
The control trials of the MSIT



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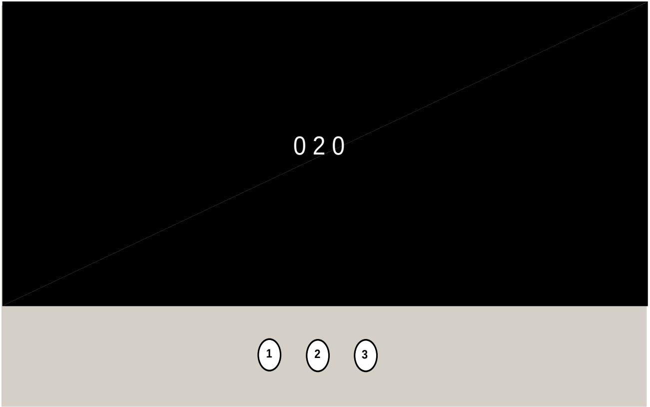

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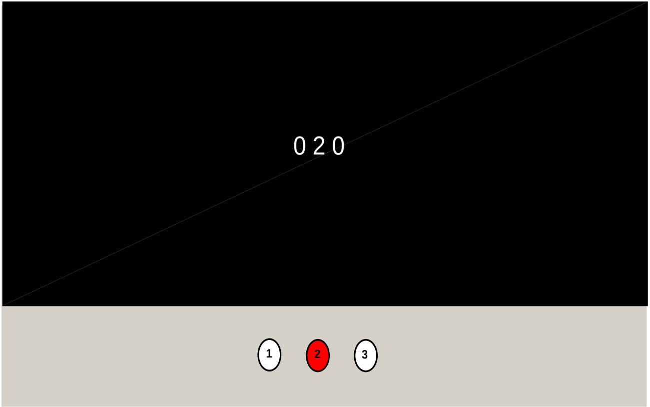

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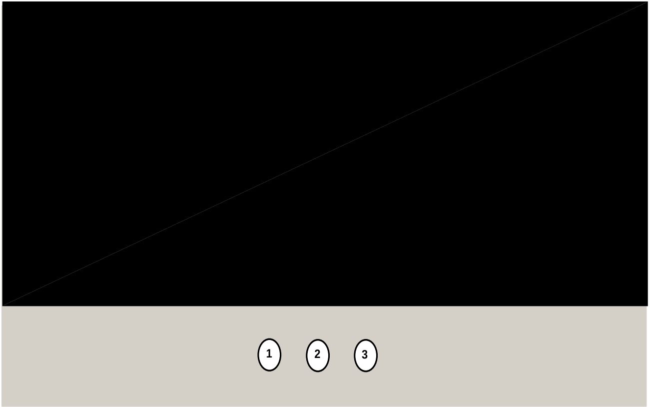
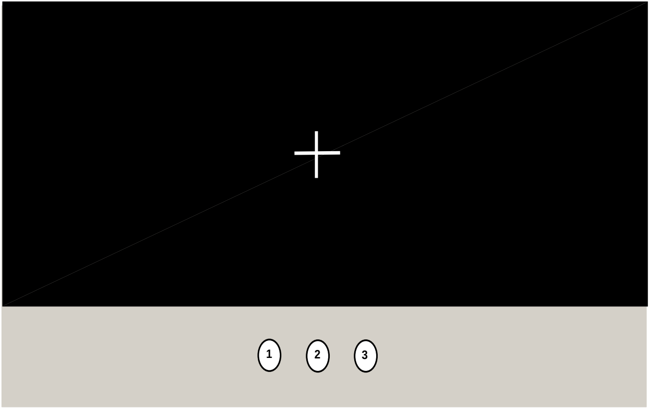


The interference trials of the MSIT



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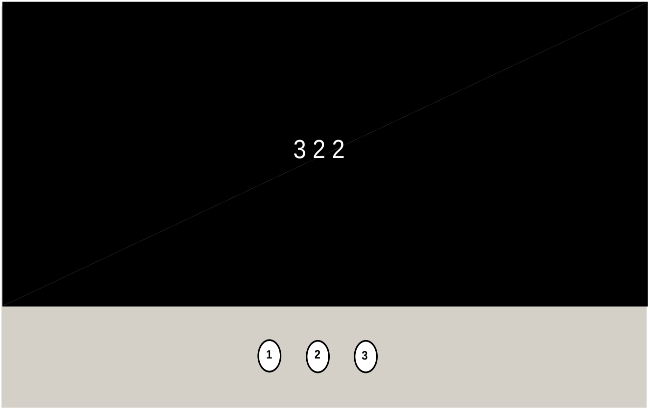

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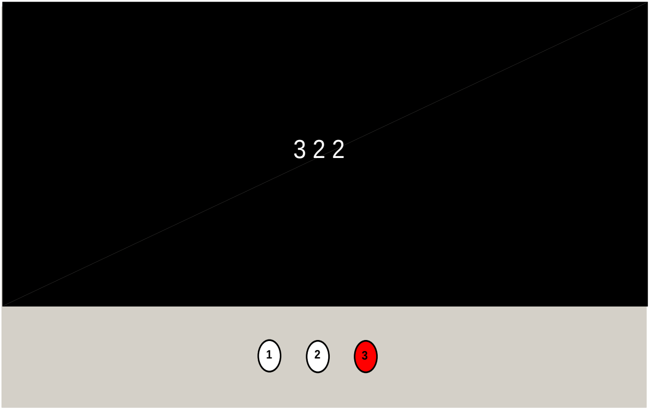
The interference trials of the MSIT



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Acquisition and Preprocessing

- The right-handed subjects (6 males, 4 females; mean age: 31) performed the MSIT.
- The brain activity was measured using a 275 channel whole-head MEG.
- Around 80 control and 80 interference trials were recorded continuously for each subject.
- Data were epoched at -3.5s to 0.5s with respect to the response at time = 0s.
- Trials with incorrect response were removed before subsequent analysis.

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Functional activities of motor cortices

1. Study the influence of the task difficulty on functional activities at the contralateral motor cortex (MIc) in the gamma frequency band ([60 - 90 Hz]).
2. Study functional couplings between the contralateral motor cortex and the ipsilateral motor cortex (MIi) in the alpha ([8 - 12 Hz]) and beta ([12 - 30 Hz]) frequency bands.

- **To improve the accuracy, we apply the proposed adaptive measures to the studies. Note that the resolution of these measures automatically adjusts to the specific frequency bands of interests.**

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Studying gamma-band activities at the MIc

- A differential minimum-variance beamformer algorithm (SAM) (60-90 Hz) was applied for source localization at the MIc.
- The motor activity at the gamma-frequency band (event-related synchronization (ERS)) was calculated based on the AST-based power spectrum for any group of MEG trials.
- The t-statistic was computed to assess the difference of gamma-band MIc activities in time between two groups of trials.
- The statistical significance was further examined using a permutation test ($N = 2000$, $\alpha = 0.05$) and corrected following the single threshold test.

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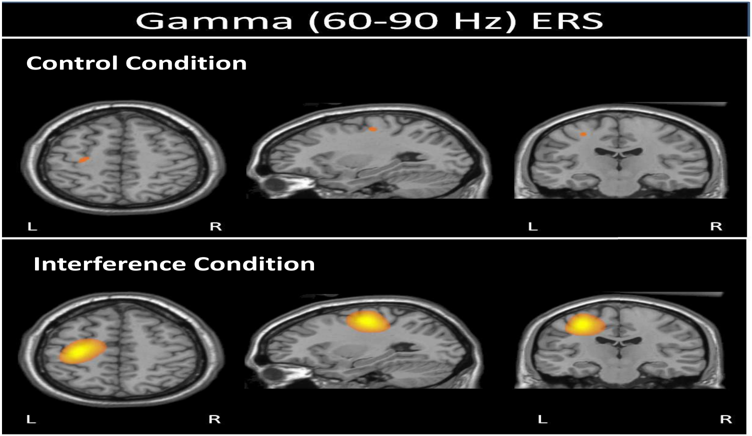
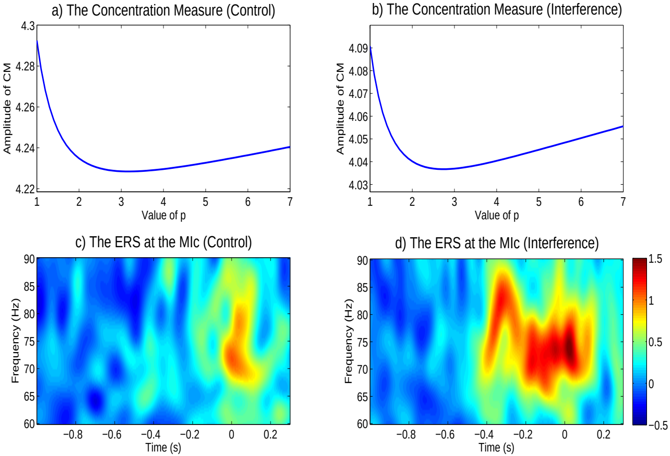
The SAM result



The TF results of a single subject



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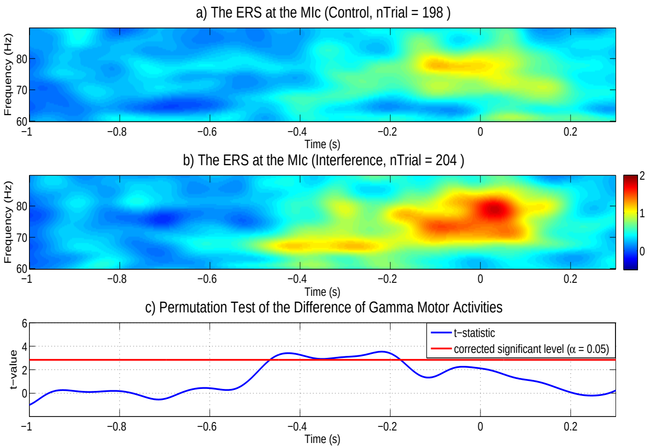
Group analysis: control trials VS interference trials



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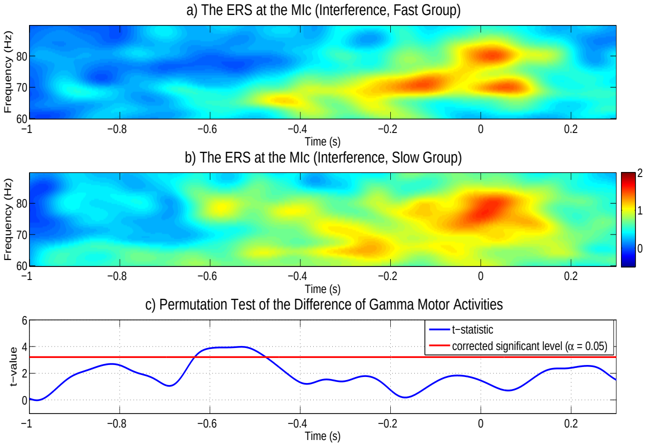
Group analysis: fast trials VS slow trials (interference)



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Our findings

- More gamma-band activities at the MIc were observed for interference trials compared to control trials,
- More gamma-band activities at the MIc were observed for slow trials compared to fast trials under the interference condition.
- Both the task condition and the RT information provide measures for the task difficulty in the MSIT. Therefore, our results suggest that more gamma-band MIc activities are needed for dealing with more difficult tasks.

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Study functional couplings between the MIc and the MIi

- A differential SAM (15 - 30 Hz) was applied for source localization around the MIc and the ipsilateral motor cortex (MIi).
- The motor activity at frequency bands of alpha and beta (event-related desynchronization (ERD)) was calculated based on the AST-based power spectrum for each group of MEG trials.
- The AST-based coherence and PLS were computed to measure the interactions between the MIc and the MIi in the time-frequency domain.
- The statistical significance was further examined using a bootstrap test ($N = 500, \alpha = 0.05$).

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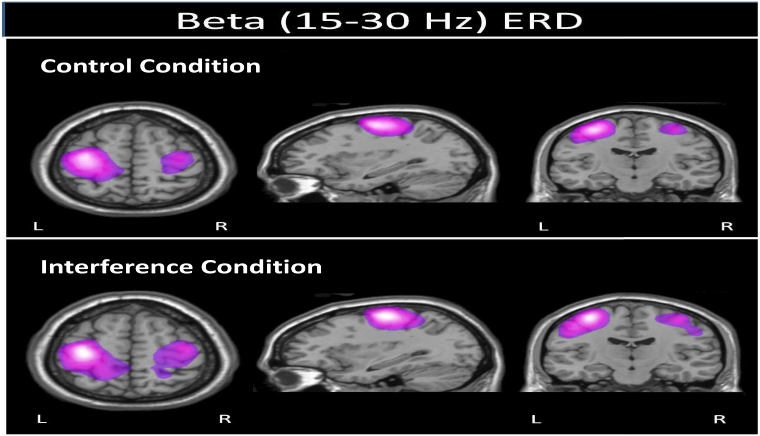
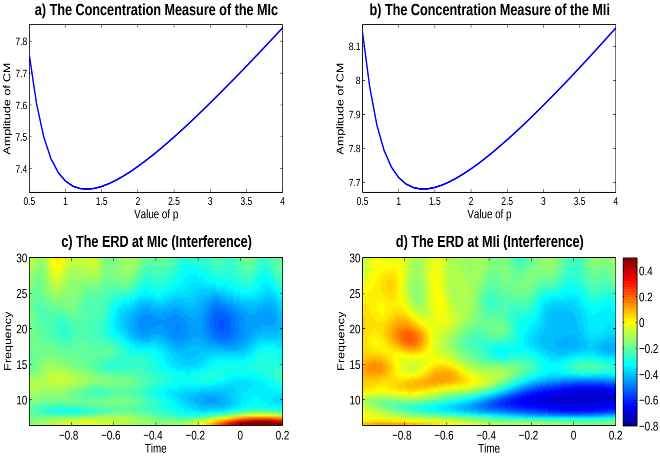
The SAM result



The TF results of a single subject (interference)



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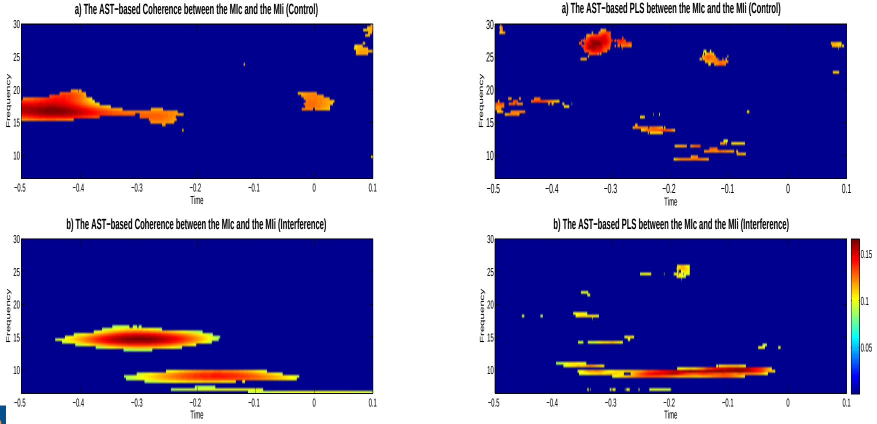
Grand averaged results: the functional coupling



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Our findings

- Both of the AST-based coherence and PLS results shown functional couplings between the MIc and the MIi, which were predominant for interference trials rather than control trials.
- Our results suggest that the functional connectivity between the MIc and the MIi at low frequencies ([8 - 15 Hz]) mainly appears when subjects perform interference trials under the MSIT.

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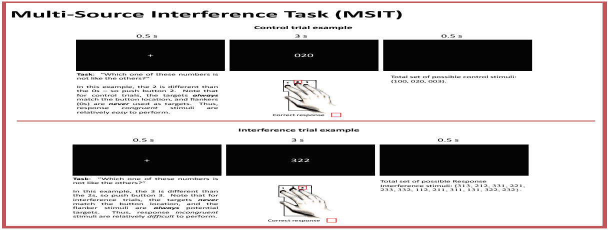
Summary

- Motivation: investigate functional activities of motor cortices using the MSIT



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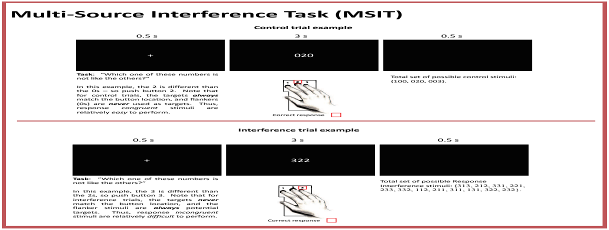
Summary

- Motivation: investigate functional activities of motor cortices using the MSIT



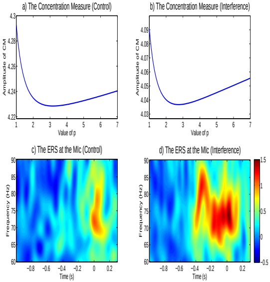
- A new time-frequency analysis tool

$$\mathrm{GST}_{\psi}^{(p,q)}(t,f) \longrightarrow \mathrm{GST}_{\mathrm{mGaussian}}^{(p,1)}(t,f) \longrightarrow \mathcal{AST}_{\mathrm{mGaussian}}^{(p_{opt},1)}(t,f)$$

KINECTRICS

Summary

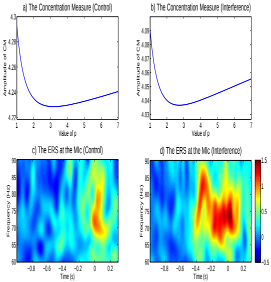
- The AST-based power spectrum:
  a time-varying power spectrum



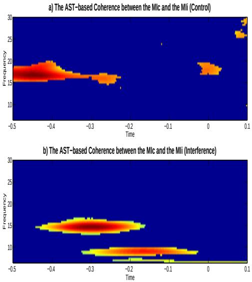
KINECTRICS

Summary

- The AST-based power spectrum:
  a time-varying power spectrum

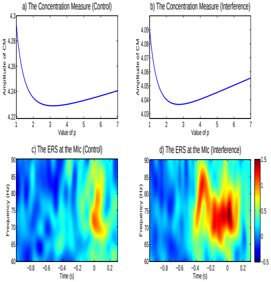


- The AST-based coherence:
  a linear interrelation measure

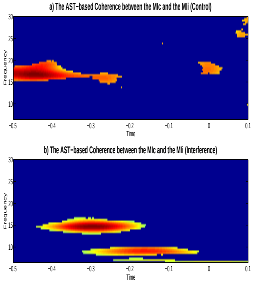


Summary

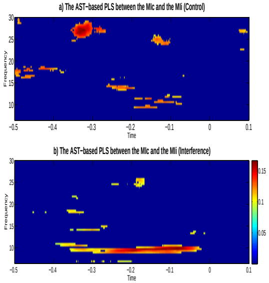
- The AST-based power spectrum: a time-varying power spectrum



- The AST-based coherence: a linear interrelation measure



- The AST-based PLS: a phase synchronization measure



Future work

1. Find energy distribution forms of the generalized Stockwell transforms with arbitrary values of q.
2. Extend the adaptive approach to multi-dimensional Stockwell transforms.
3. Develop advanced time-frequency domain measures for studying cross-frequency coupling and unidirectional interactions between brain areas.

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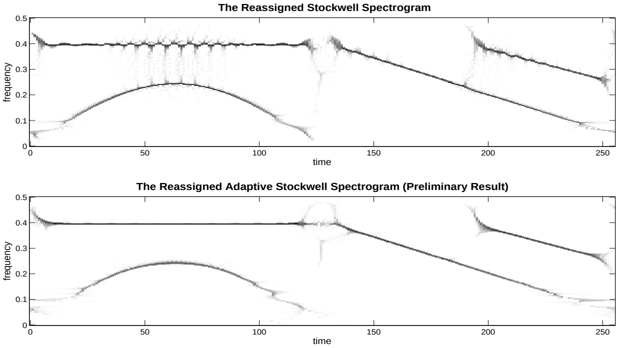
Future work

- Extract instantaneous frequencues of non-stationary signals



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Thank you

- I would like to thank **Dr. William Gaetz** from the Children's Hospital of Philadelphia for providing the experimental data and **Professor Hongmei Zhu** from York University for guiding this research work.

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- **Question ?**

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