Homomorphism Dichotomies and Graph Classes

Pavol Hell, Simon Fraser University

Fields Institute, July 13, 2011

Joint work with

Principal co-authors

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- Jing Huang
- Arash Rafiey

Interval Graphs

An interval graph H

H admits a *representation* by real intervals I_v (for $v \in V(H)$)

$$v \sim w \iff I_v \cap I_w \neq \emptyset$$

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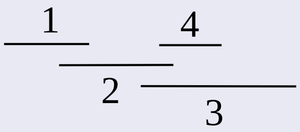
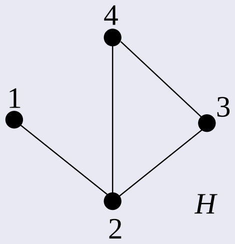
Example





Interval Graphs

Ordering characterization

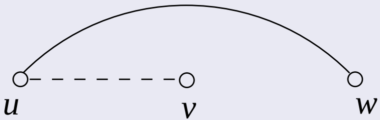
H is an interval graph

$$\iff$$

$V(H)$ can be linearly ordered by $<$ so that for $u < v < w$

$$u \sim w \implies u \sim v$$

Dotted edge cannot be absent



Interval Graphs

Forbidden structure characterization

H is an interval graph

$$\Longleftrightarrow$$

H has no induced $C_{(>3)}$ and no asteroidal triple

Lekkerkerker-Boland 1962

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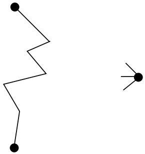
Asteroidal triple

any two joined by a path avoiding the neighbours of the third



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Lekkerkerker-Boland 1962

Clique structure characterization

H is an interval graph

$$\iff$$

the maxcliques of H can be linearly ordered so that each vertex lies in a consecutive set

Fulkerson-Gross 1965

Interval Graphs

$O(m+n)$ algorithms

- Booth-Lueker 1976
- Korte-Mohring 1989
- Habib-McConnell-Paul-Viennot 2000
- Corneil-Olariu-Stewart 2010 LexBFS x 6

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The second algorithm has been subsequently made certifying

Kratsch et al 2001

Proper Interval Graphs

Representable by a proper family ($v \neq w \implies I_v \not\subset I_w$)

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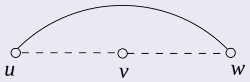

- No induced $C_{(>3)}$, net, tent, or claw Wegner 1967
- $O(m+n)$ certifying algorithm 3x LexBFS Corneil 2004 and H+Huang 2005

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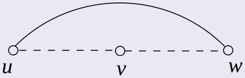

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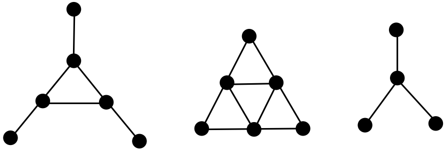
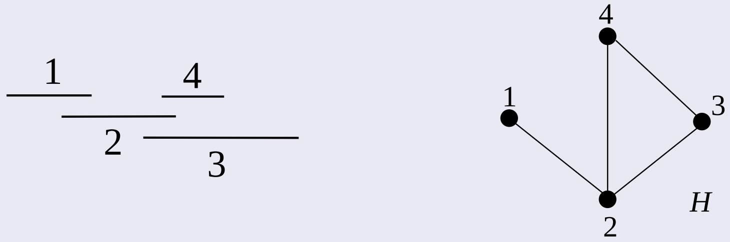


Interval Graphs

Example



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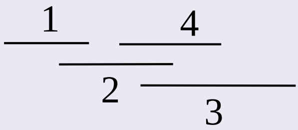
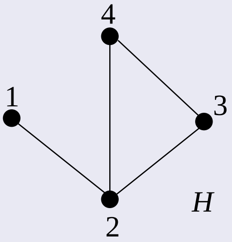


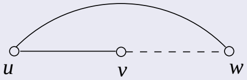


Representable by subtrees T_v of a tree T

Chordal Graphs

Representable by subtrees T_v of a tree T

- Orderable so that $u < v < w, u \sim w, u \sim v \implies v \sim w$



- No induced $C_{(>3)}$
- $O(m+n)$ certifying algorithm 1x LexBFS Booth-Lueker1976

Circular Arc Graphs

Representable by arcs A_v on a circle C

- $O(m+n)$ algorithm McConnell2003

Digraphs

Digraphs

?

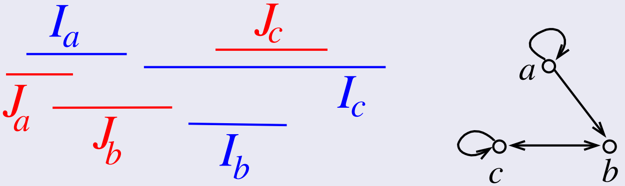
Digraphs

Interval digraphs - representable by pairs I_v, J_v

$$v \to w \iff I_v \cap J_w \neq \emptyset$$

Interval digraphs - representable by pairs I_v, J_v

$$v \rightarrow w \iff I_v \cap J_w \neq \emptyset$$



Digraphs

Not nice!

Digraphs

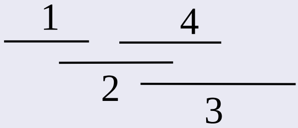
Not nice!

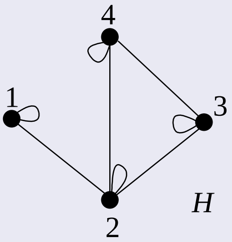
- Structural characterization not known
- Recognition polynomial but best algorithm is $O(n^2 m^7)$

Mueller 1997

Observation

Interval graphs are reflexive (have all loops)





Irreflexive Graphs

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Bigraphs (with red and blue vertices)

Each edge joins a red vertex and a blue vertex

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Bigraphs (with red and blue vertices)

Each edge joins a red vertex and a blue vertex

Equivalently, we can think of the edges as directed (red $\longrightarrow$ blue)

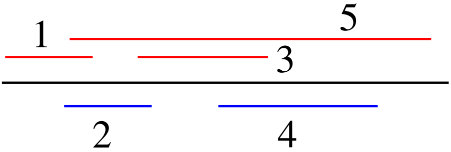
Interval bigraph H

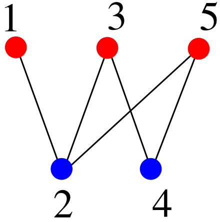
Representable by real intervals I_v (for $v \in V(H)$)

$$r \sim b \Longleftrightarrow I_r \cap I_b \neq \emptyset$$

(r is red, b is blue)

Interval Bigraphs





Interval Bigraphs

As before

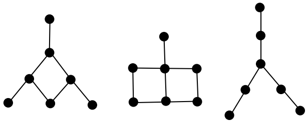
- No structural characterization
- Recognition algorithm only high degree polynomial

Representable by inclusion-free families

Proper Interval Bigraphs

Representable by inclusion-free families

- No induced $C_{(>4)}$, bi-net, bi-tent, or bi-claw
- $O(m+n)$ certifying recognition algorithm



H+Huang 2005

Homomorphisms

Given digraphs G and H

A *homomorphism* $f : G \to H$ is a mapping $f : V(G) \to V(H)$ such that $xy \in E(G) \implies f(x)f(y) \in E(H)$

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Undirected graphs are viewed as symmetric digraphs

Homomorphism Problems

Given a fixed digraph H

Does an input digraph G admit a homomorphism to H?

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Example: $H = K_t$

Does an input graph G admit a t-colouring?

Constraint Satisfaction Problems

CSP with fixed template H

H with $V(H)$ and relations $R_1(H), \dots, R_k(H)$

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CSP with fixed template H

H with $V(H)$ and relations $R_1(H), \ldots, R_k(H)$
Does an input G admit a homomorphism to H?

G has corresponding relations $R_1(G), \ldots, R_k(G)$
Homomorphisms preserve all relations

Dichotomy Conjecture

Feder - Vardi, 1993, conjectured for any template H

The CSP problem for H is polynomial or is NP-complete

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True for two-element templates Schaeffer 1978

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The CSP problem for H is polynomial or is NP-complete

True for two-element templates Schaeffer 1978
True for three-element templates Bulatov 2001

Dichotomy for Graphs versus Digraphs

If H is an undirected graph

The homomorphism problem for H is polynomial if H is bipartite or contains a loop; otherwise it is NP-complete

H+Nešetřil 1990

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If H is a digraph

If dichotomy holds for all digraph templates H then the dichotomy conjecture holds for all CSP

Feder+Vardi 1993

List Homomorphism Problems

Given a fixed digraph H

Each vertex x of the input digraph G has a *list* $L(x) \subseteq V(H)$

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Given a fixed digraph H

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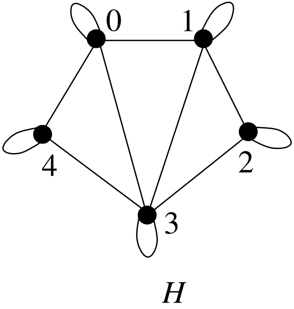
Is there a homomorphism $f : G \to H$ for which all $f(x) \in L(x)$?

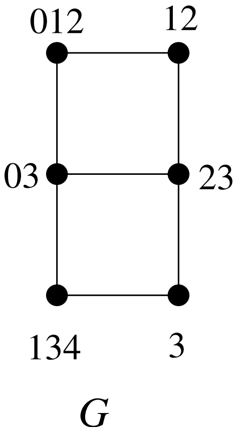
List Homomorphism Problems

Fixed graph H

Processors and connections



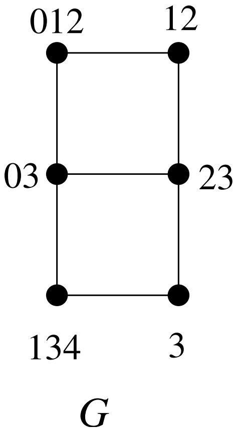
List Homomorphism Problems

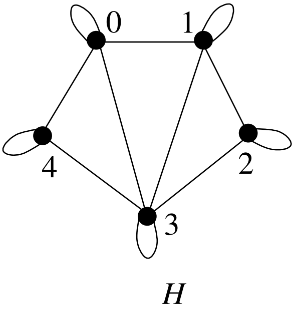


Input graph G

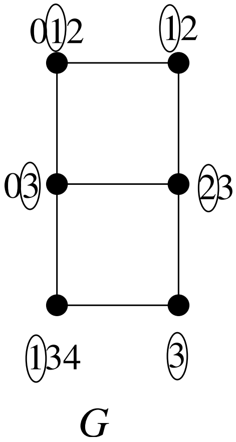
Tasks and communications

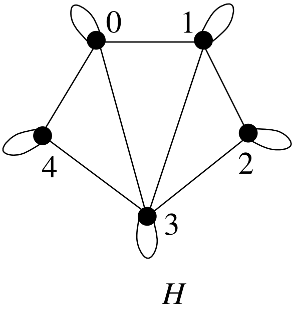
List Homomorphism Problems





List Homomorphism Problems





List Homomorphism Dichotomy to Reflexive Graphs

For a reflexive graph H

If H is an interval graph, then the problem for H is polynomial

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If H is an interval graph, then the problem for H is polynomial
Otherwise the problem is NP-complete

Feder+H 1998

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If inputs are restricted to have connected lists

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If inputs are restricted to have connected lists

If H is a chordal graph, then the problem for H is polynomial
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Feder+H 1998

Focus on Bigraphs

List homomorphisms to a bigraph H

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If $\overline{H}$ is a circular arc graph, then the problem is polynomial
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Feder+H+Huang 1999

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Feder+H+Huang 1999

Relation to interval bigraphs

A bipartite graph H is an interval bigraph if and only if

- $\overline{H}$ is a circular arc graph, and
- there exists a representation in which no two arcs cover the circle

H+Huang 2004

Co-Circular-Arc Bigraphs

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For a bigraph H

$\overline{H}$ is a circular arc graph
$\iff$ H has no induced $C_{>4}$ and no edge-asteroid

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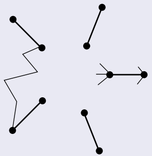

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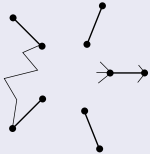

$\iff$ the *bipartite* complement of H can be two-edge-coloured so that there is no monochromatic $2K_2$

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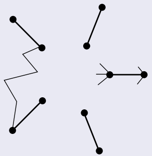

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H+Huang 2004

$O(n^2)$ algorithm

List Homomorphisms / Conservative CSP's

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Dichotomy

Every list CSP is polynomial or NP-complete

Bulatov 2003

Focus on Reflexive Digraphs

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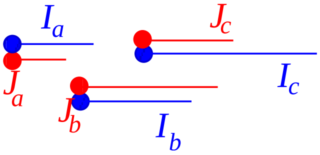
List homomorphisms to a reflexive digraph H

If H is an adjusted interval digraph, the problem is polynomial

Feder+H+Huang+Rafiey 2010



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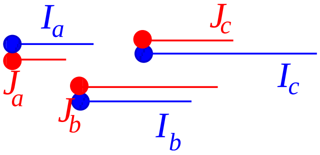

Conjecture

Otherwise it is NP-complete

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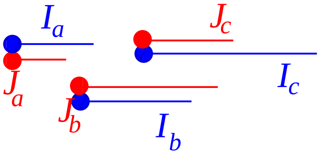


Conjecture

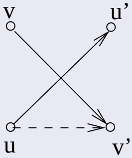
Otherwise it is NP-complete

Verified in important basic cases (trees, tournaments, etc)

An ordering characterization

H is an adjusted interval digraph if and only if $V(H)$ can be linearly ordered by $<$ so that for $u < v$ and $u' > v'$

$$u \to u' \text{ and } v \to v' \implies u \to v'$$



Adjusted Interval Digraphs

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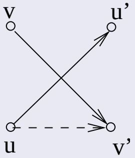


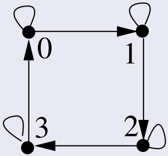
A structural characterization

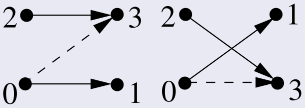
H is an adjusted interval digraph if and only if it has no invertible pair

$O(m^2)$ algorithm

Invertible Pairs

C_4 is not an adjusted interval digraph





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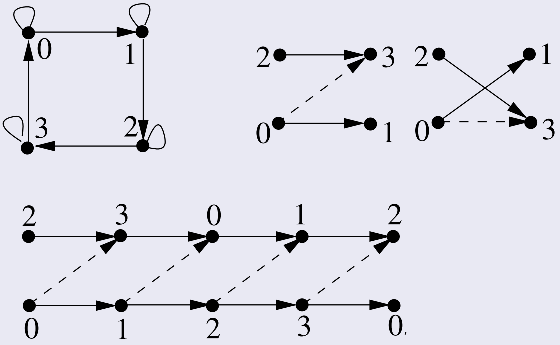
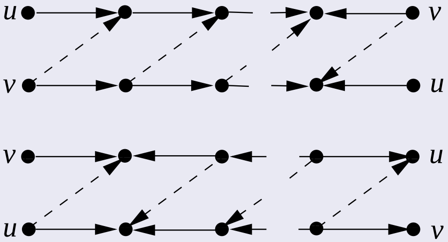


Invertible Pairs

Invertible pair u, v



Invertible Pairs

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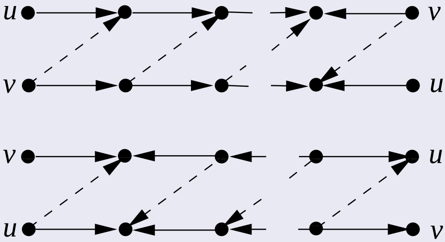

For reflexive digraphs

H is an adjusted interval digraph if and only if *H* has no invertible pair

Feder+H+Huang+Rafiey 2009

Revisit Co-Circular-Arc Bigraphs

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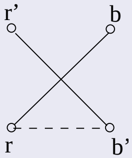
For a bigraph H

$\overline{H}$ is a circular arc graph
$\iff$ the red and the blue vertices can be linearly ordered by $<$ so that for $r < r'$ and $b > b'$

$$r \sim b \text{ and } r' \sim b' \implies r \sim b'$$



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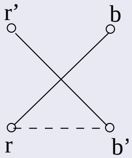

$\iff$ H has no invertible pair

H+Rafiey 2008

Dichotomy for General Graphs

List homomorphisms to a graph H

If H is a bi-arc graph, then the problem is polynomial
Otherwise the problem is NP-complete

Feder+H+Huang 2004

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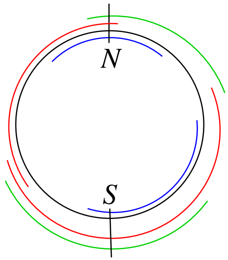

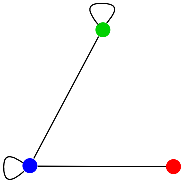

Bi-arc graphs

Generalizes both previous cases

- A reflexive H is a bi-arc graph $\iff$ it is an interval graph
- An irreflexive H is a bi-arc graph $\iff$ H is bipartite and $\overline{H}$ is a circular arc graph

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The structure of bi-arc graphs

H is a bi-arc graph if and only if the complement of $Bip(H)$ is a circular arc graph

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Feder+H+Huang 2004

This is the equivalent definition of bi-arc graphs used in the earlier talk by Benoit Larose

List homomorphisms to a digraph H

If H is an DAT-free digraph, then the problem is polynomial
Otherwise the problem is NP-complete

H+Rafiey 2011

Dichotomy for General Digraphs

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A digraph asteroidal triple (DAT) is a (somewhat technical) directed analogue of an asteroidal triple

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The presence of a DAT can be detected in polynomial time

Is there a geometric representation? (ordering characterization?)

New Graph Classes

In addition to interval graphs

- Co-circular-arc bigraphs
- Bi-arc graphs
- Adjusted interval digraphs
- DAT-free digraphs

Minimum Cost Homomorphism Problems

As in the earlier talk by Arash Rafiey

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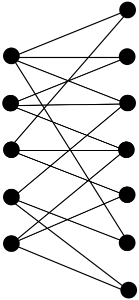

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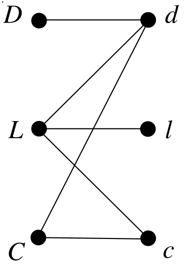

H

Minimize the overall cost

Each decision (map $x \in V(G)$ to $u \in V(H)$) has a given cost $c(x, v)$

Minimum Cost Homomorphism Problems

The nice graph classes identified

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"Proper" means $left(I_v) < left(I_w) \iff right(I_v) < right(I_w)$
(and similarly for the J's)

New Graph Classes

In addition to interval graphs and proper interval graphs and bigraphs

- Co-circular-arc bigraphs
- Bi-arc graphs
- Adjusted interval digraphs
- DAT-free digraphs
- Proper adjusted interval digraphs
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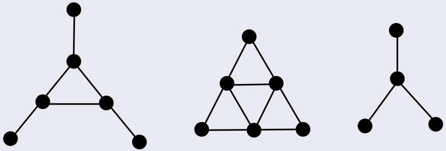
Structural Characterizations

Structural Characterizations

Forbidden for proper interval graphs



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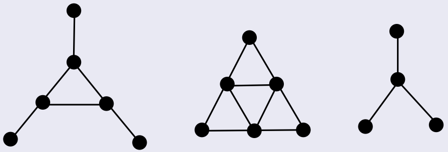
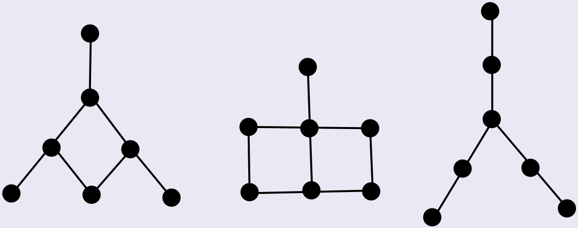


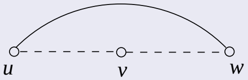
Forbidden for proper interval bigraphs



Ordering Characterizations

Reflexive graph H

H is a proper interval graph
$\iff$ it has an ordering without



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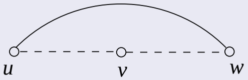
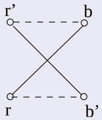


Bigraph H

H is a proper interval bigraph
$\iff$ it has two orderings (red and blue vertices separately) without



Ordering Characterizations

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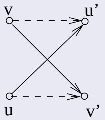

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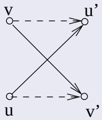

(Or a reflexive H being a proper adjusted interval digraph)

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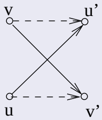

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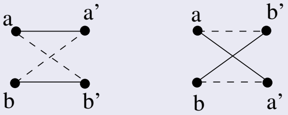
Obstruction: a symmetrically invertible pair



Structural Characterizations

Reflexive digraph H

H is a proper adjusted interval digraph
$\iff$ it has no symmetrically invertible pair

Structural Characterizations

Reflexive digraph H

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General digraph H

H is a proper monotone interval digraph
$\iff$ it has no symmetrically invertible pair and no induced directed cycle of length greater than one

H+Rafiey 2010

Obstructions

Current

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- Reflexive interval graphs: induced $C_{(>3)}$ and asteroidal triples

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- Reflexive interval graphs: induced $C_{(>3)}$ and asteroidal triples
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- Monotone proper interval digraphs: induced directed $C_{(>1)}$ and symmetrically invertible pairs

Obstructions

Unified

- Reflexive interval graphs: induced $C_{(>3)}$ and asteroidal triples
- Co-circular arc bigraphs: **invertible pairs**
- Adjusted interval digraphs: invertible pairs
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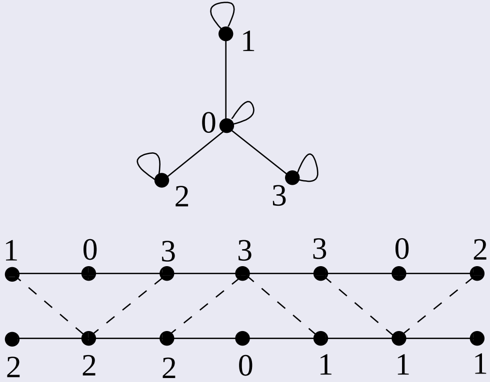
Reflexive claw is not a proper interval graph

Illustration

Reflexive claw is not a proper interval graph



Obstructions

Unified

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Revisiting Interval Graphs

Forbidden structures

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$1 \implies 2 \implies 3 \implies 1$

Taking Stock

Classes with best potential

Co-circular-arc bigraphs and adjusted interval digraphs

Taking Stock

Classes with best potential

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We are looking at these, and similar ones

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Ali Ershadi

Chronological Interval Digraphs

A chronological interval digraph H

H admits a representation by intervals I_v (for $v \in V(H)$) in which

$$v \to w \iff I_v \cap I_w \neq \emptyset$$

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$\iff$

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$O(m+n)$ recognition

Das+Francis+H+Huang 2011