# Approximating CSPs with Global Cardinality Constraints

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# Constraint Satisfaction Problems

**A classic example – Max Cut**

Given a (weighted) graph $G = (V, E)$, partition the vertices into two pieces $V = S \cup \bar{S}$ such that the number(fraction) of crossing edges $|E(S, \bar{S})|$ is maximized.

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- General Max-CSPs:
  - Domain $\{0, 1, ..., q-1\}$
  - Payoff Functions $P_i : [q]^k \mapsto [0, 1]$
  - Objective: Find an assignment that maximizes the total(average) payoff
  - Examples: Max-3SAT, Max-DiCut, Metric Labeling, Label Cover, Unique Games...

# Approximability of Max-CSPs

Following a long line of works, Raghavendra gave optimal hardness/algorithm for all Max-CSPs.

**Theorem(Raghavendra 08)**

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Raghavendra and Steurer also gave a simple and unified way to optimally round every CSP

# Beyond Local Constraints...

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  - Graph Expansion
  - Balanced Separator/Sparsest Cut
  - Densest Subgraph
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**Max-Bisection**

Given a (weighted) graph $G = (V, E)$, partition the vertices into two *equal* pieces $V = S \cup \bar{S}$ such that the number(fraction) of crossing edges $|E(S, \bar{S})|$ is maximized.

# Approximating Max-Bisection

- Approximation Ratio
  - 0.6514 [Frieze-Jerrum97]
  - 0.699 [Ye01]
  - 0.7016 [Halperin-Zwick02]
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- Almost Perfect Bisection
  - $1 - \epsilon$ v.s $1 - O(\epsilon^{1/3} \log(1/\epsilon))$

  [Guruswami-Makarychev-Raghavendra-Steurer-Zhou11]

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- Our Results
  - 0.85-approximation
  - $1 - \epsilon$ v.s $1 - O(\sqrt{\epsilon})$

# Goemans-Williamson Algorithm for Max-Cut

## SDP Relaxation

$$\max \quad \sum_{(i,j)\in E} \frac{1 - v_i \cdot v_j}{2}$$

$$s.t \quad \|v_i\| = 1$$

Rounding Scheme: Random hyperplane

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Each vertex has half probability to fall on each side of the hyperplane.

- Do we get a bisection?
- No, b/c the vertices are correlated.

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  - Can indeed be achieved
  - Similar idea is used in the SDP-based sub-exponential algorithm for Unique Games by Barak, Steurer and Raghavendra

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- $\mu_S$: Given any subset of vertices $S$ with size at most $k$, the SDP provides a probability distribution over the assignments of $S$
- $v_{\{S,\alpha\}}$: For each subset of vertices $S$ with size at most $k$ and an assignment $\alpha$ of $S$, an indicator vector vector $v_{\{S,\alpha\}}$. (In the intended solution, $v_{\{S,\alpha\}} = I$ if $S$ is assigned to be $\alpha$, $v_{\{S,\alpha\}} = 0$ otherwise)

# Lasserre's Hierarchy (cont.)

The constraints for Max-Bisection are

- For $S, T \subset V$ such that $|S \cup T| \leq k$, $\alpha \in \{0,1\}^S$, $\beta \in \{0,1\}^T$

$$v_{\{S,\alpha\}} \cdot v_{\{T,\beta\}} = \mathbb{P}_{\mu_{S\cup T}}[X_S = \alpha, X_T = \beta]$$

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$$\mathbb{E}[\mathbb{P}(X_i = 0 | X_S = \alpha)] = \frac{1}{2}$$

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The objective is to maximize

$$\mathbb{E}_{(i,j) \in E} \mathbb{P}_{\mu_{\{X_i, X_j\}}}(X_i \neq X_j)$$

# Lasserre's Hierarchy (cont.)

Remarks:

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1. The variables in Lasserre's Hierarchy are not jointly distributed (i.e, there is no way to sample the variables such that the marginal distributions are preserved)
2. Locally indistinguishable from joint distribution
3. One can *condition* on one variable and get a $(k-1)$-rounds Lasserre's solution w.r.t the conditional distribution

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- Several measure of dependence: Covariance, Correlation, Statistical Distance, Mutual Information...
- All the definitions are *local*, therefore well-defined
- We use mutual information in this work

# Information Theoretical Background

## Entropy

Let $X$ be a random variable taking value in $[q]$. The *entropy* of $X$ is defined as

$$H(X) = -\sum_{i\in[q]} \Pr(X = i) \log \Pr(X = i)$$

## Mutual Information

Let $X$ and $Y$ be two jointly distributed variables taking value in $[q]$. The *mutual information* of $X$ and $Y$ is defined as

$$I(X;Y) = \sum_{i,j\in[q]} \Pr(X = i, Y = j) \log \frac{\Pr(X = i, Y = j)}{\Pr(X = i)\Pr(Y = j)}$$

# Information Theoretical Background (cont.)

**Fact**

Mutual information $\sim 0 \Rightarrow$ Statistical distance $\sim 0$, i.e,

$$\sum_{i,j \in [q]} |\mathbb{P}(X = i, Y = j) - \mathbb{P}(X = i)\mathbb{P}(Y = j)| \sim 0$$

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## Conditional Entropy

The connection between entropy and mutual information can be formulated as:

$$H(X|Y) = H(X) - I(X;Y)$$

where $H(X|Y)$ is the *conditional entropy*.

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**Conditional Entropy**

The connection between entropy and mutual information can be formulated as:

$$H(X|Y) = H(X) - I(X;Y)$$

where $H(X|Y)$ is the *conditional entropy*.

Intuition: If the *average* mutual information is high, randomly conditioning on a variable will make some progress.

# $\alpha$-Independent Solution

We say a solution is close to being independent if the mutual information between a *random pair* of vertices is low.

**Definition**

A Lasserre's solution is $\alpha$-independent if $\mathbb{E}_{i,j}(I(X_i; X_j)) \leq \alpha$

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**Definition**

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One can construct $\alpha$-independent solution via conditioning:

1. Randomly sample $X_{i_1}, \ldots, X_{i_k}$.
2. In $t$-th step, randomly fix variable $X_i$ according to the conditional probability after the first $t-1$ fixings.
3. The algorithm terminates whenever the solution is $\alpha$-independent.

# $\alpha$-Independent Solution (cont.)

We show the algorithm terminates with an $\alpha$-independent solution w.h.p

**Proof.**

Define the potential function $\Phi$ to be the average entropy of the variables, i.e

$$\Phi = \mathbb{E}_i H(X_i)$$

In each step, the (expected) decreasing of the potential function is exactly the average mutual information

Therefore, there exists $1 \leq i \leq k$ such that the expected decreasing(average mutual information) of entropy in step $i$ is at most $1/k$

□

# Structured SDP solution

**Theorem**

For any $\alpha > 0$, we can get an $\alpha$-independent solution by conditioning on $k$-rounds Lasserre's solution for some sufficiently large $k$. ($k = poly(1/\alpha)$ suffices)

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- $v_{i,0} = P_{i,0}I + w_i$, $v_{i,1} = P_{i,1}I - w_i$ for some vector $w_i$ orthogonal to $I$

# Structured SDP solution

**Theorem**

For any $\alpha > 0$, we can get an $\alpha$-independent solution by conditioning on $k$-rounds Lasserre's solution for some sufficiently large $k$. ($k = poly(1/\alpha)$ suffices)

The SDP solution consists of:

- For each $i$, two orthogonal vectors $v_{i,0}, v_{i,1}$ such that $v_{i,0} + v_{i,1} = I$.
- $v_{i,0} = P_{i,0}I + w_i$, $v_{i,1} = P_{i,1}I - w_i$ for some vector $w_i$ orthogonal to $I$
- $w_i$ characterizes the correlation of the $i$-th variable and other variables, i.e.

  $\mathbb{P}(X_i = \alpha, X_j = \beta) - \mathbb{P}(X_i = \alpha)\mathbb{P}(X_j = \beta) = \pm(w_i \cdot w_j)$

# Rounding Algorithm

## Rounding Algorithm

1. Let $\bar{w}_i$ be the normalized $w_i$
2. Sample a standard gaussian vector $g$
3. Pick $t_i$ such that $\Phi(t_i) = P_{i,0}$
4. If $g \cdot \bar{w}_i < t_i$, assign $X_i = 0$, otherwise $X_i = 1$

Remark: the algorithm preserves the bias individually

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The analysis of the algorithm consists of two parts: balance and cut value.

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  Will show, $I(X_i; X_j) \sim 0 \Rightarrow \mathrm{Cov}(F(X_i), F(X_j)) \sim 0$

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**Proof.**

$I(X_i; X_j) \sim 0 \Rightarrow$ Statistical distance $\sim 0 \Rightarrow w_i \cdot w_j \sim 0 \Rightarrow$ $|w_i||w_j| \cos\theta(\bar{w}_i, \bar{w}_j) \sim 0$

Case 1. $\cos\theta \sim 0$

$I(g \cdot \bar{w}_i, g \cdot \bar{w}_j) \sim 0 \Rightarrow I(F(X_1), F(X_2)) \sim 0$

Case 2. $|w_i|(|w_j|) \sim 0$

The variable is highly biased, since the rounding algorithm preserve the bias, we're done

# Value of the Cut

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# Dictatorship Test Gadget

**Dictatorship Test from $\alpha$-independent SDP gap**

1. Sample an edge $e = (u, v) \in E$
2. Sample $x, y$ from the distribution $\mu_e^R$
3. Perturb each coordinate of $x, y$ independently with probability $\epsilon$
4. Add an edge between $x$ and $y$
5. Split each vertex w.r.t its weight

# Dictatorship Test Gadget (cont.)

The Dictatorship gadget satisfies:

- Completeness: Dictator cuts are bisections with value $\approx$ sdp(G)
- Soundness: If a function $F : \{\pm 1\}^R \mapsto [-1, 1]$ is a bisection (i.e, $\mathbb{E}(F(x)) = 0$) and all its influences are at most $\tau$, i.e

$$\mathsf{Inf}_k^{\mu_i} \leq \tau$$

then the value of $F$ is at most $opt(G) + C(\tau, \epsilon)$.

# Dictatorship Test Gadget (cont.)

**Proof.**

Completeness:

- Balance: SDP Constraints
- Value: Same as in [Rag08]

Soundness: can use the function to round the SDP solution

- Balance
  - Expected Balance: $\approx 1/2$ (Invariance Principle)
  - Concentration: $\alpha$-independence
- Value: Same as in [Rag08]

$\square$

# Summary

- SDP hierarchy helps when global cardinality constraints are imposed
- Simple framework to approximate CSPs with global constraints (0.92-approximation of 2-SAT)
- As an attempt to prove matching hardness, we give a construction of dictatorship test via SDP gap instance

Open Questions

- Is Max-Bisection 0.878 approximable?
- Optimal hardness/algorithm for every Max-CSP with global cardinality constraints?

# Thanks

Questions?