Affleck-Kennedy-Lieb-Tasaki states as a resource for universal quantum computation

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Refs. (1) **Wei**, Affleck & Raussendorf, PRL 106, 070501 (2011) and arXiv:1009.2840
(2) **Wei**, Raussendorf & Kwek, arXiv:1105.5635
(3) Li, Browne, Kwek, Raussendorf & **Wei**, PRL 107,060501 (2011)
(4) Raussendorf & **Wei**, to appear in *Annual Review of Condensed Matter Physics*

Fields, Aug. 8, 2011

Outline

I. Introduction

II. Cluster state quantum computation
(a.k.a. one-way or measurement-based quantum computation)

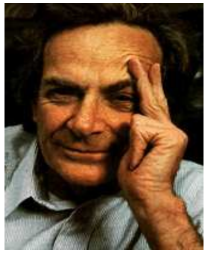
motivations

III. Resource states for quantum computation: ground states of two-body interacting Hamiltonians

- ❑ 1D AKLT states (not universal)
- ❑ 2D AKLT state on honeycomb (universal)
- ❑ 2D Cai-Miyake-Dur-Briegel state (universal)

V. Summary

Quantum computation



Feynman ('81): "Simulating Physics with (Quantum) Computers"

→ Idea of quantum computer further developed by Deutsch ('85), Lloyd ('96), …

1st conference on Physics and Computation, 1981

Quantum computation

Shor ('94): quantum mechanics enables fast factoring

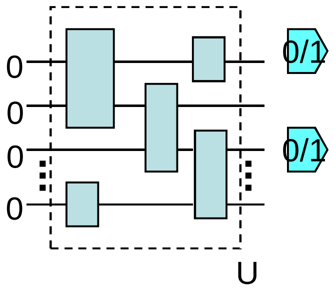
18070820886874048059516561644059055662781025167694013491701270214
50056662540244048387341127590812303371781887966563182013214880557

=(39685999459597454290161126162883786067576449112810064832555157243)
x (45534498646735972188403686897274408864356301263205069600999044599)

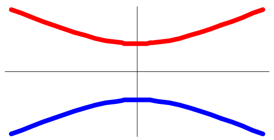
➔ Ever since: rapid growing field of quantum information & computation

❑ Quantum computational models

1. Circuit model (includes topological):

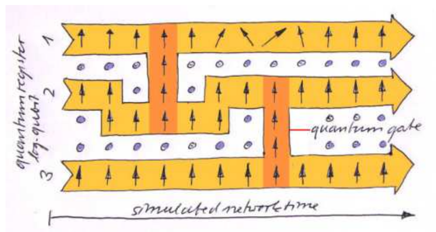


2. Adiabatic QC:



[*Farhi, Goldstone, Gutmann & Sipster* '00]

3. Measurement-based:

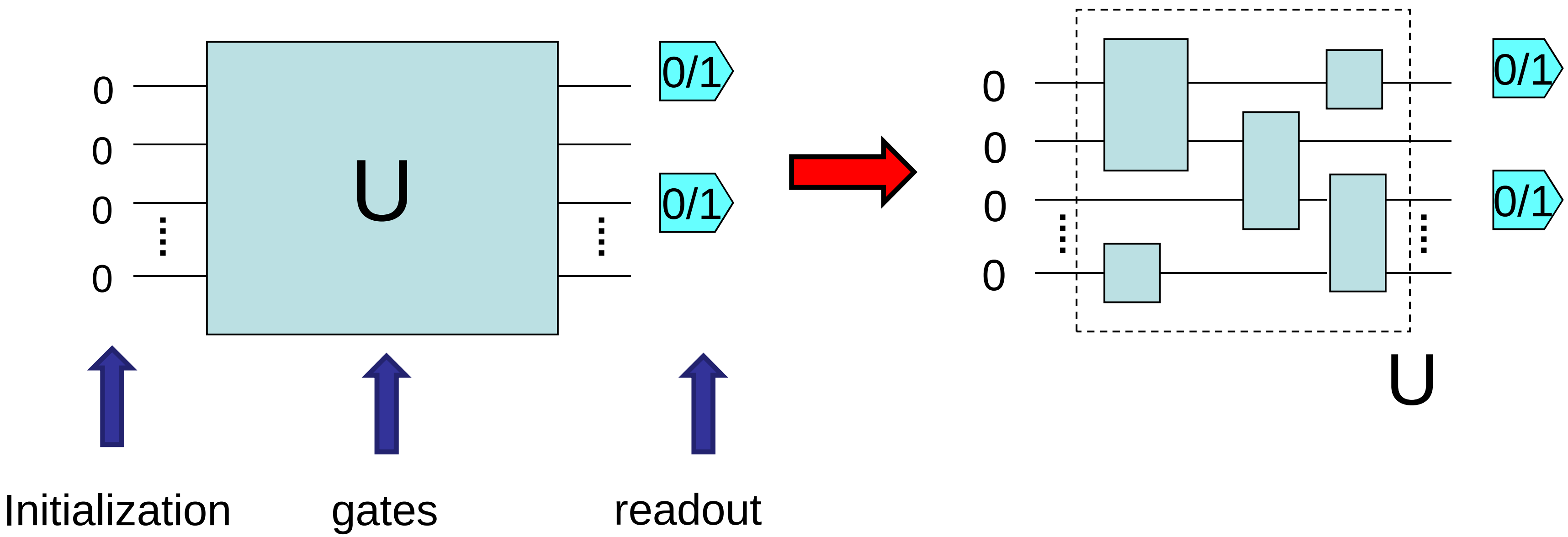


[*Raussendorf &Briegel* '01]

[*Gottesman & Chuang*, '99
Childs, Leung & Nielsen '04]

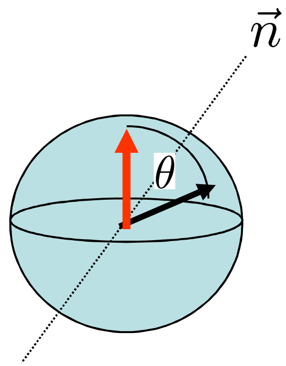
Circuit Model

- Key point: Decompose any unitary U into sequence of building blocks (universal gates): one + two-qubit gates



Single-qubit Unitary gates

$$|\psi\rangle \to e^{i\vec{n}\cdot\vec{\sigma}\theta/2}|\psi\rangle = \left(\cos\frac{\theta}{2}I + i\sin\frac{\theta}{2}\vec{n}\cdot\vec{\sigma}\right)|\psi\rangle$$



- Only need a finite set of gates:

$$H \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad S \equiv \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \qquad T \equiv \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

Two-qubit unitary gates

- Four by four unitary matrices (acting on the two qubits)

  - ✓ Control-NOT gate:
    0 0 → 0 0
    0 1 → 0 1
    1 0 → 1 1
    1 1 → 1 0

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \left(\begin{array}{c|c} I & 0 \\ \hline 0 & X \end{array}\right)$$

  - ✓ Control-Phase gate:
    0 0 → 0 0
    0 1 → 0 1
    1 0 → 1 0
    1 1 → -1 1

$$\mathrm{CP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \left(\begin{array}{c|c} I & 0 \\ \hline 0 & Z \end{array}\right)$$

- Generate entanglement

$$|+\rangle|+\rangle = \frac{(|0\rangle + |1\rangle)}{\sqrt{2}}\frac{(|0\rangle + |1\rangle)}{\sqrt{2}} \xrightarrow{\text{CP}} \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$$
$$= \frac{1}{\sqrt{2}}(|0\rangle|+\rangle + |1\rangle|-\rangle) \neq |\phi_1\rangle|\phi_2\rangle$$

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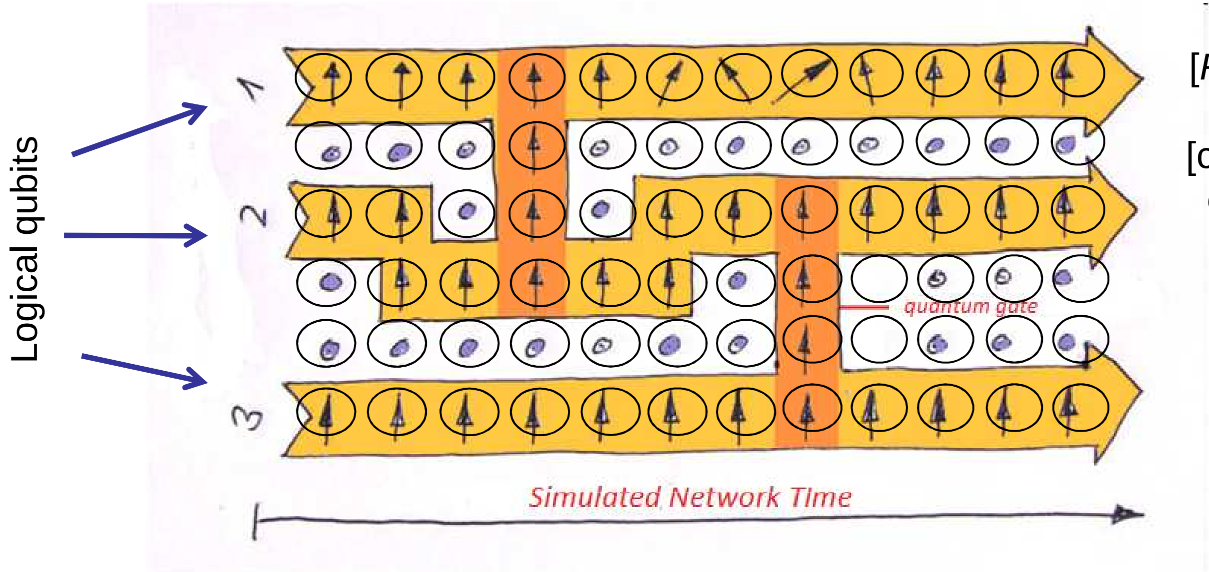
Quantum computation by measurement



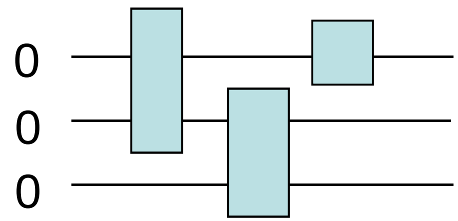
[*Raussendorf & Briegel* '01]

[c.f. *Gottesman & Chuang*, '99
Childs, Leung & Nielsen '04]

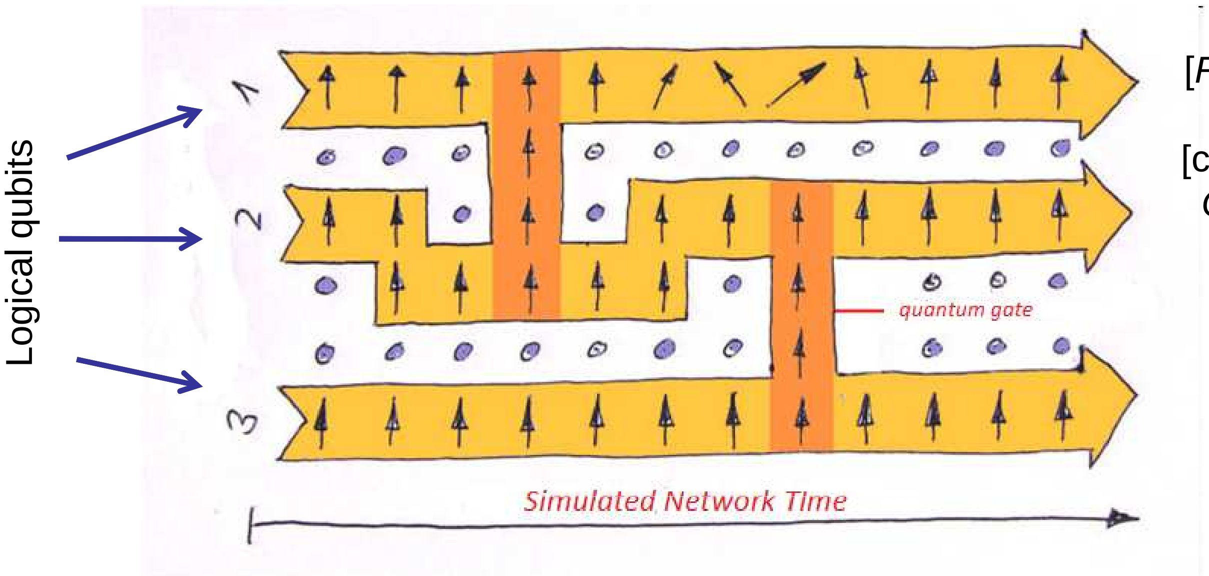
- Use cluster state $|\mathcal{C}\rangle$ as computational resource

- Information is written on to $|\mathcal{C}\rangle$, processed and read out all by ***single*** spin measurements

- Can simulate quantum computation by circuit models (i.e. universal QC)



Q Computation by measurement: intuition



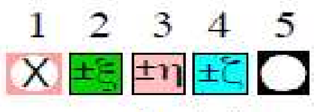
[*Raussendorf & Briegel* '01]

[c.f. *Gottesman & Chuang*, '99
Childs, Leung & Nielsen '04]

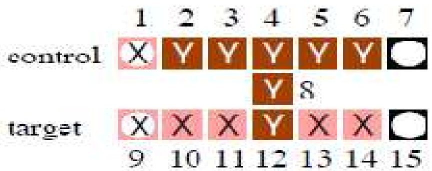
- How can single-spin measurements simulate unitary evolution?

  ➔ Entanglement (➔ state and gate teleportation)

- Key ingredients: simulating 1- and 2-qubit gates



general rotation



CNOT-gate

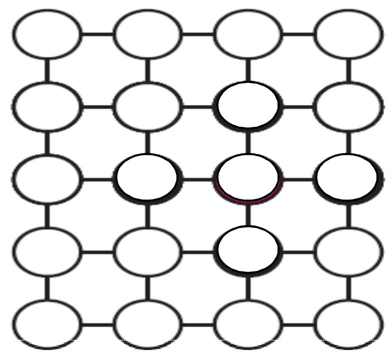
Cluster state: entangled resource

- Cluster state

[*Briegel & Raussendorf* '00]

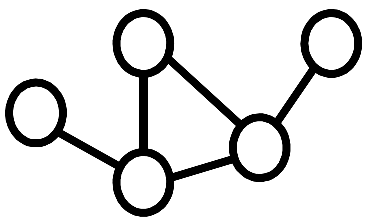
$$|\mathcal{C}\rangle = \prod_{\text{edge } \langle i,j \rangle} CP_{ij}\left(|+\rangle|+\rangle\cdots|+\rangle\right)$$

Control-Phase gate applied to pairs of qubits linked by an edge



$$CP_{ij} = |0\rangle\langle 0|_i \otimes I_j + |1\rangle\langle 1|_i \otimes \sigma_z^{[j]}$$

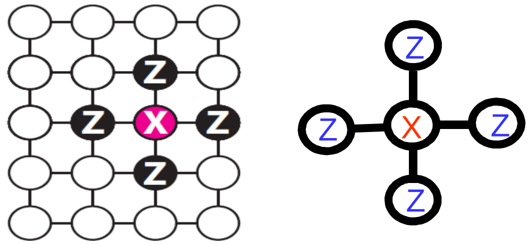
- Can be defined on any graph



➔ Resulting state is called graph state

Cluster and graph states as ground states

- Cluster state $|C\rangle$ = graph state on square lattice



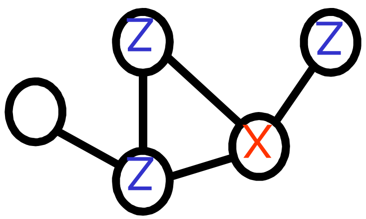
[*Raussendorf &Briegel*, 01']

$$|\mathcal{C}\rangle = |\mathcal{C}\rangle$$

$$H_G = -\sum_{\text{site } v} K_v \qquad \text{with} \qquad \boxed{K_v \equiv X_v \bigotimes_{u \in \mathrm{Nb}(v)} Z_u}$$

neighbors

- Graph state: defined on a graph [*Hein, Eisert & Briegel* 04']



➔ Graph state is the unique ground state of H_G

$$\boxed{K_v|G\rangle = |G\rangle, \ \forall \, \text{site } v}$$

Note: X, Y & Z are Pauli matrices

Creating cluster states?

1. Active coupling: to construct Control-Phase gate

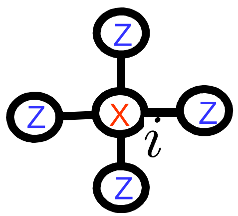
   (by Ising interaction) $$|\mathcal{C}\rangle = \prod_{\text{edge } \langle i,j\rangle} CP_{ij}\big(|+\rangle|+\rangle\cdots|+\rangle\big)$$

   $$CP_{12} = e^{-i\frac{\pi}{4}(1-\sigma_Z^{(1)})(1-\sigma_Z^{(2)})} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

   [Implemented in cold atoms: *Greiner et al.* Nature '02]

   ➔ Not necessarily have such control

2. Cooling: if cluster states are unique ground states of certain simple Hamiltonians with a gap

   $$H = -\sum_i$$ 

   - Cluster state is the unique ground state of five-body interacting Hamiltonian (cannot be that of two-body)☹

   [*Nielsen* '04]

What about other states?

Ground states as universal resource states?

- First, finding universal resource states is hard (they are rare)

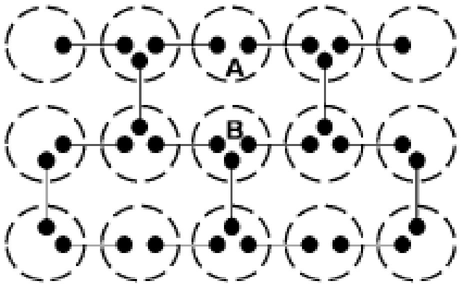
  [*Gross, Flammia & Eisert* PRL '09; *Bemner, Mora & Winter*, PRL '09]

- Second, need to construct short-ranged Hamiltonians so that they are unique ground states

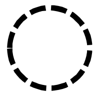
  - So finding ground states as universal resource states is hard

A tour-de-force example

❖ TriCluster state (6-level) [*Chen, Zeng, Gu, Yoshida & Chuang*, PRL'09]



 $= |00\rangle + |01\rangle + |10\rangle - |11\rangle$

 $P_{\text{Tric}} = |\tilde{0}\rangle\langle 000| + |\tilde{1}\rangle\langle 111| + |\tilde{2}\rangle\langle 100|$

$+|\tilde{3}\rangle\langle 011| + |\tilde{4}\rangle\langle 010| + |\tilde{5}\rangle\langle 101|$

$$H^{\star}_{triC} = \sum_{a} \left(h_{ab} + h_{ba} + h_{\frac{b}{a}} \right)$$

$$\begin{aligned}
h_{ab} \; = & \\
& 2(2S_{a_z}-5)(2S_{a_z}-3)(2S_{a_z}-1)(2S_{a_z}+1)(4S_{a_z}+11) \\
& (2S_{b_z}+5)(2S_{b_z}+3)(2S_{b_z}-1)(2S_{b_z}+1)(4S_{b_z}-11) \\
- \; & 75\sqrt{2}S_{a_+}(2S_{a_z}-5)(2S_{a_z}+3)(2S_{a_z}-1)(2S_{a_z}+1) \\
& (48S^4_{b_z}+64S^3_{b_z}-280S^2_{b_z}-272S_{b_z}+67) \\
+ \; & 75\sqrt{2}(48S^4_{a_z}-64S^3_{a_z}-280S^2_{a_z}+272S_{a_z}+67) \\
& S_{b_+}(2S_{b_z}-5)(2S_{b_z}-3)(2S_{b_z}-1)(2S_{b_z}+3) \\
+ \; & 4\sqrt{10}S^3_{a_+}(2S_{a_z}-1)(2S_{a_z}-3)\times \\
& (128S^5_{b_z}+560S^4_{b_z}-2840S^2_{b_z}-3848S_{b_z}+675) \\
+ \; & 4\sqrt{10}(128S^5_{a_z}-560S^4_{a_z}+2840S^2_{a_z}-3848S_{a_z}-675) \\
& S^3_{b_+}(2S_{b_z}-5)(2S_{b_z}-3)+h.c.
\end{aligned}$$

$$\begin{aligned}
h_{\frac{b}{a}} \; = & \\
& -25(2S_{a_z}-5)(2S_{a_z}-3)(2S_{a_z}+3)(2S_{a_z}+5) \\
+ \; & 25S^3_{a_+}(2S_{a_z}-5)(2S_{a_z}-1) \\
& (224S^5_{b_z}-16S^4_{b_z}-1968S^3_{b_z}+40S^2_{b_z}+3550S_{b_z}-9) \\
- \; & 12S^5_{a_+} \\
& (416S^5_{b_z}-80S^4_{b_z}-3600S^3_{b_z}+520S^2_{b_z}+5994S_{b_z}-125) \\
+ \; & h.c.+(a \Leftrightarrow b)\,,
\end{aligned}$$

Ground states as universal resource states?

- First, finding universal resource states is hard

  [*Gross, Flammia & Eisert* PRL ’09; *Bemner, Mora & Winter*, PRL ‘09]

- Second, need to construct short-ranged Hamiltonians so that they are unique ground states

- Alternatively, first find ground states of short-ranged Hamiltonians & check whether they are universal resources

  - The family of Affleck-Kennedy-Lieb-Tasaki (AKLT) states provide a good framework

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- 2D AKLT state on honeycomb (universal)
- 2D Cai-Miyake-Dur-Briegel state (universal)

V. Summary

Affleck-Kennedy-Lieb-Tasaki states

[*AKLT* '87,88]

- States of spin S=1,3/2, or higher (defined on any graph)
  - ➔ S= (# of neighboring vertices) / 2

- Unique* ground states of two-body isotropic Hamiltonians

$$H = \sum_{\langle i,j \rangle} f(\vec{S}_i \cdot \vec{S}_j)$$ *f(x)* is a polynomial

- Important progress on 1D spin-1 AKLT state for QC:

[*Gross & Eisert, PRL* '07] [*Brennen & Miyake, PRL* '09]

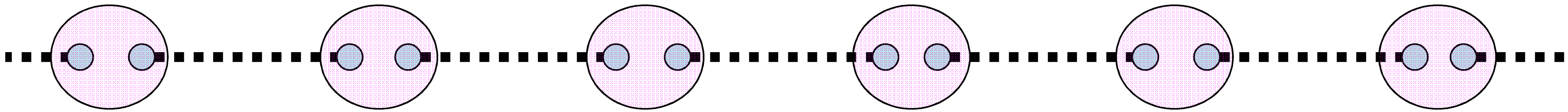
➔ Can be used to implement rotations on single-qubits

*with appropriate boundary conditions

1D spin-1 AKLT state

[*AKLT* '87,'88]

- Two virtual qubits per site (thus S=2/2)



$$\frac{1}{2}\otimes\frac{1}{2} = 0 \oplus 1 \quad \begin{cases} |00\rangle \Rightarrow |1,1\rangle \\ |11\rangle \Rightarrow |1,-1\rangle \\ (|01\rangle + |10\rangle)/\sqrt{2} \Rightarrow |1,0\rangle \end{cases}$$

Project into symmetric subspace of two spin-1/2 (qubits)

singlet

$$|01\rangle - |10\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

- Ground state of two-body interacting Hamiltonian (with a gap)

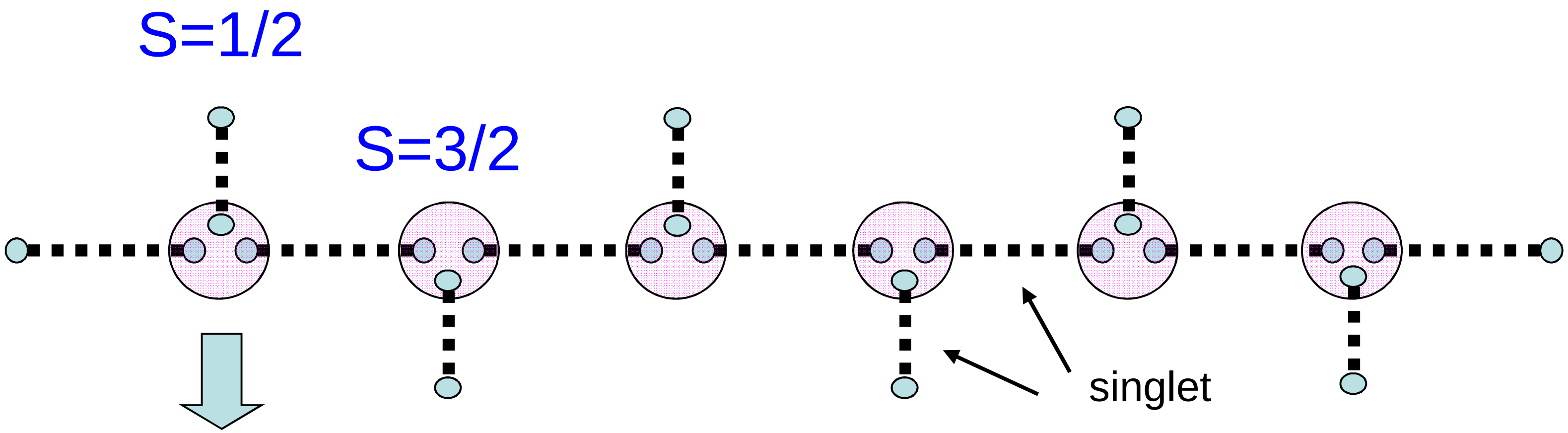
$$H = \sum_i \left[\vec{S}_i\cdot\vec{S}_{i+1} + \frac{1}{3}(\vec{S}_i\cdot\vec{S}_{i+1})^2 + \frac{2}{3}\right] = 2\sum_i \hat{P}^{(S=2)}_{i,i+1}$$

projector onto S=2

➔ Can realize rotation on one logical qubit by measurement
(not sufficient for universal QC)

[*Gross & Eisert, PRL* '07] [*Brennen & Miyake, PRL* '09]

1D mixed spin-3/2 & spin-1/2 quasichain



Project into
symmetric subspace
of three spin-1/2 (qubits)

$$|000\rangle \leftrightarrow \left|\frac{3}{2},\frac{3}{2}\right\rangle \qquad |W\rangle \equiv \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle) \leftrightarrow \left|\frac{3}{2},\frac{1}{2}\right\rangle$$

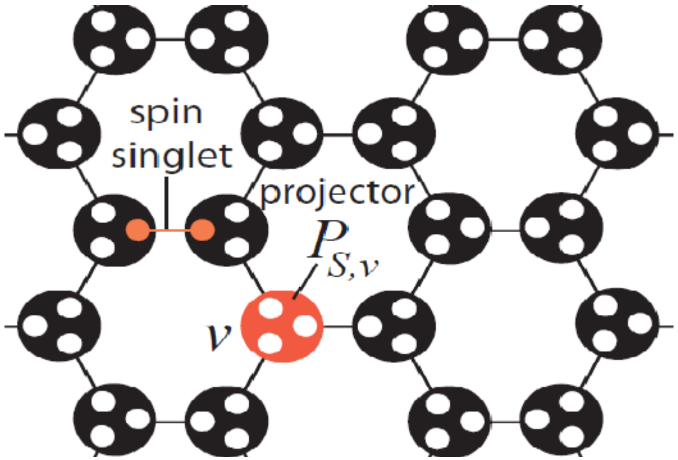
$$|111\rangle \leftrightarrow \left|\frac{3}{2},-\frac{3}{2}\right\rangle \qquad |\overline{W}\rangle \equiv \frac{1}{\sqrt{3}}(|110\rangle + |101\rangle + |011\rangle) \leftrightarrow \left|\frac{3}{2},-\frac{1}{2}\right\rangle$$

- Ground state of two-body interacting Hamiltonian (with a gap)

$$H = \sum_{i=1}^{N-1} P^{S=3}_{A_i,A_{i+1}} + \sum_{i=1}^{N} P^{S=2}_{A_i,b_i} + P^{S=2}_{A_1,b_0} + P^{S=2}_{A_N,b_{N+1}}$$

➔ Can realize rotation on one logical qubit by measurement
(not sufficient for universal QC) [*Cai et al. PRA* ‘10]

Spin-3/2 AKLT state on honeycomb lattice



- Unique ground state of

$$H = \sum_{\text{edge } \langle i,j \rangle} \left[\vec{S}_i \cdot \vec{S}_j + \frac{116}{243}(\vec{S}_i \cdot \vec{S}_j)^2 + \frac{16}{243}(\vec{S}_i \cdot \vec{S}_j)^3 + \frac{55}{108} \right]$$

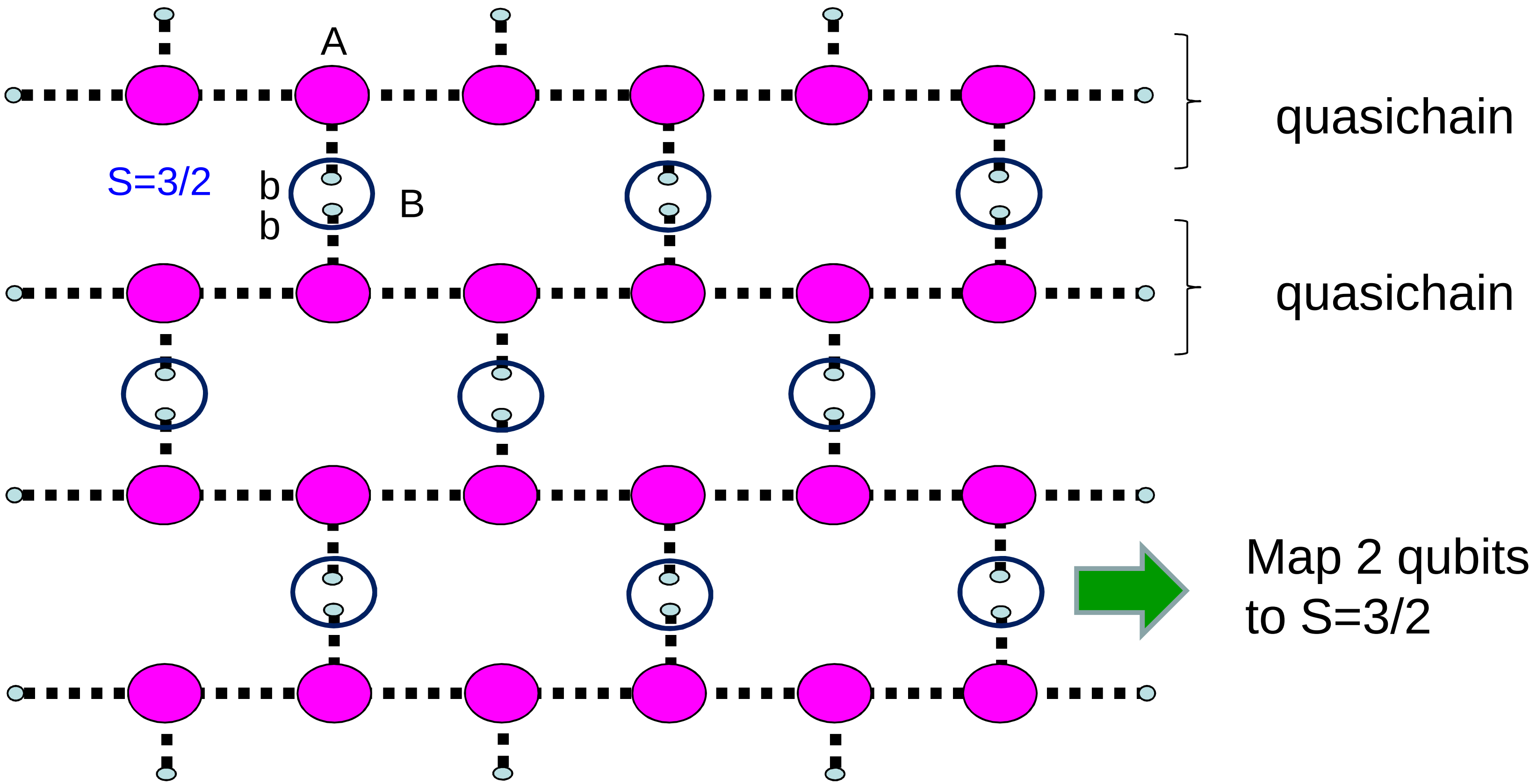
- We show that the spin-3/2 2D AKLT state on honeycomb lattice *is* a universal resource state

[*<u>Wei</u>, Affleck & Raussendorf,* PRL106, 070501 (2011)]

[Alternative proof: *Miyake, Ann Phys* (2011)]

2D Cai-Miyake-Dur-Briegel state

[*Cai, Miyake, Dür & Briegel* ',PRA'10]



➔ No longer rotationally invariant; not AKLT state

➔ But universal for quantum computation

[*Cai, Miyake, Dür & Briegel* ',PRA'10] [*Wei,Raussendorf & Kwek*,arXiv'11]

Unified understanding of these resource states

They can be locally converted to a cluster state (known resource state) in the same dimension:

➔ *Unveiling cluster states hidden in these AKLT / AKLT-like states*

- Spin 1 (2 levels) or 3/2 (4 levels) → Spin ½ (2 levels)?

  ➔ Need "projection" into smaller subspace

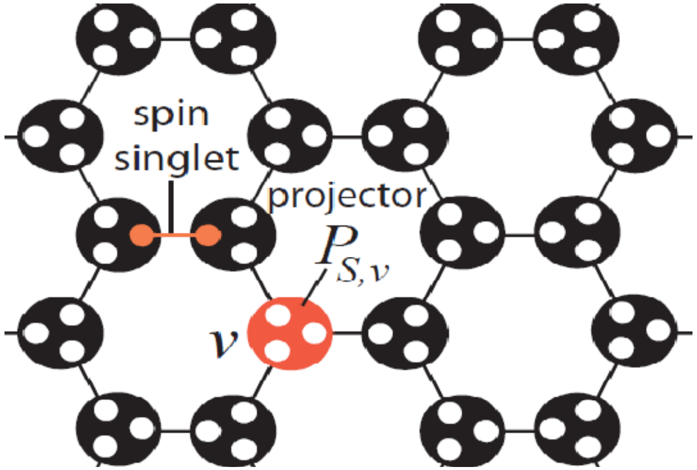
- We use generalized measurement (or POVM)

  ➔ Give rise to a graph state; but random outcome modifies the graph

- Use percolation argument (if necessary):

  ➔ typical random graph state converted to cluster state

Now focus on the spin-3/2 honeycomb case



Spin 3/2 and three virtual qubits

- Addition of angular momenta of 3 spin-1/2's

$$\frac{1}{2}\otimes\frac{1}{2}=0\oplus 1 \qquad \frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}=\frac{1}{2}\oplus\frac{1}{2}\oplus\frac{3}{2}$$

Symmetric subspace

- The four basis states in the symmetric subspace

$$|000\rangle \leftrightarrow \left|\frac{3}{2},\frac{3}{2}\right\rangle$$

$$|W\rangle \equiv \frac{1}{\sqrt{3}}(|001\rangle+|010\rangle+|100\rangle) \leftrightarrow \left|\frac{3}{2},\frac{1}{2}\right\rangle$$

$$|\overline{W}\rangle \equiv \frac{1}{\sqrt{3}}(|110\rangle+|101\rangle+|011\rangle) \leftrightarrow \left|\frac{3}{2},-\frac{1}{2}\right\rangle$$

$$|111\rangle \leftrightarrow \left|\frac{3}{2},-\frac{3}{2}\right\rangle$$

Effective 2 levels of a qubit

- Projector onto symmetric subspace

$$P_{S,v}=|000\rangle\langle 000|+|111\rangle\langle 111|+|W\rangle\langle W|+|\overline{W}\rangle\langle\overline{W}| \leftrightarrow I_{3/2}$$

Generalized measurement (POVM)

$$F_{v,z} = \sqrt{\frac{2}{3}}\left(\left|\frac{3}{2}\right\rangle\left\langle\frac{3}{2}\right|_z + \left|-\frac{3}{2}\right\rangle\left\langle-\frac{3}{2}\right|_z\right) = \frac{1}{\sqrt{6}}\left(S_z^2 - \frac{1}{4}\right)$$

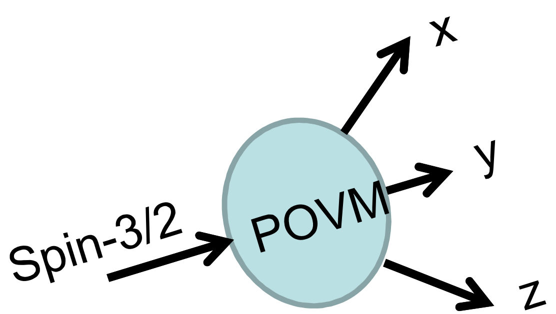
$$F_{v,x} = \sqrt{\frac{2}{3}}\left(\left|\frac{3}{2}\right\rangle\left\langle\frac{3}{2}\right|_x + \left|-\frac{3}{2}\right\rangle\left\langle-\frac{3}{2}\right|_x\right) = \frac{1}{\sqrt{6}}\left(S_x^2 - \frac{1}{4}\right)$$

v: site index
$$F_{v,y} = \sqrt{\frac{2}{3}}\left(\left|\frac{3}{2}\right\rangle\left\langle\frac{3}{2}\right|_y + \left|-\frac{3}{2}\right\rangle\left\langle-\frac{3}{2}\right|_y\right) = \frac{1}{\sqrt{6}}\left(S_y^2 - \frac{1}{4}\right)$$

[*Wei*,*Affleck* & *Raussendorf* '10; *Miyake* '10]

➔ Three elements satisfy: $F_{v,x}^{\dagger}F_{v,x} + F_{v,y}^{\dagger}F_{v,y} + F_{v,z}^{\dagger}F_{v,z} = I_v$

- POVM outcome (*x*,*y*, or *z*) is random (a$_v$ ={x,y,z} ϵ *A* for all sites v)



➔ effective 2-level system

$$\left|\frac{3}{2}\right\rangle_{a_v} \leftrightarrow \left|000\right\rangle, \quad \left|-\frac{3}{2}\right\rangle_{a_v} \leftrightarrow \left|111\right\rangle$$

➔ a_v : new quantization axis

$$\overline{Z} \equiv \left|\frac{3}{2}\right\rangle\left\langle\frac{3}{2}\right|_{a_v} - \left|-\frac{3}{2}\right\rangle\left\langle-\frac{3}{2}\right|_{a_v} \qquad \overline{X} \equiv \left|\frac{3}{2}\right\rangle\left\langle-\frac{3}{2}\right|_{a_v} + \left|-\frac{3}{2}\right\rangle\left\langle+\frac{3}{2}\right|_{a_v}$$

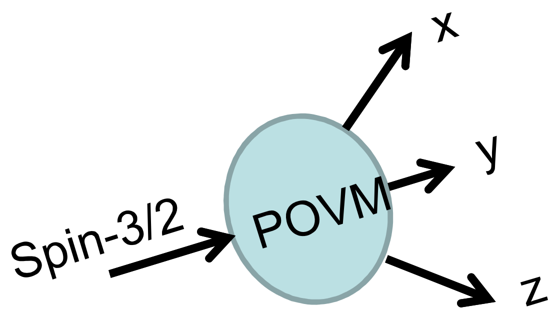
➔ state becomes $|\Phi\rangle \longrightarrow F_{v,a_v}|\Phi\rangle$

Post-POVM state

- Outcome a_v ={x,y,z} ϵ *A* for all sites v

$$|\Psi(\mathcal{A})\rangle = \bigotimes_v F_{v,a_v} |\Phi_{\mathrm{AKLT}}\rangle$$

$$\sim \bigotimes_v \left(S^2_{v,a_v} - \frac{1}{4}\right) |\Phi_{\mathrm{AKLT}}\rangle$$



[*Wei, Affleck & Raussendorf* , arxiv'10 & PRL'11]

➔ What is this state?

The random state is an encoded graph state

[*Wei, Affleck & Raussendorf* , arxiv'10 & PRL'11]

- Outcome a_v ={x,y,z} ϵ *A* for all sites v

$$|\Psi(\mathcal{A})\rangle = \bigotimes_v F_{v,a_v} |\Phi_{\text{AKLT}}\rangle \sim \bigotimes_v \left(S_{v,a_v}^2 - \frac{1}{4} \right) |\Phi_{\text{AKLT}}\rangle$$

- Encoding: effective two-level (qubit) is delocalized to a few sites

  ➔ Property of AKLT ("antiferromagnetic" tendency) gives us insight on encoding

- What is the graph? Isn't it honeycomb?

  ➔ Due to delocalization of a "logical" qubit, the graph is modified

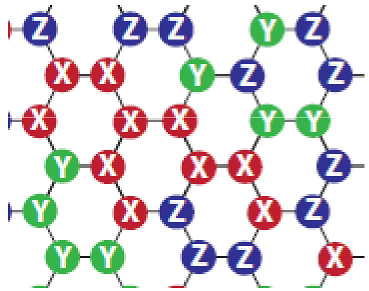
Encoding of a qubit: AFM ordering

- AKLT: Neighboring sites cannot have the same $S_a = \pm 3/2$

[*AKLT* ’87,’88]

➔ Neighboring sites with same POVM outcome *a* = x, y or z: only two AFM orderings (call these site form a ***domain***):

$$|\bar{0}\rangle \equiv \left|\frac{3}{2}, -\frac{3}{2}, \frac{3}{2}, -\frac{3}{2}, \cdots \right\rangle_a \quad \text{or} \quad |\bar{1}\rangle \equiv \left|-\frac{3}{2}, \frac{3}{2}, -\frac{3}{2}, \frac{3}{2}, \cdots \right\rangle_a$$



➔ Form the basis of a qubit

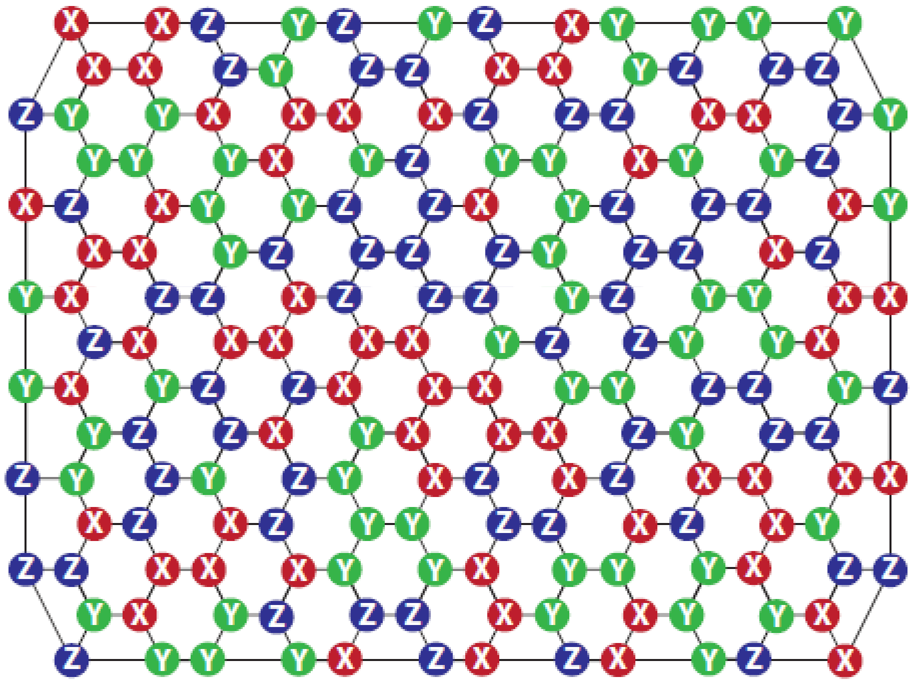
- Effective Pauli Z and X operators become (extended)

$$\overline{Z} = |\bar{0}\rangle\langle\bar{0}| - |\bar{1}\rangle\langle\bar{1}| \qquad \overline{X} = |\bar{0}\rangle\langle\bar{1}| + |\bar{1}\rangle\langle\bar{0}|$$

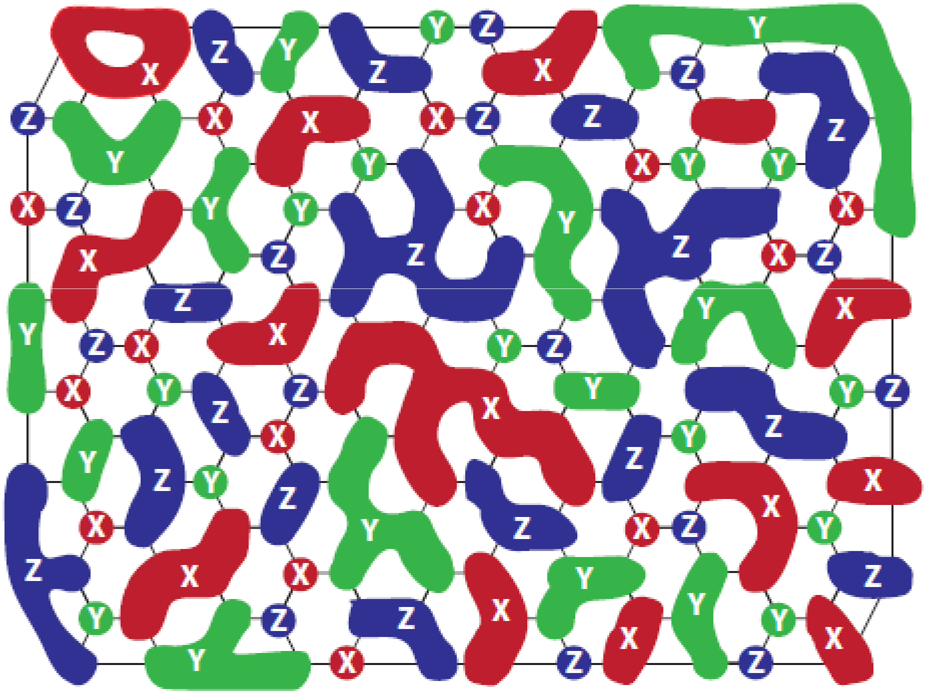
- A domain can be reduced to a single site by measurement

➔ Regard a domain as a single qubit

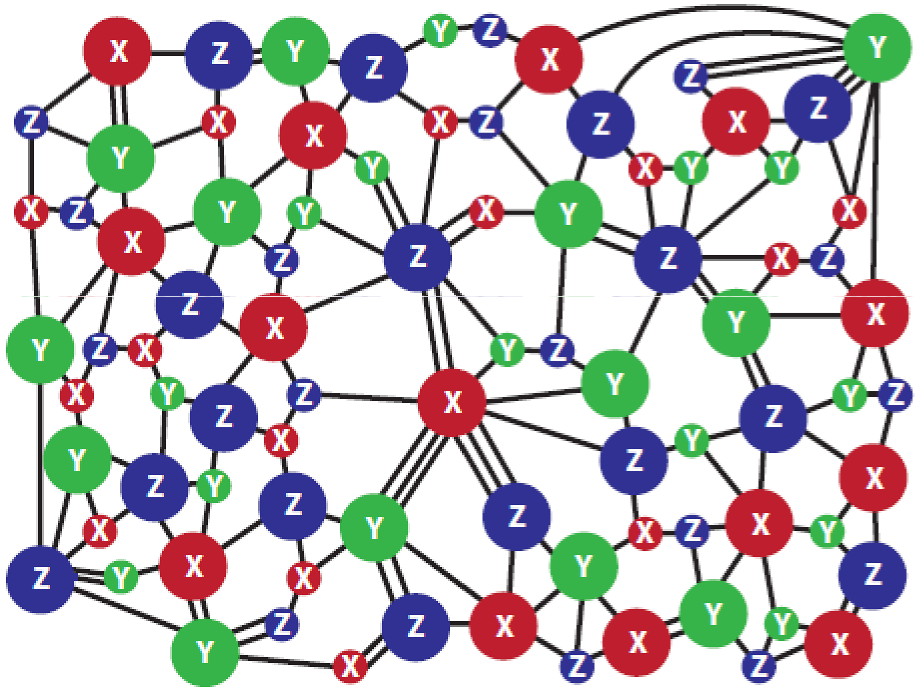
Perform generalized measurement:
mapping spin-3/2 to effective spin-1/2



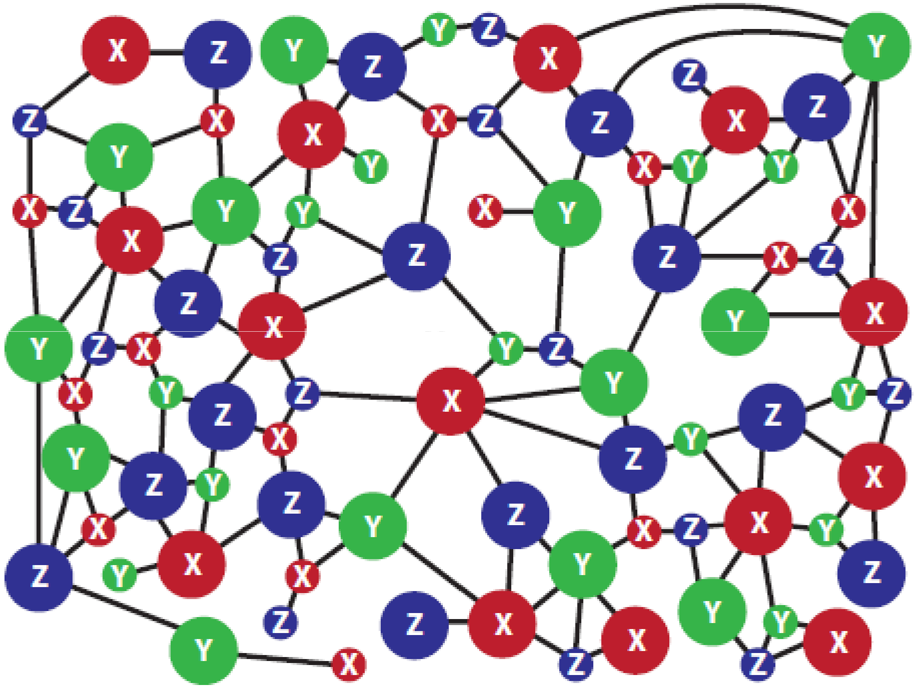
Perform generalized measurement: mapping spin-3/2 to effective spin-1/2



Perform generalized measurement: mapping spin-3/2 to effective spin-1/2

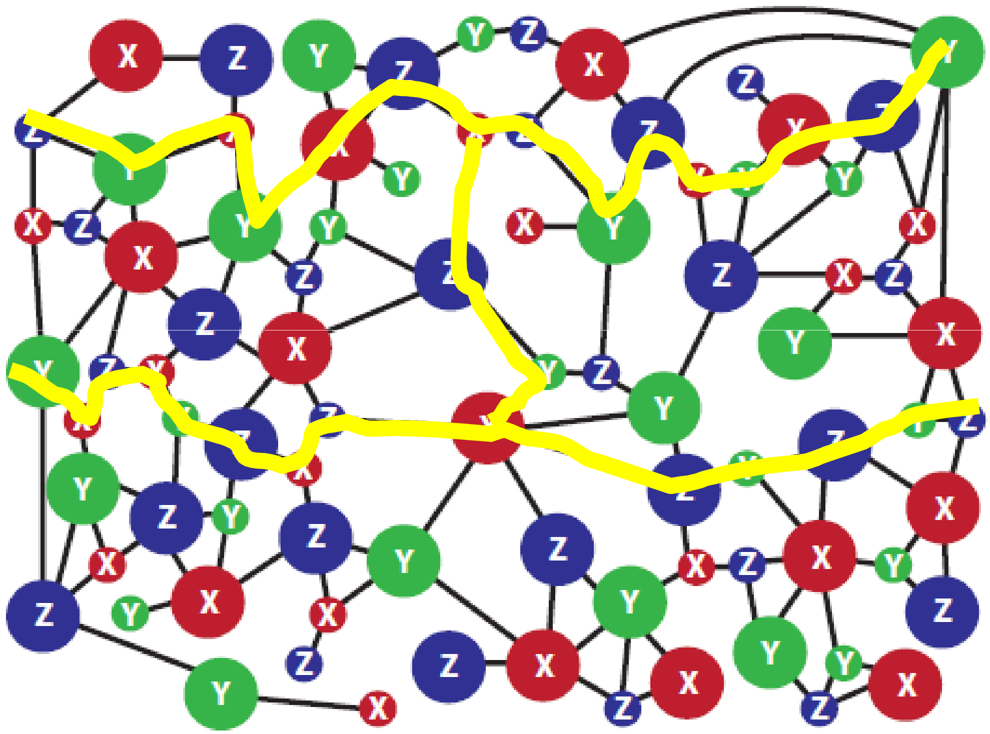


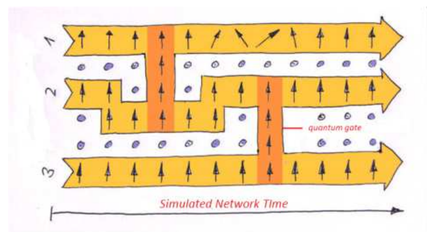
The resulting state is a “cluster” state on random graph



➔ The graph of the graph state

➔ Quantum computation can be implemented on such a (random) graph state

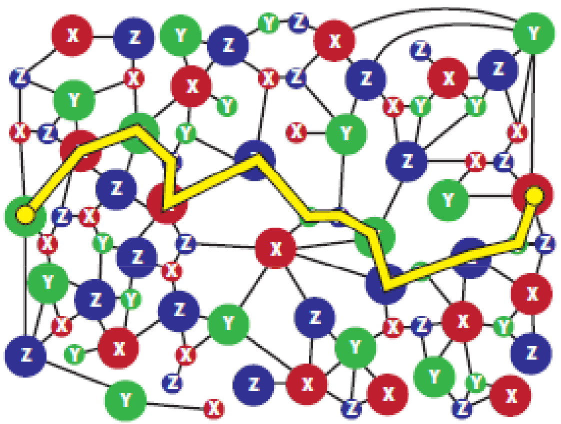


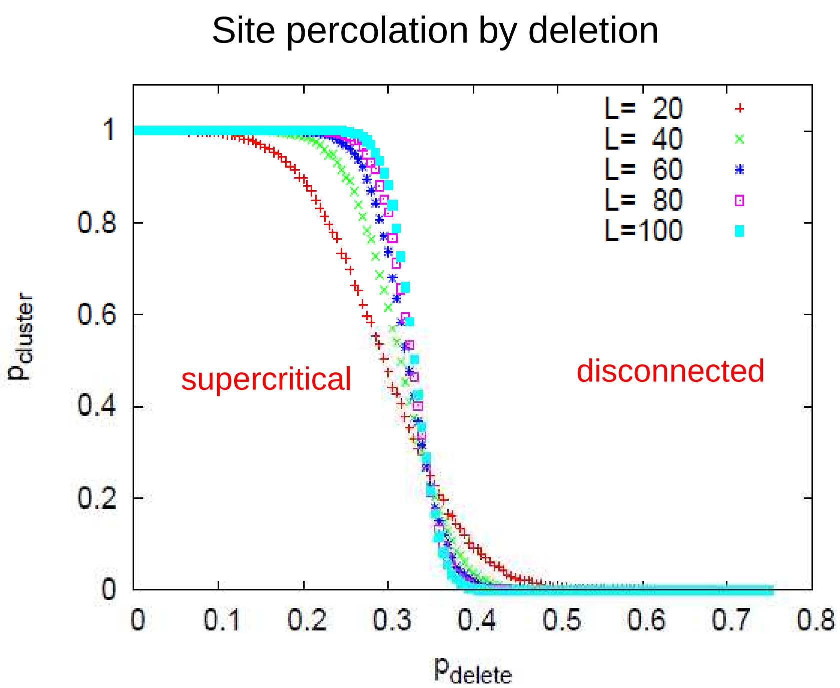


- Wires define logical qubits, links give CNOT gates
- Sufficient number of wires if graph is supercritical (percolation)

Robustness: finite percolation threshold

- Typical graphs are in percolated (or supercritical) phase



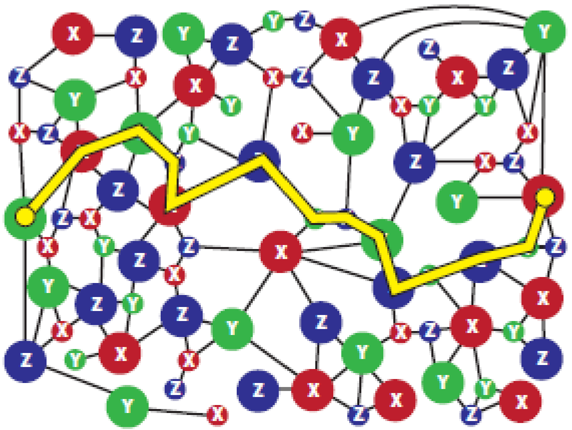


- C.f. Site perc threshold:
  Square: 0.593, honeycomb:0.697

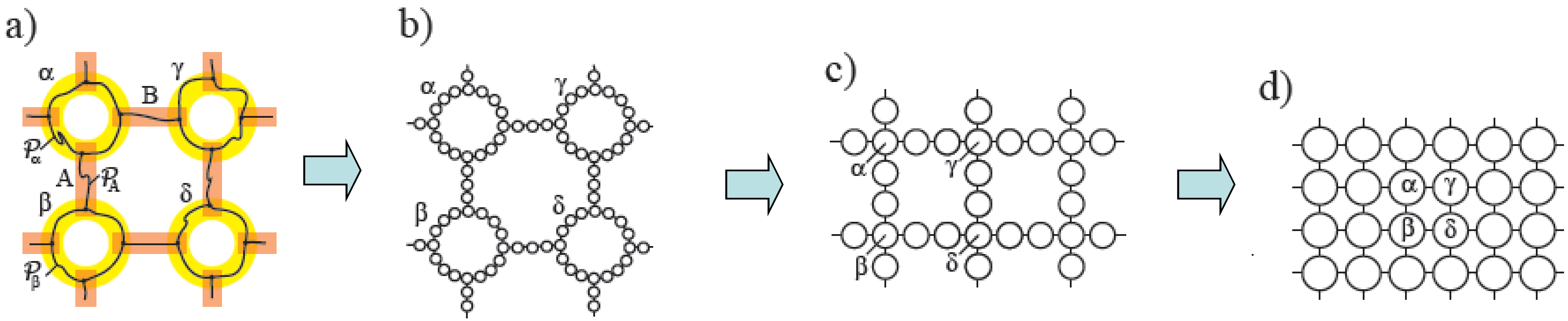
➔ threshold ≈1-0.33=0.67

➔ Sufficient (macroscopic) number of traversing paths exist

Convert graph states to cluster states



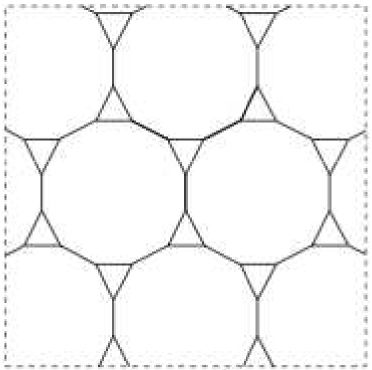
- Can identity graph structure and trim it down to square



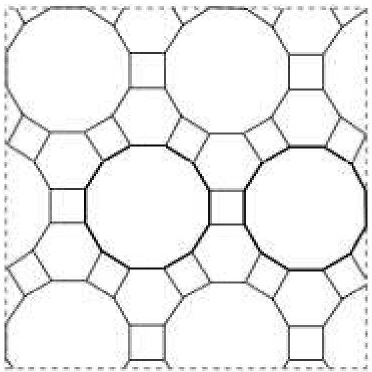
➔ Thus we have shown the 2D AKLT state on hexagonal lattice is a universal resource

Other 2D AKLT states expected to be universal resources

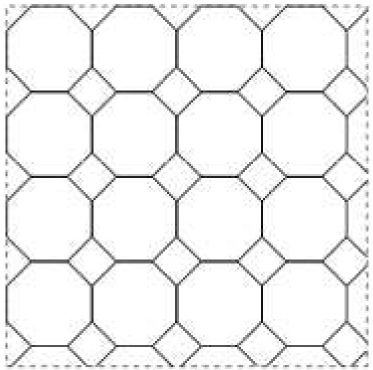
- Trivalent Achimedean lattices (in addition to honeycomb):



$(3, 12^2)$



$(4, 6, 12)$



$(4, 8^2)$

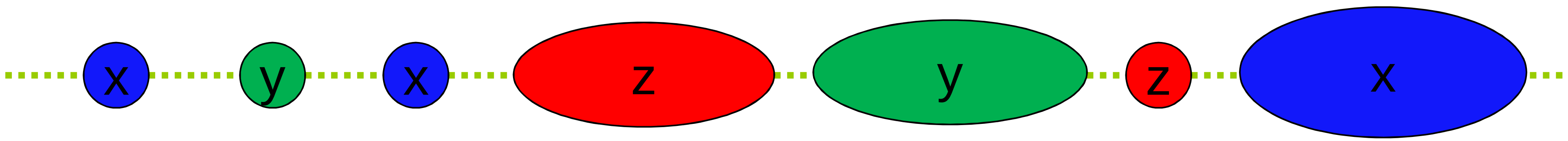
Bond percolation threshold > 2/3:	≈0.7404	≈0.694	≈0.677

1D spin-1 AKLT state ➔ cluster state

➔ POVM

$$F_{v,z} = \sqrt{\frac{1}{2}}(|+1\rangle\langle+1|_z + |-1\rangle\langle-1|_z) \quad = \quad \sqrt{\frac{1}{2}}(|00\rangle\langle 00| + |11\rangle\langle 11|)$$
$$F_{v,x} = \sqrt{\frac{1}{2}}(|+1\rangle\langle+1|_x + |-1\rangle\langle-1|_x) \quad = \quad \sqrt{\frac{1}{2}}(|++\rangle\langle++| + |--\rangle\langle--|)$$
$$F_{v,y} = \sqrt{\frac{1}{2}}(|+1\rangle\langle+1|_y + |-1\rangle\langle-1|_y) \quad = \quad \sqrt{\frac{1}{2}}(|i,i\rangle\langle i,i| + |-i,-i\rangle\langle -i,-i|)$$

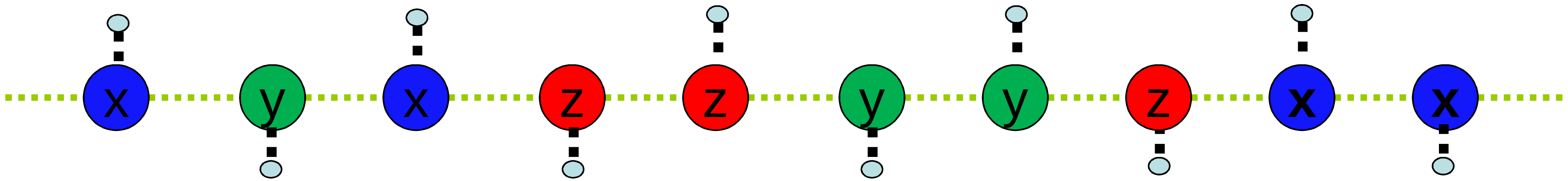
gives rise to an encoded cluster state



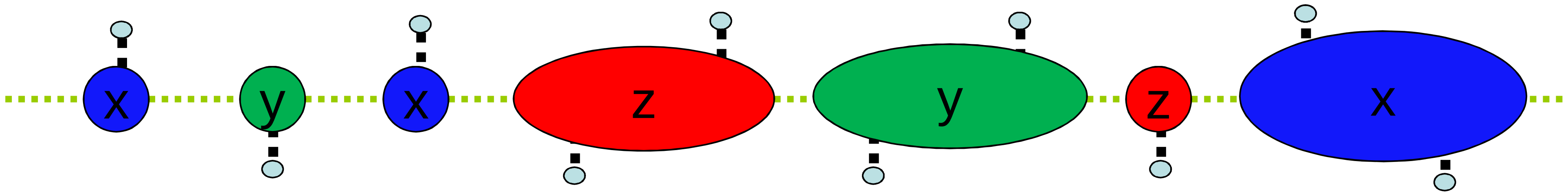
➔ In a large system, cluster state has length 2/3 of AKLT

1D mixed AKLT state ➔ cluster state

[*Wei, Raussendorf& Kwek,* arXiv '11]



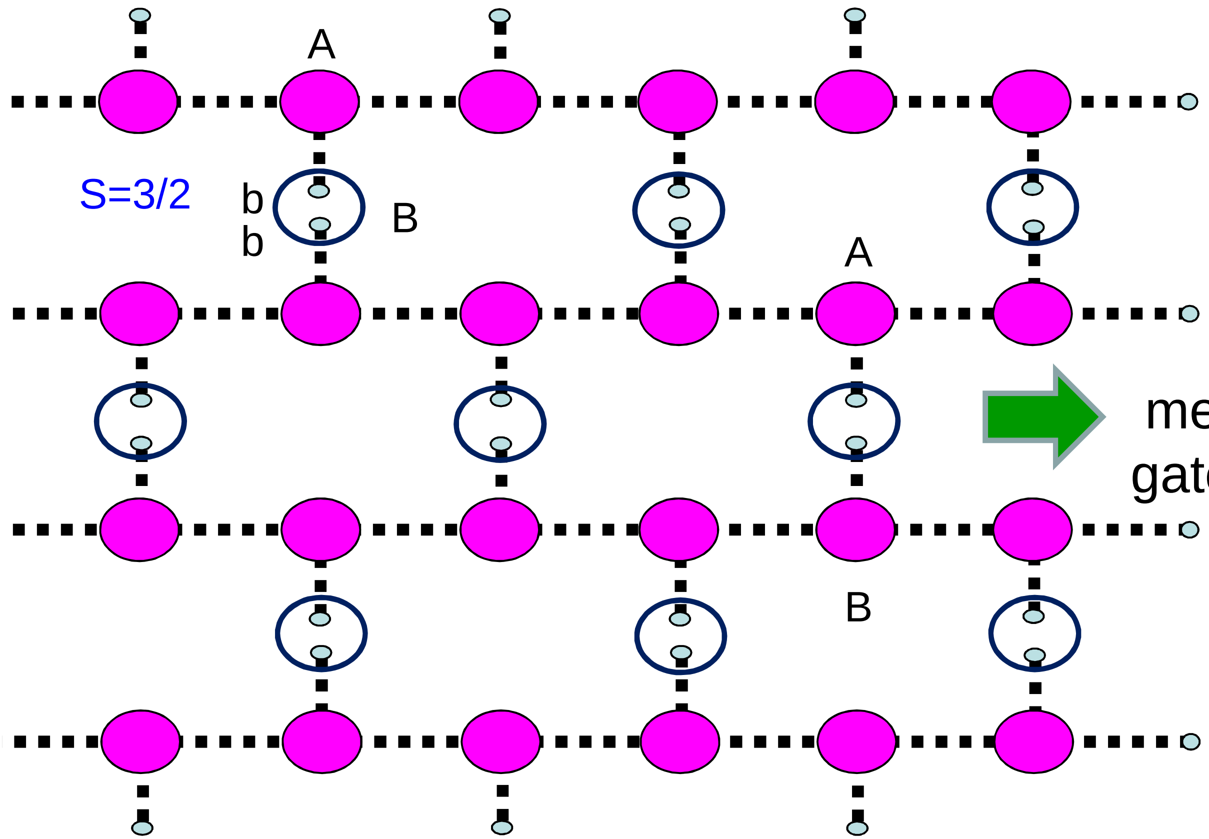
➔ POVM on spin-3/2's gives rise to an encoded cluster state



Universality of Cai-Miyake-Dur-Briegel state ➔ cluster state

[*Wei, Raussendorf& Kwek,* arXiv '11]

- POVM on A's and projective measurement on B's ➔ 2D cluster state



➔ 1D cluster state

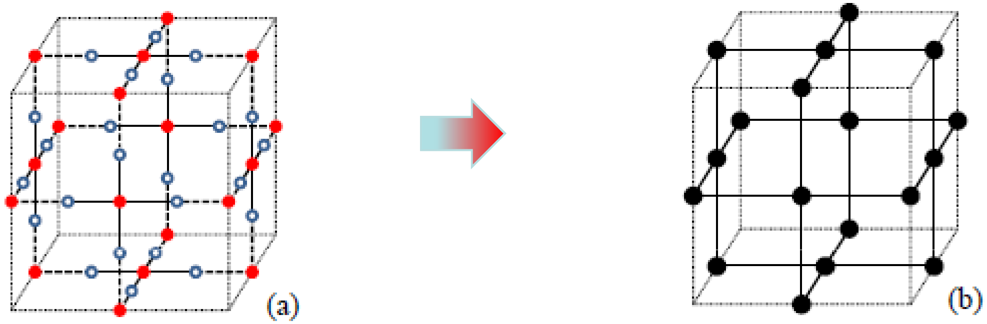
➔ 1D cluster state

measurement can induce CP gate btwn two neighboring A's

Further results

[*Li, Browne, Kwek, Raussendorf, Wei*, PRL 107,060501(2011)]

- Extending the "patching" idea to 3D



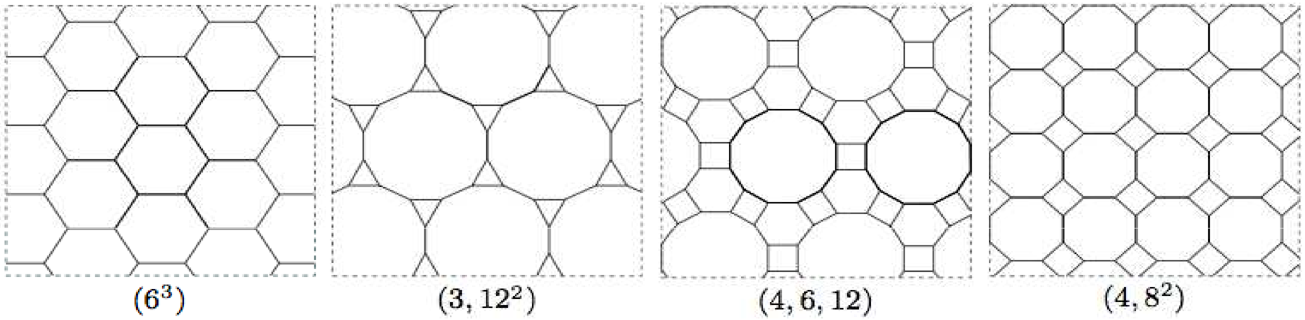
➔ Deterministic "distillation" of a 3D cluster state

➔ Allows quantum computation at finite temperature

➔ Even with the Hamiltonian always-on

Conclusion

- Spin-3/2 valence-bond ground states on some 2D lattices are universal resource for quantum computation



  - ➔ Design a generalized measurement
  - ➔ Convert to graph states and then cluster states (←universal)

- 2D structure from patching 1D AKLT quasichains also universal

- Can extend to 3D as well with thermal state and always-on interaction

Collaborators

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