Quantum gates for superconducting qubits with fixed coupling

CQIQC Toronto, 12-08-2011

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Outline

- Quantum superconducting devices
  - Superconducting artificial atoms
  - The flux qubit
- Use of superconducting quantum systems
  - Quantum computing with superconducting qubits
  - Quantum optics in the strong coupling regime
- Two qubit gate based on controlled transition matrix elements
- Application of the gate to other types of systems

Qubit implementations

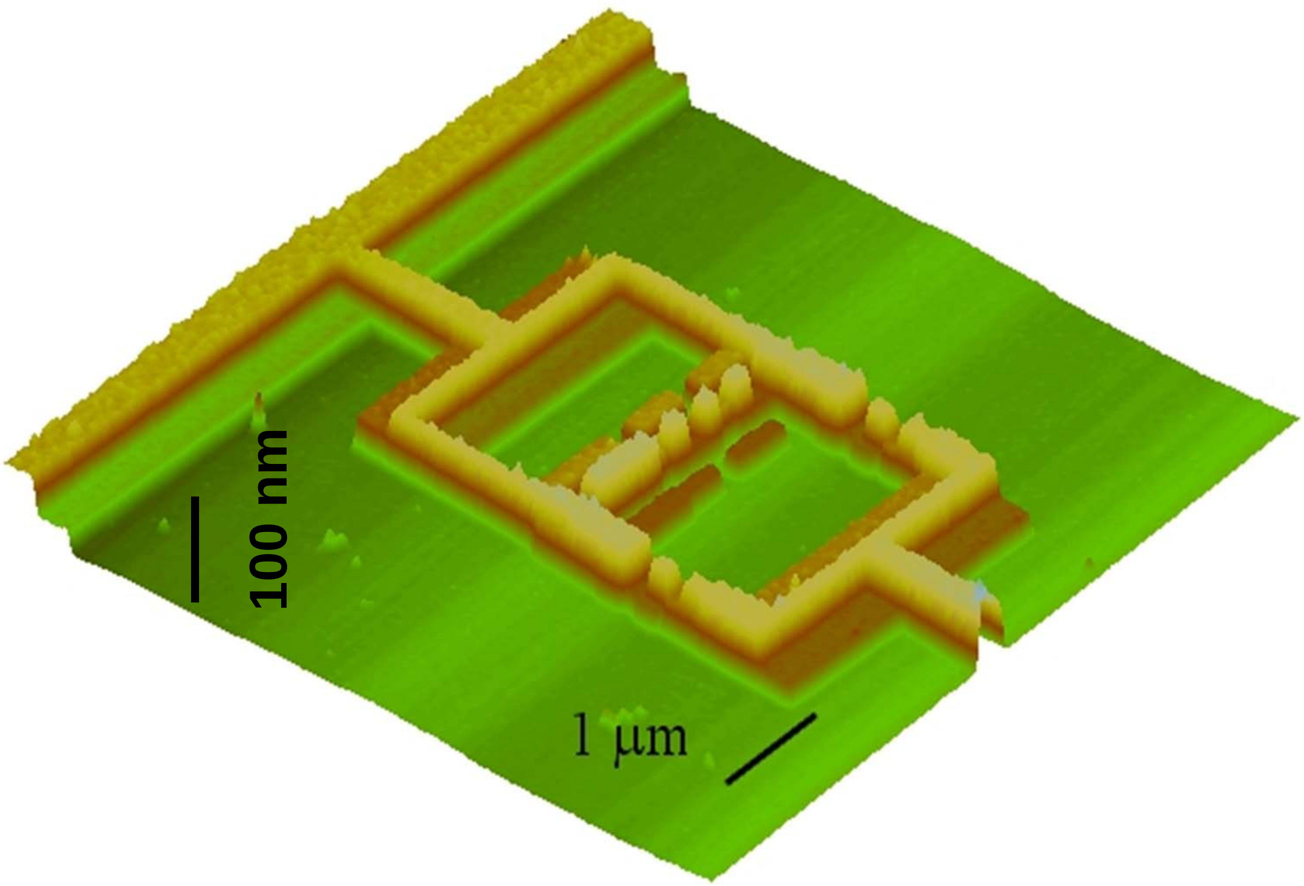
Natural systems
- Neutral atoms
- Trapped ions
- Nuclear spins
- Photons

Artificial systems
- Superconducting qubits
- Quantum dots

Mixed systems
- NV centers in diamond
- Impurities in Silicon

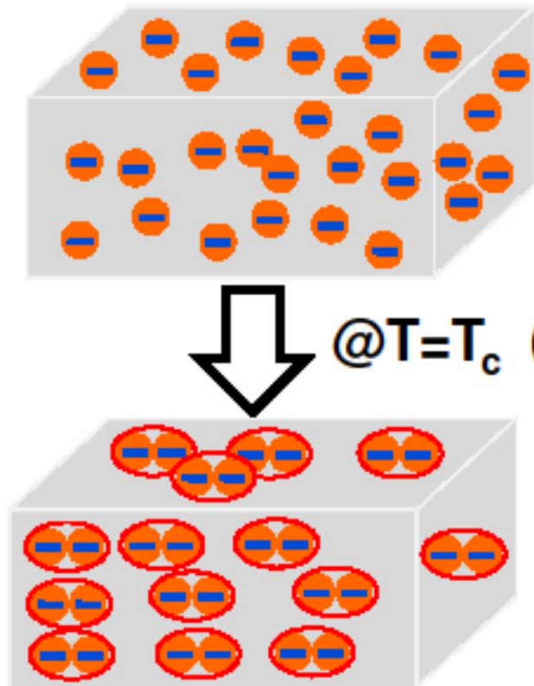
Artificial systems: top down approach to quantum system design → scalability

Example of superconducting qubit



Superconductivity

Normal state
- finite conductivity
- no electron order
- no energy gap



@T=T_c (1.2 K for Aluminum)

Superconducting state
- infinite conductivity
- electron order
- energy gap

The gap for electron-hole excitations survives in small electromagnetic fields.

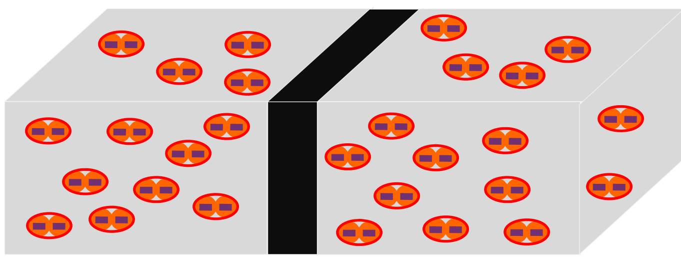
Lower energy excitations, with anharmonic spectrum

Josephson energy

charging energy

Josephson effect: description of a Josephson junction

tunnel barrier



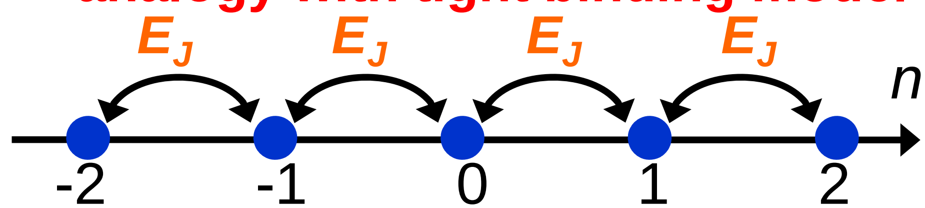
N_1 - n $\quad$ $N_2 + n$

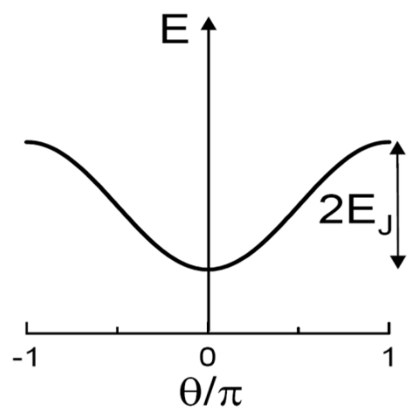
$$\hat{H}_T = -E_J \sum_n \left(|n\rangle\langle n+1| + |n+1\rangle\langle n| \right)$$

$\boxed{E_J \text{ - Josephson energy}}$

• thick barrier: states with different n are degenerate (neglecting charging energy)
• thin barrier: degeneracy is lifted

analogy with tight binding model





$$E(\gamma) = -E_J \cos(\gamma)$$

$$|\gamma\rangle = \frac{1}{\sqrt{n_{total}}} \sum_n e^{i\gamma n} |n\rangle$$

Josephson, Phys. Lett. **1**, 251 (1962) de Gennes, *Superconductivity of ..,(* 1966)

Current flow through a Josephson junction

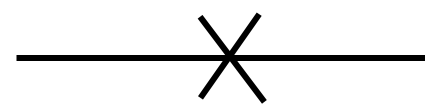
Free particle → Josephson junctions

$$x \to n,\ p \to \gamma$$

Form a wavepacket with a narrow distribution in γ

$$I = 2e < \dot{n} > = \frac{2e}{\hbar} E_J \sin\gamma$$

$$I = I_c \sin\gamma$$



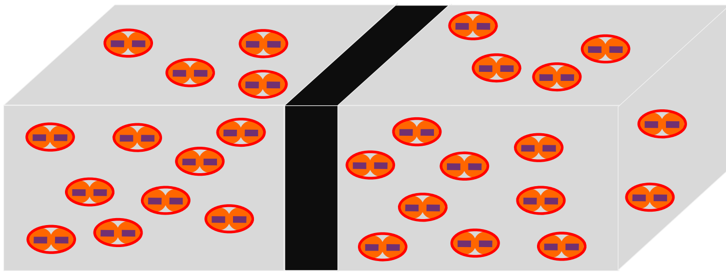
inductance

$$\boxed{I_c = \frac{2e}{\hbar} E_J\ :\ \text{critical current}}$$

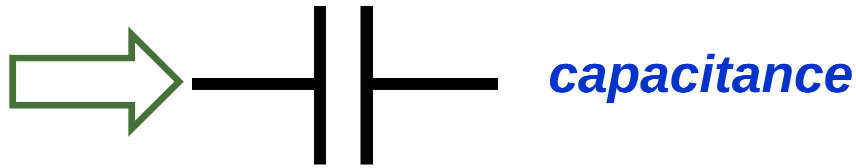
$$\boxed{E_J\ :\ \text{Josephson energy}}$$

Josephson, Phys. Lett. **1**, 251 (1962)

Charging energy of a Josephson junction



$N_1 - n$ $\quad$ $N_2 + n$



$$E = \frac{Q^2}{2C} = \frac{(2e)^2}{2C} n^2$$

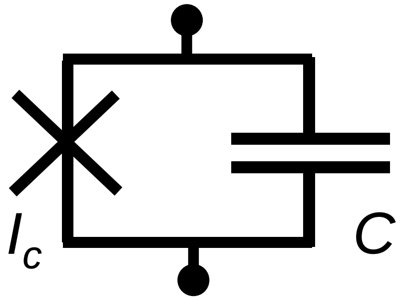
For a small system, the relocation of Cooper pairs comes at high energy cost.

$|n\rangle$ states are **NOT** degenerate

C - capacitance

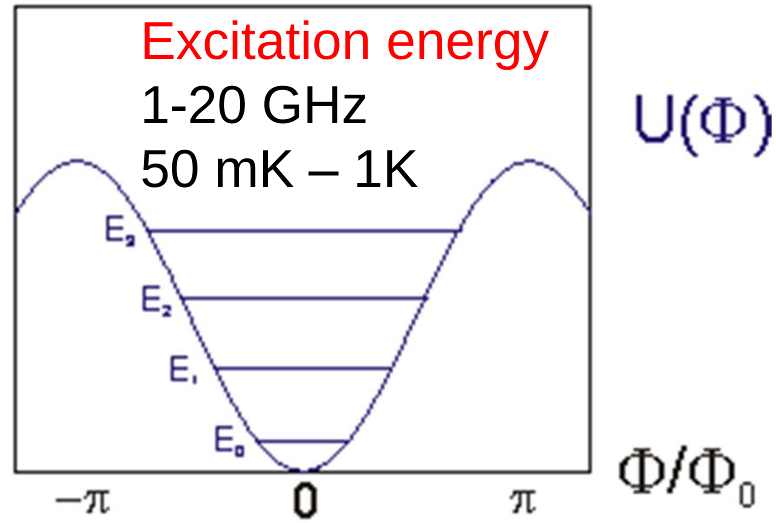
$E_C = \frac{(2e)^2}{2C}$ - charging energy

Josephson junction: quantum description



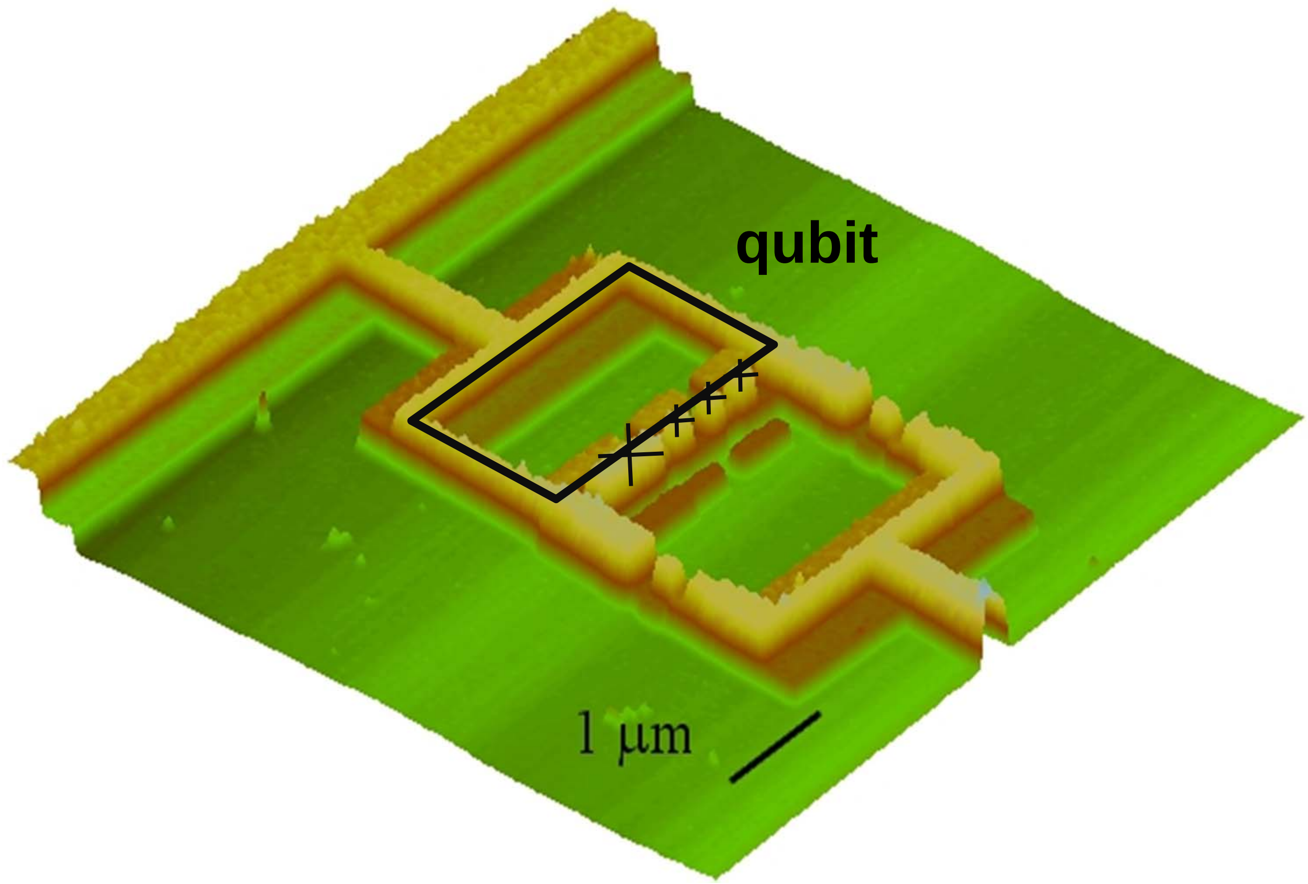
$$[\hat{\gamma}, \hat{n}] = i$$

$$\hat{H} = E_C \hat{n}^2 + E_J (1 - \cos\hat{\gamma})$$

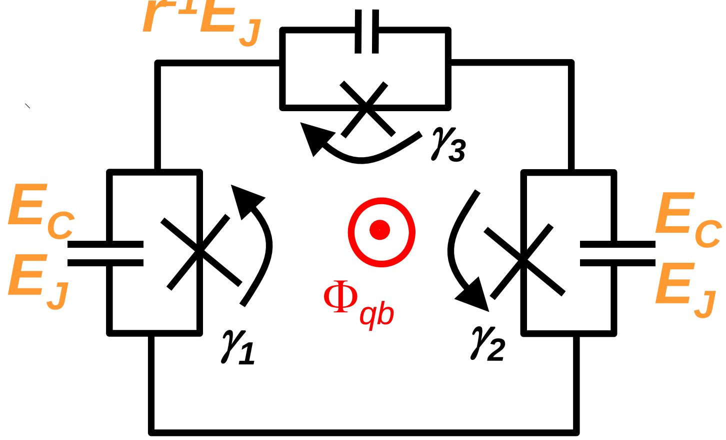


	Phase	*Charge*
Quantum variables	γ	n
Fundamental constants	$\Phi_0 = \hbar/2e$	2e
Mesoscopic constants	I_c 0.1 – 10 μA	C 1-100 fF
Energy scales	$E_J = \Phi_0 I_c/2\pi$ 50-5000 GHz **/** 2-200 K	$E_C = (2e)^2/2C$ 0.8 - 80 GHz **/** 0.04 – 4 K

Flux qubit



Electrical circuit model of the flux qubit



E_C , E_C , r - *design* (mesoscopic) parameters

Φ_{qb} – ***control*** parameter

Flux quantization condition

$$\gamma_1 + \gamma_2 + \gamma_3 = -2\pi\Phi_{qb}/\Phi_0$$

Energy in terms of variables γ_1 and γ_2

$$\hat{T} = E_C \left(\hat{n_1}^2 + \hat{n_2}^2 + r(\hat{n_1}^2 + \hat{n_2}^2)\right)$$

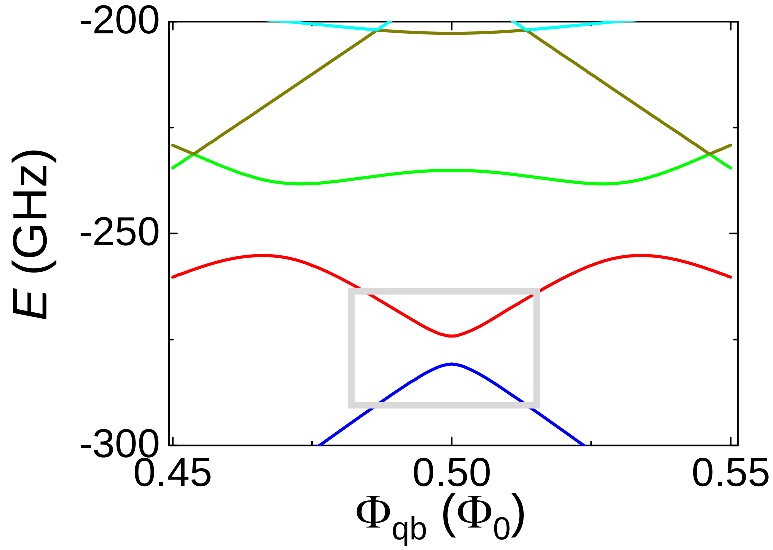
$$\hat{U} = -E_J \left(\cos\hat{\gamma_1} + \cos\hat{\gamma_2} + r\cos\left(\hat{\gamma_1} + \hat{\gamma_2} + 2\pi\frac{\Phi_q b}{\Phi_0}\right)\right)$$

Energy level structure

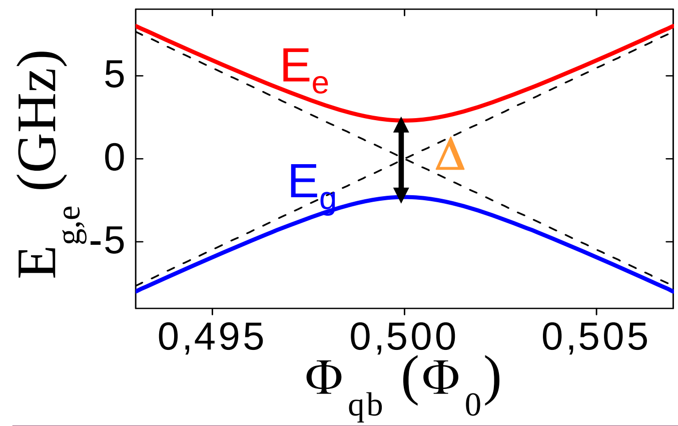
•energy levels depend on Φ_{qb} with period Φ_0

Mooij *et al.*, Science **285**, 1036 (1999)

Energy levels of the flux qubit



QUBIT MODEL



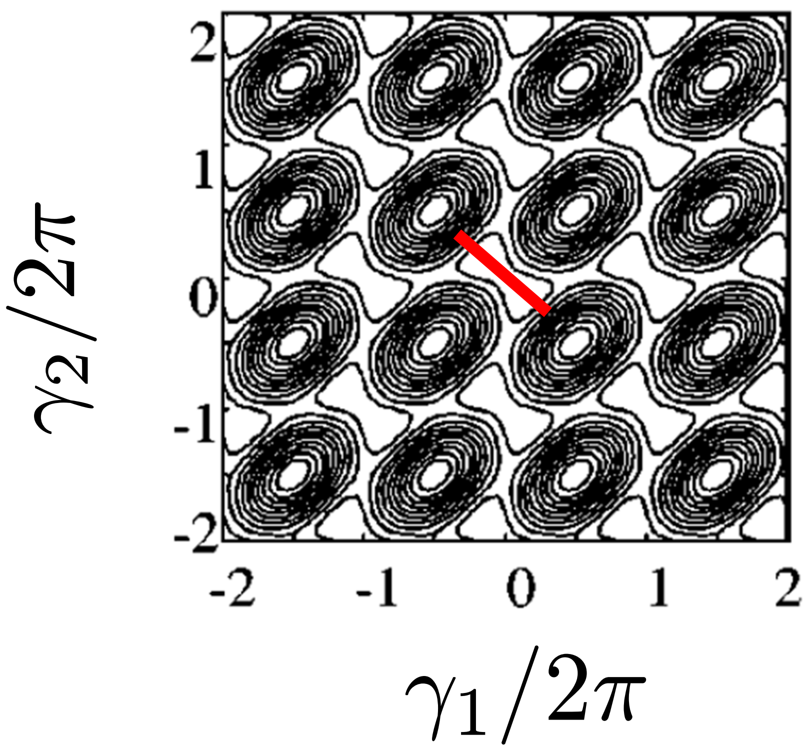
$$\hat{H} = \frac{h}{2}\left(2 I_p \left(\Phi_{qb} - \Phi_0/2\right)\hat{\sigma}_x + \Delta\hat{\sigma}_z\right)$$

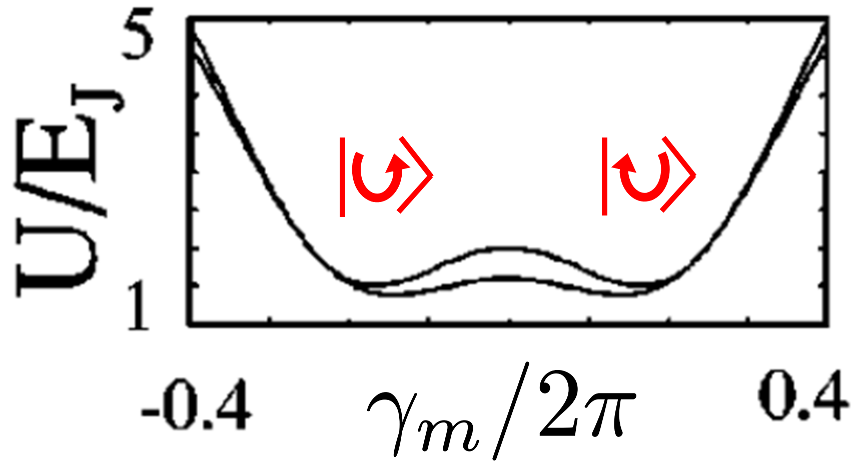
I_p, Δ: ***design*** parameters

Φ_{qb} : ***control*** parameter

Basis: $\{ |\circlearrowleft\rangle , |\circlearrowright\rangle \}$

Properties of the flux qubit





States localized in the potential wells:
well defined phase → well defined current/flux

Orlando *et al.*, PRB **60**, 15398 (1999)

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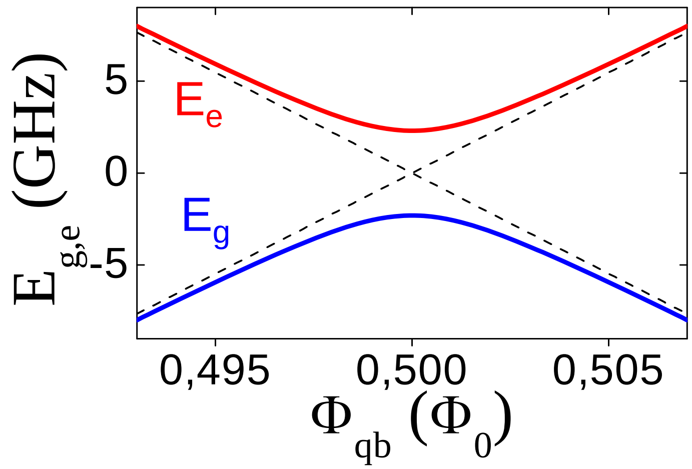
Implementation of QC with superconducting qubits

DiVincenzo criteria

1. **a scalable physical system with well characterized qubits**
2. **the ability to initialize the qubits in a well defined state**
3. **long decoherence times**
4. **a universal set of quantum gates**
5. **a qubit specific measurement capability**

DiVincenzo, Fortschr. Phys. **48**, 771 (2000)

Qubit initialization



Energy level splitting

$E_{ge} = E_e - E_g \sim$ 2-20 GHz

Temperature

T < 30 mK (i.e. 0.6 GHz)

Energy relaxation (typically over microseconds) → ground state preparation.

Fidelity of preparation: $\dfrac{1}{1+e^{-hE_{ge}/k_B T}}$ **exceeds in most cases 99%.**

Universal set of quantum gates

- We need
  - 1-qubit gates
  - 2-qubit gates

1-qubit gates
A discrete set is sufficient, but in general continuous sets available

2-qubit gates
C-NOT is sufficient

$$
\begin{aligned}
|11\rangle &\to |10\rangle \\
|10\rangle &\to |11\rangle \\
|01\rangle &\to |01\rangle \\
|00\rangle &\to |00\rangle
\end{aligned}
$$

Nielsen and Chuang, *Quantum Computation and Quantum Information* (2000)

Universal set of quantum gates: 1 - qubit gate

- State rotations: resonant driving

$$\hat{H} = -\tfrac{1}{2}\Delta\hat{\sigma}_z + a\cos\left(\omega t + \phi\right)\hat{\sigma}_z$$

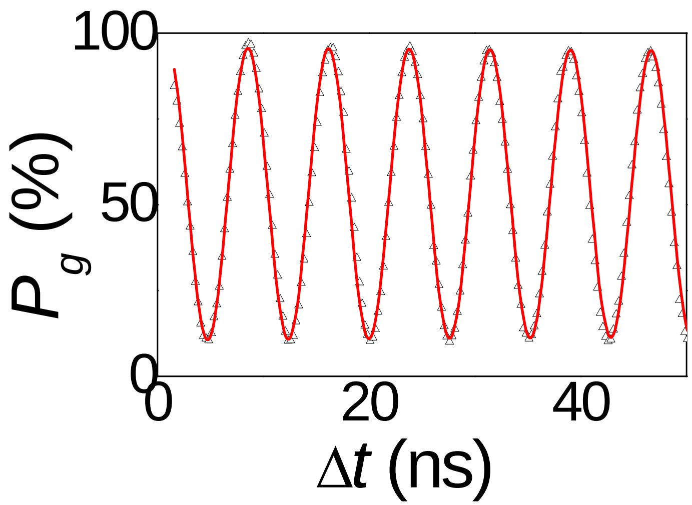
- Undriven (a=0) case: superposition of 0 and 1 acquires phase Δt ω_{01}
- Driven case: additional Rabi oscillations

$$\hat{U}(t) = \cos\tfrac{at}{2}\,\hat{I} + i\sin\tfrac{at}{2}\left(\cos\phi\hat{\sigma}_x + \sin\phi\hat{\sigma}_y\right)$$

$$\phi = 0$$

$$\hat{U}(\pi/a) = i\hat{\sigma}_x$$

$$\hat{U}(\pi/a)|0\rangle = i|1\rangle$$



A. Lupascu *et al.*, PRL **96**, 127003 (2006)

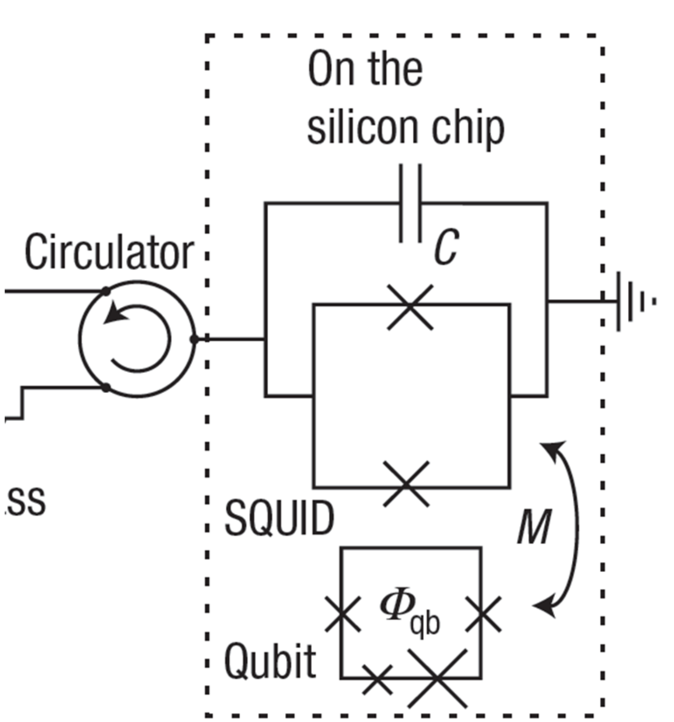
Decoherence

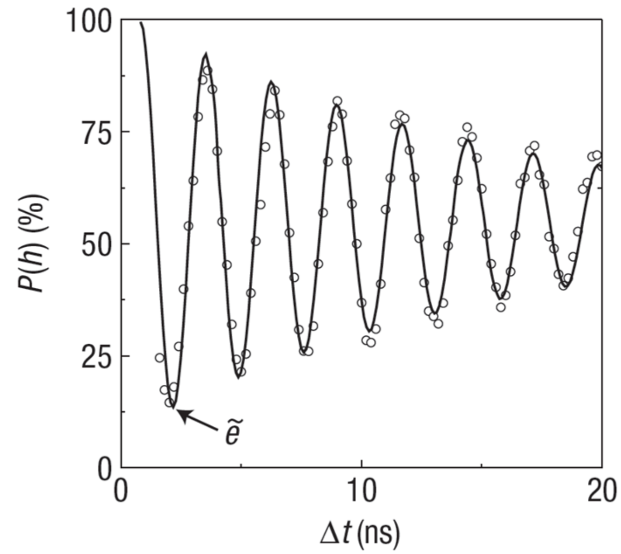
- Long coherence times observed: see eg Bylander *et al.*, Nature Physics (2011)

- Open questions
  - Origin of low frequency noise (dephasing)
  - Relaxation time: microscopic origin, reproducibility
- Best coherence time: requires operation at noise insensitive point

Qubit readout

- Example: a qubit state dependent resonance of a SQUID based circuit

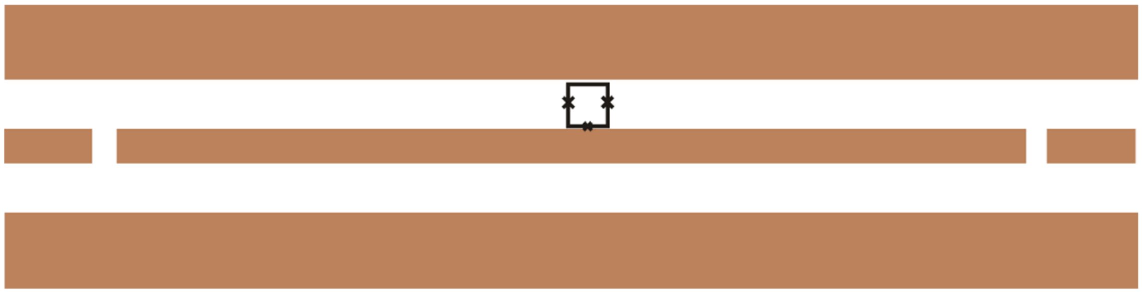




Large fidelity/projective measurements can be achieved (implementation dependent)

Lupascu *et al.*, Nature Physics 3, 119 (2007)

Qubit-resonator system: Jaynes-Cumming model



Jaynes-Cumming model

$$H = \hbar\omega_{res}a^{\dagger}a + \hbar\omega_{qb}/2\sigma_z + \hbar g\left(\sigma^{+}a + \sigma^{-}a^{\dagger}\right)$$

Strong coupling regime

$$g \gg \kappa, \gamma$$

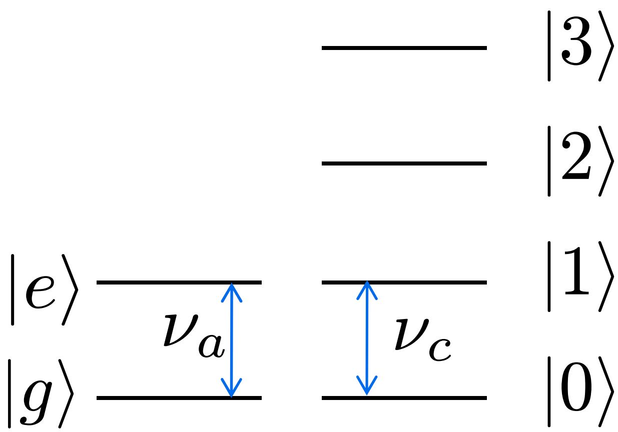
κ, γ - cavity/qubit decay rate

Resonant and dispersive regimes

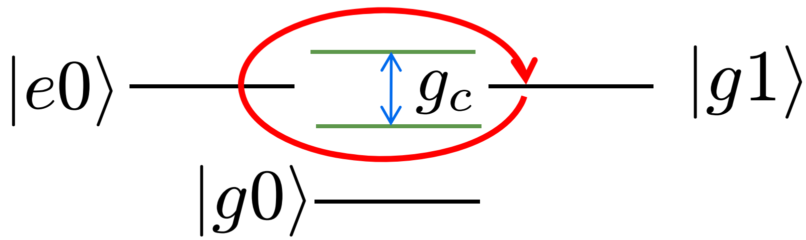
Resonant coupling

- Equal atom and cavity frequencies $\nu_a = \nu_c$

actual eigenstates $\frac{1}{\sqrt{2}}(|e0\rangle + / - |g1\rangle)$



vaccum Rabi oscillations



Wallraff *et al.*, Nature **431**, 162 (2004)

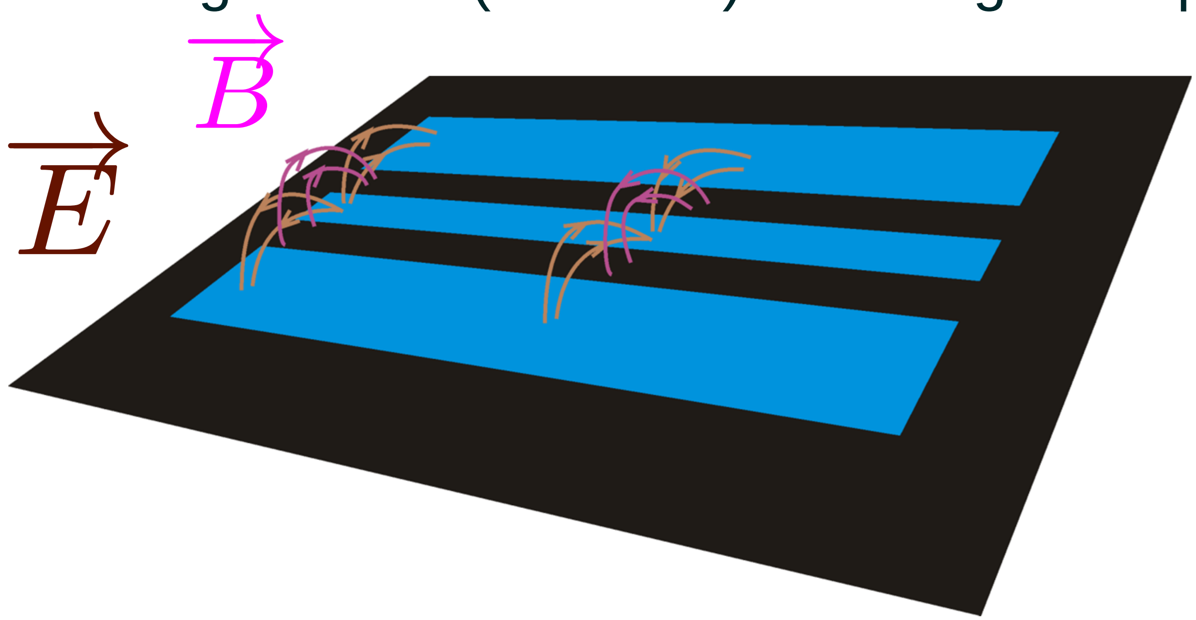
Dispersive coupling

- Detuning much larger than coupling $\Delta \gg g$

$$H \approx \hbar\left[\omega_{res} + \frac{g^2}{\Delta}\sigma_z\right] a^\dagger a + \frac{\hbar}{2}\left[\omega_{qb} + \frac{g^2}{\Delta}\right]\sigma_z$$

Single atoms and photons

- Single atom - single photon mode interaction: cavity QED (quantum electrodynamics)
- Circuit QED:
  - domain of EM frequencies usually not considered light: ~10^{10} Hz compared to ~10^{14} for visible light
  - More confined light mode (1d vs 3d) → stronger coupling



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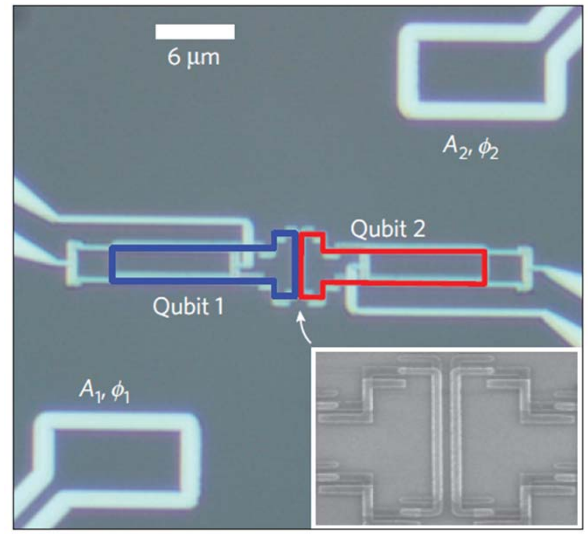
Coupled flux qubits

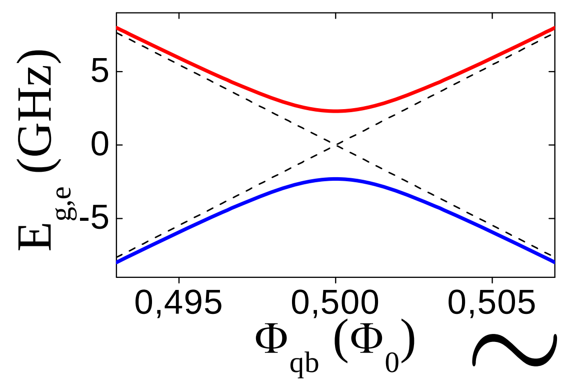
- Coupling mechanism: each qubit senses the magnetic flux of the other qubit
- Formal description

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{H}_{int}$$
$$\hat{H}_1 = -\frac{1}{2}\Delta_1\hat{\sigma}_{z1} - \frac{1}{2}\epsilon_1\hat{\sigma}_{x1}$$
$$\hat{H}_2 = -\frac{1}{2}\Delta_2\hat{\sigma}_{z2} - \frac{1}{2}\epsilon_2\hat{\sigma}_{x2}$$
$$\hat{H}_{int} = J\hat{\sigma}_{x1}\hat{\sigma}_{x2}$$

- Orders of magnitude
  - J $\lesssim \Delta_1$, Δ_2
  - ϵ tunable



de Groot *et al.*,
Nature Physics **6**, 763 (2010)



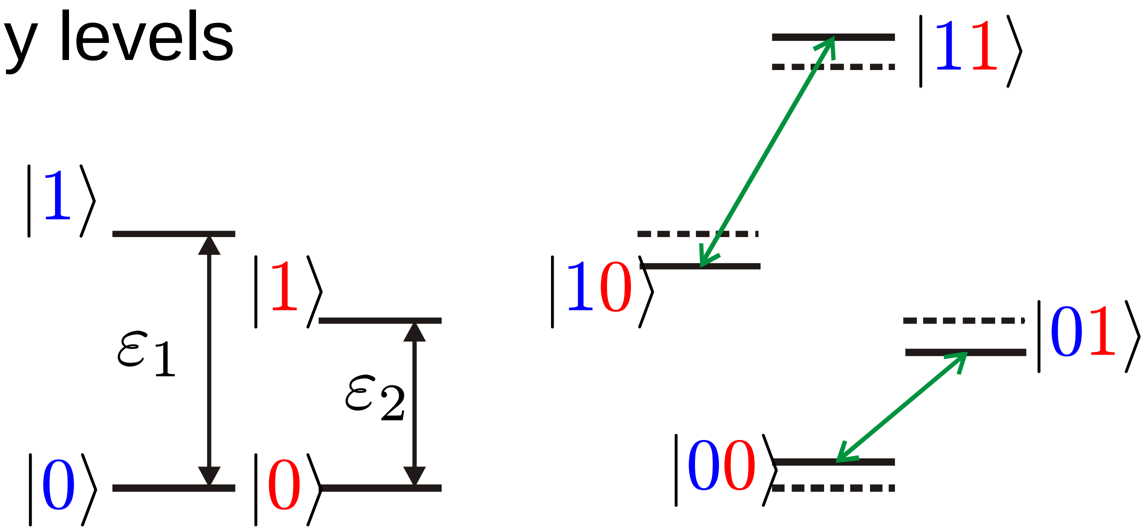
$\sim \epsilon$

Away from the symmetry point

- $\epsilon_i \gg \Delta_i$, i=1,2
- Hamiltonian

$$\hat{H} \simeq -\frac{1}{2}\epsilon_1\hat{\sigma}_{x1} - \frac{1}{2}\epsilon_2\hat{\sigma}_{x2} + J\hat{\sigma}_{x1}\hat{\sigma}_{x2}$$

- Energy levels

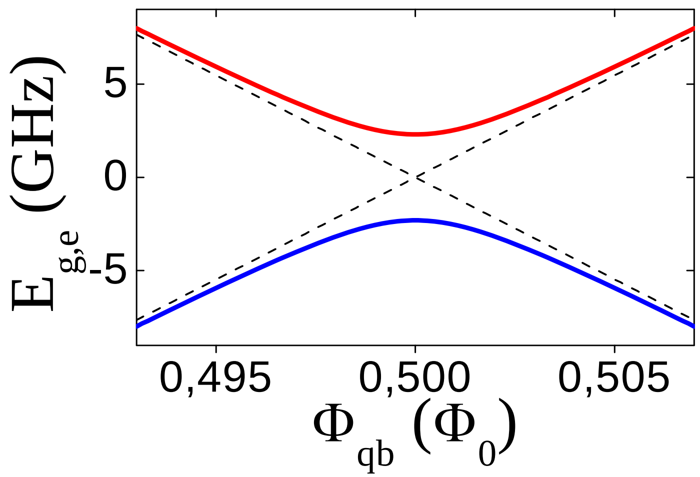


- Transition frequency $\omega_{10,11}$ different of all the other transition frequencies → CNOT

Plantenberg *et al.*, Nature **447**, 836 (2007)

Decoherence away from the symmetry point

- Sensitivity to flux fluctuations is minimum at the symmetry point



- Strong “pure dephasing” away from the symmetry point

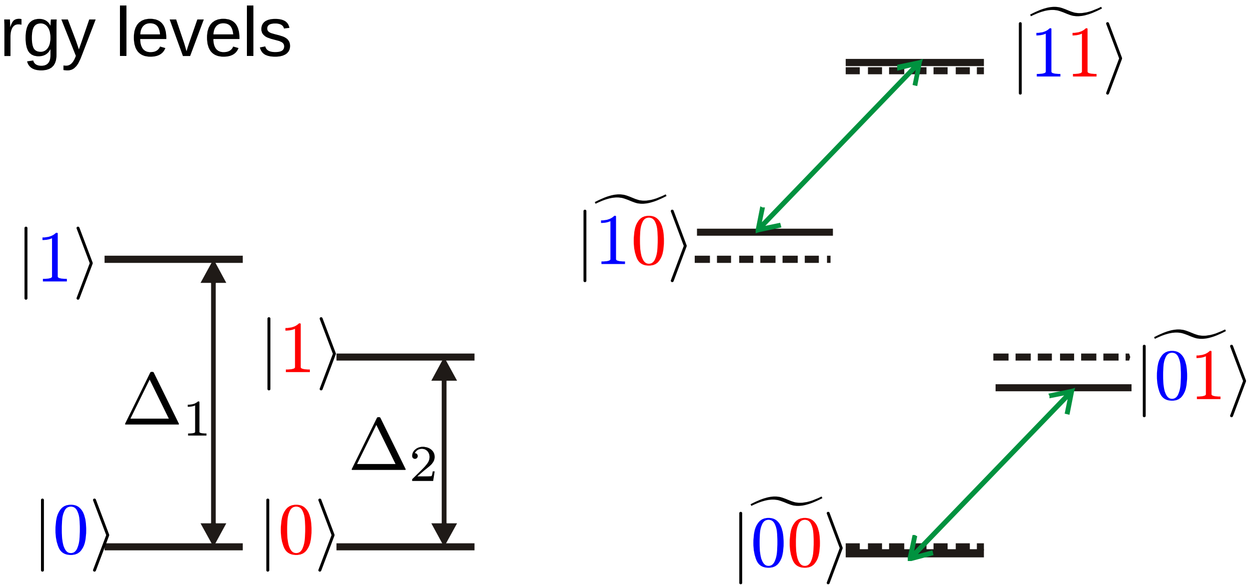
Yoshihara *et al.*, Phys. Rev. Lett. **97**, 167001 (2006)

At the symmetry point

- ϵ_i =0, i=1,2
- Hamiltonian

$$\hat{H} = -\frac{1}{2}\Delta_1\hat{\sigma}_{z1} - \frac{1}{2}\Delta_2\hat{\sigma}_{z2} + J\hat{\sigma}_{x1}\hat{\sigma}_{x2}$$

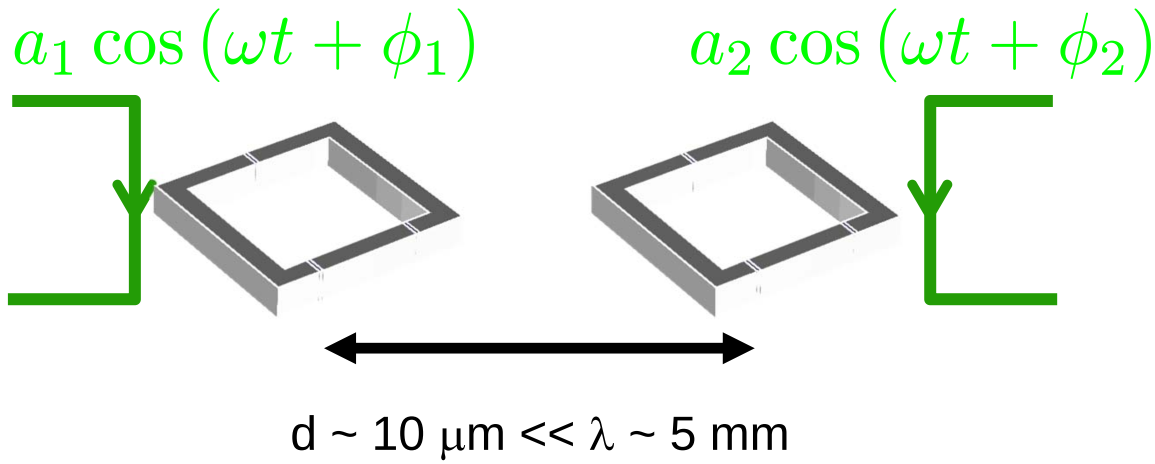
- Energy levels



- Impossible to use frequency selection to implement a CNOT gate: $\omega_{10,11} = \omega_{00,01}$

Way out: independent driving

- Distance between qubits is << λ, but large enough to apply near field microwaves



- Hamiltonian with driving

$$\hat{H} = \sum_{i=1,2} \left[-\frac{1}{2}\Delta\hat{\sigma}_{z,i} + a_i \cos(\omega t + \phi_i)\hat{\sigma}_{x,i} \right] + J\hat{\sigma}_{x,1}\hat{\sigma}_{x,2}$$

de Groot *et al.*, Nature Phys. **6**, 763 (2010)

Driving both qubits: transition matrix elements

- Matrix element calculation
  - Resonance: $\omega = \omega_{kl} - \omega_{ij} \;\; (\omega_{kl} > \omega_{ij})$
  - Result (in rotating frame)

$$T_{kl,ij} = \langle \widetilde{kl} | \frac{a_1}{2} e^{i\phi_1} \hat{\sigma}_{x1} + \frac{a_2}{2} e^{i\phi_2} \hat{\sigma}_{x2} | \widetilde{ij} \rangle$$

  - Results when $J \ll |\Delta_1 - \Delta_2|$ (for simplicity)

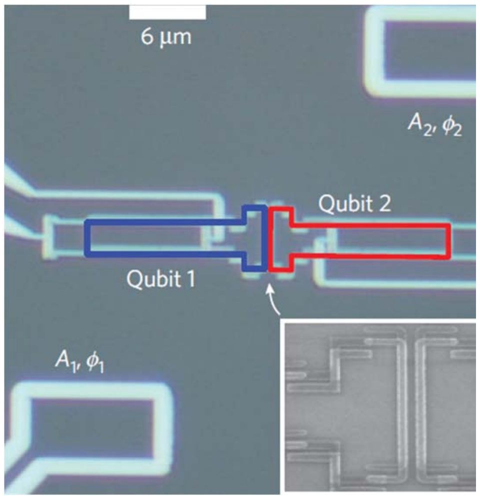
$$\begin{aligned} |\widetilde{00}\rangle &= |00\rangle \\ |\widetilde{01}\rangle &= |01\rangle - \frac{J}{\Delta_1 - \Delta_2} |10\rangle \\ |\widetilde{10}\rangle &= |10\rangle + \frac{J}{\Delta_1 - \Delta_2} |01\rangle \\ |\widetilde{11}\rangle &= |11\rangle \end{aligned}$$

$$\begin{aligned} T_{01,00} &= -\frac{J}{\Delta_1 - \Delta_2} \frac{a_1}{2} e^{i\phi_1} + \frac{a_2}{2} e^{i\phi_2} \\ T_{11,10} &= \frac{J}{\Delta_1 - \Delta_2} \frac{a_1}{2} e^{i\phi_1} + \frac{a_2}{2} e^{i\phi_2} \end{aligned}$$

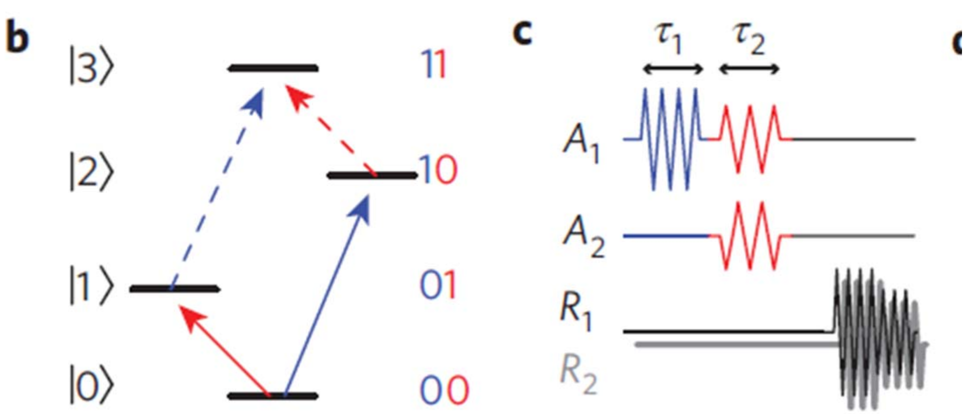
- Cancellation condition

$$\frac{a_2}{a_1} = \frac{J}{\Delta_1 - \Delta_2} \quad \Rightarrow \quad \begin{aligned} T_{01,00} &= 0 \\ T_{11,10} &= a_1 \frac{J}{\Delta_1 - \Delta_2} \end{aligned}$$

Experimental results



- Pulse 1: creates a superposition of states $|0\rangle$ and $|1\rangle$ of qubit 1
- Pulse 2: rotation of qubit 2, dependent of state of qubit 1



- **Oscillations**

  $$|00\rangle \leftrightarrow |01\rangle$$

  and

  $$|10\rangle \leftrightarrow |11\rangle$$

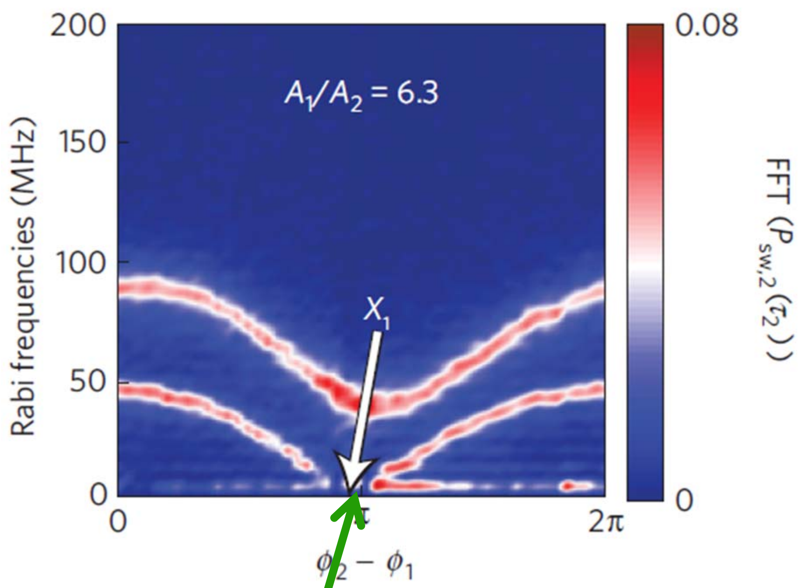
  should occur at different rates

de Groot *et al.*, Nature Physics **6**, 763 (2010)

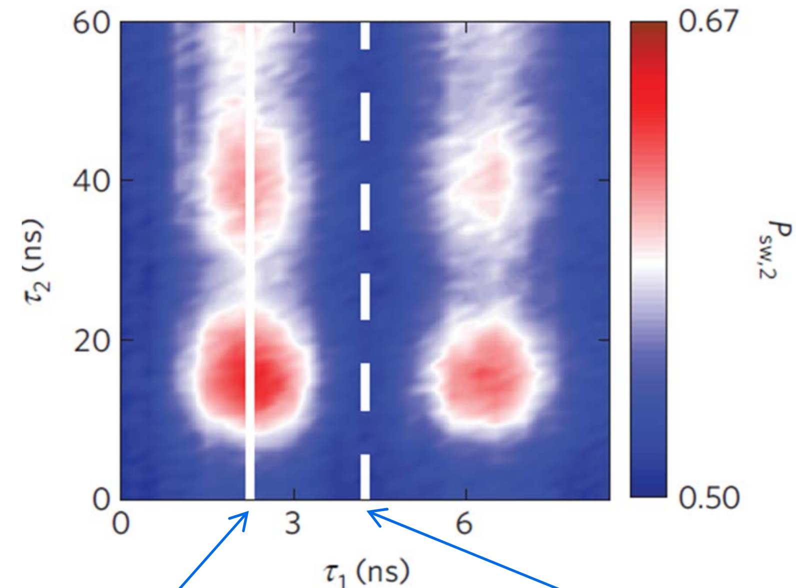
Experimental results

- Prepare qubit 1 in $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
- Measure oscillation frequency of qubit 2

- Demonstration of CNOT gate



$$|00\rangle \not\leftrightarrow |01\rangle$$

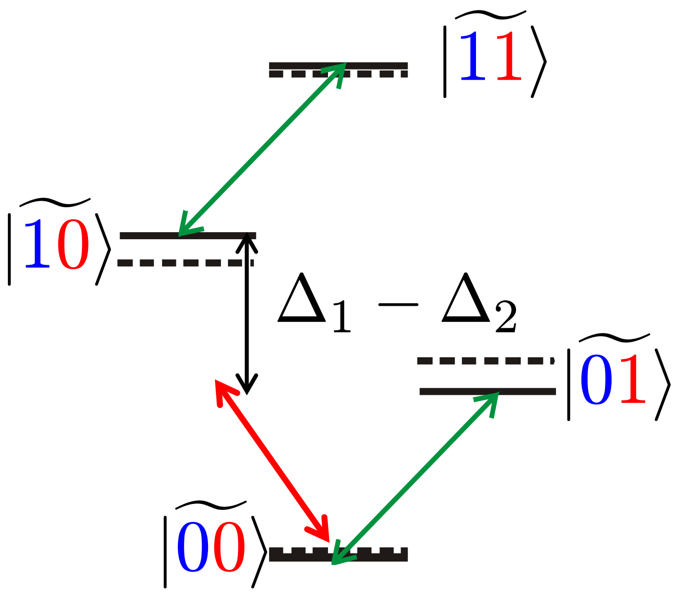


$$|10\rangle \leftrightarrow |11\rangle \qquad |00\rangle \leftrightarrow |01\rangle$$

de Groot *et al.*, Nature Physics **6**, 763 (2010)

Gate speed

- Limited by off resonance driving



$$\frac{a_2}{a_1} = \frac{J}{\Delta_1 - \Delta_2}$$

$$T_{01,00} = 0$$
$$T_{11,10} = a_1 \frac{J}{\Delta_1 - \Delta_2}$$

$$T_{10,00} = \frac{a_1}{2}\left[1 + \left(\frac{J}{\Delta_1 - \Delta_2}\right)^2\right] \sim \frac{a_1}{2}$$

- Leakage effects when $T_{10,00} \sim \Delta_1 - \Delta_2$
- Maximum gate speed $T_{11,10} < J$

Other schemes for operation at a symmetry point

Parametric modulation

Niskanen *et al.*, Science **316**, 723 (2007)

FLIC-FORQ

Rigetti *et al*, Phys. Rev. Lett. **94**, 240502 (2007)

Cross resonance

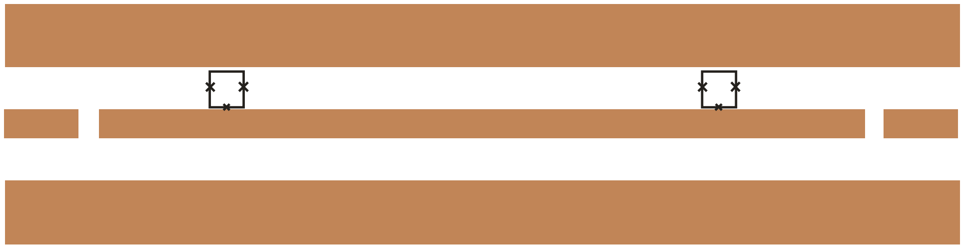
Rigetti and Devoret, PRB **80**, 134507 (2010) – **related recent theoretical work**

Matrix element gate vs other gates for transverse coupling

- Speed gate: similar (limited by the coupling strength J)
- Advantages
  - No additional coupler element is required (unlike parametric modulation)
  - Tolerant to parameters
- Problems with **present** implementation
  - Low coherence times, due to low frequency noise coupled through detectors
  - Direct coupling: limited range

Improvement of the matrix element gate

- Qubits in cavities



Blais *et al*,
Phys. Rev. A **75**, 032329 (2007)

- System model

$$\begin{aligned} H =& h\nu_{cav} a^\dagger a \\ &+ h\Delta_1 \sigma_{z1} + hg_1 (a\sigma_1^+ + a^\dagger \sigma_1^-) \\ &+ h\Delta_2 \sigma_{z2} + hg_2 (a\sigma_2^+ + a^\dagger \sigma_2^-) \end{aligned}$$

- Effective qubit-qubit interaction

$$H_{int} = \frac{g_1 g_2 ((\Delta_1 - \nu_{cav}) + (\Delta_2 - \nu_{cav}))}{(\Delta_1 - \nu_{cav})(\Delta_2 - \nu_{cav})} \sigma_{x1} \sigma_{x2}$$

- The scheme is applicable to many qubits in a cavity

- Quantum superconducting devices
  - Superconducting artificial atoms
  - The flux qubit
- Use of superconducting quantum systems
  - Quantum computing with superconducting qubits
  - Quantum optics in the strong coupling regime
- Two qubit gate based on controlled transition matrix elements
- Application of the gate to other types of systems

Applicability of the gate to other solid state qubits

- Requires interaction of form $\hat{\sigma}_{x1}\hat{\sigma}_{x2}$ - generic. All cQED implementations have this term
- Used recently in experiments with transmons

  Filipp *et al.*, arxiv 1107.2078 (2011)

- Applicable to coupled quantum dots

Application to atomic systems

- Difficulties
  - Phase difficult to enforce for short wavelength optical fields
  - Position control
- Potential application: molecular ensembles coupled by resonator: see arxiv 1108.1412

Summary

- The dark transition gate is a flexible and generic method for two qubit gates in superconducting circuits
- Cavity QED architecture for experiments with many flux qubits
- The gate is applicable to other systems with fixed coupling: quantum dots, molecular ensembles

Acknowledgement

- SQD group
  - Jean-Luc Orgiazzi
  - Chunqing Deng
  - Florian Ong
  - Mustafa Bal





- Collaboration
  - Delft group
  - Jonathan Baugh
  - Sahel Ashhab

NSERC CRSNG

Canada Foundation for Innovation

Fondation canadienne pour l'innovation

Ontario MINISTRY OF RESEARCH & INNOVATION

ALFRED P. SLOAN FOUNDATION

UNIVERSITY OF WATERLOO

IQC Institute *for* Quantum Computing