

Scavenging Quantum Information: Multiple Observations of Quantum Systems

PETER RAPČAN

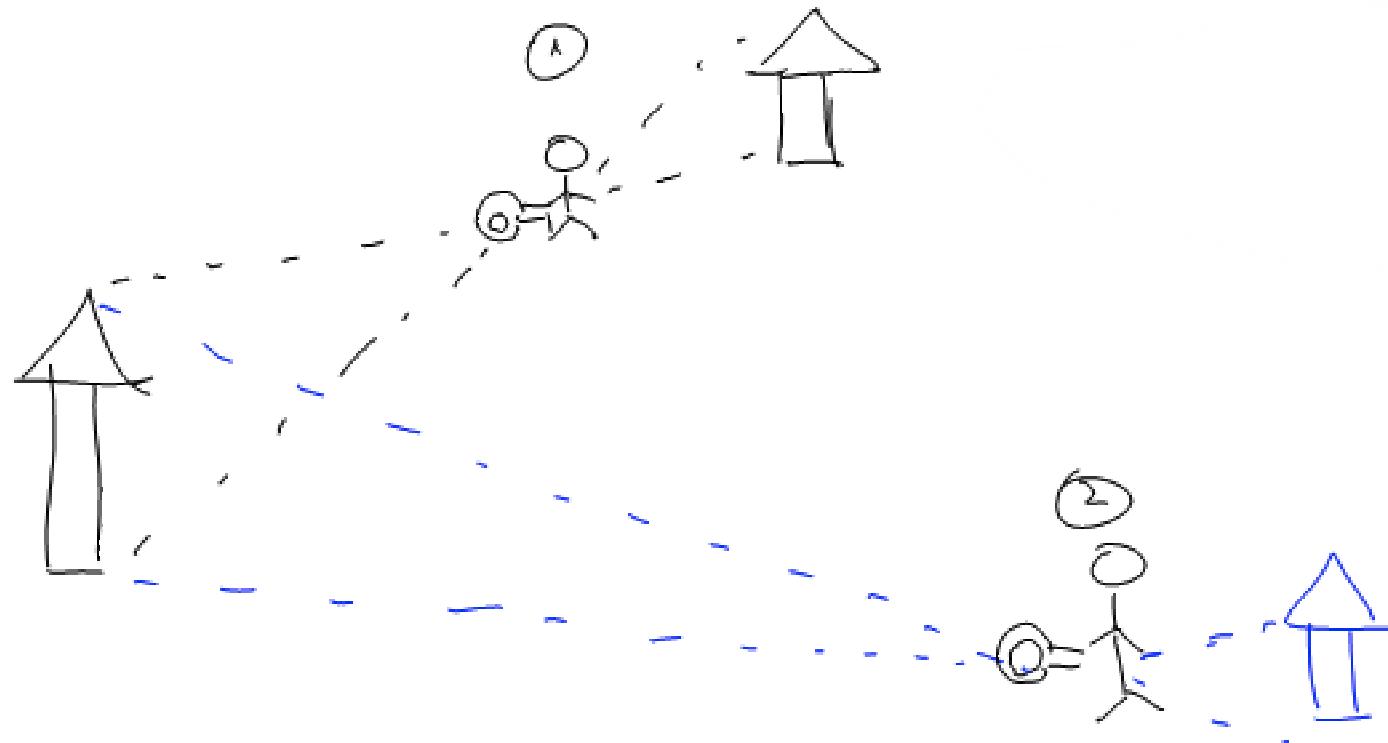
(RCQI, Institute of Physics, Slovak Academy of Sciences)

joint work with
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CQIQC IV, Toronto, August 10, 2011.



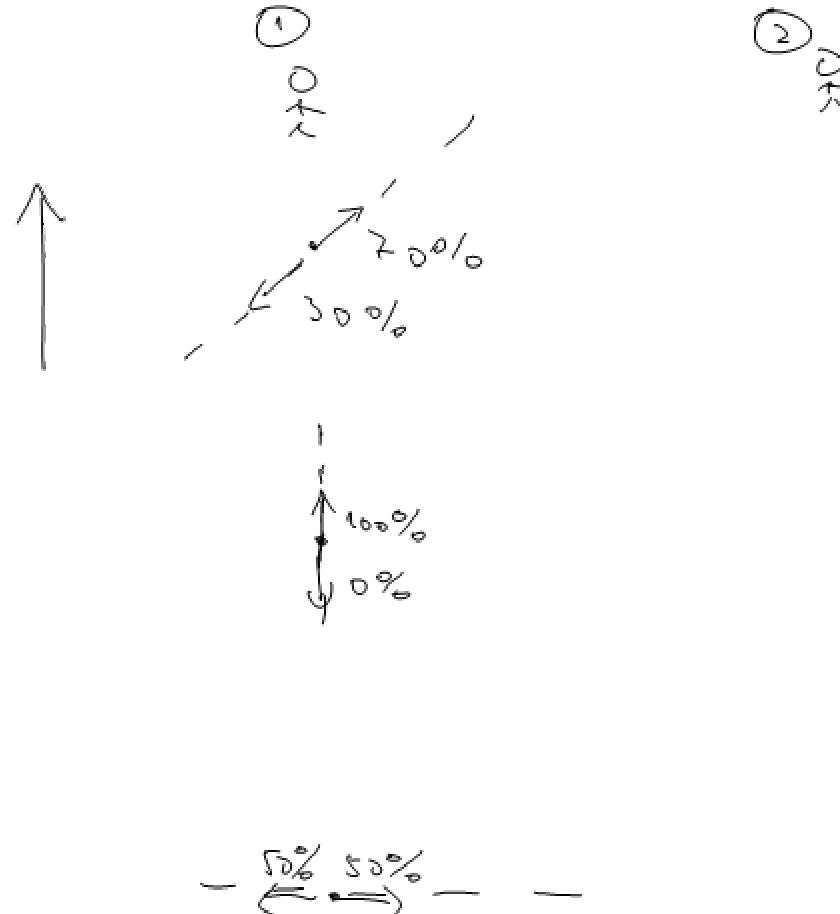
Motivation: observing classical systems



- Classical systems are “perfectly recyclable” for observations
 - **fidelity of estimation = 1** (complete information, all properties)
 - **state_{in} = state_{out}**
 - any number of observers see the same properties of an object

Motivation: observing quantum systems

single pure spin-1/2, polarization direction uniformly distributed:



- Quantum systems:
 - contain limited amount of extractable information (1 qubit up to 1 bit)
 - non-orthog. set → (typically) disturbed by a measurement – “information-disturbance trade-off”

Motivation: quantum → classical



$$N = \infty$$

$$k = \infty$$

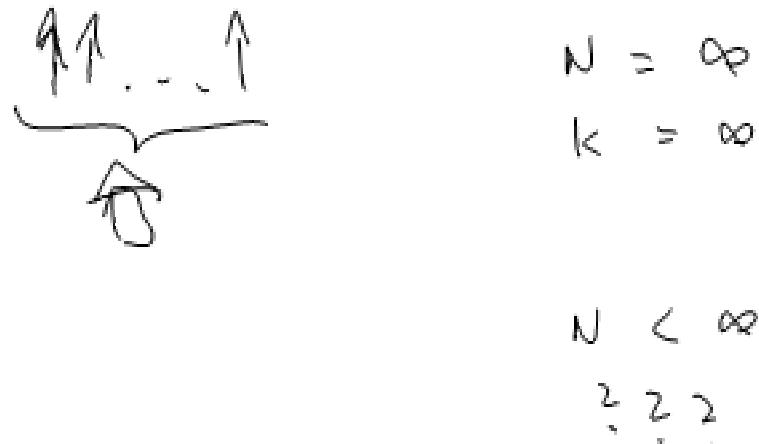
$$N < \infty$$

? ? ?

notation

- N : number of elementary systems
- K : number of observers
- k : tally number of an observer

Motivation: quantum → classical



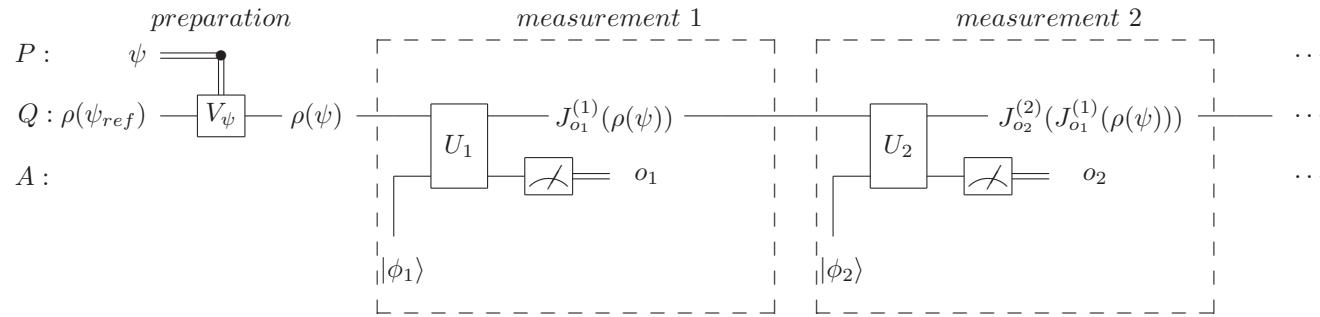
notation

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encoding: $\varrho: \mathcal{S}(\mathcal{H}_d) \rightarrow \mathcal{S}(\mathcal{H}_D)$, $\psi \mapsto \varrho(\psi)$

- system that we encode – pure qudit: $\psi = |\psi\rangle\langle\psi|$, $|\psi\rangle \in \mathcal{H}_d$, $d = \dim(\mathcal{H}_d)$
- system to which we encode: \mathcal{H}_D , $D = d^N$ (N systems of the same type)
- examples:
 - $N = 1$: nothing done, into a discrete set, covariant, ...
 - $N > 1$: into copies $\uparrow \mapsto \uparrow\uparrow$, antiparallel $\uparrow \mapsto \uparrow\downarrow$, covariant, ..., arbitrary

The problem



- interpretation: $o_i \rightarrow \psi_i$ (can be probabilistic)

The problem

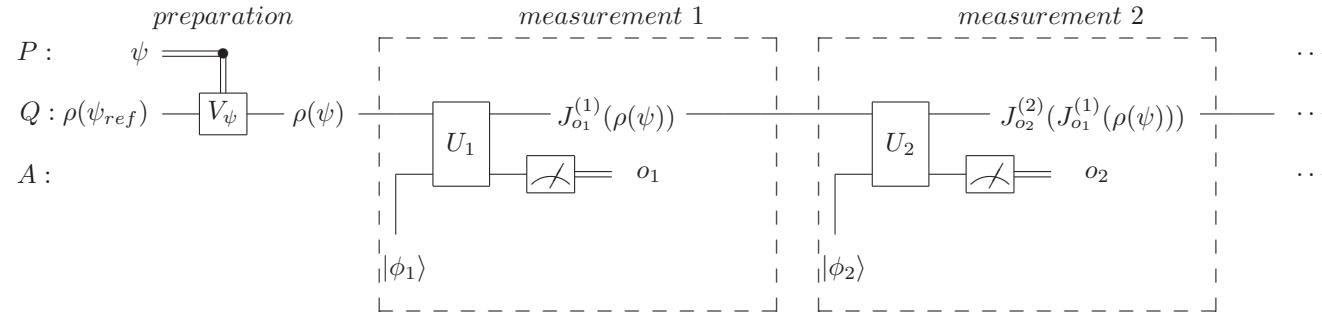


Figure of merit

$$F_k = \int_{\text{events}} d\mu_k(\psi_k, \psi) f(\psi_k, \psi) = \int \int d\psi_k d\psi \tilde{p}_k(\psi_k | \psi) f(\psi_k, \psi),$$

$$f(\psi', \psi) = \text{Tr}[\psi' \psi] = |\langle \psi' | \psi \rangle|^2 = \frac{1}{d}(1 + (d-1)\mathbf{n}(\psi) \cdot \mathbf{n}(\psi'))$$

The problem

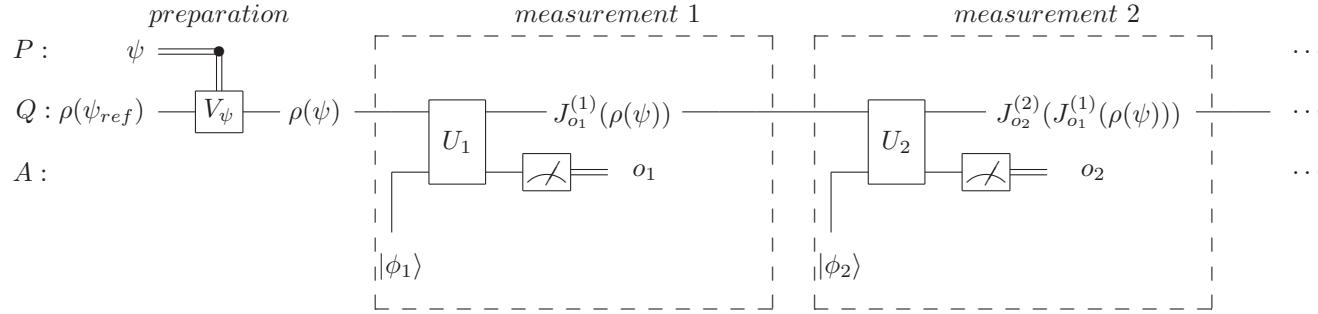


Figure of merit

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We search for (strategies s.t.)

$$\mathcal{F}_k(d, N) = \max F_k(d, N),$$

- maximization is over strategies (encoding, measurements up to k , interpretation: outcome \mapsto estimate)
- scenarios:
 - greedy observers
 - equalitarian observers
 - privileged observer
 - maximize F_1 , then F_2 given F_1 remains maximal, etc.
 - maximize F_k given the condition $F_k = F_j \quad \forall k, j = 1, \dots, K$
 - maximize F_K given all observers use the **same** apparatus

The problem (cntd) – strategies

Quantum instruments

- introduced by Davies & Lewis, 1970
- outcome-dependent update-rule + probability of outcome
- mapping: outcome (set) \mapsto quantum operation
- given input state & outcome, provides:
 - post-measurement state
 - $\text{prob}(\text{outcome}) = \text{Tr}[\text{post-measurement state}]$ – includes POVM description

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Independence of observers

- no communication of estimates
- operations $\$(\cdot)$ and $U(g)\$(U(g)^\dagger \cdot U(g))U(g)^\dagger$ equiprobable $\forall g \in SU(d)$, $U(g) := g^{\otimes N}$
- effective transformation $\$_s(\hat{\rho}^{(N)}) = \int dg U(g)\$(U(g)^\dagger \hat{\rho}^{(N)} U(g))U(g)^\dagger$

Simplifications

- outcome → estimate – incorporated in the apparatus description (re-labeling, re-defining)

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Simplifications due to independence of observers

- suffices to consider covariant encodings (ϱ), POVMs (\mathcal{M}), Instruments (\mathcal{I}), Channels (χ):
(covariance with respect to $SU(d)$ and its symmetric representation U)

$$\varrho(g\psi g^{-1}) = U_g \varrho(\psi) U_g^\dagger$$

$$\tilde{\mathcal{M}}_{g\psi g^{-1}} = U_g \tilde{\mathcal{M}}_\psi U_g^\dagger$$

$$\tilde{\mathcal{I}}_{g\psi g^{-1}}(\hat{\rho}) = U_g \tilde{\mathcal{I}}_\psi(U_g^\dagger \hat{\rho} U_g) U_g^\dagger$$

$$\chi(U_g \hat{\rho} U_g^\dagger) = U_g \chi(\hat{\rho}) U_g^\dagger$$

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Simplifications due to independence of observers

- suffices to consider covariant encodings (ϱ), POVMs (\mathcal{M}), Instruments (\mathcal{I}), Channels (χ)
(covariance with respect to $SU(d)$ and its symmetric representation U)
- reminder:

$$F_k = \int d\psi_k d\psi \tilde{p}_k(\psi_k | \psi) f(\psi_k, \psi),$$

- then:

$$\tilde{p}(\psi_k | \psi_0) = \int d\psi_{k-1} \dots \int d\psi_1 \text{Tr} \left[\widetilde{\mathcal{M}}_{\psi_k}^{(k)} \left(\widetilde{\mathcal{I}}_{\psi_{k-1}}^{(k-1)} \circ \dots \circ \widetilde{\mathcal{I}}_{\psi_1}^{(1)} \right) (\varrho_0(\psi_0)) \right],$$

or

$$\tilde{p}(\psi_k | \psi_0) = \text{Tr} \left[\widetilde{\mathcal{M}}_{\psi_k}^{(k)} \chi_{k-1} \circ \dots \circ \chi_1 (\varrho_0(\psi_0)) \right],$$

Greedy observers

- most informative measurements \Rightarrow 'factorized' form

$$\tilde{p}(\psi_k | \psi_0) = \int d\psi_{k-1} \tilde{p}(\psi_k | \psi_{k-1}) \dots \int d\psi_0 \tilde{p}(\psi_1 | \psi_0),$$

with

$$\tilde{p}(\psi_i | \psi_{i-1}) = \text{Tr}[\tilde{\mathcal{M}}^{(i)}(\psi_i) \varrho_{i-1}(\psi_{i-1})].$$

$$\varrho_{i-1}(\psi_{i-1}) := \frac{\tilde{\mathcal{I}}_{\psi_{i-1}}^{(i-1)}(\hat{\rho})}{\text{Tr}\left[\tilde{\mathcal{I}}_{\psi_{i-1}}^{(i-1)}(\hat{\rho})\right]}$$

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-

$$F_k = \frac{1}{d} \left(1 + (d-1) \prod_{i=1}^k \Delta_i \right).$$

where Δ_i – number characterizing fidelity of estimation of a state ψ_{i-1} encoded by the $(i-1)$ th apparatus.

-

$$\mathcal{F}_k = \frac{1}{d} [1 + (d-1)\Delta^k],$$

$$\Delta = \frac{\mathcal{F}_1 d - 1}{d - 1}.$$

$\uparrow \dots \uparrow$ N copies of a qudit

•

$$\mathcal{F}_k^{\text{par}} = \frac{1}{d} \left[1 + (d-1) \left(\frac{N}{N+d} \right)^k \right] \xrightarrow{N \gg 1} \frac{1}{d} \left[1 + (d-1) \left(1 - \frac{d}{N} \right)^k \right].$$

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N qubits - the optimal encoding

- for even N

$$\mathcal{F}_k^{\text{opt}} = \frac{1}{2} \left[1 + x_{N/2+1}^k \right] \xrightarrow{N \gg 1} \frac{1}{2} \left[1 + \left(1 - \frac{2\xi_0^2}{N^2} \right)^k \right]$$

- $x_{N/2+1}$ is the largest zero of the Legendre polynomial $P_{N/2+1}(x)$.
- $\xi_0 \doteq 2.4$ is the first zero of the Bessel function $J_0(x)$.

[Bagan et. al, Optimal encoding and decoding of a spin direction. *Phys. Rev. A*, 63(5):052309, Apr 2001.]

Discussion

How large a quantum system must be to be considered “classical”?

- “classicality”

-

$$\forall k \quad \mathcal{F}_k \rightarrow 1.$$

- for fixed N never happens
 - what scaling $N(k) = ck^\alpha, c > 0$ gives

$$\lim_{k \rightarrow \infty} \mathcal{F}_k(N) = 1$$

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- $\uparrow \dots \uparrow$ parallel encoding (qudits)

$$\alpha > 1$$

- optimal encoding (qubits)

$$\alpha > 1/2$$

Summary and references

Summary

Presented

- emergence of classical-like recyclability of q. systems even under **most informative** measurements of single-pure-qudit parameters
- quantitative results for fidelity degradation, optimal strategy vs. copy-encoding strategy

Not presented

- weak measurements \Rightarrow channel approach
 - egalitarian distribution of information among K observers
 - distribution of information using a fixed apparatus, favoring a privileged observer
- implications for the longevity of a directional reference (coherent strategies)

More details

- arXiv:1105.5326 (accepted to PRA)

Thank you for your attention.