Planning for retirement: sustainability versus legacy

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October 30, 2011

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  Call this a *SWiP*: Systematic Withdrawal Plan
- Buy an immediate or deferred annuity.
  Call this an *LPIA*: Lifetime Payout Income Annuity.

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The problem is that there isn't necessarily an "optimal" choice. There are tradeoffs, as retirees have multiple conflicting objectives. My goal is to show you a framework for making decisions about these and other strategies, that make the tradeoffs clear, and allow individuals to make rational decisions about them.

Pros and Cons

SWIP

- Preserves liquidity.
- Leaves an inheritance for the kids (eg. if you die early).
- Inflation protected (via capital growth).
- Preserves upside potential.
- Risk of investment losses.
- Longevity risk: may live so long your money runs out.

LPIA

- No longevity risk: income for life.
- Enhanced returns (mortality credits).
- Downside protection (no risk of loss).
- No upside potential from a bull market.
- Fixed income: erodes with inflation.
- No inheritance: when you die, the money is gone.
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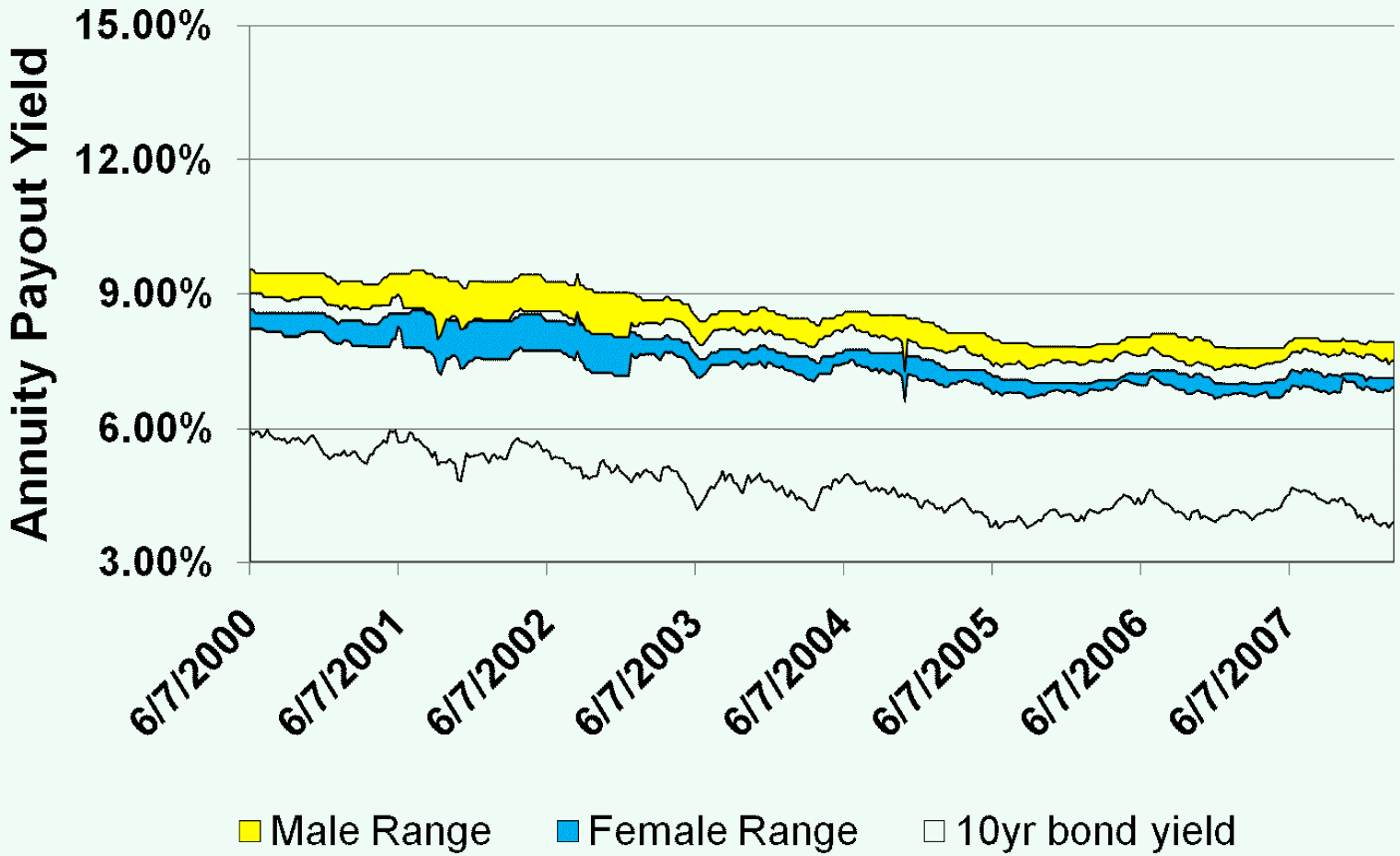
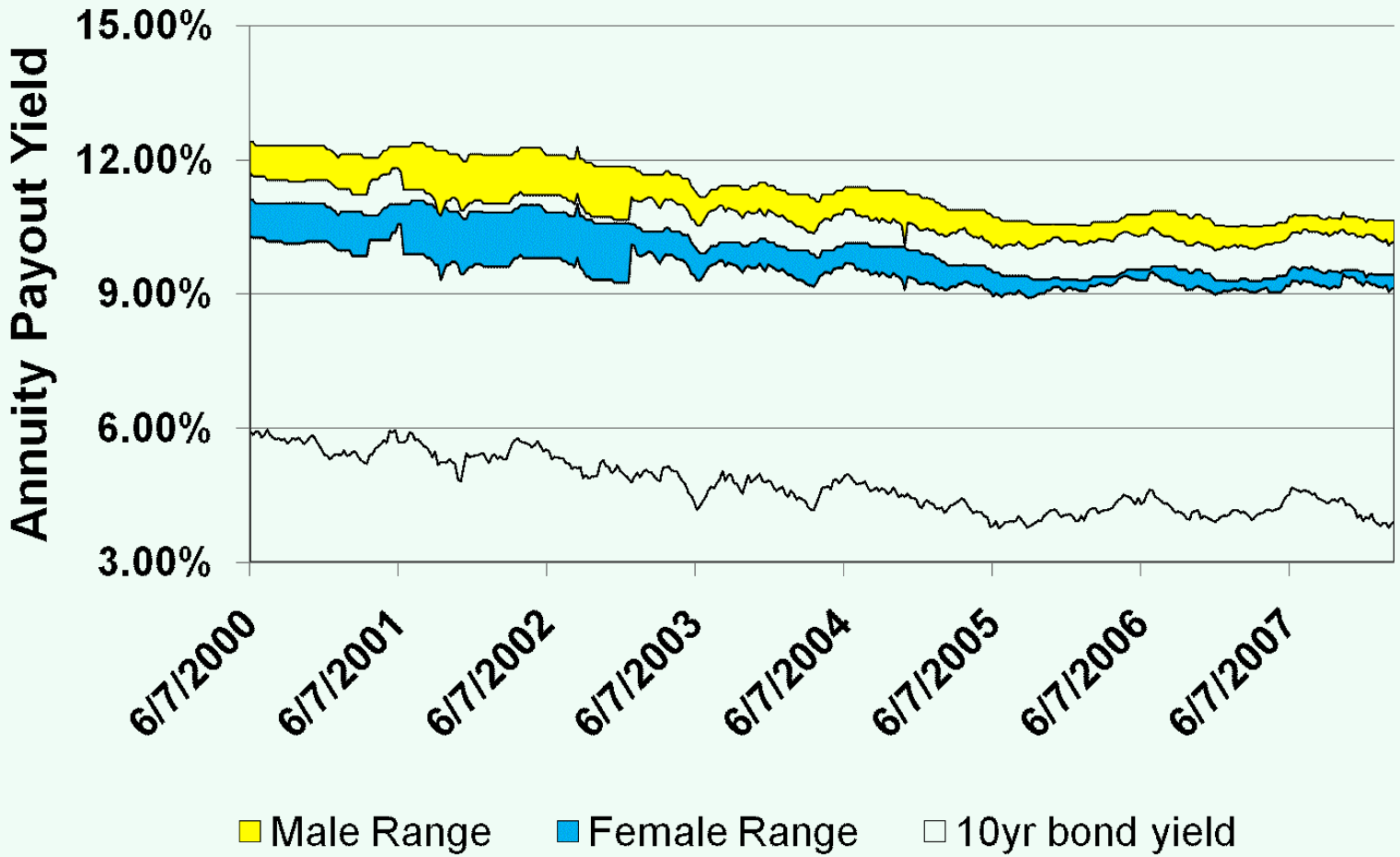
Annuity Yields in Canada: **Age 65**



 Source: CANNEX, Bank of Canada, the IFID Centre, June 2000 - February 2008

Annuity Yields in Canada: **Age 75**



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Tradeoffs

Retiree decision making will typically consider:

- Consumption – How much do you need to live on each year?
- Sustainability – How confident are you that there will be funds to support that consumption for as long as you live?
- Legacy (or liquidity) – Will there be something to leave the kids when you die?

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Typically these considerations compete with each other.
Eg an LPIA wins on sustainability but loses on legacy.
A SWiP preserves legacy but may not be sustainable.

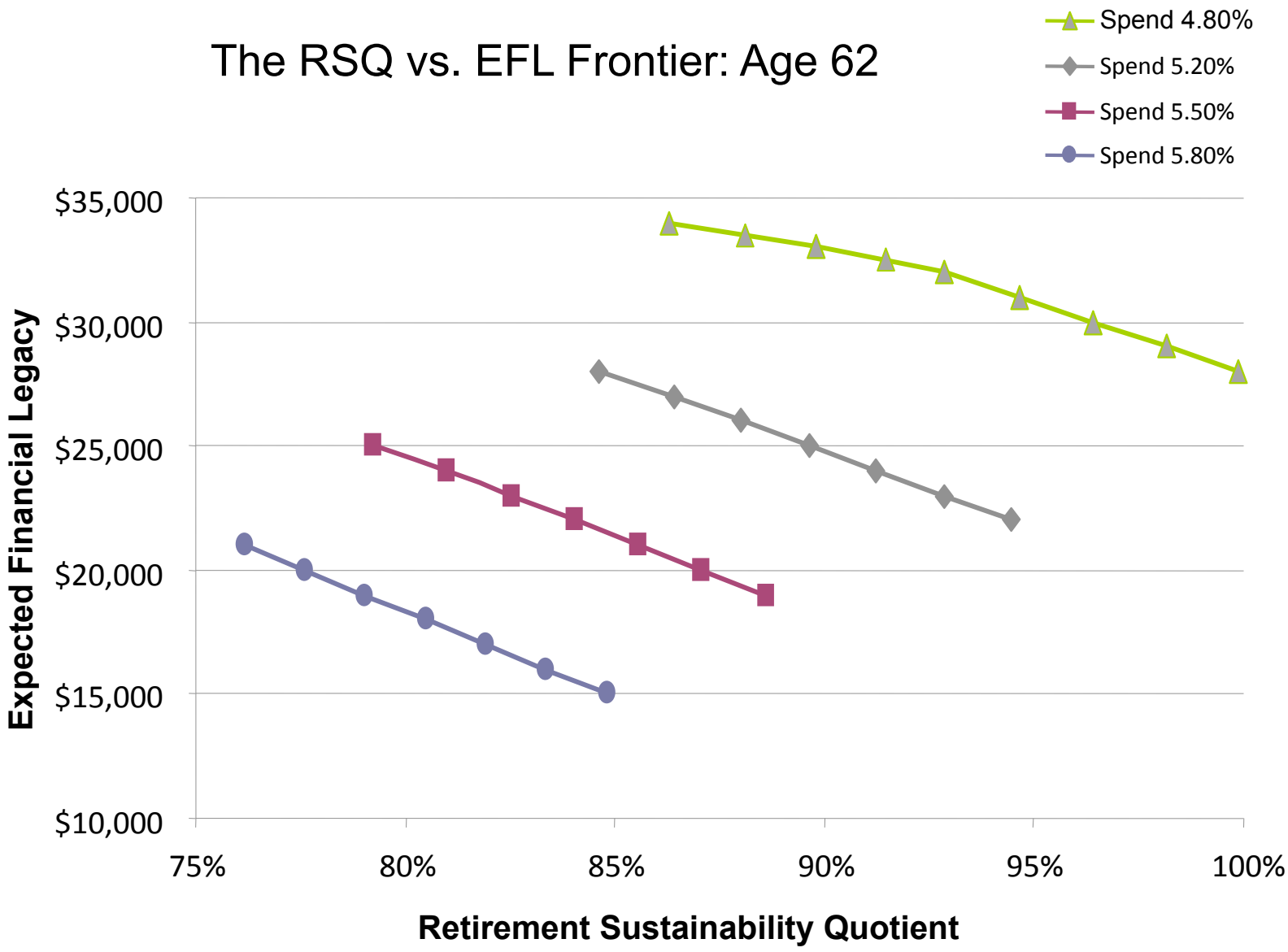
The planning process involves confronting this tradeoff head-on, and providing diagnostics that allow individuals to assess both sustainability and legacy.

Risk measures

What we need are two appropriate risk measures, to evaluate each objective

- RSQ: *Retirement Sustainability Quotient.*
  This will be a number $\in [0, 1]$ measuring the sustainability of the portfolio.
- EFL: *Expected Financial Legacy.*
  This will be a dollar figure, measuring the mean **current dollar** value of the financial portion of the eventual legacy.

If either risk measure seems irrelevant (eg. no kids, or great wealth) then maybe an extreme strategy is appropriate. But most people must pick a point on the RSQ/EFL efficient frontier.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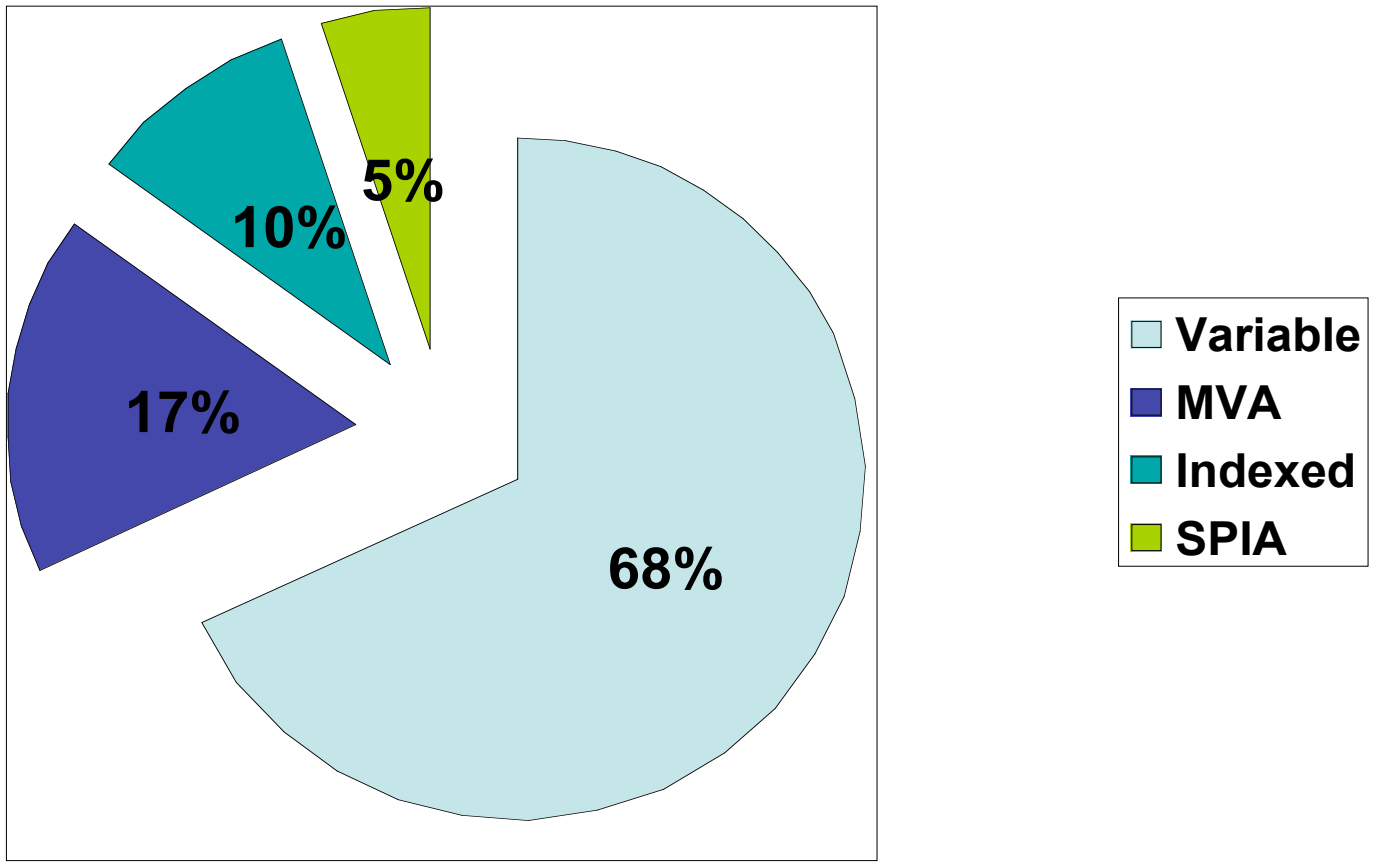
Retirement Sustainability Quotient

GLWB

- We expand the universe of products, to include a GLWB: *Guaranteed Lifetime Withdrawal Benefit*
- This blends features of an LPIA and a SWiP.
- Acts like a SWiP (but with fees) until the account ruins. Then converts to an annuity. In other words, withdrawals (at a contracted rate) are guaranteed to continue for life, or until the client lapses the account.
- A retirement savings product, very popular in the U.S., Japan, U.K. and elsewhere (eg new markets in Europe, Australia).
- Entered Canada in 2006 and have been wildly popular here too. Worked exactly as planned during the downturn (from the client side anyway).

Sales of Annuities in Year 2007: approximately $220 Billion U.S.D.



Source: National Underwriter (16 March 2008)

Product Allocation

- Advisors are used to thinking about *asset allocation*
- But the assets supporting a GLWB and a SWiP are essentially the same.
- Choosing between them is not a question of asset allocation – it is a question of what products are the appropriate choice to house those.
- We call this choice *Product Allocation*

Retirement Sustainability Quotient (RSQ)

- You may split your nest egg between P/LPIA/SWiP/GLWB.
- P $\leftrightarrow$ exogenous pension (including CPP at a minimum)
- Each component has a different sustainability and legacy – LPIA is fully sustainable, but has no legacy. GLWB is also fully sustainable, and has partial legacy (fees have an effect). SWiP has highest legacy, but least sustainability.
- We weight the sustainability by the importance of the income stream represented by that component; If a component is less important, it should impact RSQ less.
- Let c be the consumption rate, $p \geq 0$ the rate pension is paid, $a \geq 0$ the rate paid by the LPIA, and $g \geq 0$ the rate paid by the GLWB. So $w = c - p - a - g$ is the rate of SWiP withdrawals. Then we define

$$\text{RSQ} = \frac{p}{c} \cdot q^{\text{P}} + \frac{a}{c} \cdot q^{\text{LPIA}} + \frac{g}{c} \cdot q^{\text{GLWB}} + \frac{w}{c} \cdot q^{\text{SWiP}},$$

where the q's are probabilities of sustainability.

RSQ and SWiP dynamics

- Here $q^{\text{LPIA}} = 1 = q^{\text{GLWB}} = q^{\text{P}}$.
- And $q^{\text{SWiP}} = P(W_\zeta > 0)$, where W_t is the SWiP account value at time t, and ζ is the lifetime of the client.
- SWiP dynamics in the simplest model are that

$$dW_t = \mu W_t \, dt + \sigma W_t \, dB_t - w \, dt$$

  where μ are the SWiP investment growth rate and volatility (depend on the asset allocation within the SWiP).
  Must use an actuarial model for ζ (eg Gompertz).
- q^{SWiP} is calculated by solving a PDE.

PDE for RSQ

For a suitable function $f(t,x)$ we must calculate

$$P(W_\zeta > 0 \mid \mathcal{F}_t) = \begin{cases} f(t,W_t), & t < \zeta \\ 1, & t \geq \zeta, W_\zeta > 0 \\ 0, & t \geq \zeta, W_\zeta = 0. \end{cases}$$

This is a martingale, with a jump at ζ. Let R be the ruin time. Compensating,

$$f(t,W_t) + \int_0^t \lambda(s)[1 - f(s,W_s)]\, ds$$

is a continuous martingale when stopped at $R \wedge \zeta$. Applying Ito,

$$f_t + [\mu x - w]f_x + \frac{1}{2}\sigma^2 x^2 f_{xx} + \lambda(t)(1-f) = 0.$$

Then $q^{\text{SWiP}} = f(0,w_0)$, $w_0 =$ initial SWiP account value.

Expected Financial Legacy (EFL)

- Define *Expected Financial Legacy* (EFL) as the expected amount of current financial wealth that is destined to fund the eventual legacy left the heirs.
- In other words, it is denominated in current dollars.
- **Eg 1:** Suppose a client segregates current wealth into two pieces $a + b$, uses a to purchase an LPIA that funds consumption, and invests b to entirely fund the legacy. Then the EFL is simply b, regardless of what the actual bequest is or what assets make up the investment portfolio.
  (Just as if it was gifted immediately.)
- The point is, we bring everything to present value terms, to make comparisons as fair and clear as possible.

Expected Financial Legacy (EFL)

- **Eg 2.** Suppose a client's entire wealth is invested in an asset class, that is the sole funder of BOTH consumption and legacy. After death, one could go back and determine what portion Z of initial wealth eventually funded the legacy. It isn't observable at time 0, but we can still compute EFL $= E[Z]$ & use this as a risk measure.
- The longer we live, the lower Z gets, so EFL is monotone for mortality (as it should be).
- This focuses information on the element we must decide about (the anticipated allocation to the kids) rather than on things we can't control (future investment returns).

Portfolio EFL

- EFL is therefore measured *relative to a reference asset*.
- If wealth is invested in multiple streams, with different return characteristics, one may calculate an EFL for each stream, relative to its reference asset. Then add the resulting expectations for an overall EFL:

$$\begin{aligned}\text{EFL} &= \text{EFL}^{\text{P}} + \text{EFL}^{\text{LPIA}} + \text{EFL}^{\text{GLWB}} + \text{EFL}^{\text{SWiP}} - \text{EFL}^{\text{RUIN}} \\ &= \text{EFL}^{\text{GLWB}} + \text{EFL}^{\text{SWiP}} - \text{EFL}^{\text{RUIN}}\end{aligned}$$

  (since the pension and annuity leave no legacy).
- Will explain EFL^{RUIN} later.

EFL dynamics

- The first two terms come from solving PDEs.
- We treat the GLWB here – the SWiP term is the same (possibly with different parameters and asset allocations).
- For simplicity, we treat a simple GLWB (no ratchet or lapsation or bonusing).
- Let S_t = value at time t of \$1 invested at time 0 in the absence of consumption. Let X_t = the asset value remaining at time t in the presence of consumption. Legacy = X_ζ, so the portion of initial wealth that funded that legacy is $Z = X_\zeta / S_\zeta$. In other words, X_ζ / S_ζ at time 0 grows to X_ζ at time ζ.

EFL dynamics

- Set $Z_t = X_t/S_t$, so $X_t = Z_t S_t$.
- If μ and σ represent the growth rate and volatility for the reference asset, then $dS_t = \mu S_t\,dt + \sigma S_t\,dB_t$ and $dX_t = \mu X_t\,dt + \sigma X_t\,dB_t - g\,dt$.
- Z_t will be of bdd variation, so also $dX_t = Z_t\,dS_t + S_t\,dZ_t = \mu X_t\,dt + \sigma X_t\,dB_t + S_t\,dZ_t$.
- In other words, $dZ_t = -\frac{g}{S_t}\,dt$ with $Z_0 = x_0$.
  (The initial GLWB account value is x_0.
  After ruin, Z_t stays at 0.)
- We want to calculate $E[Z_\zeta]$.

EFL PDE

For a suitable function $h(t, y, z)$ we must calculate

$$E[Z_\zeta \mid \mathcal{F}_t] = \begin{cases} h(t, S_t, Z_t), & t < \zeta \\ Z_\zeta, & t \geq \zeta. \end{cases}$$

As with RSQ, this is a martingale, with a possible jump at ζ. Compensating,

$$h(t, S_t, Z_t) + \int_0^t \lambda(s)[Z_s - h(s, S_s, Z_s)]\, ds$$

is a continuous martingale when stopped at ζ. Applying Ito,

$$h_t + \mu y h_y + \frac{1}{2}\sigma^2 y^2 h_{yy} - \frac{g}{y} h_z + \lambda(t)(z - h) = 0.$$

Solve. Then $\text{EFL}^{\text{GLWB}} = E[Z_\zeta] = h(0, 1, x_0)$.

EFL under Ruin

- Recall the term $-\text{EFL}^{\text{RUIN}}$.
- This term comes from the possibility of ruin before death. In that case, SWiP funds are no longer sufficient to make up the gap in consumption left by the pension, LPIA, GLWB.
- To maintain consumption, the client must therefore turn to other sources to finance it. Eg. non-financial assets (maybe take out a reverse mortgage on the family home). Or the heirs might be called upon to fund consumption.
- In either case, this produces a negative legacy.
- Since no investment assets are directly involved, we model this as the hedging cost a firm would demand for insuring a collection of lifetime consumption streams against the ruin of a reference portfolio used to finance them.
- Elsewhere we call this a *Ruin Contingent Life Annuity* (RCLA).

RCLA dynamics

- Recall that w is the SWiP withdrawal rate. Let σ be the SWiP asset voltatility and r the risk free rate.
- Set X_t be the SWiP value at time t, and $R =$ the ruin time for the SWiP.
- Then the hedging cost is a risk-neutral expectation $E^Q[1_{\{R<\zeta\}} \cdot \int_R^\zeta we^{-rt}\, dt]$

RCLA

Again, get a martingale from

$$E^Q[1_{\{R<\zeta\}} \cdot \int_R^\zeta e^{-rs}\, ds \mid \mathcal{F}_t]$$

$$= \begin{cases} \phi(t, X_t), & t < R \wedge \zeta \\ \int_R^t e^{-rs}\, ds + A(t), & R \le t < \zeta \\ \int_R^\zeta e^{-rs}\, ds, & R \le \zeta \le t \\ 0, & t \ge \zeta, R \ge \zeta \end{cases}$$

where $A(t) = E[\int_t^\zeta e^{-rs}\, ds]$ is basically an annuity price. There is potentially a jump at ζ, so compensating,

$$\phi(t, X_t) - \int_0^t \lambda(s)\phi(s, X_s)\, ds$$

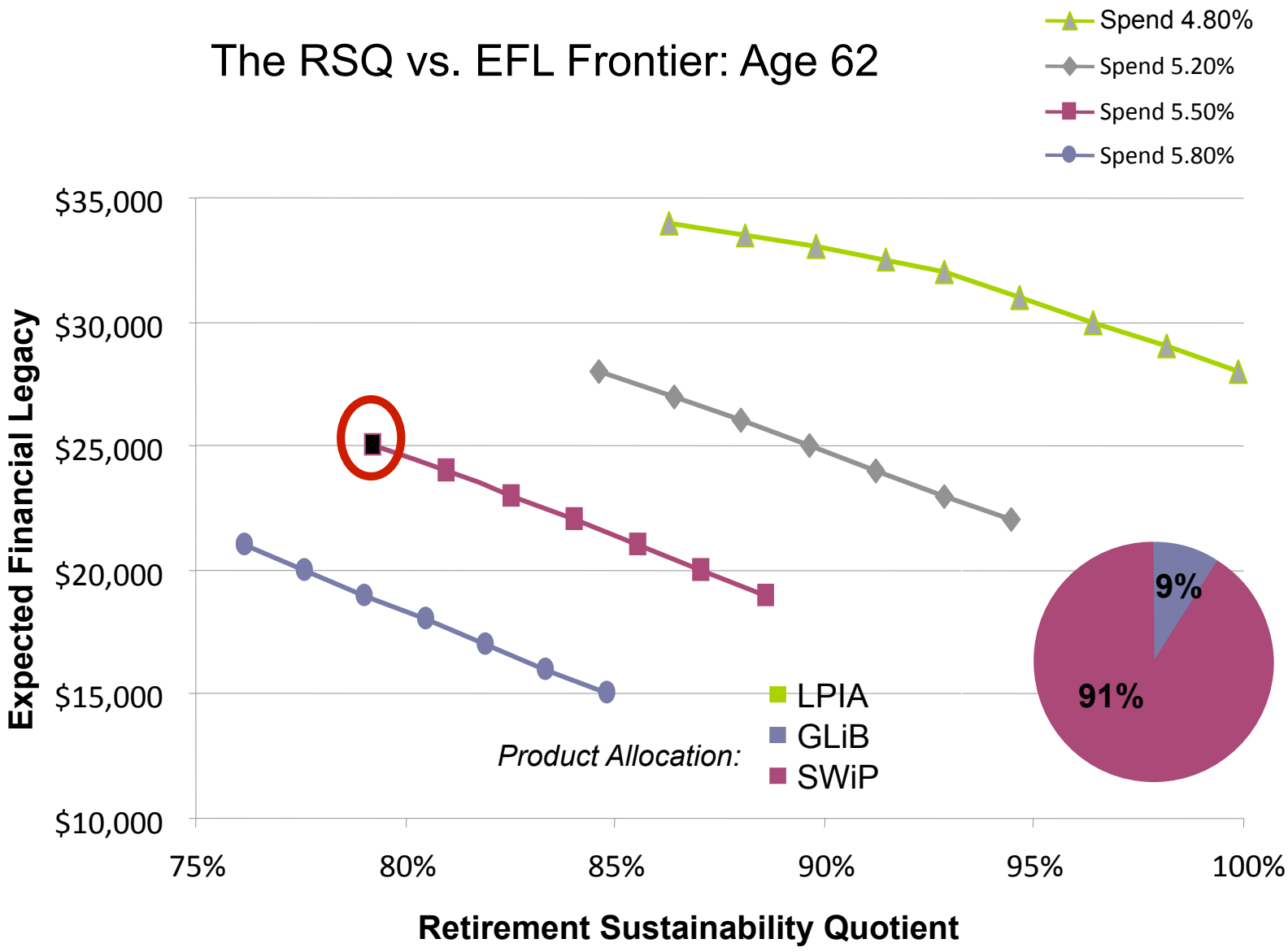
is a martingale when stopped at $\zeta \wedge R$. So

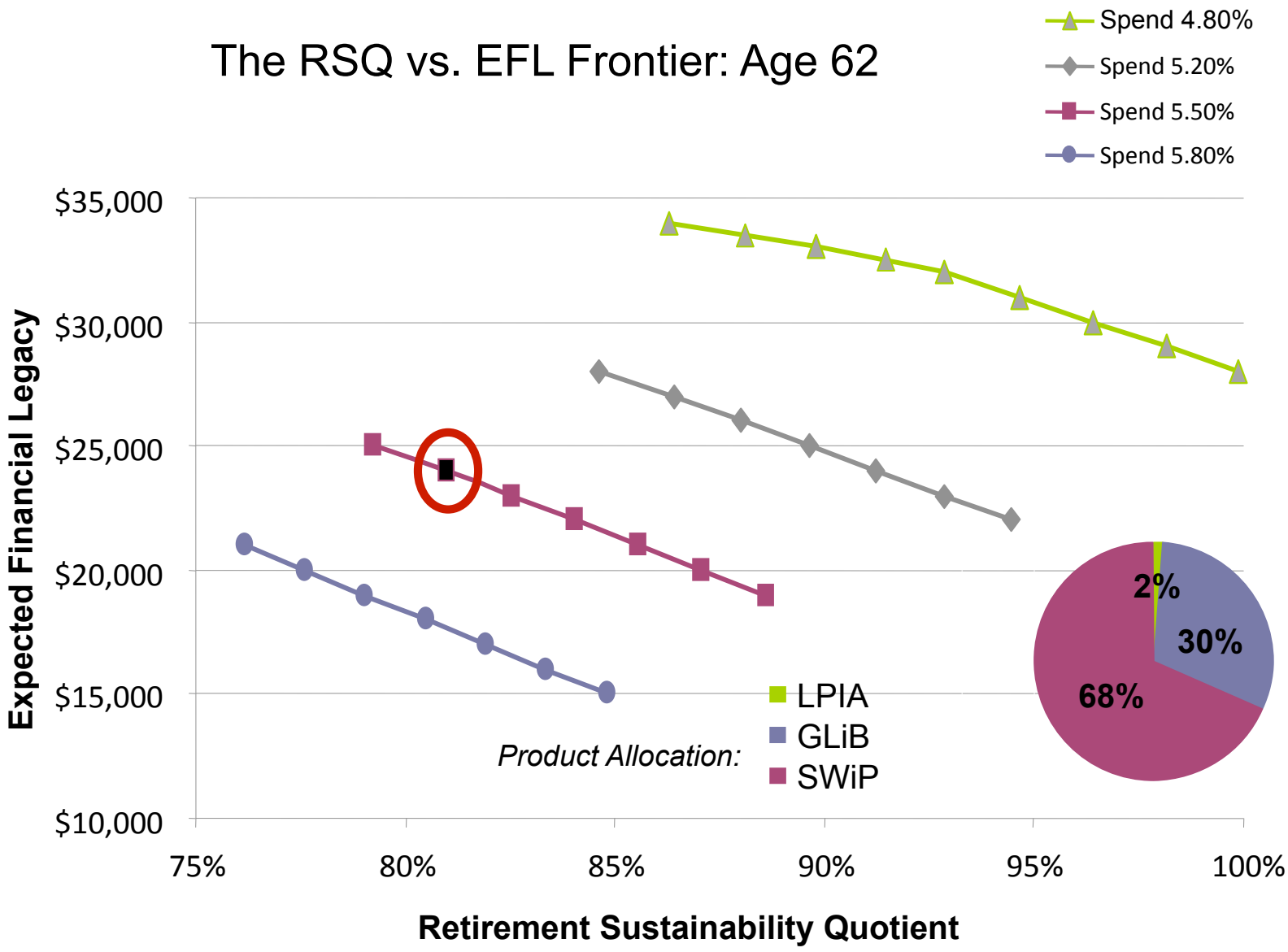
$$\phi_t + (rx - w)\phi_x + \tfrac{1}{2}\sigma^2 x^2 \phi_{xx} - \lambda(t)\phi = 0.$$

Solve, and $\text{EFL}^{\text{RUIN}} = w\phi(0, w_0)$.

Efficient Frontier

- Now that RSQ and EFL are defined, can evaluate them for any given product and asset allocation.
- For a given consumption and RSQ, one can likewise **optimize** the product and asset allocations to maximize EFL. That is how the efficient frontier is obtained.
- This also yields a plot of product allocations along the efficient frontier.


The RSQ vs. EFL Frontier: Age 62
Spend 4.80%
Spend 5.20%
Spend 5.50%
Spend 5.80%
$35,000
$30,000
$25,000
$20,000
$15,000
$10,000
Expected Financial Legacy
75%
80%
85%
90%
95%
100%
Retirement Sustainability Quotient
Product Allocation:
LPIA
GLiB
SWiP
9%
91%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75%
80%
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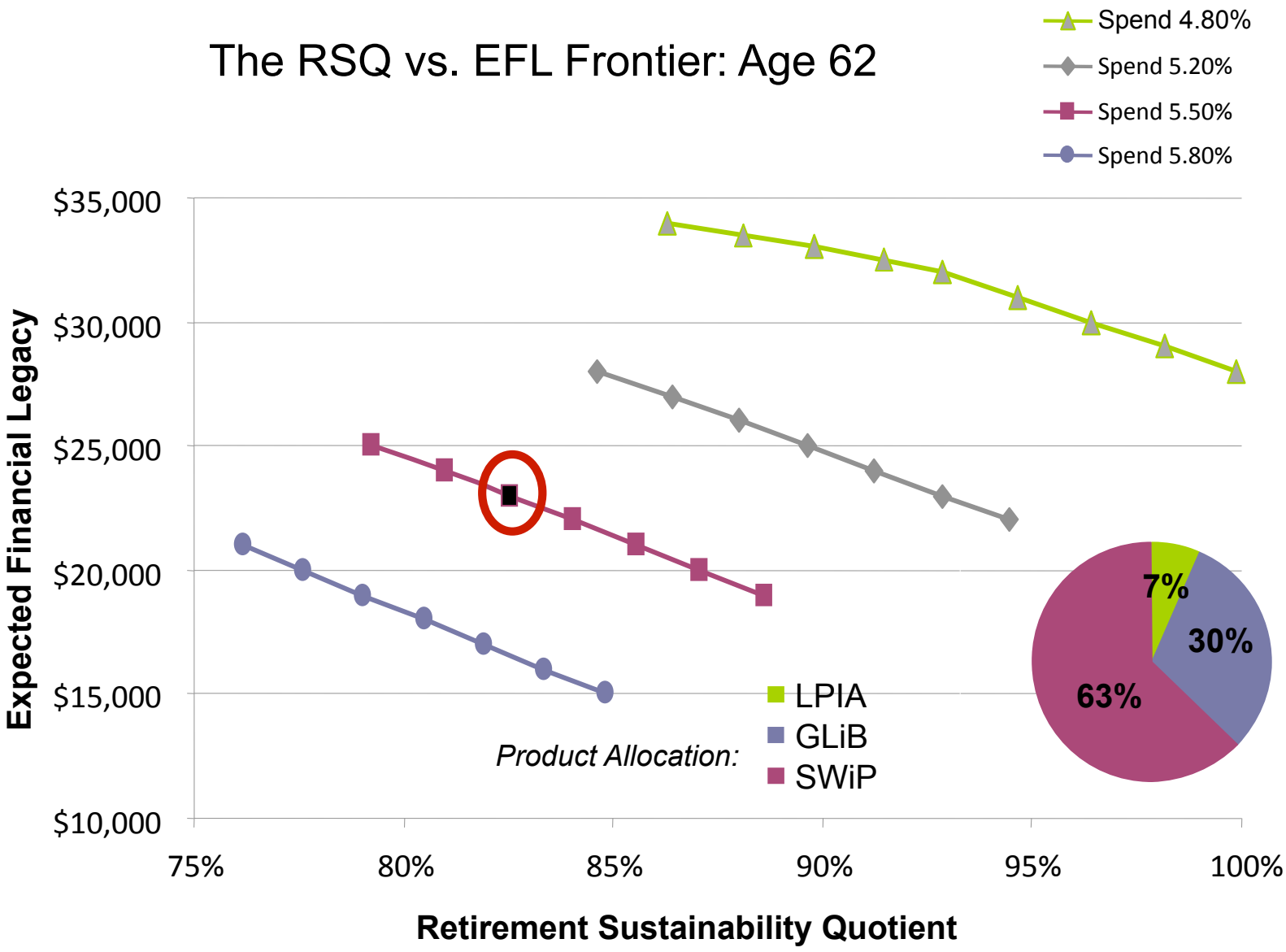
95%
100%
Retirement Sustainability Quotient
Product Allocation:
LPIA
GLiB
SWiP
2%
30%
68%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Retirement Sustainability Quotient
Product Allocation:
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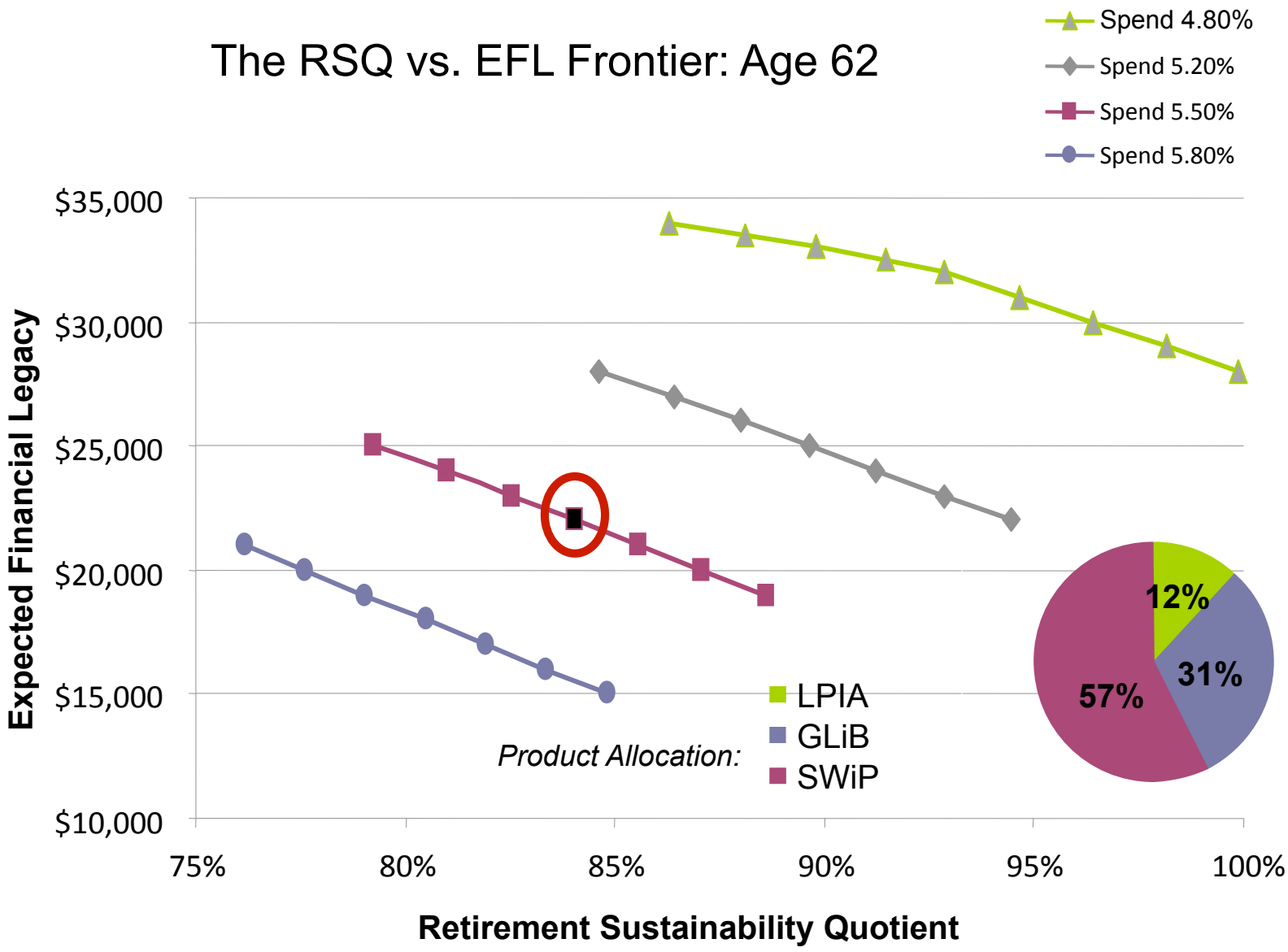
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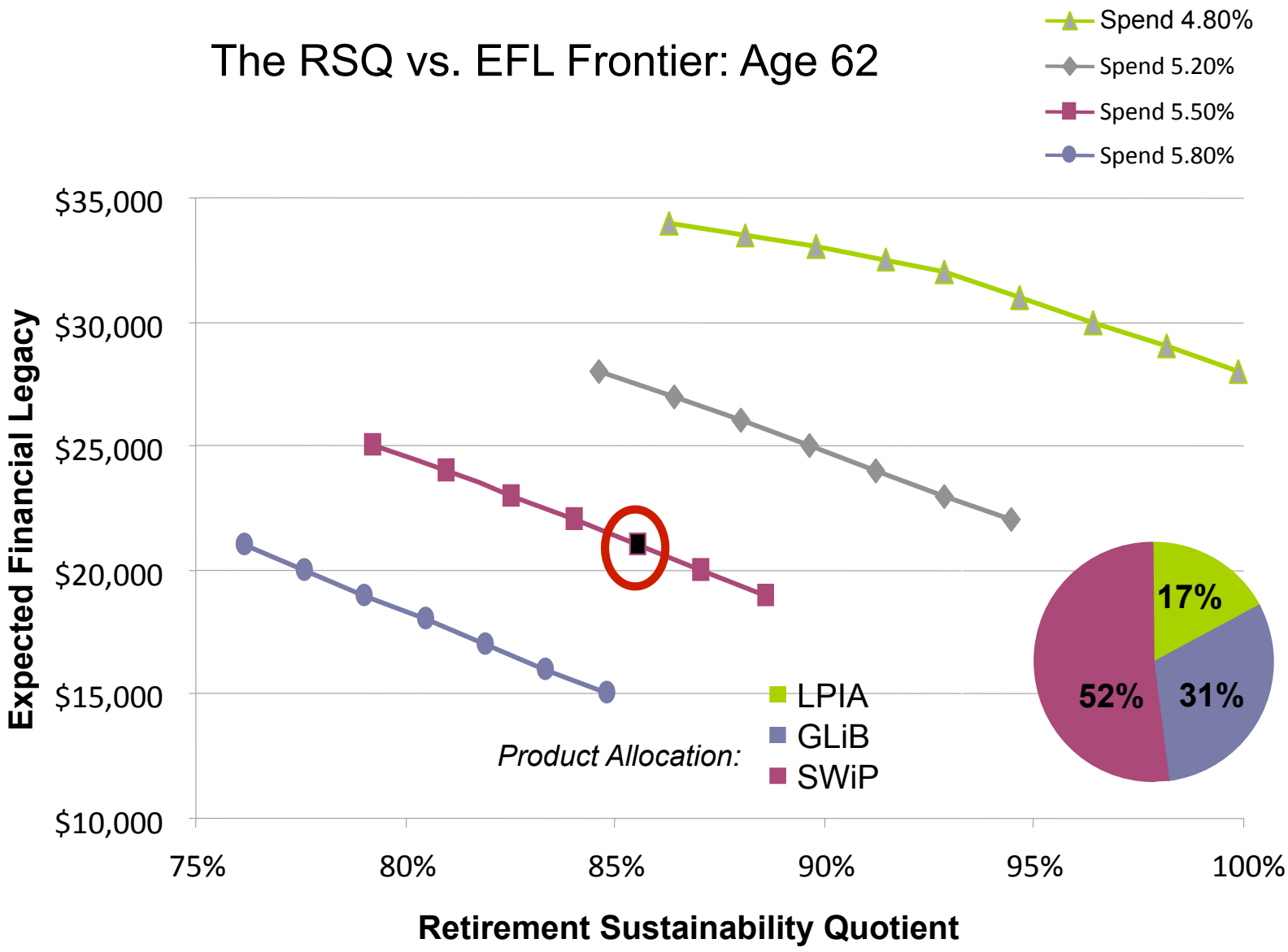
Retirement Sustainability Quotient
Product Allocation:
LPIA
GLiB
SWiP
12%
31%
57%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Retirement Sustainability Quotient
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SWiP
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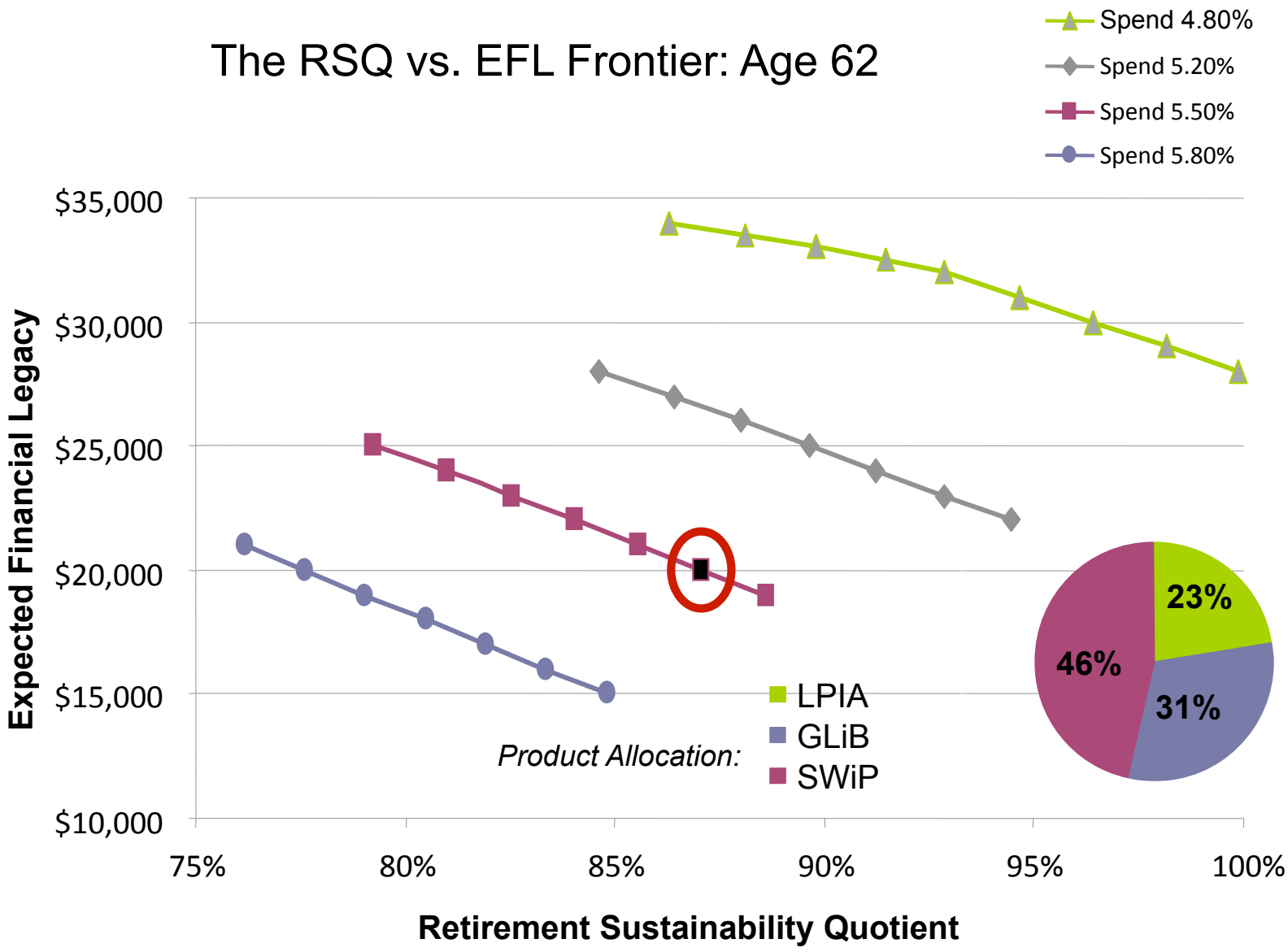
52%
31%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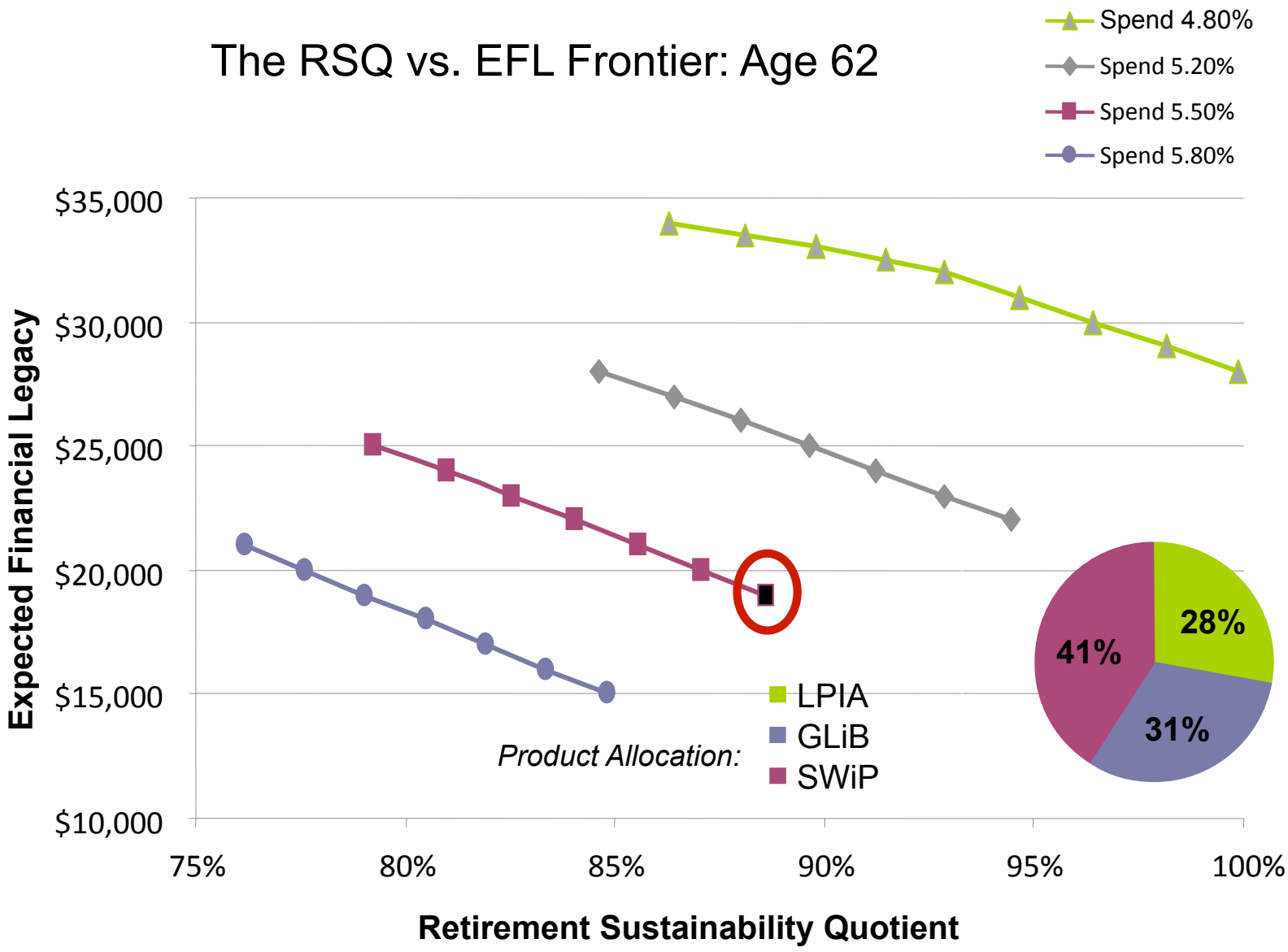
31%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41%

Implementation

- *QWeMA group inc.* has implemented this in software.
  See either *Product Allocation for Retirement Income* (PrARI) or *Optimal Product Allocation and Depletion* (oPAD).
- A real implementation has to cope with many other issues, eg. variable pension income streams, variable consumption, inflation, ratchets and inhomogeneous income from the GLWB, and a host of other technical issues.
- But the basic framework remains as described above.