What can we learn from EU ETS experience? Recommendations for Effective Design and Efficient Trading

Walid Mnif
wmnif@uwo.ca

Matt Davison
mdavison@uwo.ca

The University of Western Ontario, Canada

The First 3-C Risk Forum & 2011 International Conference on Engineering and Risk Management (ERM)

Toronto, Canada, October 2011.

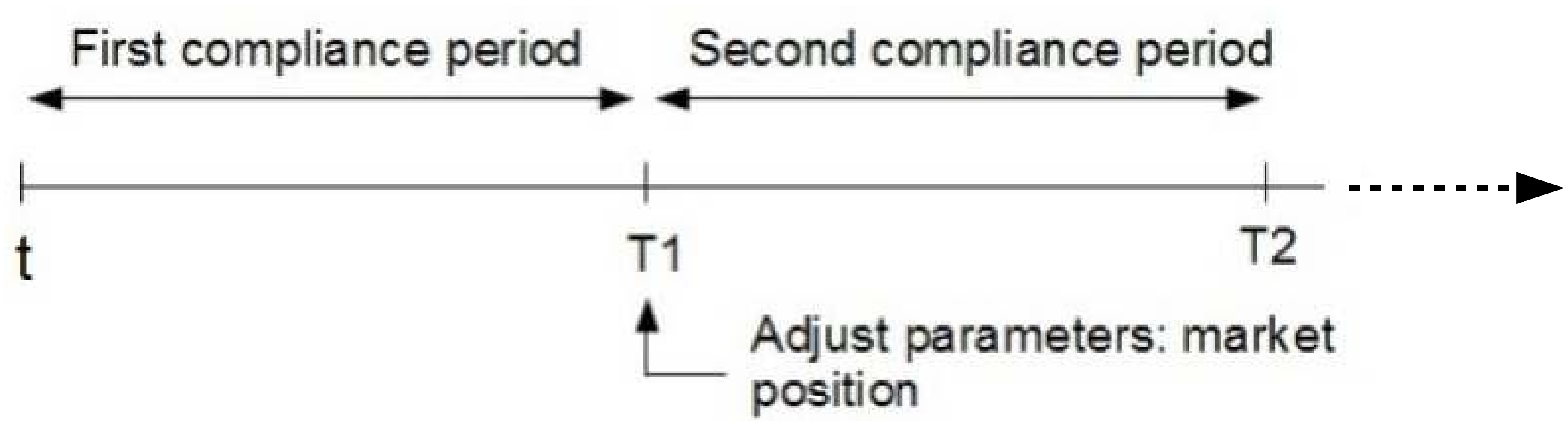
Carbon Emissions Market

- Cap-and-Trade Market Mechanism
  - ▷ Ceiling for emissions
  - ▷ Compliance period
  - ▷ Market: price to comply with emission target
  - ▷ Least cost: internal abatement or acquisition of allowances
  - ▷ Trading between: Regulated emitters, Non-regulated emitters, Non-emitters

Futures Allowance Dynamics

- S_t: d-dimensional vector of discounted futures allowance price,

$S_t^i, t \leq T_i$ used for compliance at T_i



Futures Allowance Dynamics

- Discrete time framework with $(\Omega, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$
- If the market is complete:

$$S_t^i = \xi_t^i S_{t-1}^i, \forall t \geq 1, \tag{1}$$

  ξ_t^i is a $\mathcal{F}_t-$measurable process.

- However:
  - ▷ Market is incomplete
  - ▷ Information set describing expected market position strongly affects the prices

Futures Allowance Dynamics

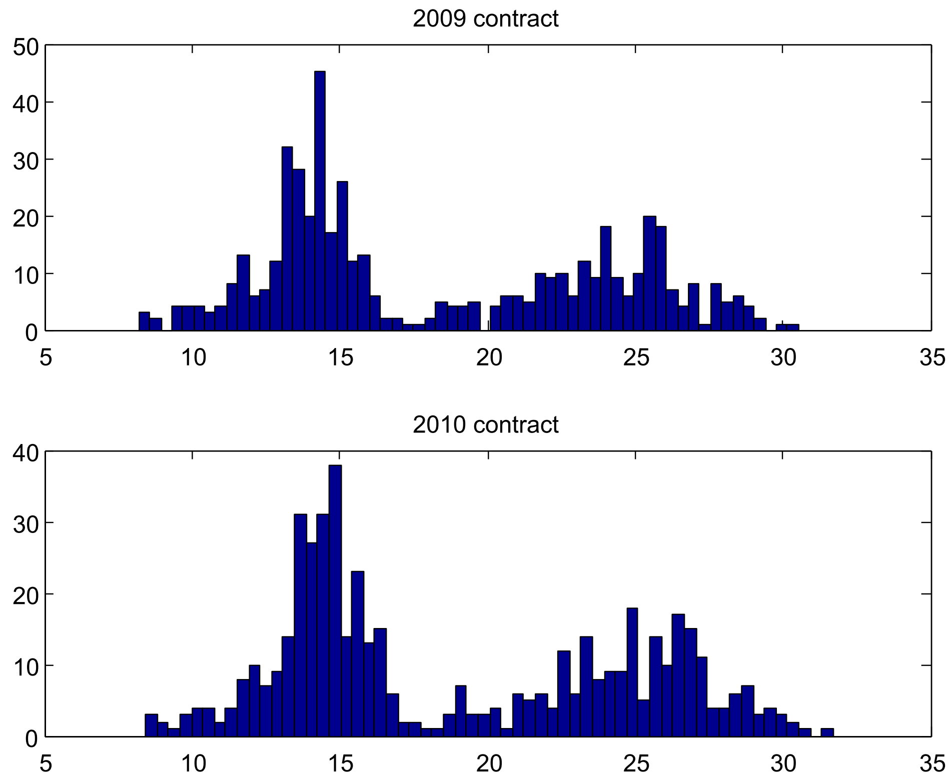


Figure 1: Price distribution over time of Dec-2009 and Dec-2010 contracts

Futures Allowance Dynamics

- So we model:

$$S_t^i = f_{t-1}^i(\xi_t^i, Y_{t-1}) S_{t-1}^i, \forall t \geq 1, \tag{2}$$

  where

  ▷ f_{t-1}^i: $\mathcal{F}_{t-1}-$measurable return function

  ▷ Y_t: non-observable process reflecting the implied investors market expectation position at time t for the subsequent compliance dates.

Futures Allowance Dynamics

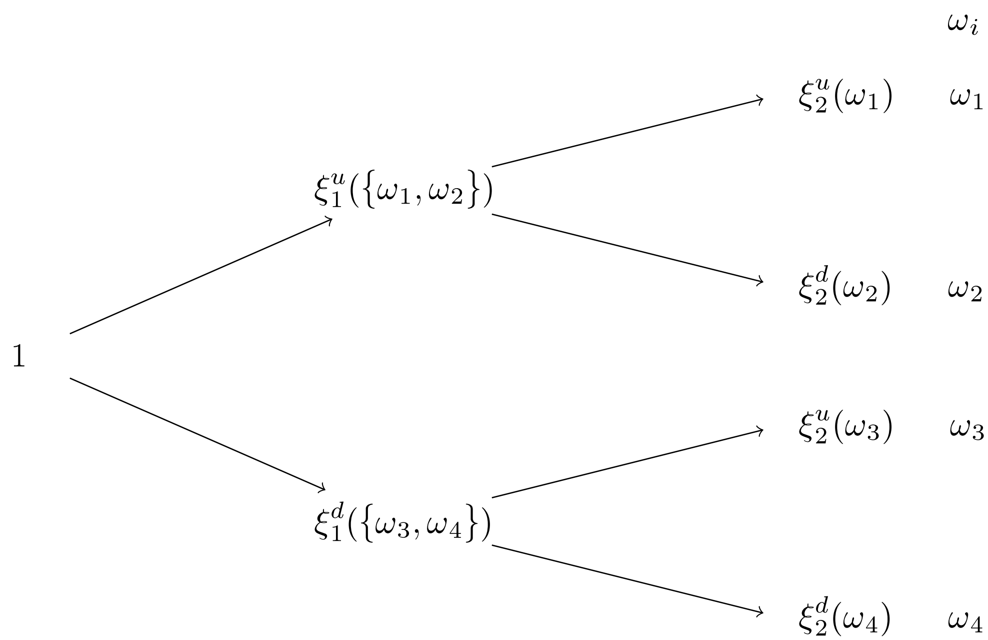


Figure 2: Returns tree of the traded asset in absence of Y_t per time step.

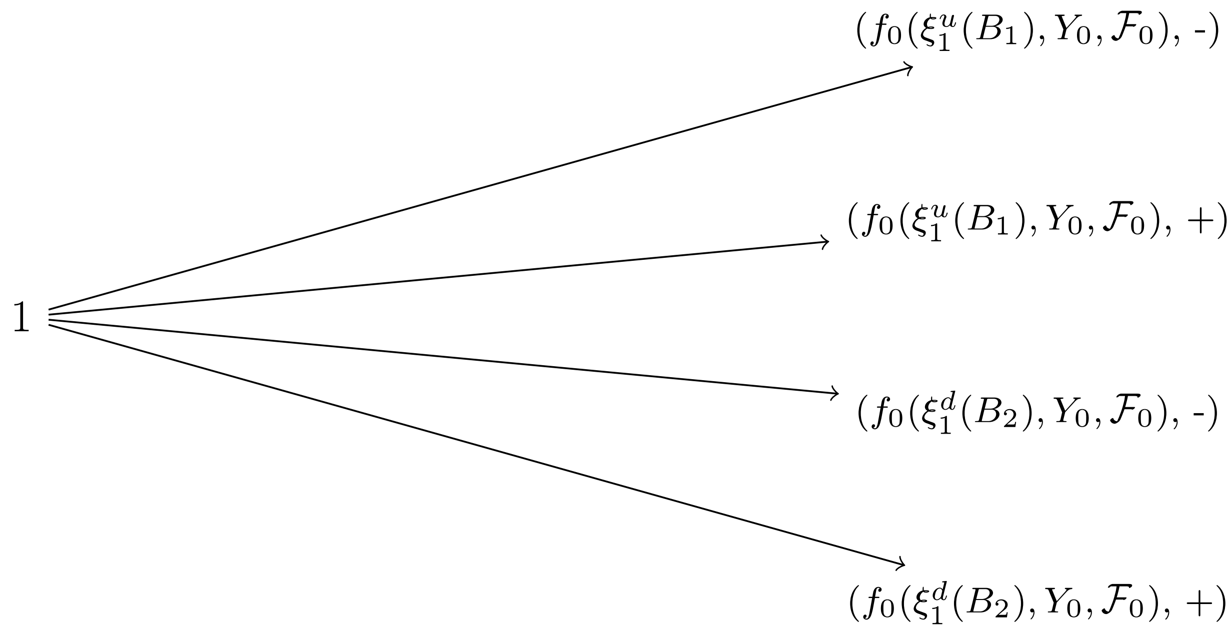


Figure 3: Returns in presence of Y_t at t=1, $B_1 = \{\omega_i, i = 1, ..., 8\}$, and $B_2 = \{\omega_i, i = 9, ..., 16\}$. "-": the market is expected to be short, "+": the market is anticipated to be long.

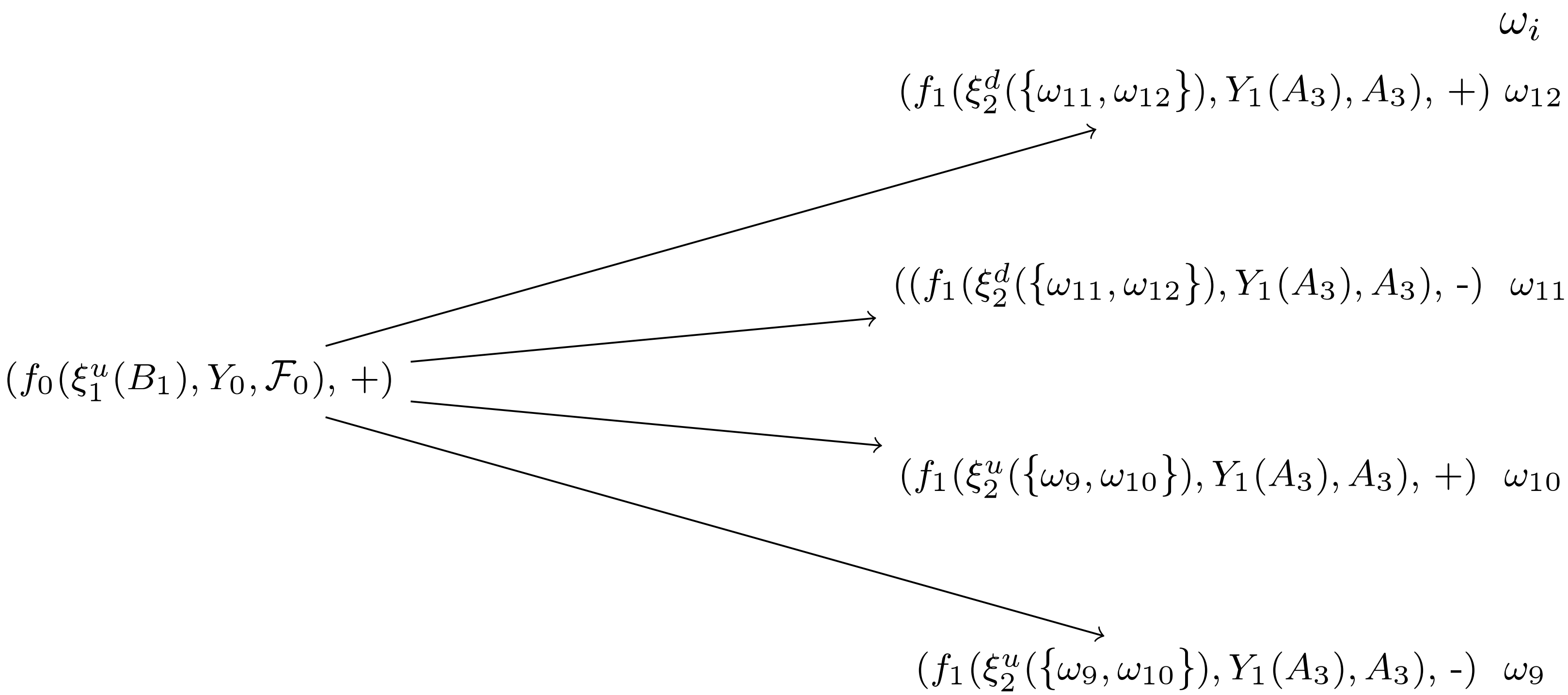


Figure 4: Returns tree in presence of Y_t at t=2, where $A_j = \{\omega_{(j-1)*4+i}, i = 1, 2, 3, 4.\}, j = 1, 2, 3, 4.$

Futures Allowance Dynamics

- Parameter estimation
  - ▷ Focus on Dec-2009 (S_t^1) and Dec 2010 (S_t^2) contracts
  - ▷ January 2008- December 2009
  - ▷ Dec-2009 provides information about current market expected position
  - ▷ Assume $\xi_t^i = \xi^i =$ Empirical average

- We consider a special case for f_{t-1}^i
  - ▷ $f_{t-1}^1(\xi_t^1, Y_{t-1}) = \xi^1 + Y_{t-1}^1$
  - ▷ $f_{t-1}^2(\xi_t^2, Y_{t-1}) = \xi^2 + Y_{t-1}^2$

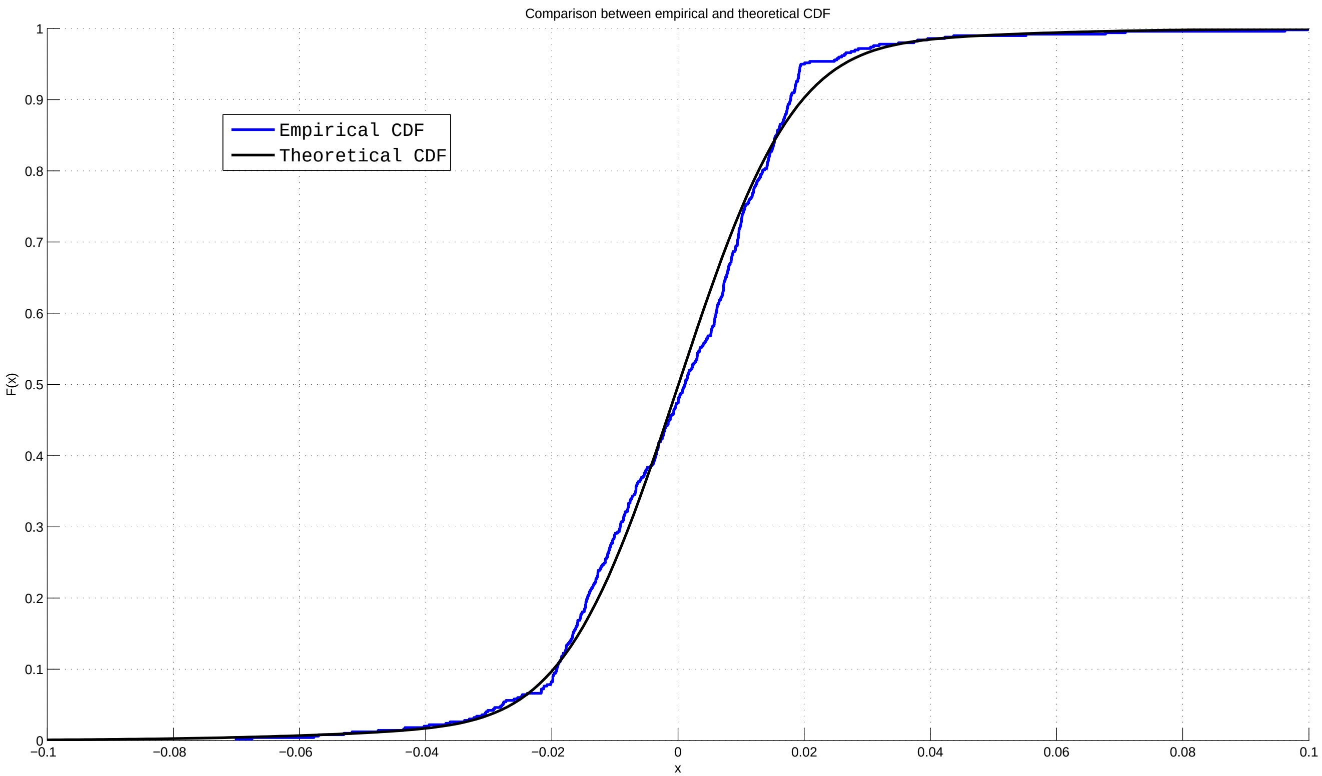
Futures Allowance Dynamics

- Estimated upward and downward movement returns:

$$\xi^{1u} = 1.021 \quad \xi^{1d} = 0.980$$
$$\xi^{2u} = 1.038 \quad \xi^{2d} = 0.982$$

- Y_t^1 i.i.d.:
  - Follows a Gaussian mixture distribution.
  - Kolmogorov-Smirnov test is accepted at a significance level of 99%.

Figure 5: Comparison of empirical CDF of Y_t^1 with a Gaussian mixture CDF.



Futures Allowance Dynamics

- $Y_t^2 = g(Y_t^1) + I_t + u_t$
  - ▷ u_t i.i.d such that $E[u_t \mid (Y_t^1, I_t)] = 0$
  - ▷ I_t represents the impact of the expected market position at time T_1 on Y_t^2.

- I_t is not observed
  - ▷ $I_t = h_0 + h_1 MS_t + \upsilon_t$
  - ▷ MS_t: Expected market position $= Sign(Y_t^1)$.
  - ▷ $E[Y_t^2 \mid (Y_t^1, I_t, MS_t)] = E[Y_t^2 \mid (Y_t^1, I_t)]$

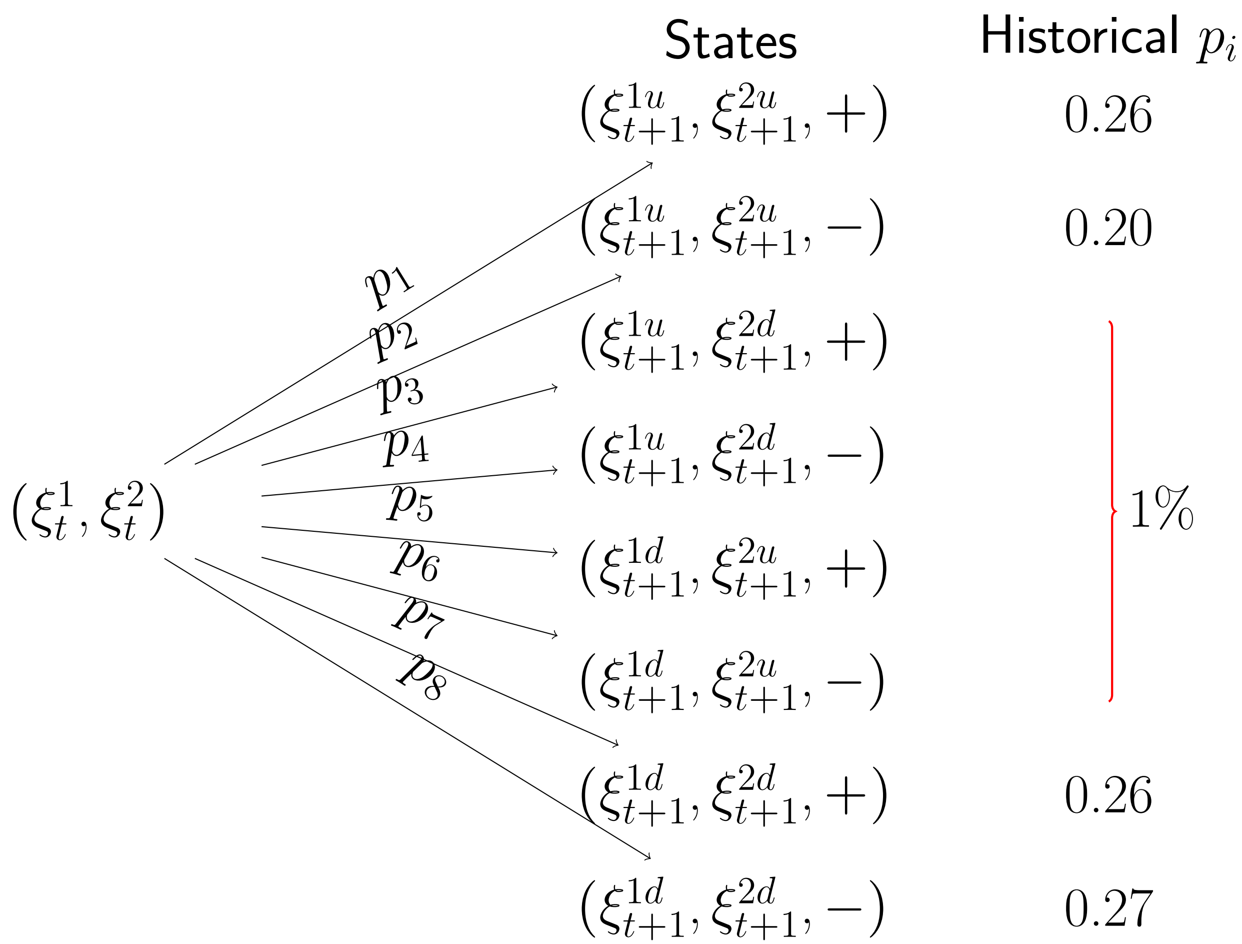
- Regression: $Y_t^2 = (h_0 + a_0) + \Sigma_{k=1}^{p} a_k (Y_t^1)^k + h_1 MS_t + \epsilon_t$

Futures Allowance Dynamics

	With MS_t		I_t omitted	
	R^2	a_1	R^2	a_1
p=0	68.21%	-	-	-
p=1	73.86%	0.541	60.35%	1.17
p=3	74.98%	0.278	62.67%	1.34
p=9	76.34%	0.467	70.34%	2.05
p=10	76.34%	0.467	70.34%	2.05

Table 1: Parameters resulting from the OLS estimator as function of the polynomial degree p in both cases where I_t is omitted or approximated by the proxy variable MS_t.

Figure 6: States of nature generated at time $t+1$ by a knot at time t.



Pricing Framework: Investor Side

- Given random variable $H \in \mathcal{L}^2(\mathcal{F}_T, P)$ describing the payoff

$$(V_0, \zeta) = \underset{(c,\vartheta)\in\mathbb{R}\times\Theta}{\arg\min} \; E_P[(H - c - G_T(\vartheta))^2], \tag{3}$$

where

$$\Theta := \{\text{predictable processes } \vartheta | \vartheta_k' \Delta S_k \in \mathcal{L}^2(P)\}, \tag{4}$$

$$G_T(\vartheta) := \sum_{j=1}^{T} \vartheta_j' \Delta S_j. \tag{5}$$

Pricing Framework: Investor Side

Definition 1. *Non-Degeneracy Condition (ND): Suppose* $\delta \in (0, 1)$*. The process* $(S_t)_{t\in\mathcal{T}} \in \mathcal{L}_d^2(P)$ *meets the non-degeneracy condition, if* $\forall k = 1, ..., T$*, the random matrix*

$$\delta E[\Delta S_k^2|\mathcal{F}_{k-1}] - (E[\Delta S_k|\mathcal{F}_{k-1}])^2 \tag{6}$$

is positive-semidefinite P-a.s.

Intuition: in 1-dimension, (6) corresponds to:

$$Var(\Delta S_k^2|\mathcal{F}_{k-1}) > 0$$

i.e. we need some randomness or it all falls apart.

Pricing Framework: Investor Side

Proposition 2. *Assume a probability space* $(\Omega, \mathbb{F}, P)$ *and stochastic process* $(S_t)_{t\in\mathcal{T}} \in \mathcal{L}_d^2(P)$ *adapted to the filtration* $\mathbb{F} = (\mathcal{F}_t)_{t\in\mathcal{T}}$ *such that* $E[\Delta S_k^2|\mathcal{F}_{k-1}]$ *is invertible and satisfies (ND). Therefore, there exists a unique solution* (V_0, ζ) *solving* (3)*, where:*

$$\zeta_k = \varrho_k - \beta_k(V_0 + G_{k-1}(\zeta)), \tag{7}$$

$$V_0 = E_{\widetilde{P}}[H], \tag{8}$$

$$\varrho_k = \left(E\left[\Delta S_k^2 \prod_{j=k+1}^{T}(1-\beta_j'\Delta S_j)^2|\mathcal{F}_{k-1}\right]\right)^{-1} E\left[H\Delta S_k \prod_{j=k+1}^{T}(1-\beta_j'\Delta S_j)|\mathcal{F}_{k-1}\right], \tag{9}$$

$$\beta_k = \left(E\left[\Delta S_k^2 \prod_{j=k+1}^{T}(1-\beta_j'\Delta S_j)^2|\mathcal{F}_{k-1}\right]\right)^{-1} E\left[\Delta S_k \prod_{j=k+1}^{T}(1-\beta_j'\Delta S_j)|\mathcal{F}_{k-1}\right], \tag{10}$$

$$\frac{d\widetilde{P}}{dP} = \frac{\widetilde{Z}_0}{E[\widetilde{Z}_0]}, \tag{11}$$

$$\widetilde{Z}_0 = \prod_{j=1}^{T}(1-\beta_j'\Delta S_j). \tag{12}$$

Furthermore, the unhedgeable risk defined by (3) is:

$$V_0^2 E\left[\widetilde{Z}_0\right] - 2V_0 E\left[H\widetilde{Z}_0\right] + E\left[(H - \sum_{j=1}^{T} \varrho_j' \Delta S_j \prod_{l=j+1}^{T} (1-\beta_l' \Delta S_l))^2\right] \tag{13}$$

Proof.

- ▷ Same as presented in Schweizer (1996) for a 1-dimension processes
- ▷ We present the generalization of Rémillard and Rubenthaler (2009) for a multidimensional framework under very mild conditions

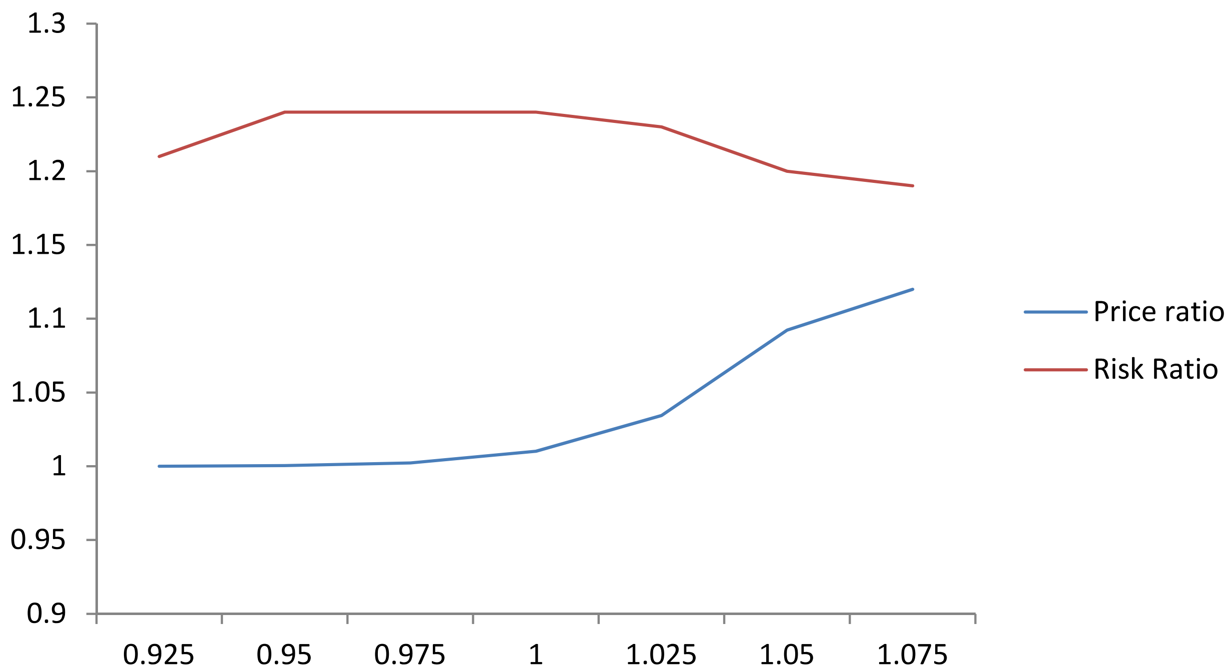
■

Pricing Framework: Investor Side

- Price a derivative written on S_t^1
  - ▷ Usual markets: Dynamic hedging position on S_t^1
  - ▷ Figure 6: Returns of S_t^1 and S_t^2 have same dynamic pattern
  - ▷ Spread between prices: Evaluates the uncertainty of the expected market position
- Consider portfolios:
  - ▷ A: trading on S^1
  - ▷ B: trading on both S^1 and S^2
- Short position scenario: prices quoted on 4/4/2008:

$$S_0^1 = €23.96 \text{ and } S_0^2 = €24.61.$$

Figure 7: Comparison between strategies A and B to price 5 period (5 day) calls written on S_t^1 for different strike prices. Market is initially assumed to be short but expected to be long. Price ratio $= \frac{\text{Price A}}{\text{Price B}}$, Risk ratio $= \frac{\text{Unhedged risk A}}{\text{Unhedged risk B}}$



Pricing Framework: Investor Side

Proposition 3. *Assume a probability space* $(\Omega, \mathbb{F}, P)$, $H \in \mathcal{L}^2(\mathcal{F}_T, P)$, *and stochastic process* $(S'_t, \Xi)'_{t\in\mathcal{T}} \in \mathcal{L}^2_{d+1}(P)$ *adapted to the filtration* $\mathbb{F} = (\mathcal{F}_t)_{t\in\mathcal{T}}$ *such that* $E[\Delta S_k^2|\mathcal{F}_{k-1}]$ *and* $E[(\Delta S'_k, \Delta\Xi_k)'^2|\mathcal{F}_{k-1}]$ *are invertible and satisfy the non-degeneracy condition.*

If $P(E[(H - V_0 - G_T(\zeta, S))\Delta\Xi_T|\mathcal{F}_{T-1}] \neq 0) > 0$, *then hedging with* $(S'_t, \Xi_t)_{t\in\mathcal{T}}$ *is more efficient than hedging with* $(S_t)_{t\in\mathcal{T}}$.

Pricing Framework: Investor Side

- Interdependency between compliance periods:
  - ▷ Need a multiperiod pricing framework
  - ▷ Reduce market position risk
  - ▷ Reduce liquidity: Non-emitters, who fear long term regulatory changes, may not trade
- An equivalent solution to encourage non-emitters to trade:
  - ▷ Need regulator's intervention
  - ▷ Provide new tradeable asset besides the right to emit

Pricing Framework: Regulator Side

- New tradeable asset G
  - ▷ Allow some of the intrinsic market risk to be hedged
  - ▷ Exogenous to market participants
  - ▷ Consider the social wealth of market parameters Γ, initially set up by the regulator
  - ▷ Consistent with arbitrage free theory

- Indifference pricing
  - ▷ $U(X^{x,\alpha}, \Gamma)$: Utility function of the representative agent
  - ▷ x: Initial wealth; α: trading strategy
  - ▷ The price $\nu_t(G_{T_1})$ of G is given via:

$$\sup_{\alpha} E_{\mathbb{P}} \left[U(X^{x,\alpha}, \Gamma) \right] = \sup_{\alpha} E_{\mathbb{P}} \left[U(X^{x+\nu_t(G_{T_1}),\alpha} - G, \Gamma) \right] \tag{14}$$

Pricing Framework: Regulator Side

- Exponential utility

$$U(x) = -e^{-\gamma x}, \quad \forall x \in \mathbb{R} \text{ and } \gamma > 0, \tag{15}$$

γ is a parameter selected by the regulator (Elliot and Vander Hoek (2009)).

- Price obtained by moving backward

$$\nu_t(G_{T_1}) = \mathcal{E}_{\mathbb{Q}}^{(t,t+1)}(\nu_{t+1}(G_{T_1})), \tag{16}$$

$$\mathcal{E}_{\mathbb{Q}}^{(s,s+1)}(L_{s+1}) = E_{\mathbb{Q}}\left(\frac{1}{\gamma_s}\log\left(E_{\mathbb{P}}(e^{\gamma_s Z_{s+1}}|\mathcal{F}_s \vee \mathcal{F}^S{}_{s+1})\right)|\mathcal{F}_s\right), \tag{17}$$

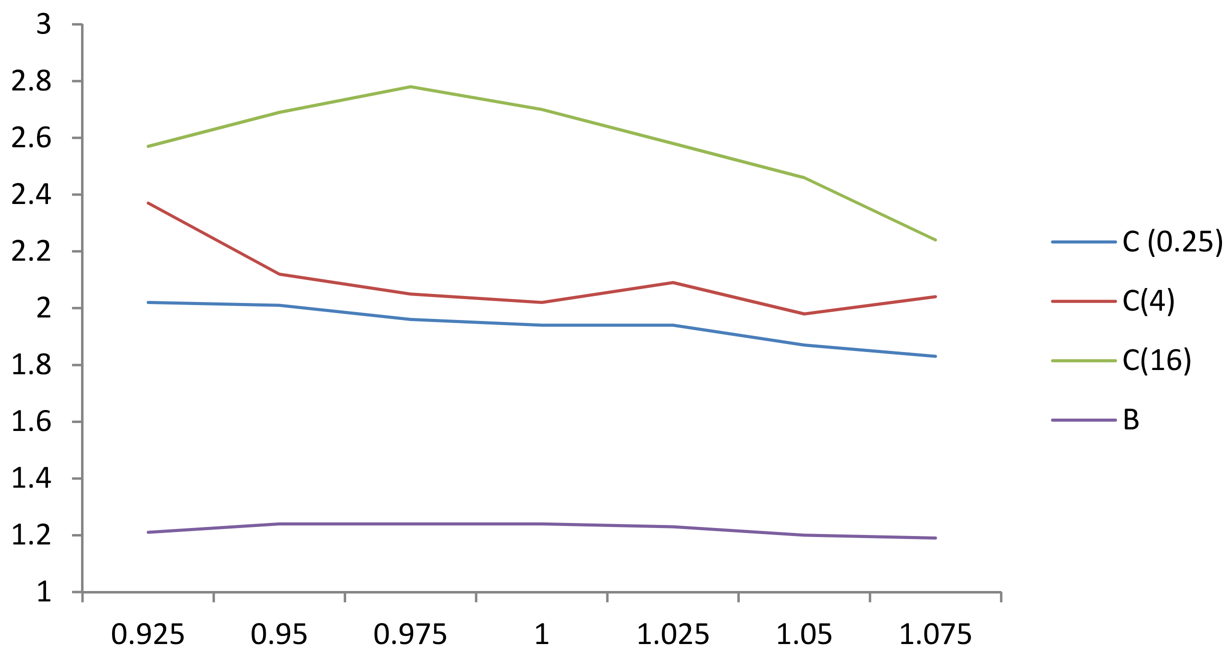
$$\mathcal{E}_{\mathbb{Q}}^{s,s}(L_s) = L_s, \quad \nu_{T_1}(G_{T_1}) = G_{T_1} \tag{18}$$

$$\mathcal{F}^S = \sigma\{S^1\}, \quad Q \text{ is the } S^1 \text{ equivalent martingale measure.} \tag{19}$$

Pricing Framework: Regulator Side

- Example: Digital option
- Pays out a certain amount if a predefined event happens at future time T
- Regulator announcement about the market position at time t: set of Y_t values observable
- Example: Regulator pays 1 unit if he announces the market is short and expected to remain short at the next compliance date
- Payment occurs if $\omega \in \{\omega_1, \omega_5, \omega_9, \omega_{13}\}$ happens
- Strategy C: Investor holds position on S_t^1 and ν_t.

Figure 8: Risk ratio between strategy A and C for different values of $\gamma - C(\gamma) -$ to price 5 period (5 day) calls for different strike prices.



References

1. Elliott, R. J. and J. Vander Hoek, 2009 , "Duality Methods," *Indifference pricing : theory and applications* (R. Carmona), 9, 321-386, *Princeton Series in Financial Engineering*, Princeton University Press, Princeton and Oxford.

2. Motoczyński, M., 2000, "Multidimensional Variance-Optimal hedging in Discrete-Time Model – A General Approach," *Mathematical Finance*, 10, 243- 257.

3. Musiela, M. and T. Zariphopoulou, 2004, "A Valuation Algorithm for Indifference Prices in Incomplete Markets," *Finance and Stochastics*, 8, 399-414.

4. Rémillard, B. and S. Rubenthaler, 2009, "Optimal Hedging in Discrete and Continuous Time," *Technical report*, G-2009-77, Gerad.

5. Schweizer, M., 1996, “Approximation Pricing and the Variance-Optimal Martingale Measure,” *Annals of Probability*, 24, 206-236.