Quantile Risk Management in Equity-Linked Life Insurance with Stochastic Interest Rate

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Introduction
Theoretical and Practical Motivation

Insurance company can issue a *mixed* contract for the period $[0,T]$, where the payoff function is a function of stock prices $S_0,\ldots,S_T$ and $T(x)$ (*future lifetime* of a client of age x).

- **Terminology**
  - Segregated funds (Canada),
  - Variable annuities (USA),
  - Equity (unit)-linked insurance (Europe).
- **Numbers**

According The Insured Retirement Institute (IRI, formerly, NAVA): "Variable annuity sales for the 3-rd quarter were $34 billion. Year-to year quarterly sales of variable annuities were up, posting a 9.7% increase from third quarter 2009 sales".

Introduction
Theoretical and Practical Motivation

- **Types and Features of Contracts**
  - Death and maturity guarantees.
  - Term of the guarantee.
  - Resets (policyholder is allowed to reset the guarantee at the current fund value).
  - Fund switching (policyholder has the right to switch investments between various funds).
- **Types of Hedging**
  - *Perfect* hedging:

$$P\{X_T^{\pi^*} \geq H\} = 1$$

  - *Mean-variance* hedging:

$$E(X_T^{\pi^*} - H)^2 \text{ is minimal}$$

Introduction
Theoretical and Practical Motivation

- *Efficient* hedging:

$$E\{l\,(H - X_T^{\pi^*})^{+}\} \text{ is minimal}$$

where l is some loss function.

In particular, for *quantile* hedging, we have $l(x) = I_{(0,\infty)}(x)$ and $P\{X_T^{\pi^*} \geq H\}$ is maximized.

Introduction
Theoretical and Practical Motivation

- Using Hedging for Pricing and Risk Management: construct a strategy that exactly replicates the cash flows of a contingent claim

- Exact replication is not possible: find a strategy with a cash flow “close enough” to the payoff of the contingent claim in some probabilistic sense

- Equity-linked life insurance contracts have a mortality component => the exact replication is not possible

Introduction
References on Equity-linked Life Insurance

- Brennan and Schwartz (1976, 1979)
- Boyle and Schwartz (1977)
- Bacinello and Ortu (1993)
- Aase and Person (1994)
- Ekern and Person (1996)
- Moeller (1998, 2001)
  - Contracts with fixed or deterministic guarantees
  - Reduced them to call/put options
  - Apply perfect or mean-variance hedging to calculate prices

Introduction
References on Equity-linked Life Insurance

- Melnikov (2004)
- Melnikov and Skornyakova (2005)
  - Contracts with stochastic guarantee
  - Reduce them to embedded option $\left(S_T^1 - S_T^2\right)^+$
  - Apply quantile hedging to calculate prices

- Quansheng Gao, et.al (2010)
  - Guarantee of the contracts grows exponentially over time
  - Stochastic interest rate
  - Diffusion model

Introduction
Contents

- Briefly review the work of Melnikov and Skornyakov (2005)
- Consider a two-factor jump-diffusion model to describe a financial market and HJM framework for stochastic interest rate
- Study equity-linked pure endowment contracts with a stochastic guarantee
- Use quantile hedging technique for pricing these contracts
- Illustrate our results with actual data

Review: Financial settings Melnikov and Skornyakova (2005)

- Non-risky asset $B_t = \exp(rt),\ t \geq 0,\ r \geq 0$
- Risky assets S_t^1 and S_t^2 on $(\Omega, \mathbf{F}, P)$, prices follow the jump-diffusion model

$$dS_t^i = S_{t-}^i(\mu_i dt + \sigma_i dW_t - \nu_i d\Pi_t),\ i = 1,2$$

- Market is complete, the unique risk-neutral probability has the density

$$Z_T = \exp\left\{ \alpha^* W_T - \frac{(\alpha^*)^2}{2} T + (\lambda - \lambda^*)t + \ln \frac{\lambda^*}{\lambda} \cdot \Pi_t \right\}$$

(α^*, λ^*) are the unique solutions to $\begin{cases} \mu_1 - r = -\sigma_1 \alpha^* + \nu_1 \lambda^*, \\ \mu_2 - r = -\sigma_2 \alpha^* + \nu_2 \lambda^*,\ \lambda^* > 0 \end{cases}$

- Conditions:

$$\mu_1 > \mu_2,\ \sigma_1 > \sigma_2,\ \nu_1, \nu_2 < 1,\quad \begin{cases} \sigma_2 \nu_1 - \sigma_1 \nu_2 \neq 0, \\ \dfrac{(\mu_1 - r)\sigma_2 - (\mu_2 - r)\sigma_1}{\sigma_2 \nu_1 - \sigma_1 \nu_2} > 0 \end{cases}$$

Review: Insurance settings
Melnikov and Skornyakova (2005)

- $T(x)$ on $(\widetilde{\Omega}, \widetilde{\mathbf{F}}, \widetilde{P})$ is the remaining life time of a person at age x
- ${}_T p_x = \widetilde{P}\{T(x) > T\}$ is a survival probability
- Assumption: $(\Omega, \mathbf{F}, P)$ and $(\widetilde{\Omega}, \widetilde{\mathbf{F}}, \widetilde{P})$ are independent
- Mortality risk arises from the dependence of the payoff on the survival status of a client at maturity $H(T(x)) = H \cdot I_{\{T(x) > T\}}$
- The payoff for the model: $H(T(x)) = \max\{S_T^1, S_T^2\} \cdot I_{\{T(x) > T\}}$

Main Results: Theorem

- Financial market is described by two-factor jump-diffusion model
- Equity-linked life insurance contract with a flexible guarantee
- On a set $\{\Pi_T = n\}$, if

$$\frac{(\mu_1 - r)\nu_1 - (\mu_2 - r)\nu_2}{\sigma_2\nu_1 - \sigma_1\nu_2} = \frac{\sigma_1 \ \sigma_2}{\sigma_2 + (\sigma_1 - \sigma_2)^2}$$

Then

$${}_T p_x = 1 - \frac{\sum_{n=0}^{\infty} p^*_{n,T}\left(\upsilon^1_{n,T}\Phi\left(b_+\left(\upsilon^1_{n,T}, d_n\upsilon^2_{n,T}, T\right)\right) - \upsilon^2_{n,T}\Phi\left(b_-\left(\upsilon^1_{n,T}, d_n\upsilon^2_{n,T}, T\right)\right)\right)}{\sum_{n=0}^{\infty} p^*_{n,T}\left(\upsilon^1_{n,T}\Phi\left(b_+\left(\upsilon^1_{n,T}, \upsilon^2_{n,T}, T\right)\right) - \upsilon^2_{n,T}\Phi\left(b_-\left(\upsilon^1_{n,T}, \upsilon^2_{n,T}, T\right)\right)\right)}$$

where $\upsilon^i_{n,T} = (1-\nu_i)^n e^{\nu_i}, \ i = 1, 2, \qquad p^*_{n,T} = \exp\{-\lambda^* T\}\frac{(\lambda^* T)^n}{n!}$

Description of the model: Financial Setting

- Dynamics of risky assets on a filtered probability space $\left(\Omega, \mathbb{F}, \left(F_t\right)_{0\le t\le T}, P\right)$

$$dS_t^i = S_{t-}^i\left(\mu_i dt + \sigma_i dW_t - \nu_i d\pi_t\right)$$

where $\mu_i \in R, \sigma_i > 0, \nu_i < 1$ and $\sigma_1 > \sigma_2$

- HJM framework
  - $f(t,T)$: instantaneous forward interest rate

$$f(t,T) = f(0,T) + \int_0^t \alpha(s,T)ds + \int_0^t \sigma(s,T)dW_s + \int_0^t \beta\left[d\Pi_s - \lambda ds\right]$$

  - $r(t)$: spot interest rate

$$r(t) = f(t,t) = f(0,t) + \int_0^t \alpha(s,t)ds + \int_0^t \sigma(s,t)dW_s + \int_0^t \beta\left[d\Pi_s - \lambda ds\right]$$

Description of the model: Financial Setting

- $P(t,T)$: price of a default-free discount zero-coupon bond at time t with maturity time T $\quad P(t.T) = \exp\left(-\int_t^T f(t,s)\,ds\right)$
- $B(t)$:accumulated money account $\quad B(t) = \exp\left(\int_0^t r(s)\,ds\right)$

- Market is complete if

$$\frac{(\mu_1 - r(t))\sigma_2 - (\mu_2 - r(t))\sigma_1}{\sigma_2 \nu_1 - \sigma_1 \nu_2} > 0 \quad \text{and} \quad \sigma_2 \nu_1 - \sigma_1 \nu_2 \neq 0$$

- The unique martingale measure P^* has the local density

$$Z_t = \left.\frac{dP^*}{dP}\right|_t = \exp\left\{\int_0^t \phi_s dW_s - \frac{1}{2}\int_0^t \phi_s^2 ds + \int_0^t (\lambda - \lambda_s^*)\,ds + (\ln \lambda_t^* - \ln \lambda)\Pi_t\right\}$$

(ϕ_t, λ_t^*) are the solutions for the equations $\begin{cases} \mu_1 - r(t) + \phi_t \sigma_1 - \nu_1 \lambda_t^* = 0 \\ \mu_2 - r(t) + \phi_t \sigma_2 - \nu_2 \lambda_t^* = 0 \end{cases}$

Description of the model: Financial Setting

- Explicit representations of $B(t)$ and S_t^i in terms of the parameters of the systerm:

$$B(t) = \frac{1}{P(0,t)} \exp\left\{ \frac{1}{2} \int_0^t \left(\sigma^*(s,T)\right)^2 ds + \int_0^t \left[e^{-\beta(T-s)} - 1 \right] \lambda_s^* ds + \int_0^t \sigma^*(s,T)\, dW_s^* - \int_0^t \beta(T-s)\, d\Pi_s \right\} ,$$

$$S_t^i = S_0^i B(t) \exp\left\{ \sigma_i W_t^* + \Pi_t \ln(1-\nu_i) + \int_0^t \left(\nu_i \lambda_s^* - \frac{1}{2}\sigma_i^2 \right) ds \right\}$$

Description of the model: Insurance Setting

- Assumption: $(\Omega, \mathbb{F}, P)$ and $(\tilde{\Omega}, \tilde{F}, \tilde{P})$ are independent.
- A single premium equity-linked life insurance contract with payoff

$$C(t) = \max\left(S_t^1, S_t^2\right)$$

S^1 : maximal size of future profit

S^2 : stochastic guarantee for the insured

- The initial price of the contract: (Brennan-Schwartz price)

$$H(0) = E^*\left\{\tilde{E}\left[C(T)B^{-1}(T)I\{T_x > T\}\right]\right\} = E^*\left[C(T)B^{-1}(T)\right]\cdot {}_T p_x$$

- Perfect hedging is not possible due to a budget constraint

$$H(0) < E^*\left[C(T)\cdot B^{-1}(T)\right]$$

- Find a strategy that will hedge successfully with the maximal probability

Quantile Hedging
Definitions

- Self-financing strategy π has a budget constraint

$$H_0^{\pi} \leq H_0 < E^* C_T e^{-rT}$$

- $A(H_0^{\pi}, \pi) = \{\omega : H_T^{\pi}(H_0^{\pi}) \geq C_T\}$ is a *successful hedging set*

- π^* is a *quantile hedge* if

$$P\{A(H_0^{\pi^*}, \pi^*)\} = \max_{\pi : H_0^{\pi} \leq H_0} P\{A(H_0^{\pi}, \pi)\}$$

- How to construct the quantile hedge π^* and the successful hedging set $A(H_0^{\pi^*}, \pi^*)$?

Quantile Hedging Methodology

The answer is given in the following fact (Foellmer and Leukert (1999))

Let $A^* \in F_T$ be a solution to the problem $P(A^*) = \max_{A \in F_T : E^*(C_T \cdot I_A) \le H_0} P(A)$

Then the quantile hedge π^*

- does exist
- is unique
- is a perfect hedge for a modified claim $C_{A*} = C_T \cdot I_{A*}$

The structure of a maximal successful hedging set is $A^* = \left\{ Z_T^{-1} > a \frac{C_T}{B_T} \right\}$, where

A constant a is defined $a = \inf \left\{ c : P^* \left(\frac{dP}{dP^*} > c \right) \le \alpha \right\}$

Equity-Linked Life Insurance
Connecting Financial and Insurance Risks

- Due to the structure of the price, $H(0) = E^{*}\left[C(T)B^{-1}(T)\right] \cdot {}_{T}p_{x}$

  we apply quantile hedging, $H(0) = E^{*}\left[C(T)I_{\{A^{*}\}} \cdot B^{-1}(T)\right]$

- Key formulae connecting financial and insurance risks

$$ {}_{T}p_{x} = \frac{E^{*}\left[C(T)I_{\{A^{*}\}} \cdot B^{-1}(T)\right]}{E^{*}\left[C(T) \cdot B^{-1}(T)\right]} $$

Application of Quantile Hedging Preliminary Calculations

- Maximal successful hedging sets

$$A^* = \left\{ Z_T^{-1} \geq a_n \frac{C_T}{B(T)} \right\} = \left\{ Z_T^{-1} \geq a_n \frac{\max\left(S_T^1, S_T^2\right)}{B(T)} \right\}$$

- The price of the contract $H(0) = E^*\left[\max\left(S_T^1, S_T^2\right) B^{-1}(T) I_{\{A^*\}}\right]$

$$= E^*\left[\frac{S_T^1}{B_T} I_{\{A^*\}} \cdot I_{\{S_T^1 > S_T^2\}}\right] + E^*\left[\frac{S_T^2}{B_T} I_{\{A^*\}} \cdot I_{\{S_T^1 \leq S_T^2\}}\right]$$

- Further analysis relies on change of measure technique and properties of jump-diffusion processes

Main Results
Theorem 1

- Financial market is described by two-factor jump-diffusion model
- Stochastic interest rate is in HJM framework
- Equity-linked life insurance contract with a flexible guarantee

Then the Brennan-Schwartz price of the contract is

$$H(0)=\sum_{n=0}^{\infty} p_{n,T}^{*}\Big[e^{\nu_1\int_0^T \lambda_t^* dt}(1-\nu_1)^n S_0^1 \Psi_1(\Gamma_1,\Gamma_2)$$

$$+e^{\nu_2\int_0^T \lambda_t^* dt}(1-\nu_2)^n S_0^2 \Psi_2(\tilde{\Gamma}_1,\tilde{\Gamma}_2)\Big]$$

where
$$p_{n,T}^{*} = e^{-\int_0^T \lambda_t^* dt}\frac{\left(\int_0^T \lambda_t^* dt\right)^n}{n!}$$

Main Results
Theorem 1

$$\Gamma_1 = -\ln \frac{a_n \cdot S_0^1 \left(\lambda_T^1\right)^n}{\lambda^n} - \frac{1}{2}\delta_1^2 - \int_0^T \left(\lambda - \lambda_s^1\right) ds$$

$$\Gamma_2 = \ln \frac{S_0^1 \left(1 - v_1\right)^n}{S_0^2 \left(1 - v_2\right)^n} - \int_0^T \lambda_s^* \left(v_2 - v_1\right) ds + \frac{1}{2}\delta_2^2$$

$$\tilde{\Gamma}_1 = -\ln \frac{a_n \cdot S_0^2 \left(\lambda_T^2\right)^n}{\lambda^n} - \frac{1}{2}\tilde{\delta}_1^2 - \int_0^T \left(\lambda - \lambda_s^2\right) ds$$

$$\tilde{\Gamma}_2 = \ln \frac{S_0^2 \left(1 - v_2\right)^n}{S_0^1 \left(1 - v_1\right)^n} - \int_0^T \lambda_s^* \left(v_1 - v_2\right) ds + \frac{1}{2}\tilde{\delta}_2^2$$

$$\lambda_t^1 = \lambda_t^* \left(1 - v_1\right) \qquad \lambda_t^2 = \lambda_t^* \left(1 - v_2\right)$$

Main Results
Theorem 2

The survival probability of an insured is as following:

$$ {}_T p_x = \frac{\sum_{n=0}^{\infty} p_{n,T}^{*}\left[p\Psi\left(\Gamma_1,\Gamma_2;\rho,\delta_1,\delta_2\right)+q\Psi\left(\tilde{\Gamma}_1,\tilde{\Gamma}_2;\tilde{\rho},\tilde{\delta}_1,\tilde{\delta}_2\right)\right]}{\sum_{n=0}^{\infty} p_{n,T}^{*}\left[p\Phi\left(\frac{d_1}{(\sigma_1-\sigma_2)\sqrt{T}}\right)+q\Phi\left(\frac{d_2}{(\sigma_1-\sigma_2)\sqrt{T}}\right)\right]} $$

Where $d_1 = \ln\frac{S_0^1(1-v_1)^n}{S_0^2(1-v_2)^n} - \int_0^T \lambda_s^*(v_2-v_1)\,ds + \frac{1}{2}\delta_2^2$, $p_n = e^{v_1\int_0^T \lambda_t^* dt}(1-v_1)^n S_0^1$

$$ d_2 = \ln\frac{S_0^2(1-v_2)^n}{S_0^1(1-v_1)^n} - \int_0^T \lambda_s^*(v_1-v_2)\,ds + \frac{1}{2}\tilde{\delta}_2^2 \quad , \quad q_n = e^{v_2\int_0^T \lambda_t^* dt}(1-v_2)^n S_0^2 $$

Remark:
How to determine the constant a

- Fix a probability ε of failure to hedge on each set $\{\Pi_T = n\}$
  Or equivalently, fix the probability of successful hedging as

$$P\left(A^*\,\middle|\,\pi_T = n\right) = 1 - \varepsilon = \Phi\left(\Delta\right) \qquad \Delta = \Delta_1 \cup \Delta_2$$

Where

$$\Delta_1 = \frac{-\ln \dfrac{a \cdot S_0^1 \left(\lambda_T^1\right)^n}{\lambda^n} - \dfrac{1}{2}\delta_1^2 - \int_0^T \left(\lambda - \lambda_s^1\right) ds}{\delta_1}$$

$$\Delta_2 = \frac{-\ln \dfrac{a \cdot S_0^2 \left(\lambda_T^2\right)^n}{\lambda^n} - \dfrac{1}{2}\tilde{\delta}_1^2 - \int_0^T \left(\lambda - \lambda_s^2\right) ds}{\tilde{\delta}_1}$$

- Using the log-normality of the conditional distribution to estimate a

Numerical Illustration
Inputs

- Contracts with flexible guarantee: Russell 2000 and the S&P 500
- Transfer to One factor Vasicek-Hull-White model
- Estimated parameters for two-factor jump diffusion model (from monthly observations from 09/1987 to 09/2010)

$$\mu_1 = 0.2763 \qquad \mu_2 = 0.2898 \qquad \sigma_2 = 0.15 \qquad \sigma_1 = 0.19$$

$$\nu_1 = -0.27 \qquad \nu_2 = -0.2 \qquad \lambda = 0.17$$

- $S_0 = 1000$ is an initial investment
- $T = 1, 3, 5, 10, 15, 20$ years are terms of the contracts
- Fix different level of financial risks $\varepsilon = 0.01, 0.025, 0.05$

Numerical Illustration

- Table 1: Survival probabilities ${}_T p_x$ with flexible guarantee

T	ε =0.01	ε =0.025	ε =0.05
1	0.9885	0.9718	0.9447
3	0.9878	0.9705	0.9426
5	0.9874	0.9697	0.9413
10	0.9867	0.9684	0.9391
15	0.9859	0.9667	0.9364
20	0.9853	0.9656	0.9345

Numerical Illustration

- Table 2: Age of insured with flexible guarantee

T	ε =0.01	ε =0.025	ε =0.05
1	58	69	78
3	45	55	63
5	39	48	56
10	23	39	46
15	12	31	39
20	6	24	33

Numerical Illustration from Melnikov and Skornyakova (2005)

- Table 1: Survival probabilities with flexible guarantee

T	ε =0.01	ε =0.025	ε =0.05
1	0.9447	0.8774	0.7811
3	0.9511	0.8910	0.8041
5	0.9549	0.9108	0.8387
10	0.9605	0.9108	0.8378

- Table 2: Age of Insured with flexible guarantee

T	ε =0.01	ε =0.025	ε =0.05
1	78	87	94
3	61	71	79
5	53	63	71
10	41	50	58

Numerical Illustration Conclusions

- Whenever the financial risk level ε increases, the survival probability ${}_T p_x$ decreases, so that the clients' age increases in the same period. The insurance company should attract older clients for the contract with flexible guarantee to compensate increasing financial risk.
- With longer contract maturities, the company is able to attract younger clients while maintaining the same financial risk exposure, as a survival probability is decreasing over time.
- For fixed contract maturity, as the financial risk ε increases, the change of survival probabilities ${}_T p_x$ is not dramatic as the corresponding results in Melnikov and Skornyakova (2005).
- For fixed financial risk ε, as the contract maturity increases, the change of survival probabilities ${}_T p_x$ decreases in small percentage while the results in Melnikov and Skornyakova (2005) shows the opposite way, which is increasing.

Further Developments Discussion

- Mortality Modeling
  - Use theoretical models of mortality Gompertz, Makeham, Lee-Carter
  - Allows to take into account new tendencies in mortality
- Modeling with other Risk Measures
  - Conditional Tail Expectation:

$$CTE_{1-\varepsilon}(\pi) = E\left(H - X_T^{\pi}(\pi) \middle| H - X_T^{\pi}(\pi) \geq VaR_{1-\varepsilon}(\pi)\right)$$

  - Rockafellar and Uryasev (2002):

If $\overline{z}$ is a solution to $RU(z) = z + \frac{1}{\varepsilon}\left[\inf_{\pi} E\left(H - X_T^{\pi}(x) - z\right)^{+}\right] \to \min$

then $VaR_{1-\varepsilon}(\pi^*) = \overline{z}$, $CTE_{1-\varepsilon}(\pi^*) = RU(\overline{z})$

Further Developments Discussion

- Shortfall minimization problem for the claim $H(z) = (H - z)^+$

$$E\left(H - X_T^\pi(x) - z\right)^+ = E\left((H - z)^+ - X_T^\pi(x)\right)^+ \to \inf$$

over all strategies with initial budget constraints

- $\left(\widetilde{X}_0(z), \widetilde{\pi}(z)\right)$ solution to this problem (Foellmer and Leukert (2000)),

where $\widetilde{\pi}(z)$ is a perfect hedge to a modified claim

$$\widetilde{H}(z) = H(z) I_{\{Z_T^{-1} > \widetilde{a}(z)\}},$$

$$\widetilde{a}(z) = \inf\left(a \geq 0 : E^* H(z) I_{\{Z_T^{-1} > a\}} \leq X_0\right)$$

- The function $RU(z)$ has the following structure

$$RU(z) = z + \frac{1}{\varepsilon} E I_{\{Z_T^{-1} \leq \widetilde{a}(z)\}} H(z), \quad z < z^* \quad , \qquad RU(z) = z, \quad z \geq z^*$$

where $E^* H(z^*) = X_0$

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