Valuing Risky Projects Based on Managerial Cash Flow Estimates: A Real Options Approach

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Motivation

- To develop a real options approach to value R&D type projects
- **Theory**: Cash flows determined by GBM

$$df_t = \mu f_t dt + \sigma f_t dW_t$$

- **Practice**: Cash flows estimated as low, medium, high

	F'0	F'1	F'2	F'3	F'4
Economic Profit (Optimistic)	80	120	150	180	200
Economic Profit (Likely)	50	70	75	80	90
Economic Profit (Pessimistic)	20	25	25	20	20

Real Options

- Why Real Options?
  - Superior to discounted cash flow (DCF) analysis for capital budgeting / project valuation
  - Accounts for the inherent value of managerial flexibility
  - Adoption rate ~12% in industry (Brock (2007))
- What is required
  - Consistency with financial theory
  - Intuitively appealing
  - Practical to implement

Standard DCF Method

- Discount cash flows at the WACC
- For all equity firm (w/o loss of generality), CAPM:

$$R_{wacc} = \mathbb{E}[R_E] = R_f + \beta_C(\mathbb{E}[R_{MP}] - R_f)$$

$$\beta_C = \frac{\rho\sigma_C}{\sigma_{MP}}$$

- Some of the assumptions:
  - Returns are normally distributed
  - No managerial flexibility / optionality imbedded in the project
  - financial risk profile of the value of the cash-flows matches that of the average project of the company

Standard DCF Method

$$\beta_C = \frac{\rho \sigma_C}{\sigma_{MP}}$$

- Assumptions regarding β_C
  - Market volatility, σ_{MP}, is known
  - Cash flow volatility, σ_C, is known ?
  - Correlation of the cash flows to the market, ρ, is known ?

Real Options Models Used in Practice

Real Option Approaches*

	Intuitive	Practical / Easy to Implement	Financially Consistent	Minimal Subjectivity
Classic Approach	✓	✗	✓	-
Subjective Approach	✓	✓	-	✗
Market Asset Disclaimer	✓	✓	✗	✗
Revised Classic Approach	✓	✗	✓	-
Integrated Approach	✓	✗	✓	✓

*This classification was introduced by Borison, A. (2005)

Real Option Approaches: Classical

- Cash flows are closely linked to a traded asset

or

- Cash flows are assumed to be closely linked to a traded asset
- Use the traded assets parameters to model the value of the cash flows
- Strengths:
  - –Intuitive, objective and financially consistent
- Weaknesses
  - –Difficult to find an appropriate traded asset
  - –Volatility of a company is likely less than of a project (note that this is an issue with DCF analysis as well)

Real Option Approaches: Subjective

- Use managerial / expert experience to estimate parameters
- Strengths:
  - Intuitive, easy to implement
- Weakness
  - **Subjective**

Real Option Approaches: MAD

- MAD (Market Asset Disclaimer)
- Brief outline of procedure
  - Develop a cash flow spreadsheet
  - Use Monte Carlo procedure based on managerial supplied uncertainty values to determine a histogram of cash flow returns
  - Use the histogram to estimate volatility
- Strengths:
  - Intuitive, **easy to implement**
- Weaknesses
  - **Financially inconsistent** (see Brandao)
  - Assumes that the value of the cash flows is traded and follows a GBM – leads to erroneous results where as the volatility increases, the real option value always increases

Real Option Approaches: Revised Classic

- Projects are either:
  - Project value primarily derived from exogenous (market) factors
    - Thus use Classical approach
  - Project value primarily derived from endogenous (private/company) factors
    - Thus apply classical decision analysis methods (e.g. decision trees)
- Strengths:
  - Intuitive, financially consistent
- Weakness
  - “All or nothing” nature of the approach
  - Unclear what discount factor to use for endogenous projects (r_f or WACC?)

Real Option Approaches: Integrated

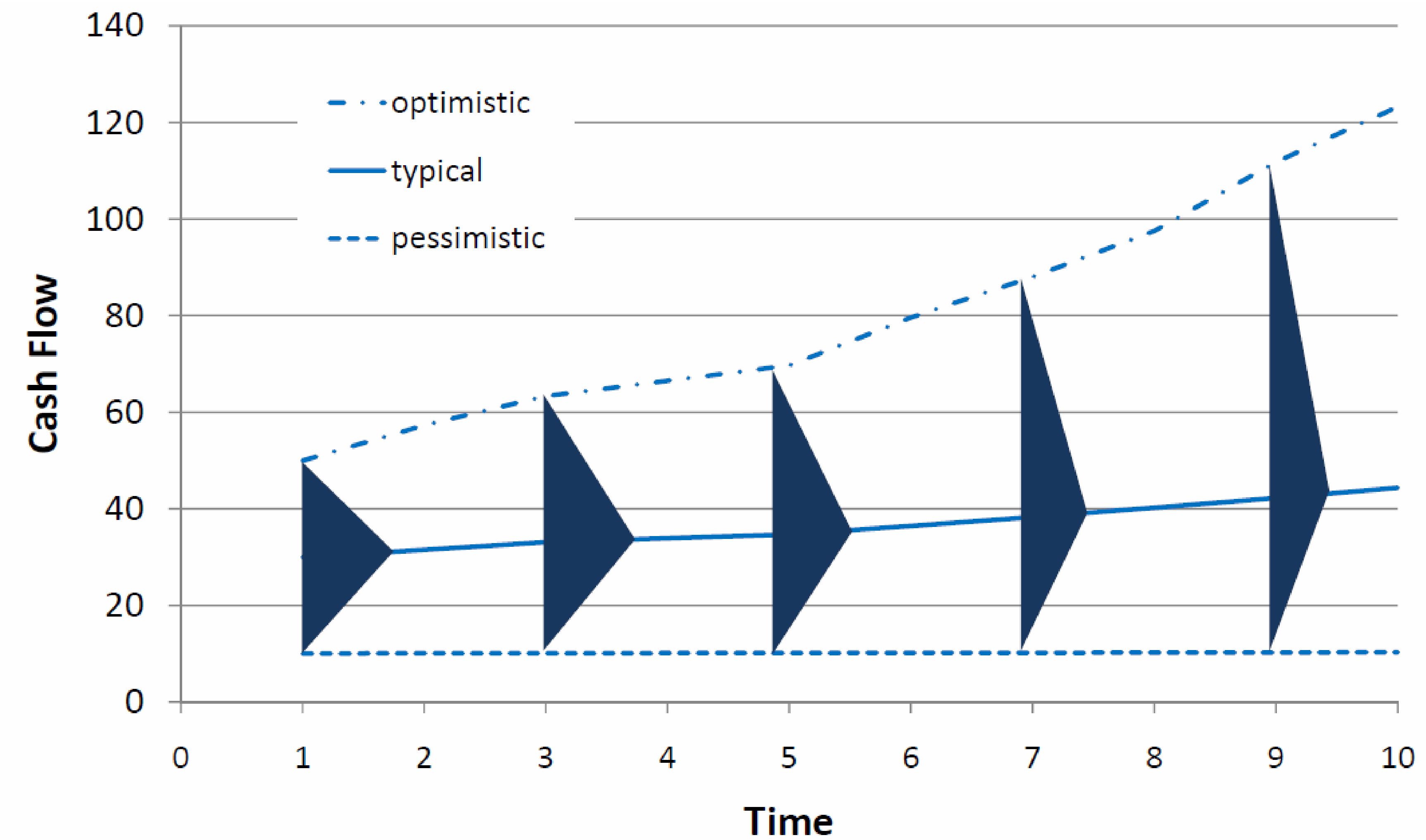
- Most projects have both exogenous and endogenous aspects
- Market risk is valued through appropriate hedging while private risk is discounted at the risk-free rate
- Projects are either:
  - Project value primarily derived from exogenous (market) factors
    - Thus use Classical approach
  - Project value primarily derived from endogenous (private/company) factors
    - Thus apply classical decision analysis methods (e.g. decision trees)
- Strengths:
  - Intuitive, **financially consistent** and objective
- Weakness
  - **Models are difficult to “fit” to reality**
- Solution: **Matching Method**

Goals of Proposed Methodology

- Practical to implement
  - Matches cash flow estimates provided by managers
  - Requires minimal subjectivity with respect to parameter estimation
- Consistent with financial theory
  - Is completely consistent with theory
    - Properly accounts for market and private risk
    - Ensures that cash flows are appropriately correlated among time periods
    - Uses established martingale measures which minimizes hedging error variance
    - Replicates manager specified distributions

Matching Method

Matching Method



Market Sector Indicator

- Assume there exists a *market sector indicator*

$$dS_t = \nu S_t dt + \eta S_t dW_t$$

- *Market sector indicator*
  - does not need to be traded
  - could represent market size / revenues
  - is not constrained to a GBM process
- Assume *market sector indicator* is correlated to a traded index / asset

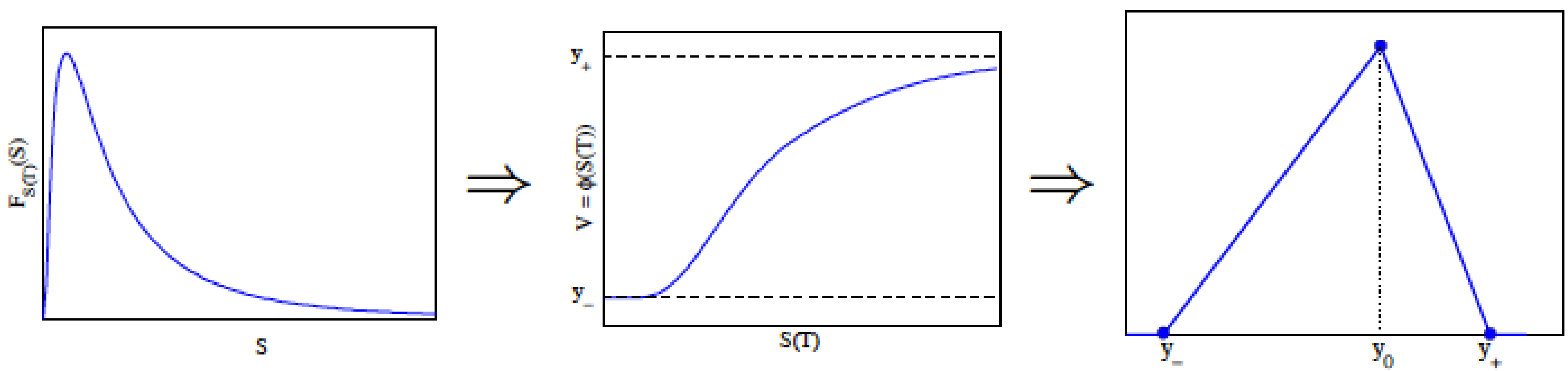
$$dI_t = \mu I_t dt + \sigma I_t \left(\rho dW_t + \sqrt{1-\rho^2} dW_t^{\perp} \right)$$

Match Cash Flow Payoff

- Each cash flow is effectively an option on the *market sector indicator*, $V_T = \varphi(S_T)$

- Matching probabilities

$$P(V_T < v) = F_A(v) \Rightarrow P(\varphi(S_T) < v) = F_A(v)$$



Calculation of φ(·)

We seek $\varphi(.)$ such that $\mathbb{P}(\varphi(S_T) \le v|\mathcal{F}_0) = F^*(v)$. Since,

$$S_T|_{\mathcal{F}_0} \stackrel{d}{=} S_0 \exp\left\{(\nu - \tfrac{1}{2}\eta^2)T + \eta\sqrt{T}Z\right\} \quad \text{where} \quad Z \underset{\mathbb{P}}{\sim} \mathcal{N}(0,1),$$

we have that

$$\mathbb{P}(\varphi(S_T) \le v|\mathcal{F}_0) = \Phi\left(\frac{\ln\frac{\varphi^{-1}(v)}{S_0} - (\nu - \frac{1}{2}\eta^2)T}{\eta\sqrt{T}}\right) \triangleq F^*(v).$$

Consequently, if $F^*(.)$ is invertible then

$$\varphi(S) = F^{*-1}\left(\Phi\left(\frac{\ln\frac{S}{S_0} - (\nu - \frac{1}{2}\eta^2)T}{\eta\sqrt{T}}\right)\right)$$

Payoff Function

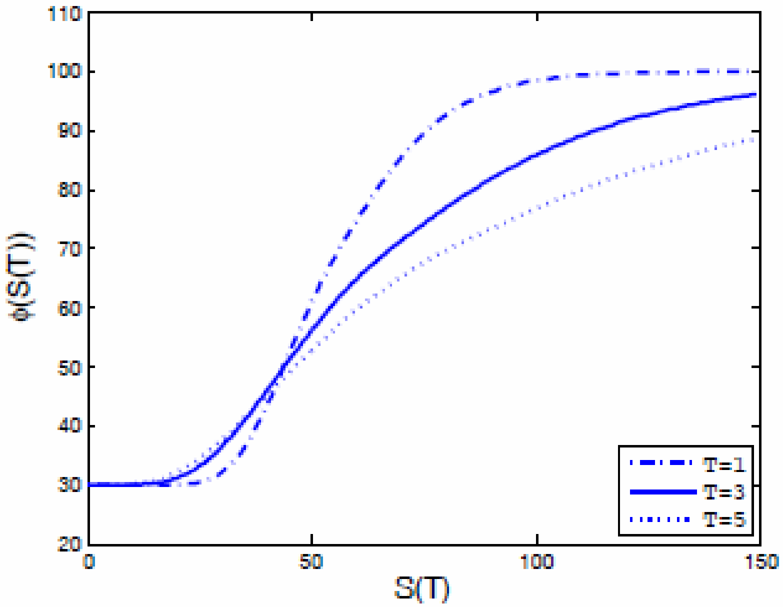
- Replicating payoff function

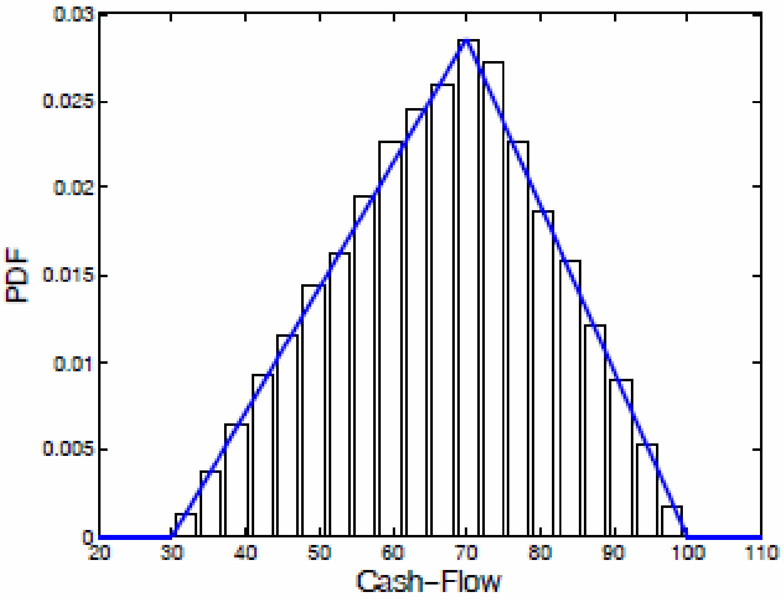
$$\varphi(S) = F_A^{*-1}\left(\Phi\left(z(S)\right)\right) = \begin{cases} y_- + \sqrt{(y_+ - y_-)(y_0 - y_-)\,\Phi\left(z(S)\right)}, & S \le S_c \\ y_+ - \sqrt{(y_+ - y_-)(y_+ - y_0)\,(1 - \Phi\left(z(S)\right))}, & S > S_c \end{cases}$$

$$z(S) = \frac{1}{\eta\sqrt{T}} \ln\frac{S}{S_0} - \frac{\left(\nu - \frac{1}{2}\eta^2\right)}{\eta}\sqrt{T}$$

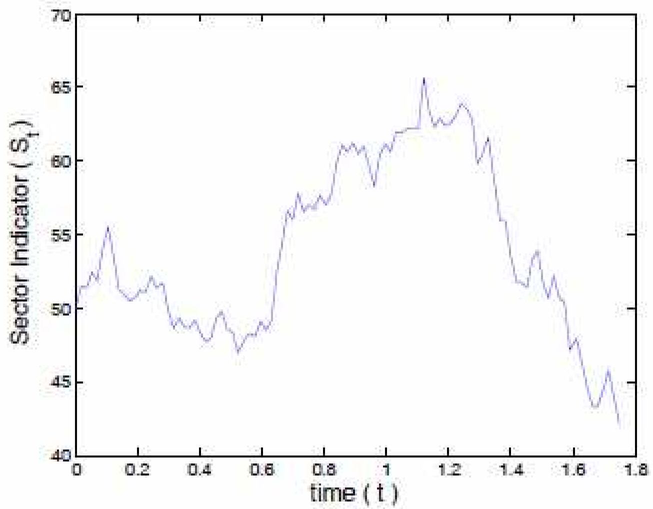
$$S_c = S_0 \exp\left\{(\nu - \tfrac{1}{2}\eta^2)T + \eta\sqrt{T}\,\frac{y_0 - y_-}{y_+ - y_-}\right\}$$

Replicating Payoff Function

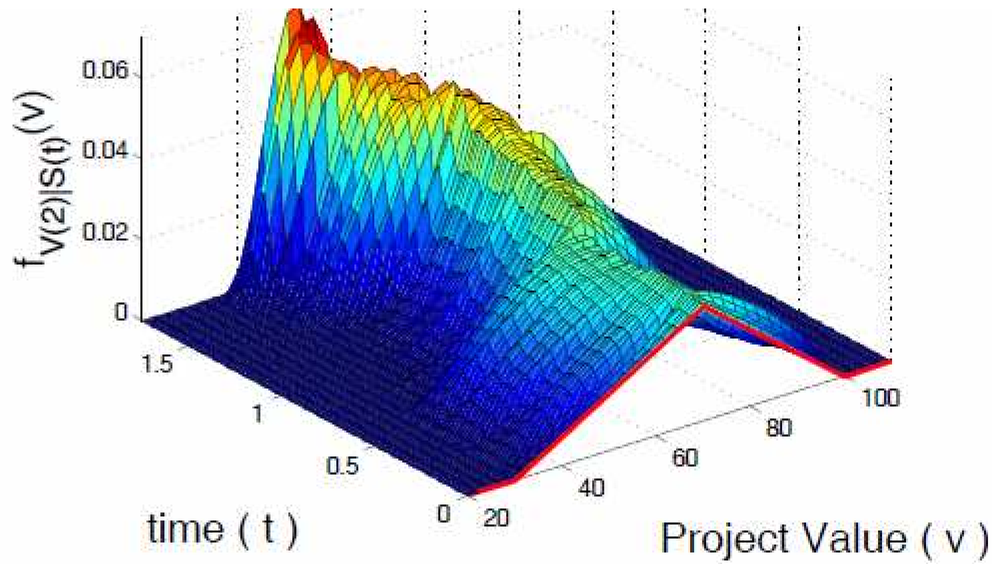




Information Distortion



(a) Market Indicator Flow



(b) Distorted Distribution Flow

Option Value

- Risk-neutral process for traded index

$$dI_t = rI_t dt + \sigma I_t \left(\rho d\tilde{W}_t + \sqrt{1-\rho^2}\, d\tilde{W}_t^{\perp} \right)$$

- Risk-neutral process for the *market sector indicator*

$$dS_t = \bar{r} S_t dt + \eta S_t d\tilde{W}_t$$

$$\bar{r} = \nu - \frac{\rho\eta}{\sigma}(\mu - r)$$

- The value of the option

$$RO_0 = e^{-rt} \tilde{E}\left[\max\left(V_t - K, 0 \right) \right]$$

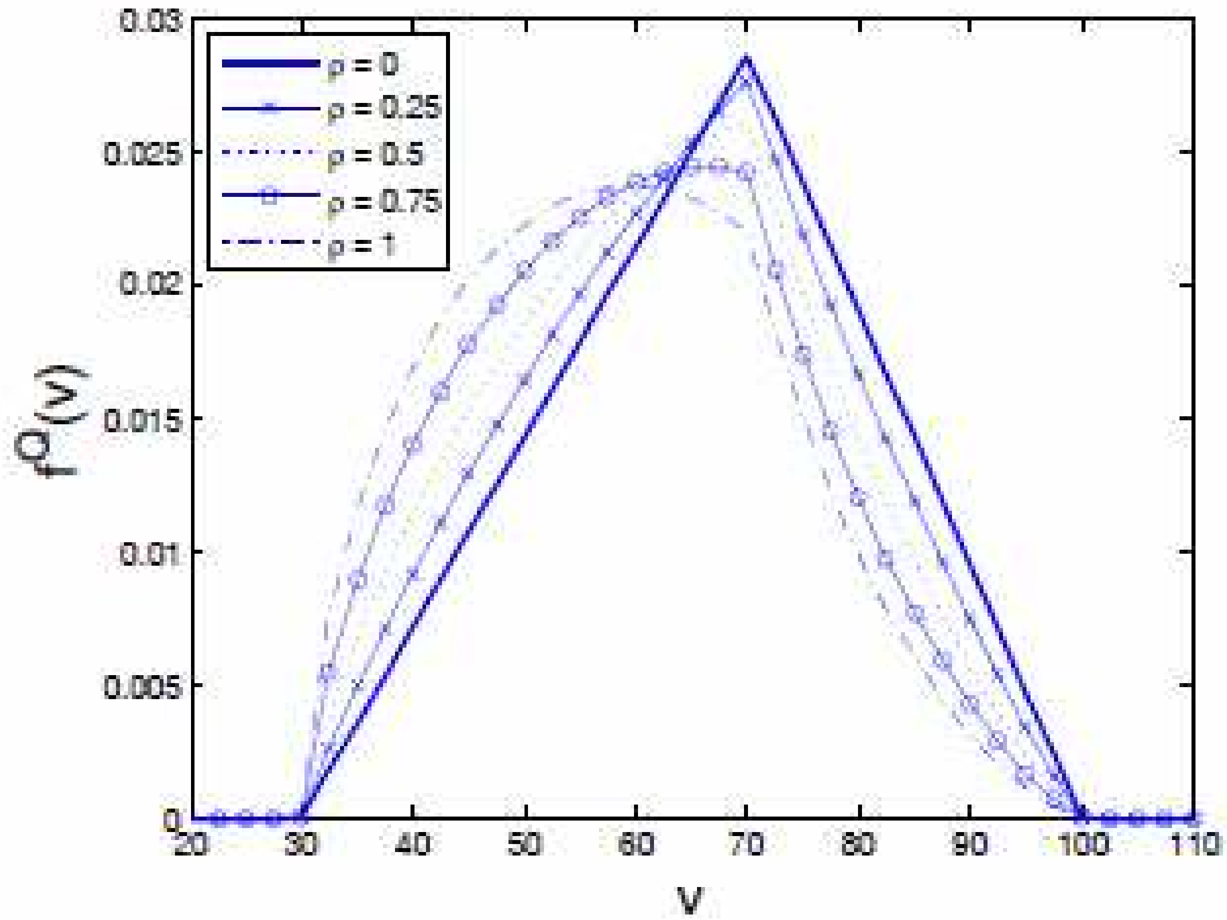
Risk-Neutral Measure

$$\begin{aligned}
\widehat{F}_{v_k|S_t}(v) &= \mathbb{Q}\left(\varphi_k(S_{T_k}) \le v \,|\, S_t = S\right) \\
&= \mathbb{Q}\left(\varphi_k\left(Se^{(\widehat{\nu}-\frac{1}{2}\eta^2)(T_k-t)+\eta\sqrt{T_k-t}\,Z}\right) \le v\right) \\
&= \mathbb{Q}\left(\Phi\left(\frac{\ln(S/S_0)+(\widehat{\nu}-\frac{1}{2}\eta^2)(T_k-t)-(\nu-\frac{1}{2}\eta^2)T_k}{\eta\sqrt{T_k}} + \sqrt{\frac{T_k-t}{T_k}}Z\right) \le F_k(v)\right) \\
&= \mathbb{Q}\left(Z \le \sqrt{\frac{T_k}{T_k-t}}\Phi^{-1}\left(F_k(v)\right) - \widehat{\lambda}_k(t,S)\right)
\end{aligned}$$

$$\widehat{F}_{v_k|S_t}(v) = \Phi\left(\sqrt{\frac{T_k}{T_k-t}}\Phi^{-1}\left(F_k(v)\right) - \widehat{\lambda}_k(t,S)\right)$$

$$\widehat{\lambda}_k(t,S) = \frac{1}{\eta\sqrt{T_k-t}}\ln\frac{S}{S_0} + \frac{\widehat{\nu}-\frac{1}{2}\eta^2}{\eta}\sqrt{T_k-t} - \frac{\nu-\frac{1}{2}\eta^2}{\eta}\frac{T_k}{\sqrt{T_k-t}}$$

Risk-Neutral Density



Project Value

- Value of the cash flows at time, t

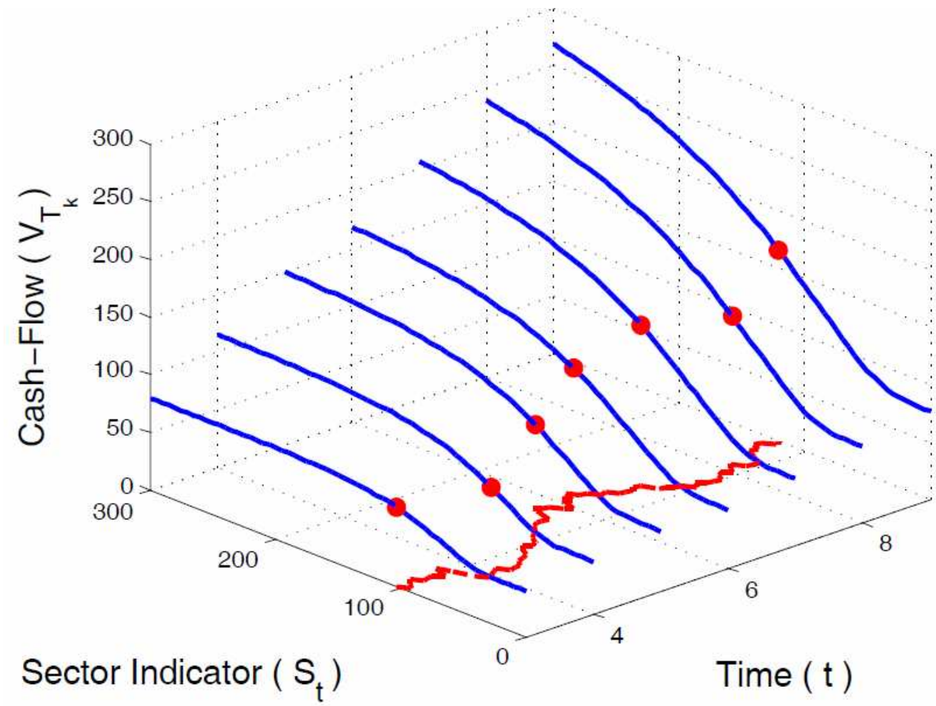
$$V_t = \sum_{i=1}^{n} e^{-r(t_i - t)} E\left[V_{t_i} \middle| F_t \right]$$

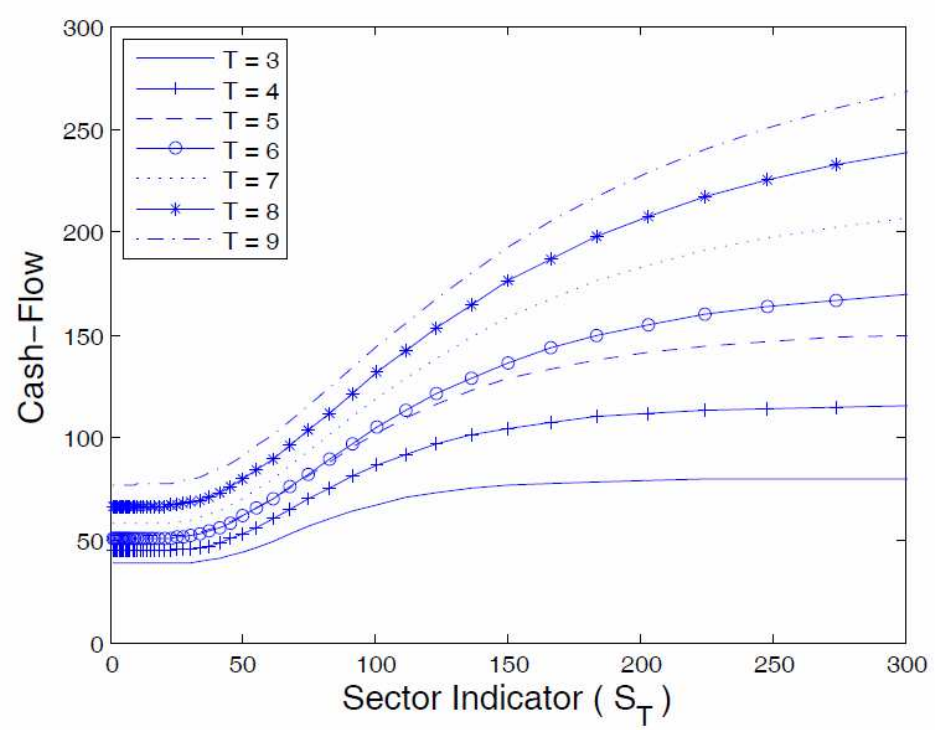
- Project value

$$V_{proj} = e^{-rt_K} \int_{-\infty}^{\infty} \left(\sum_{i=1}^{n} \left(e^{-r(t_i - t_K)} \int_{-\infty}^{\infty} \varphi_i \left(S_{t_i} \right) \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy \right) - K, 0 \right)_+ \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx$$

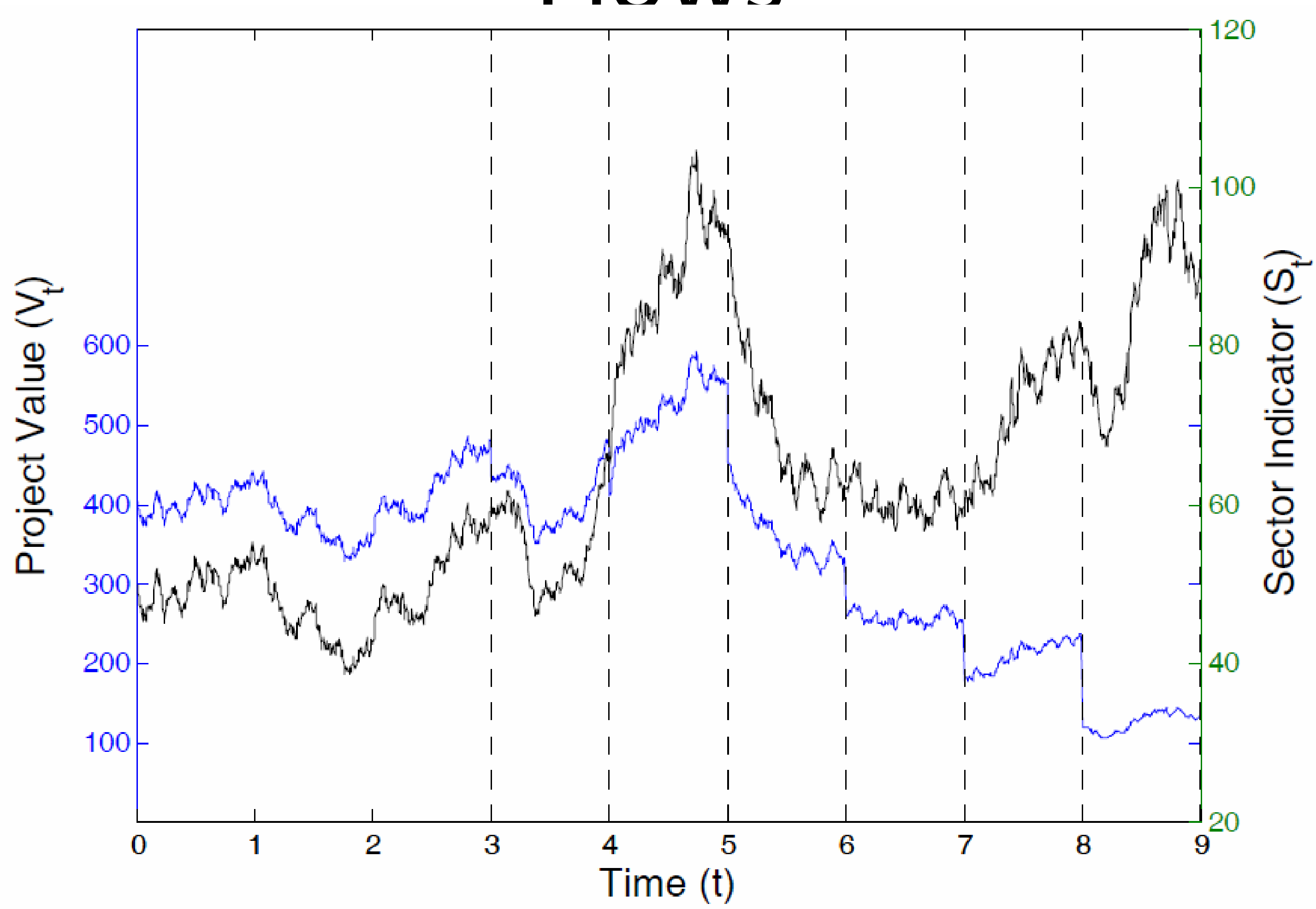
$$S_{t_i} = S_0 e^{(\bar{r} - \frac{1}{2}\eta^2) t_i + \eta(\sqrt{t_K}\, x + \sqrt{t_i - t_K}\, y)}$$

Cash-Flow and Market Sector Indicator

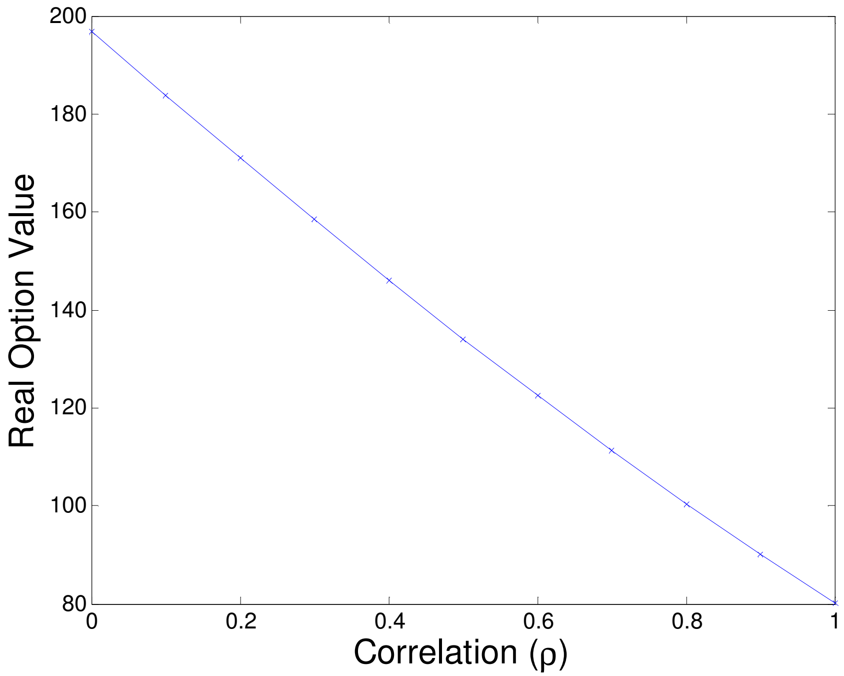




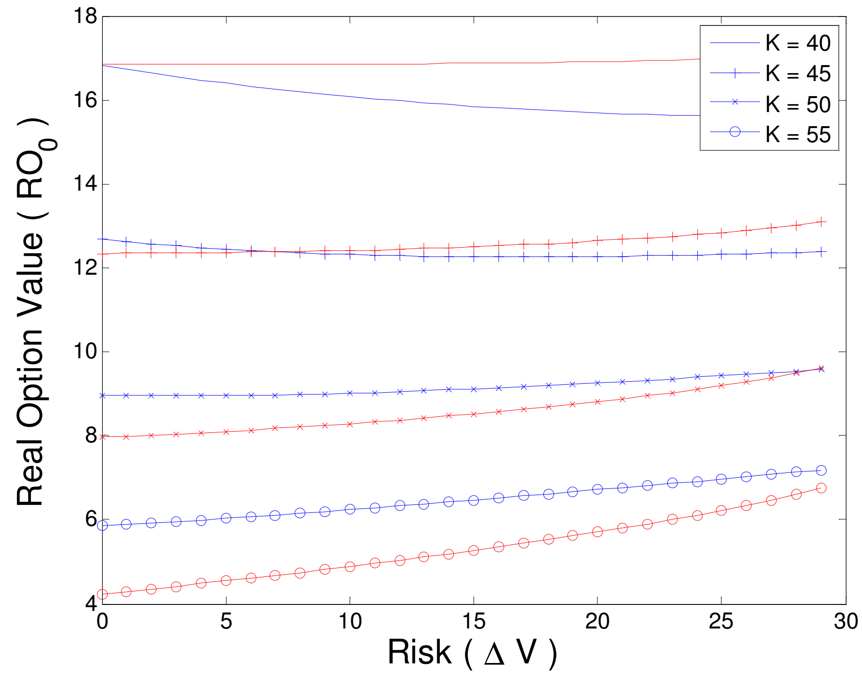
Sample Path of Value of Cash Flows



Option Value: Correlation and Risk



Real option value versus correlation.

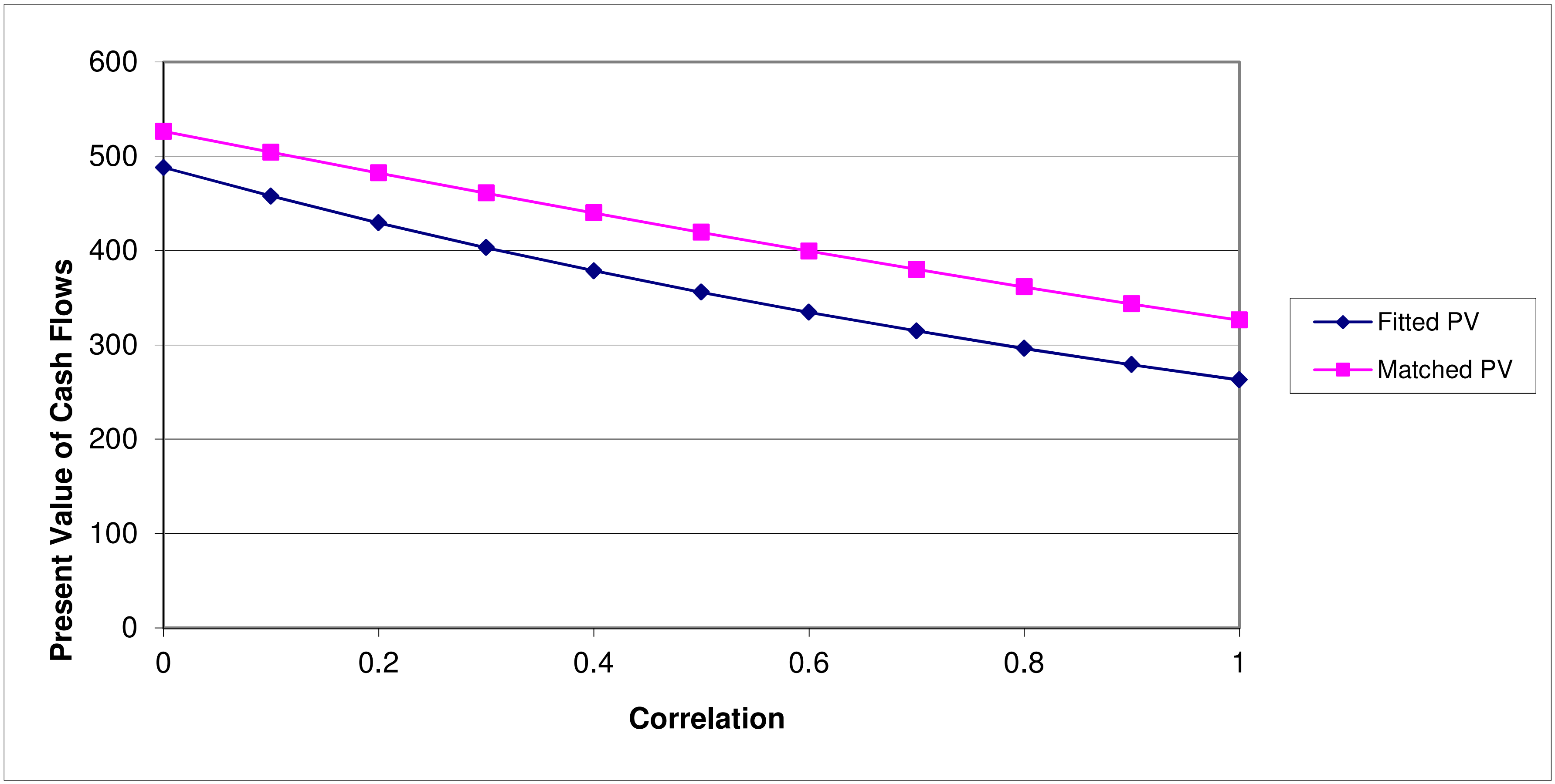


Real option value sensitivity to risk:
30 - ΔV, 70, 100 + ΔV
Matching method in BLUE
MAD method in RED

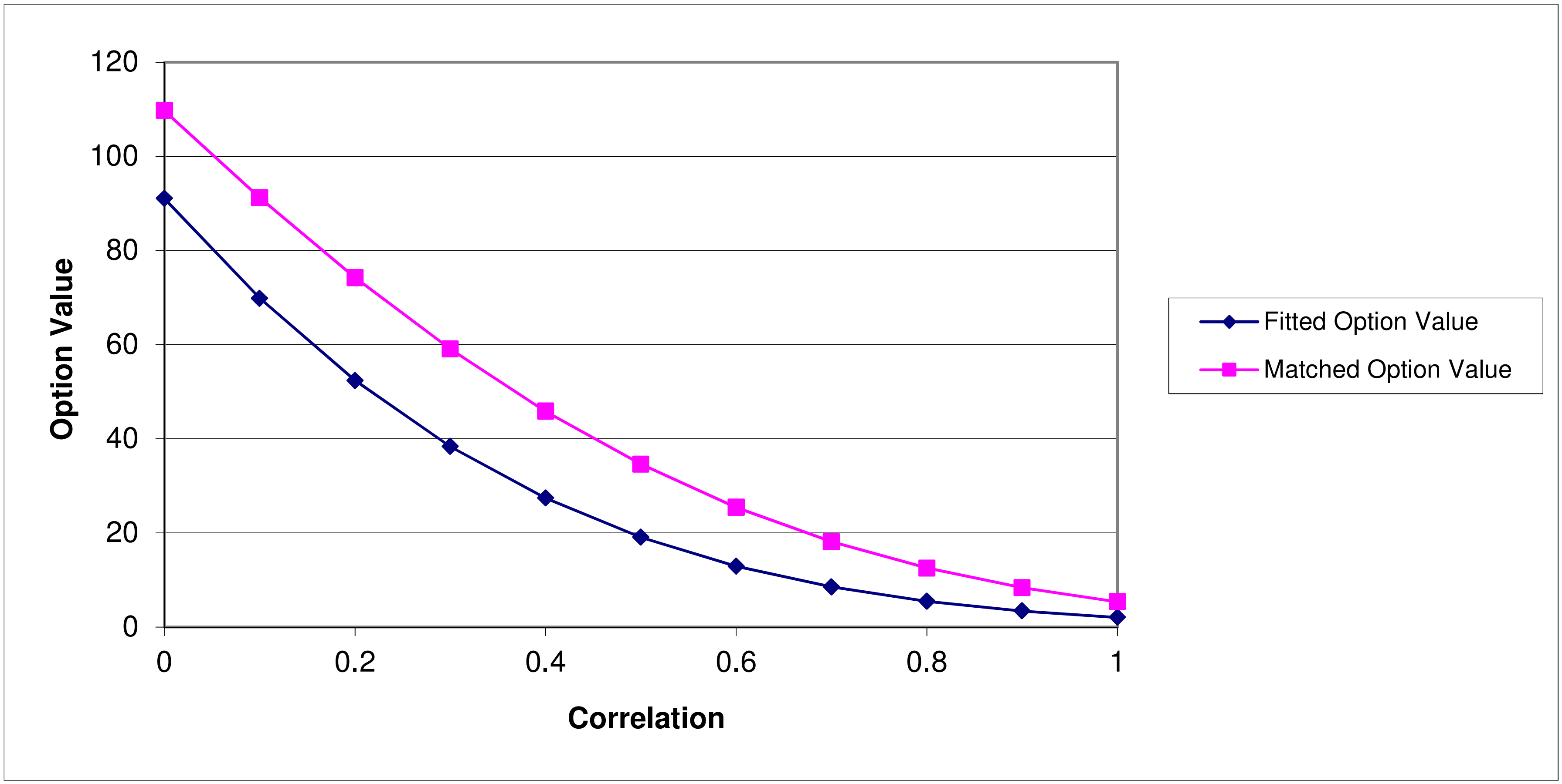
Practical Implementation

	Expected Cash Flows per Year								
Scenario	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Optimistic	0	0	80	120	150	180	200	220	250
Most likely	0	0	50	70	75	80	90	100	110
Pessimistic	0	0	20	25	25	20	20	20	20
Investment		450							

Present Value of Cash Flows



Option Value



Hedging the Real Option

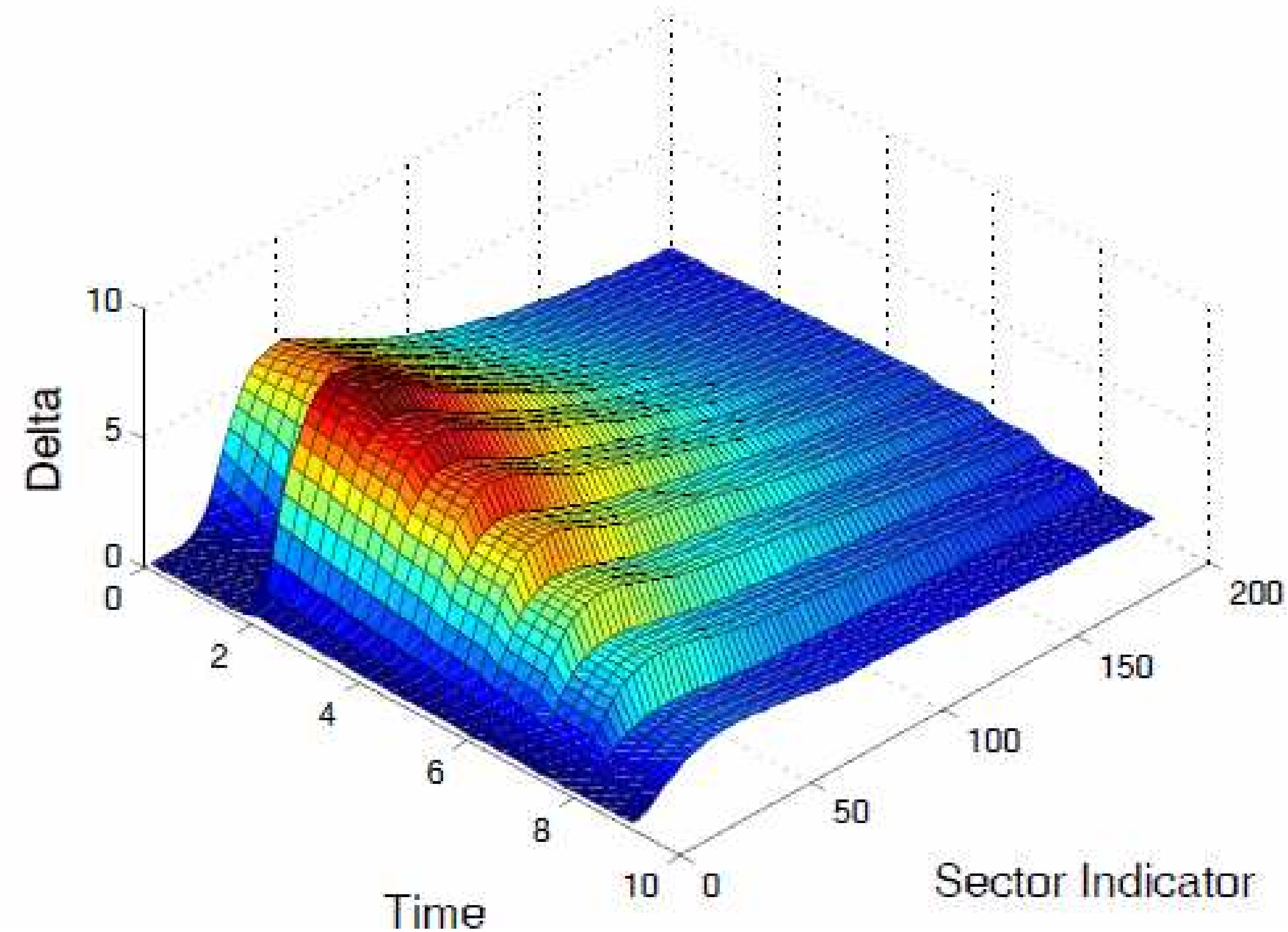
- Hedging strategy that minimizes variance:

$$\Delta_t = \begin{cases} \rho \frac{\eta\, S_t}{\sigma\, I_t} \partial_S RO_t(S_t), & t \le T_0, \\ \rho \frac{\eta\, S_t}{\sigma\, I_t} \partial_S V_t(S_t), & T_0 < t \le T_n \end{cases}$$

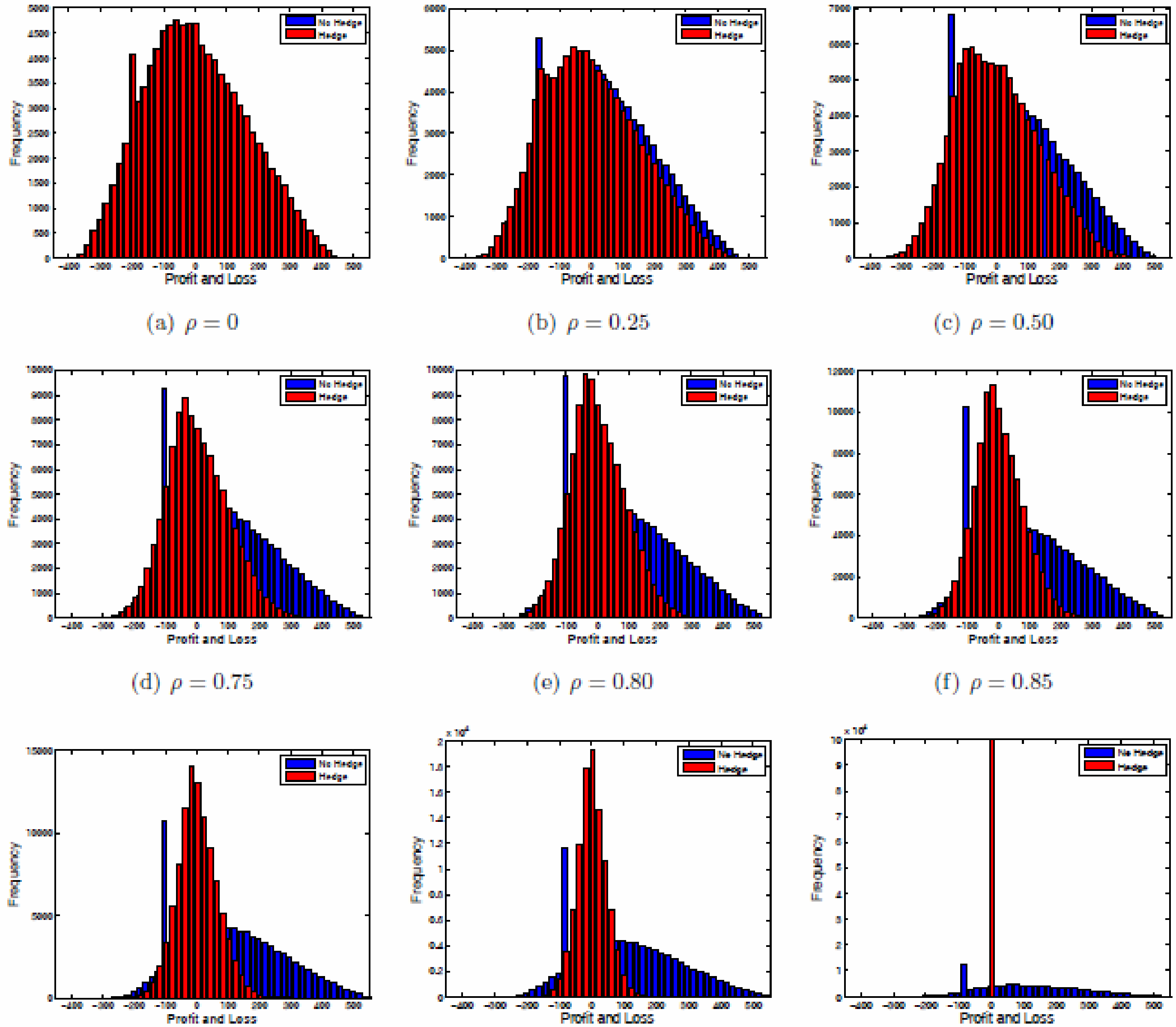
- where

$$V_t(S) = \sum_{k=\min\{m:T_m \ge t\}}^{n} e^{-r(T_k - t)} \int_0^\infty \left(1 - \widehat{F}_{v_k|S}(v)\right) dv$$

Hedging Delta



Hedging Results



(a) $\rho = 0$ (b) $\rho = 0.25$ (c) $\rho = 0.50$

(d) $\rho = 0.75$ (e) $\rho = 0.80$ (f) $\rho = 0.85$

(g) $\rho = 0.90$ (h) $\rho = 0.95$ (i) $\rho = 1$

Enhanced Matching Method

Matching Revenues and GM%

Practical Considerations

- It is more likely that a sector indicator will be correlated to revenues than cash flows
- Cash flows are estimated based on:
  - Estimated revenues
  - Estimated gross margin percent
  - Other fixed and variable costs
- Assumptions
  - Revenues are stochastic and driven by a sector indicator
  - GM% values are stochastic and correlated to revenues
  - Other fixed and variable costs are not stochastic

Managerial Estimates

Scenario	End of Year Sales / Margin						
	3	4	5	6	7	8	9
Optimistic	80	116	153	177	223	268	314
	(50%)	(60%)	(65%)	(60%)	(60%)	(55%)	(55%)
Most Likely	52	62	74	77	89	104	122
	(30%)	(40%)	(40%)	(40%)	(35%)	(35%)	(35%)
Pessimistic	20	23	24	18	20	20	22
	(20%)	(20%)	(20%)	(20%)	(15%)	(10%)	(10%)
SG&A*	10%	5%	5%	5%	5%	5%	5%
Fixed Costs	30	25	20	20	20	20	20

* Sales / General and Administrative Costs

Practical Implementation

- Traded index:

$$dI_t = \mu I_t dt + \sigma I_t \, dB_t$$

- Sales sector *indicator* used to drive revenues

$$dX_t = \rho_{SI} dB_t + \sqrt{1-\rho_{SI}^2}\, dW_t^S$$

- Revenues partially drive GM% *indicator*

$$dY_t = \rho_{SM} dX_t + \sqrt{1-\rho_{SM}^2}\, dW_t^M$$

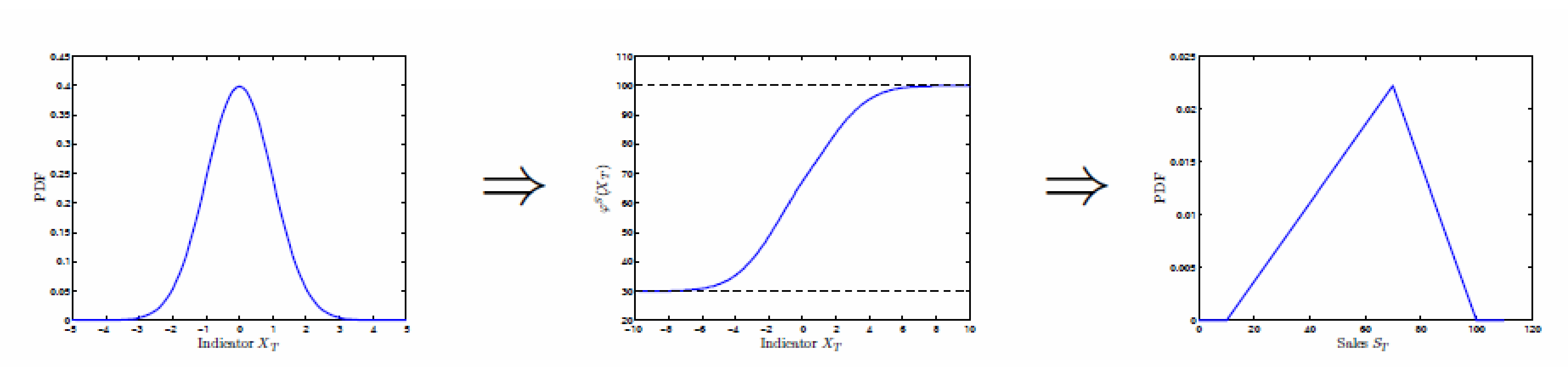
Match Sales and GM%

- Sales and GM% processes are driven by their respective indicators

$$S_k = \varphi_k^S(X_{T_k}) \text{ and } M_k = \varphi_k^M(Y_{T_k})$$

- Matching probabilities

$$P(S_T < s) = P(\varphi^S(X_T) < s) = F(s) \text{ and } P(M_T < m) = P(\varphi^M(Y_T) < m) = G(m)$$



Payoff Function

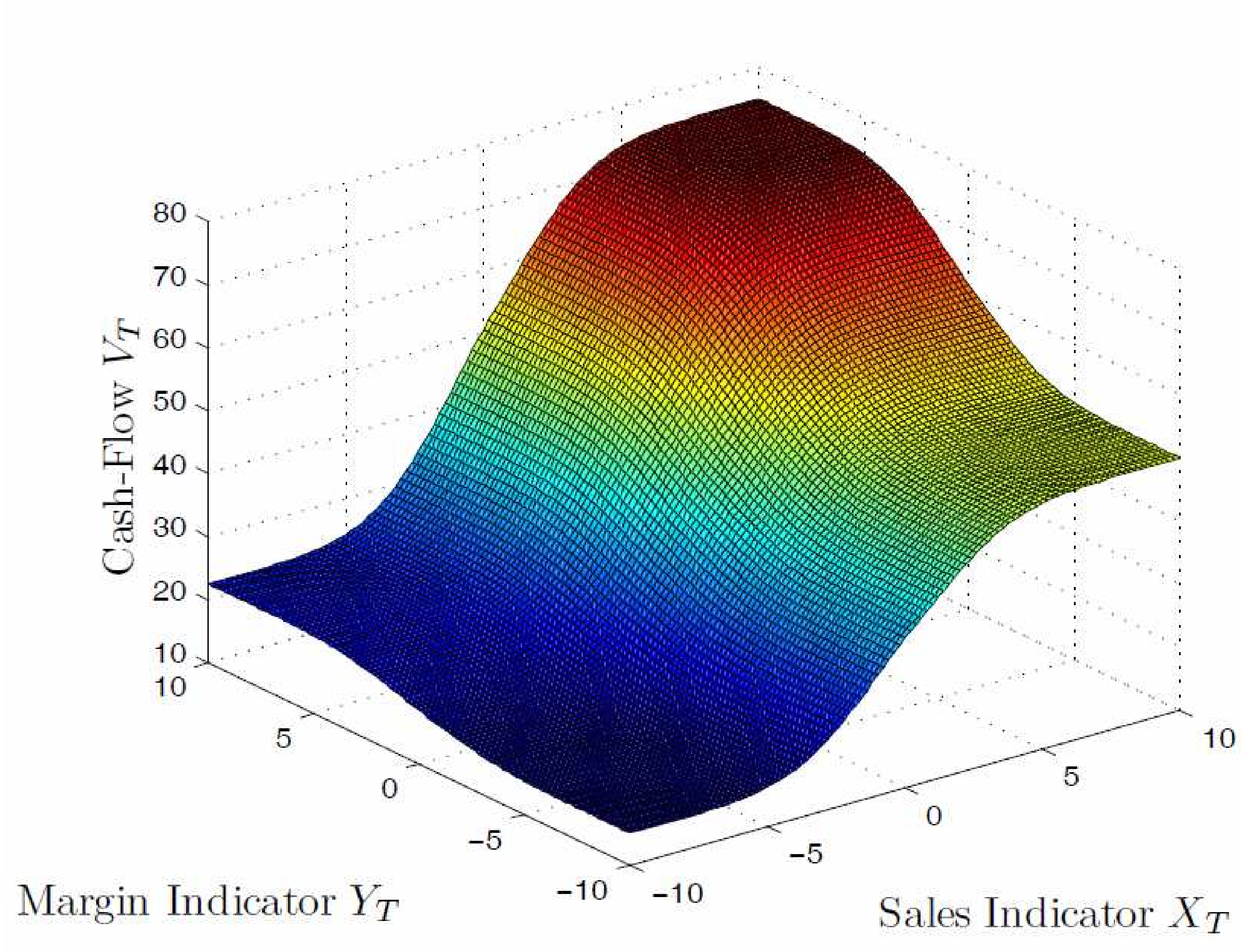
- The payoff function $\varphi^S(x)$ which produces the manager specified distribution F(v) for the sales at time T, when the underlying driving uncertainty X_t is a BM, is given by

$$\varphi^S(x) = F^{-1}\left(\Phi\left(\frac{x - x_0}{\sqrt{T}}\right)\right)$$

- Cash flow

$$V_k = (1 - \kappa_k)\,\varphi_k^S \varphi_k^M - \alpha_k$$

Cash Flow Profile



Real Option Value

- Value of the cash flows

$$V_{T_0}(S_{T_0}, M_{T_0}) = \sum_{k=1}^{n} e^{-r(T_k - T_0)}\, \mathbb{E}^{\mathbb{Q}}\left[v_k \mid S_{T_0}, M_{T_0} \right]$$
$$= \sum_{k=1}^{n} e^{-r(T_k - T_0)}\, \mathbb{E}^{\mathbb{Q}}\left[\varphi_k(S_{T_k}, M_{T_k}) \mid S_{T_0}, M_{T_0} \right]$$

- Real option value

$$RO_t(S, M) = e^{-r(T_0 - t)}\, \mathbb{E}^{\mathbb{Q}}\left[\left(V_{T_0}(S_{T_0}, M_{T_0}) - K\right)_+ \middle| S_t = S,\, M_t = M \right]$$

Risk-Neutral measure

$$
\begin{aligned}
dX_t &= \widehat{\nu}\,dt + \rho_{SI}\,d\widehat{B}_t + \sqrt{1-\rho_{SI}^2}\,d\widehat{W}_t^S,\\
dY_t &= \widehat{\gamma}\,dt + \rho_{SI}\rho_{SM} d\widehat{B}_t + \rho_{SM}\sqrt{1-\rho_{SI}^2}\,d\widehat{W}_t^S + \sqrt{1-\rho_{SM}^2}\,d\widehat{W}_t^M,\\
\frac{dI_t}{I_t} &= r\,dt + \sigma\,d\widehat{B}_t
\end{aligned}
$$

$$\boxed{\widehat{\nu} = -\rho_{SI}\,\frac{\mu-r}{\sigma} \qquad \text{and} \qquad \widehat{\gamma} = -\rho_{SI}\rho_{SM}\,\frac{\mu-r}{\sigma}}$$

Solution Methods

- Numerical integration
- Simulation
- Three dimensional trees
- PDE

$$rG = \frac{\partial G}{\partial t} + \widehat{\nu}\frac{\partial G}{\partial x} + \widehat{\gamma}\frac{\partial G}{\partial y} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2} + \frac{1}{2}\frac{\partial^2 G}{\partial y^2} + \rho_{SM}\frac{\partial^2 G}{\partial x \partial y}$$

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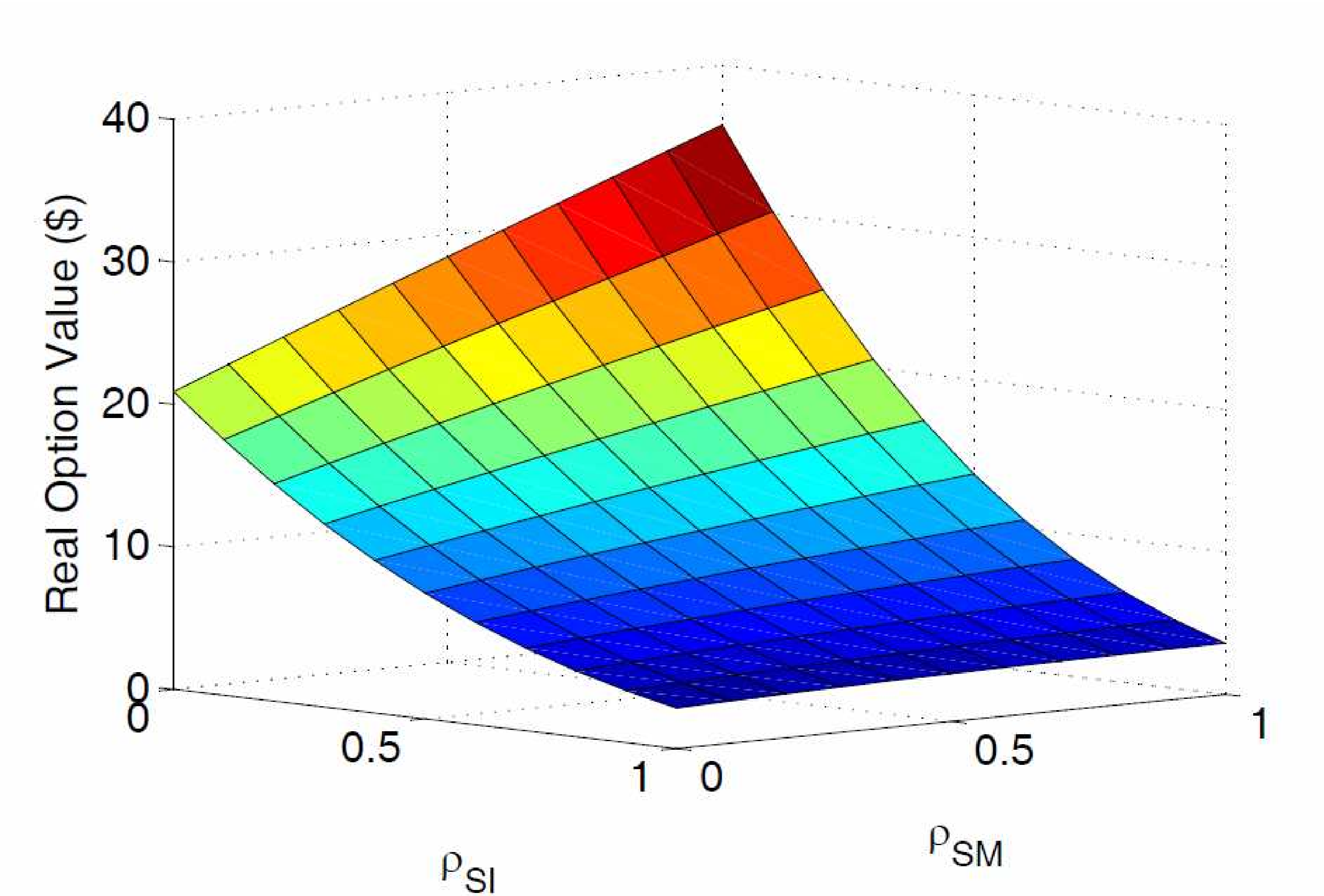
Real Option Value



Model Implementation

- Valuation Eg

47

Conclusion: Proposed Method

- Practical to implement
  - Matches estimates provided by managers
  - Requires minimal subjectivity with respect to parameter estimation
    - Required market parameters: r, μ, σ
    - Required project parameters: ρ_{SI}, ρ_{SM}
- Consistent with financial theory
  - Is generally consistent with theory
  - Specifically:
    - Properly accounts for market and private risk
    - Ensures that cash flows are appropriately correlated among time periods

Managerial Risk Aversion and Real Options

Real Options in R&D Type Applications

- Managers provide cash flow estimates

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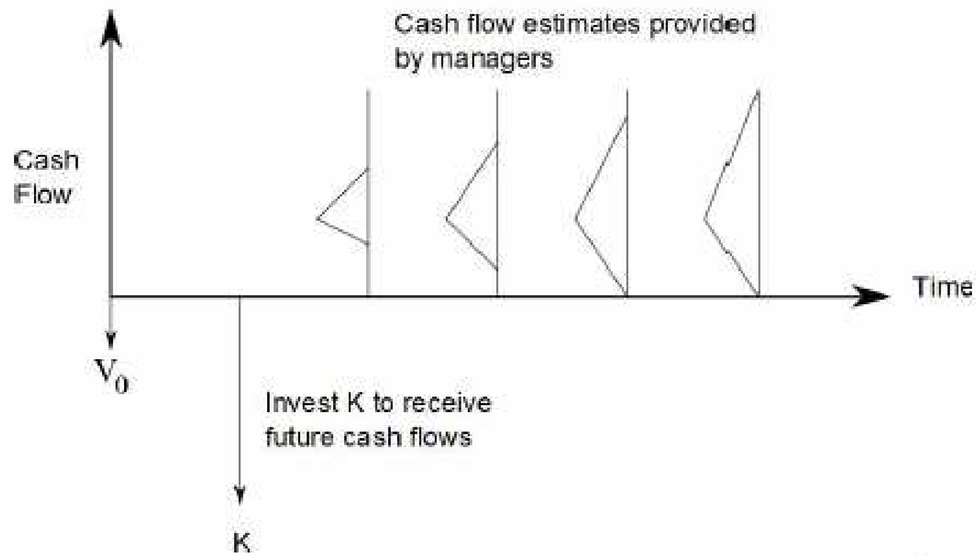


Real Options in R&D Type Applications

- Problem:
  - How should we value the cash flows?
  - How should we account for managerial risk aversion?
- Approach:
  - Apply "matching method" with MMM to value cash flows
  - Apply indifference pricing to determine value with manager's risk aversion
- Traded index / asset

$$dI_t = \mu I_t dt + \sigma I_t dW_t$$

- Assume there exists a *Market Sector Indicator* correlated to the traded index

$$dS_t = \nu S_t dt + \eta S_t(\rho dW_t + \sqrt{1-\rho^2} dW_t^{\perp})$$

Indifference Pricing: Problem Definition

- Goal: to maximize expected terminal utility of discounted wealth
- Case I: Invest in market only, with π_t invested in risky asset

$$V(t, X) = \sup_{\pi} \mathbb{E}_t \left[-\frac{1}{\gamma} e^{-\gamma X_T} \right]$$

$$dX_t = (\mu - r)\pi_t dt + \sigma \pi_t dW_t$$

- Case II: Invest in project (with option)

$$U(t, X, S) = \sup_{\pi} \mathbb{E}_t \left[-\frac{1}{\gamma} e^{-\gamma X_T} \right]$$

$$dX_t = (\mu - r)\pi_t dt + \sigma \pi_t dW_t, \ \ t \notin [T_0, T_1, ..., T_n]$$

$$X_{T_0} = X_{T_0^-} - K e^{-rT_0} \mathbf{1}_{\mathcal{A}}$$

$$X_{T_j} = X_{T_j^-} + \varphi(S_j) e^{-rT_j} \mathbf{1}_{\mathcal{A}}, \ j \in [1, 2, ..., n]$$

Indifference Pricing: HJB

- Indifference price, f, determined by $U(t, x-f, S) = V(t,x)$, and $U(t,x,S)$ satisfies HJB

$$
\begin{aligned}
&\frac{\partial U}{\partial t} + \nu S \frac{\partial U}{\partial S} + \tfrac{1}{2}\eta^2 S^2 \frac{\partial^2 U}{\partial S^2} \\
&+\mu \frac{\partial U}{\partial I} + \tfrac{1}{2}\sigma^2 \frac{\partial^2 U}{\partial I^2} + \sigma\eta\rho S \frac{\partial^2 U}{\partial S \partial I} \\
&+ \sup_{\pi} \left[(\mu - r)\pi \frac{\partial U}{\partial x} + \tfrac{1}{2}\sigma^2 \pi^2 \frac{\partial^2 U}{\partial x^2} + \sigma\eta\rho\pi S \frac{\partial^2 U}{\partial x \partial S} \right] = 0 \\
&U(T, x, S_T) = -\frac{1}{\gamma} e^{-\gamma(x + \varphi_n(S_T) e^{-rT})} \\
&(t, \omega) \in (T_{n-1}, T_n] \times \mathcal{A}(\omega)
\end{aligned}
$$

Indifference Pricing: Numerical Simulation

- With $U(t,x,S) = V(t,x)\,H^{\beta}(t,S)$, where $\beta = \frac{1}{1-\rho^2}$

$$\frac{\partial H}{\partial t} + \bar{\nu} S \frac{\partial H}{\partial S} + \tfrac{1}{2}\eta^2 S^2 \frac{\partial^2 H}{\partial S^2} = 0$$
$$H(T_n, S_T) = e^{-\gamma\varphi_n(S_T)e^{-rT_n}}$$

- At strike time, $t = T_0$, invest if $U(T_0,x,S) > V(T_0,x)$, but

$$\begin{aligned} U(T_0,x,S) &= U(T_0^+, x - Ke^{-rT_0}, S) \\ &= V(T_0,x)H^{\beta}(T_0^+,S)e^{\gamma K e^{-rT_0}} \end{aligned}$$

- Therefore invest if

$$H^{\beta}(T_0^+,S)e^{\gamma K e^{-rT_0}} \leq 1$$

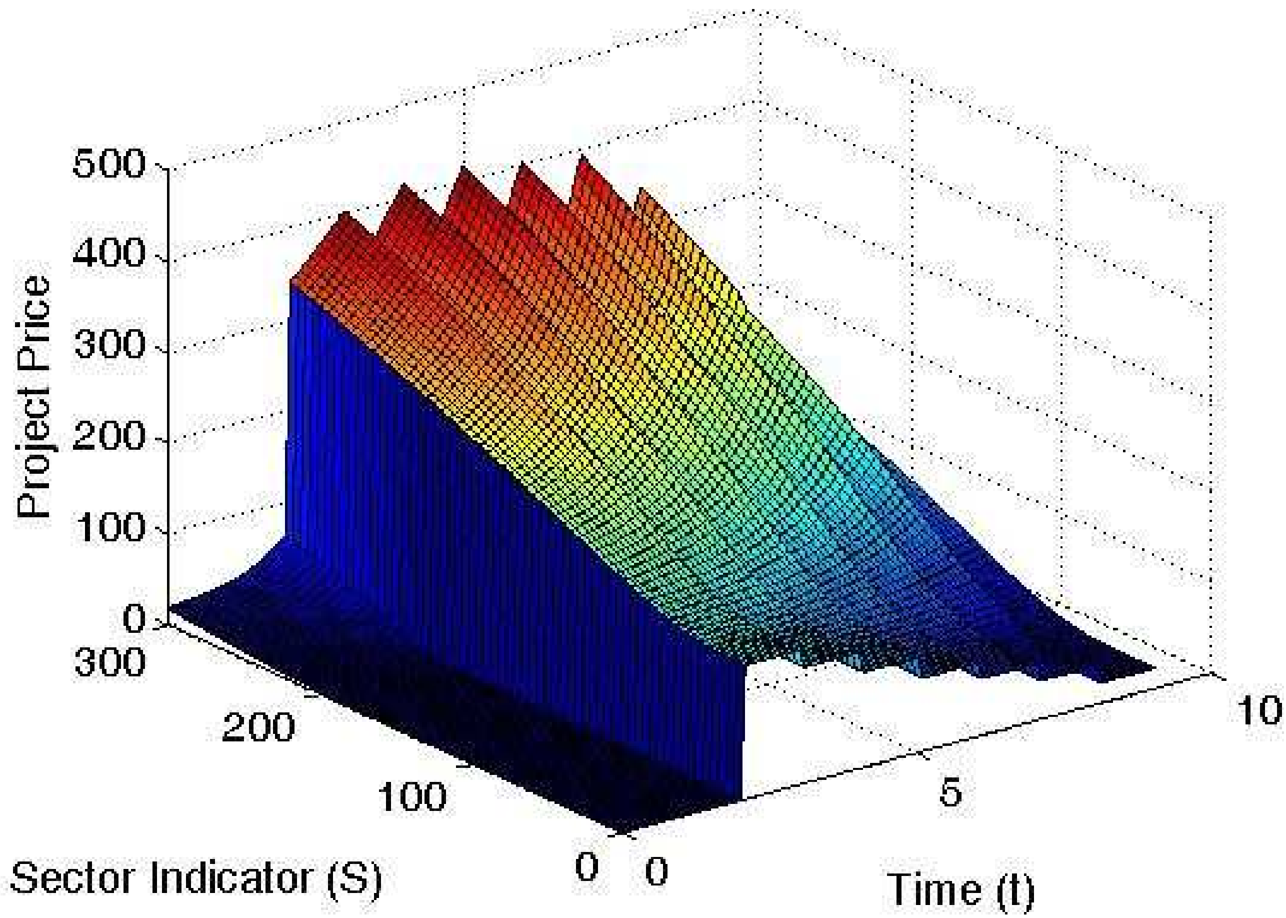
- Equivalent to the cash flow indifference price being less than the strike

Indifference Pricing: Numerical Simulation

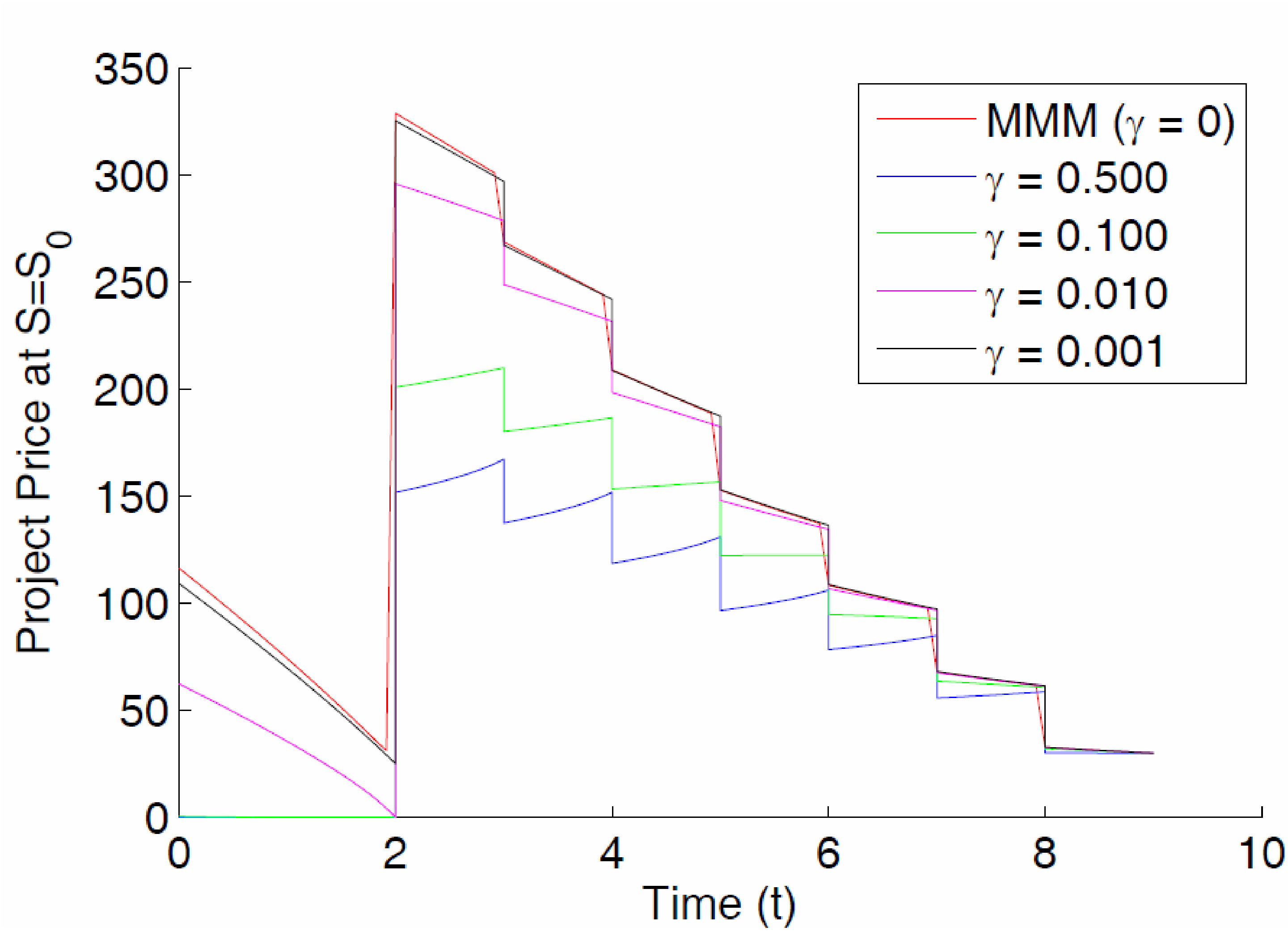
- Finally,

$$
\begin{aligned}
U(t, x-f, S) &= V(t, x) \\
V(t, x-f)H^{\beta}(t, S) &= V(t, x) \\
-\frac{1}{\gamma}e^{-\gamma(x-f)}H^{\beta}(t, S) &= -\frac{1}{\gamma}e^{-\gamma(x)} \\
f &= -\frac{\beta}{\gamma}\ln\left(H^{\beta}(t, S)\right)
\end{aligned}
$$

Results: Indifference Price



Results: Price for Varying γ



Conclusions

- Matching method provides a link between financial theory and practical implementation
- We now have a tool to show managers how risk-aversion can impact decision making based on their own cash flow estimates