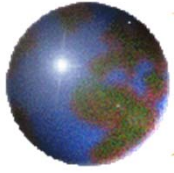


# *CVA and Wrong Way Risk*

John Hull

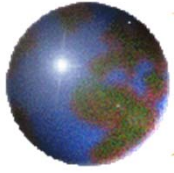
3C Risk Forum, October 2011



# *Agenda*

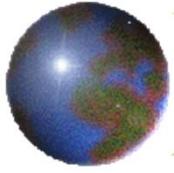
- ⊕ Calculation of CVA
- ⊕ Basel III requirements
- ⊕ A simple model of wrong-way/right-way risk
- ⊕ Numerical results

See “CVA and Wrong Way Risk” by John Hull and Alan White on [www.rotman.utoronto.ca/hull](http://www.rotman.utoronto.ca/hull)



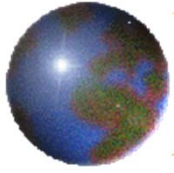
## *Background*

- ✚ CVA is present value of expected default losses on a derivatives portfolio with a particular counterparty
- ✚ Dealers must calculate one CVA for each counterparty
- ✚ CVAs are themselves derivatives
- ✚ They are more complicated than any other derivative in a dealer's book



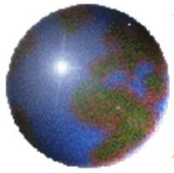
## *Background continued*

- ✚ We are concerned with transactions that are cleared bilaterally
- ✚ Usually transactions are governed by an ISDA Master Agreement
- ✚ Transactions are netted
- ✚ Often there is a collateralization agreement (CSA)
- ✚ Failure to post collateral as required gives the dealer the right to terminate transactions



## *Definitions*

- ⊕ Threshold is the unsecured credit exposure to counterparty that dealer is prepared to bear
- ⊕ Independent amount is an initial margin that must be maintained by the counterparty in addition to dealer's exposure
- ⊕ The cure period is the period assumed to elapse between the counterparty failing to post collateral and an early termination notice

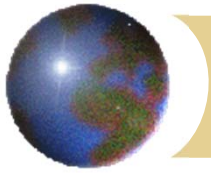


## *CVA Calculation*

If  $T$  is the life of the longest transaction with the counterparty

$$CVA = (1 - R) \int_{t=0}^T q(t) v(t) dt$$

where  $R$  is the recovery rate,  $v(t)$  is the value of a derivative that pays off the net exposure (after collateral) of the dealer on its portfolio with the counterparty at time  $t$ , and  $q(t)$  is the (risk-neutral) probability density function of the time to default



## *CVA continued*

- ✚ The exposure at time  $t$  (if no collateralization) is

$$E_{NC}(t) = \max(w(t), 0)$$

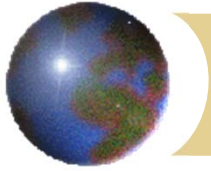
where  $w(t)$  is the value of portfolio at time  $t$

- ✚ When there is a threshold  $K$  and the cure period is  $c$ , the collateral available at time  $t$  is

$$C(t) = \max(w(t - c) - K, 0)$$

- ✚ An independent amount can be treated as a negative  $K$
- ✚ The net exposure at time  $t$  is therefore

$$E_{NET}(t) = \max(E_{NC}(t) - C(t), 0)$$



# *Numerical Approximation* *(assuming $q$ and $v$ independent)*

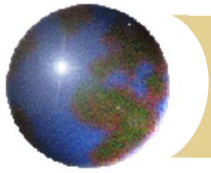
$$\text{CVA} = (1 - R) \sum_{i=1}^n q_i v_i$$

where  $0 = t_0 < t_1 < t_2 < \dots < t_n = T$

$q_i$  is the unconditional risk-neutral probability of default between times  $t_{i-1}$  and  $t_i$ ,

$v_i = v(t_i^*)$  where  $t_i^*$  is the mid point of the  $(t_{i-1}, t_i)$  interval

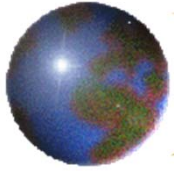




## *Estimating $q_i$*

If  $s_i$  is the credit spread for a maturity of  $t_i$  then the average hazard rate up to  $t_i$  is  $s_i/(1-R)$  and an estimate of  $q_i$  is

$$q_i = \exp\left(-\frac{s_{i-1}t_{i-1}}{1-R}\right) - \exp\left(-\frac{s_i t_i}{1-R}\right)$$

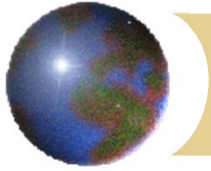


# *Monte Carlo Simulation*

- ✚ Simulate the behavior of underlying market variables in a risk-neutral world and calculate the value of the portfolio at times

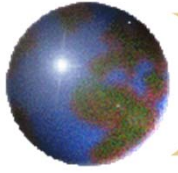
$$t_i^* \text{ and } t_i^* - c \quad (1 \leq i \leq n)$$

- ✚ Some approximations are likely to be necessary. Valuation model used in MC simulation may be simpler than a) the dealer's MTM model and b) the model used to simulate market variables
- ✚ Store data so that impact of a proposed new transaction can be calculated relatively easily



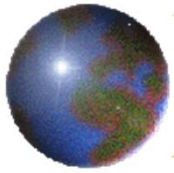
## *DVA (more controversial than CVA)*

- ✚ DVA is an estimate of the cost to the counterparty of a default by the dealer
- ✚ Same formulas apply except that  $v$  is counterparty's exposure to dealer and  $u$  is dealer's probability of default
- ✚ Value of transactions with counterparty =  
No default value – CVA + DVA

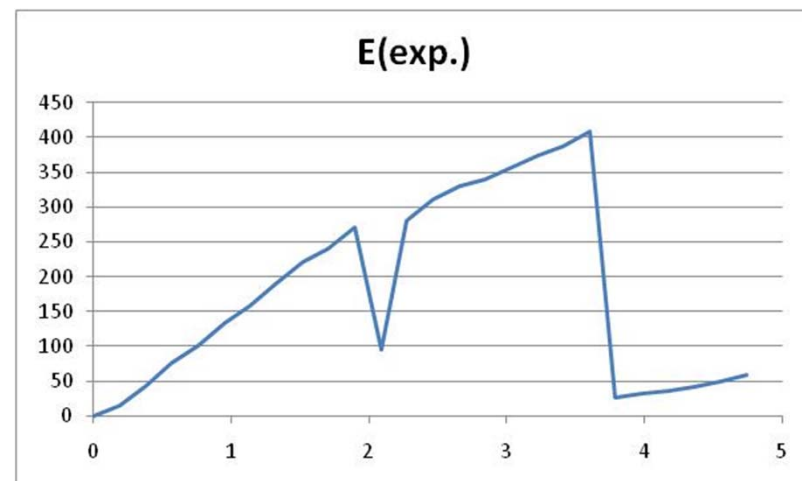
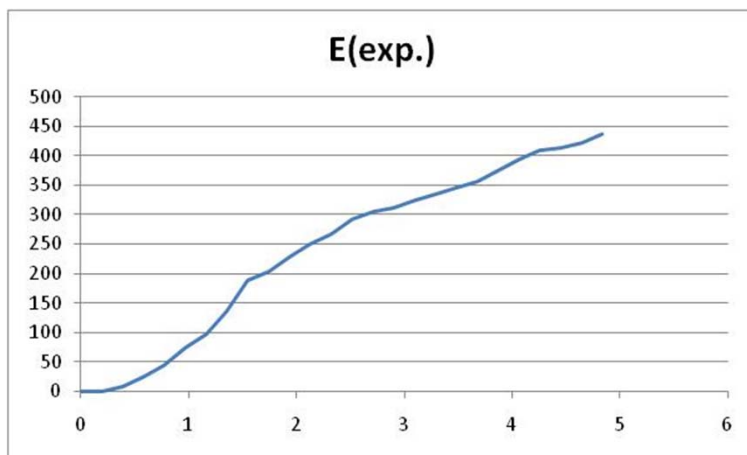
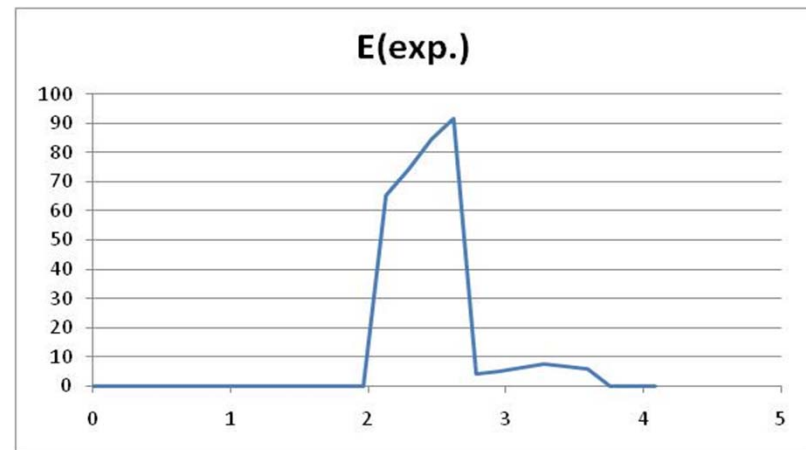
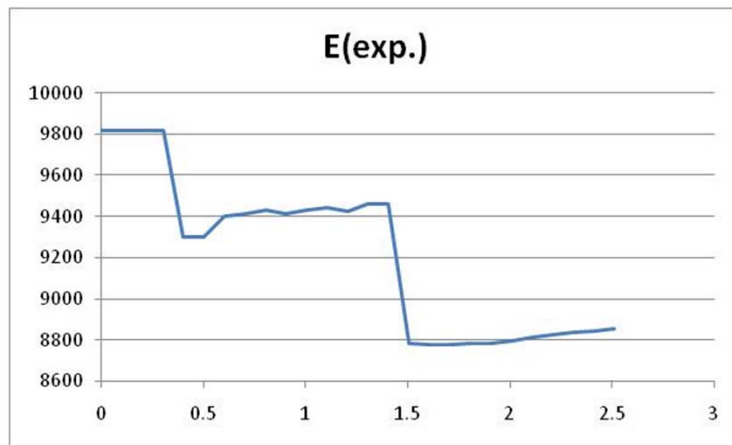


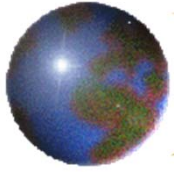
## *CVA Risk*

- ⊕ The CVA for a counterparty can be regarded as a complex derivative
- ⊕ Increasingly dealers are managing it like any other derivative
- ⊕ Two sources of risk:
  - ⊞ Changes in counterparty spreads
  - ⊞ Changes in market variables underlying the portfolio



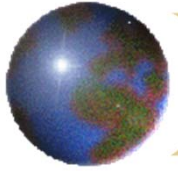
# *Profiles of Expected Exposure*





## *Basel III (2010)*

- ❖ Basel III requires CVA risk arising from a parallel shift in the term structure of counterparty credit spreads to be included in the calculation of capital for market risk
- ❖ It does not require banks to include CVA risk arising from the underlying market variables

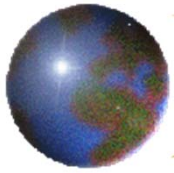


## *Calculation of CVA and CVA exposure to Spreads (Advanced Approach)*

$$\text{CVA} = (1 - R) \sum_{i=1}^n q_i v_i$$

$$q_i = \exp\left(-\frac{s_{i-1} t_{i-1}}{1 - R}\right) - \exp\left(-\frac{s_i t_i}{1 - R}\right)$$

$$\begin{aligned} \Delta(\text{CVA}) &\approx \sum_{i=1}^n \left[ t_i \exp\left(-\frac{s_i t_i}{1 - R}\right) - t_{i-1} \exp\left(-\frac{s_{i-1} t_{i-1}}{1 - R}\right) \right] v_i \Delta s \\ &+ \frac{1}{2(1 - R)} \sum_{i=1}^n \left[ t_{i-1}^2 \exp\left(-\frac{s_{i-1} t_{i-1}}{1 - R}\right) - t_i^2 \exp\left(-\frac{s_i t_i}{1 - R}\right) \right] v_i (\Delta s)^2 \end{aligned}$$

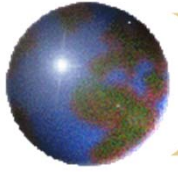


## *Wrong Way/Right Way Risk*

$$CVA = (1 - R) \int_{t=0}^T q(t) v(t) dt$$

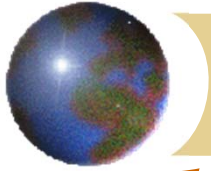
- Simplest assumption is that probability of default is independent of exposure. In other words,  $q(t)$  is independent of  $v(t)$
- Wrong-way risk occurs when  $q(t)$  is positively dependent on  $v(t)$
- Right-way risk occurs when  $q(t)$  is negatively dependent on  $v(t)$





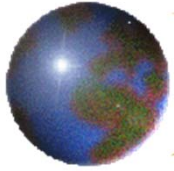
## *Examples*

- ❖ Wrong-way risk typically occurs when
  - ❑ Counterparty is selling credit protection
  - ❑ Counterparty is a hedge fund taking a big speculative positions
- ❖ Right-way risk typically occurs when
  - ❑ Counterparty is buying credit protection
  - ❑ Counterparty is partially hedging a major exposure



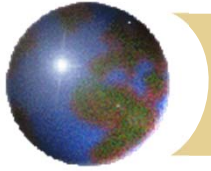
## *Problems in Estimating Wrong Way/Right Way Risk*

- ✚ Knowing trades counterparty is doing with other dealers
- ✚ Knowing how different market variables influence the fortunes of the counterparty
- ✚ Do counterparties become more likely to default when interest rates are high or low? The evidence is mixed and so we do not know whether receiving or paying fixed generates wrong way risk
- ✚ Even when there appears to be right-way risk liquidity problems can lead to a company being unable to post collateral (e.g Ashanti)



## *Allowing for Wrong-Way risk*

- ✚ One common approach is to use the “alpha” multiplier to increase the  $\nu$ 's
- ✚ Estimates of 1.07 to 1.1 for alpha obtained from banks
- ✚ Basel II sets alpha equal to 1.4 or allows banks to use their own models, with a floor of 1.2



## *Two Simple Models*

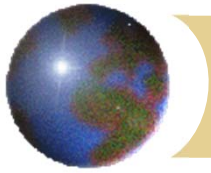
$$\ln h(t) = a(t) + bw(t) \qquad h(t) = \ln[1 + \exp(a(t) + bw(t))]$$

$h(t)$ : Hazard rate at time  $t$

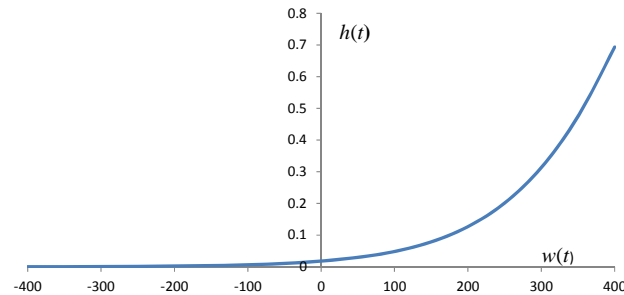
$w(t)$ : Value to dealer of outstanding derivatives at time  $t$

$b$ : Parameter measuring sensitivity of hazard rate to  $w(t)$

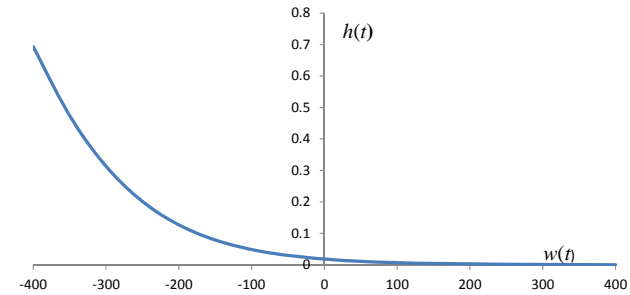
$a(t)$ : Parameter chosen to ensure that survival probabilities are matched



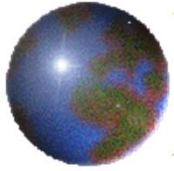
## *The Second Model $a(t)=-4$*



$$b=+0.01$$

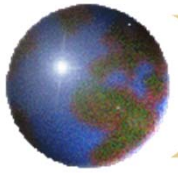


$$b=-0.01$$



## *Estimating $b$*

- ✚ Suppose current value of portfolio with counterparty is \$3 million, the counterparty's credit spread is 300 bp, and the recovery rate is 40%.
- ✚ We then know that, when  $w(0)=3$ ,  $h(0)=5\%$
- ✚ Dealer must answer a question of the form: What would happen to the spread if the value of the portfolio increased to \$20 million?
- ✚ We then have two equations in two unknowns to estimate  $a(0)$  and  $b$



# *Modifying the Monte Carlo to Incorporate WWR or RWR*

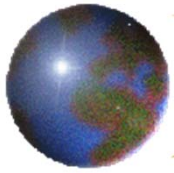
- ✚ At each time  $t_i^*$  it is necessary to do an iterative search to determine  $a(t_i^*)$
- ✚ Equations that must be solved are

$$\frac{1}{m} \sum_{j=1}^m \left[ \sum_{i=1}^k \exp(-h_{ij} \Delta t) \right] = \exp\left(-\frac{s_k t_k}{1-R}\right) \quad \text{for } 1 \leq k \leq n$$

and

$$h_{ij} = \exp\left(a(t_i^*) + bw_{ij}\right) \quad \text{or} \quad h_{ij} = \ln\left[1 + \exp\left(a(t_i^*) + bw_{ij}\right)\right]$$

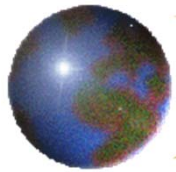
where  $m$  is the number of simulations,  $h_{ij}$  and  $w_{ij}$  are the hazard rate and portfolio value on the  $j$ th simulation trial at time  $t_i^*$



# *Wrong-Way Risk Results for Single Long Forward Contract*

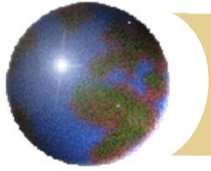
	No Collateral	$K = 10$ $c = 15$	$K = 0$ $c = 15$	$K = -5$ $c = 15$
CVA (\$ millions) for $b = 0$	0.048	0.036	0.011	0.002
Impact of $b = 0.03$ per \$mm on:				
CVA	54.8%	41.7%	37.3%	53.5%
Delta wrt Exch Rate	32.0%	15.6%	12.8%	39.3%
Gamma wrt Exch Rate	2.6%	-25.4%	17.7%	-0.7%
Delta wrt Spread	53.8%	41.2%	36.8%	52.8%
Gamma wrt Spread	181.8%	124.3%	122.8%	184.3%





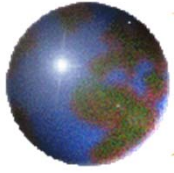
# *Wrong-Way Risk Results for Single Short Forward Contract*

	No Collateral	$K = 10$ $c = 15$	$K = 0$ $c = 15$	$K = -5$ $c = 15$
CVA (\$ millions) for $b = 0$	0.048	0.039	0.011	0.001
Impact of $b = 0.03$ per \$mm on:				
CVA	40.5%	34.0%	27.6%	28.9%
Delta wrt Exch Rate	16.2%	7.7%	-1.9%	-341.9%
Gamma wrt Exch Rate	-7.0%	-21.4%	16.4%	26.5%
Delta wrt Spread	40.0%	33.7%	27.4%	28.8%
Gamma wrt Spread	114.8%	91.0%	77.0%	70.7%



## *Further Tests*

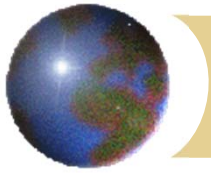
- ⊕ Generate portfolios of derivatives randomly
- ⊕ Test the impact of a non-zero  $b$  on CVA
- ⊕ Test the impact of collateralization for different thresholds and cure periods



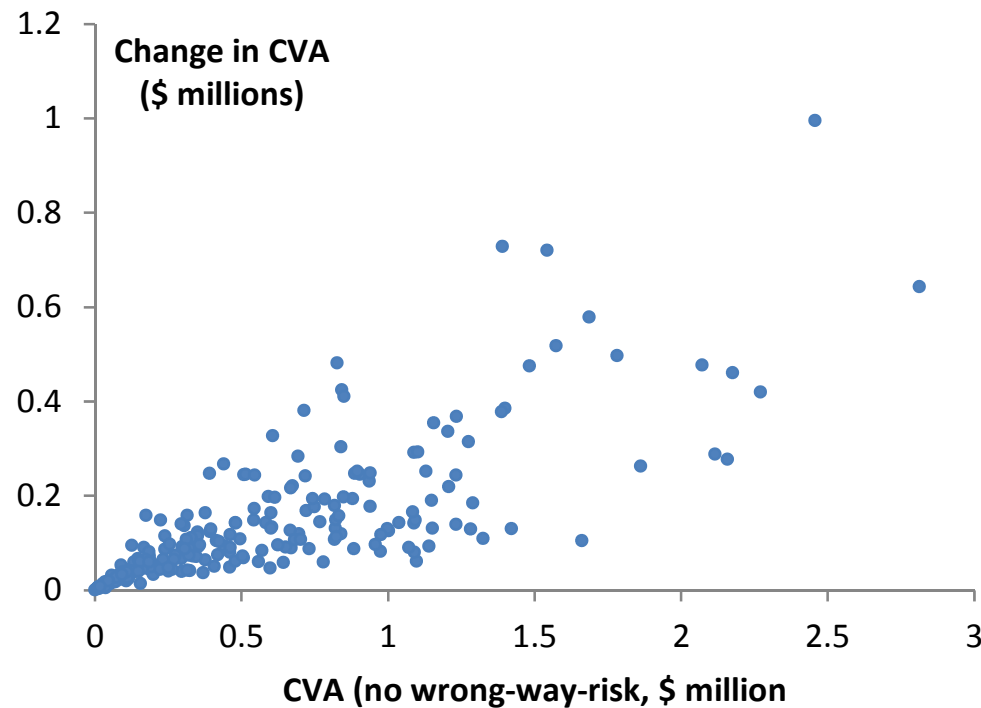
# *Experiment*

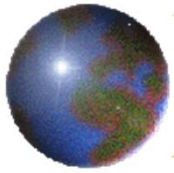
- ✚ Randomly generate 250 portfolios consisting of 25 option positions (long or short) in calls and puts on five different assets
- ✚ Principal underlying all options the same
- ✚ Time to maturity between zero and five years
- ✚ Strike price within 30% of asset price

Similar results for other experiments involving interest rate derivatives, larger portfolios and multiple underlyings



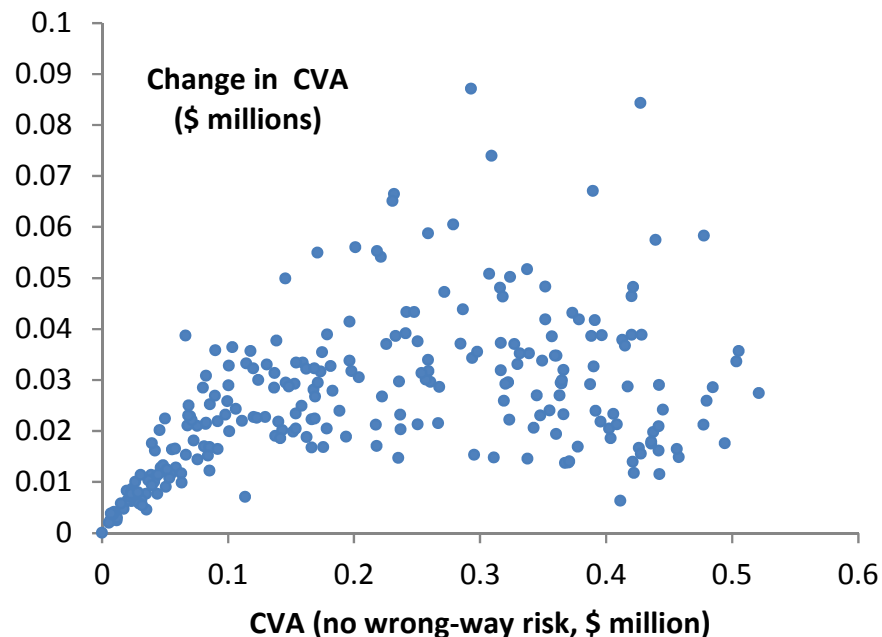
# *Impact of Wrong Way Risk (No Collateral)*

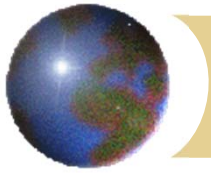




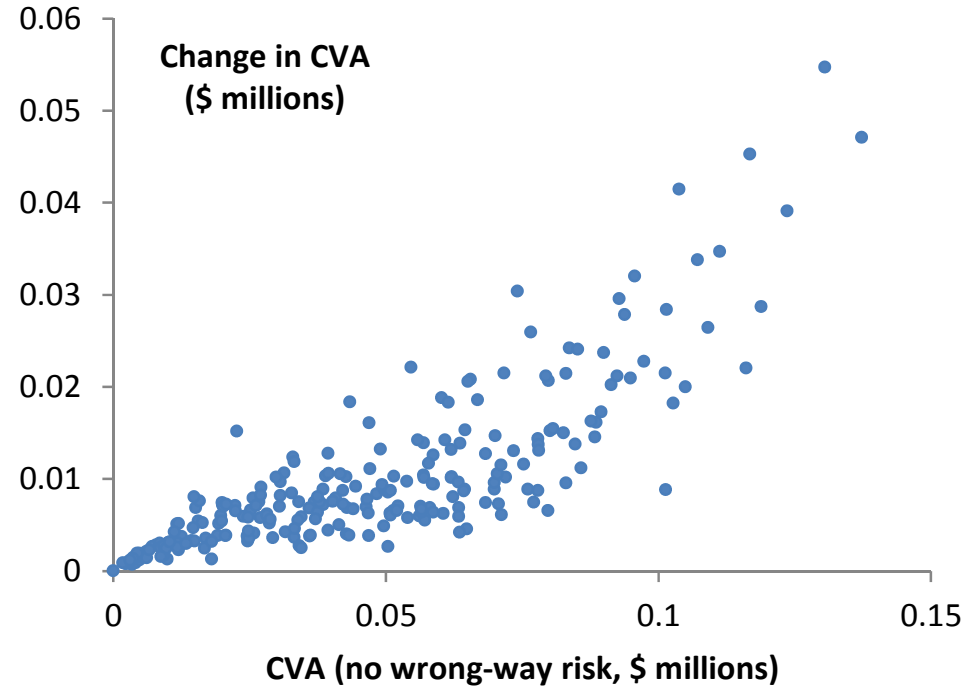
# *Impact of Wrong Way Risk*

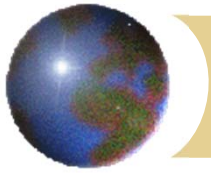
## *Threshold=10, cure period =15 days*



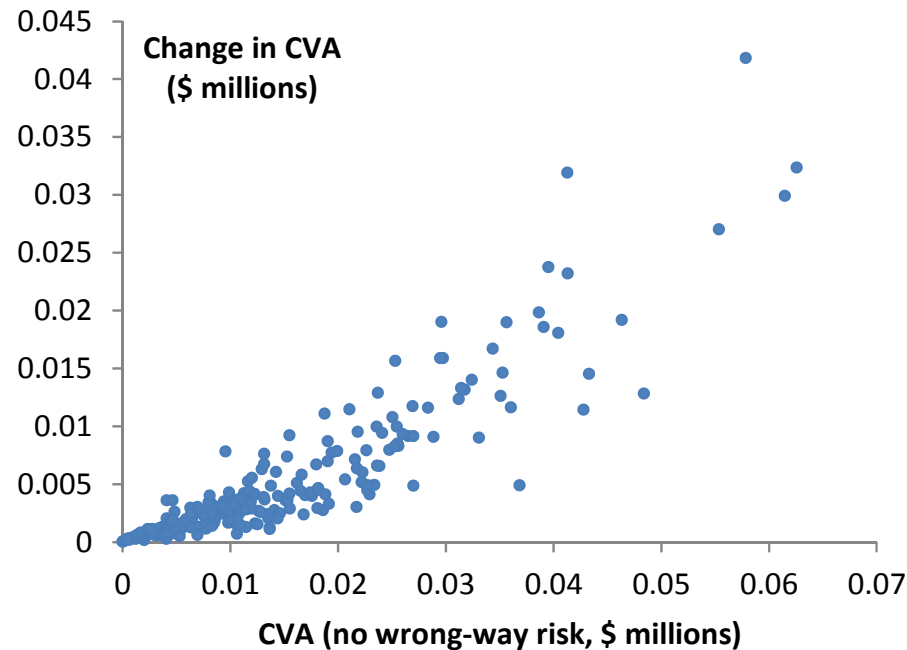


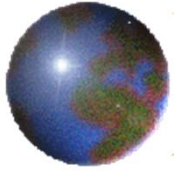
# *Impact of Wrong Way Risk Threshold=0*





# *Impact of Wrong Way Risk. Independent Amount=5*





## *Conclusions*

- ✚ We have developed a simple model for incorporating wrong-way and right-way risk into CVA calculations
- ✚ It requires a single estimate of the sensitivity of credit spread to portfolio value
- ✚ Wrong-way risk has a pronounced effect on CVA Greeks as well as on CVA itself
- ✚ Impact of wrong-way risk depends on whether there is a CSA and the terms of the CSA