Energy Storage: A problem at the intersection of Finance and Optimization
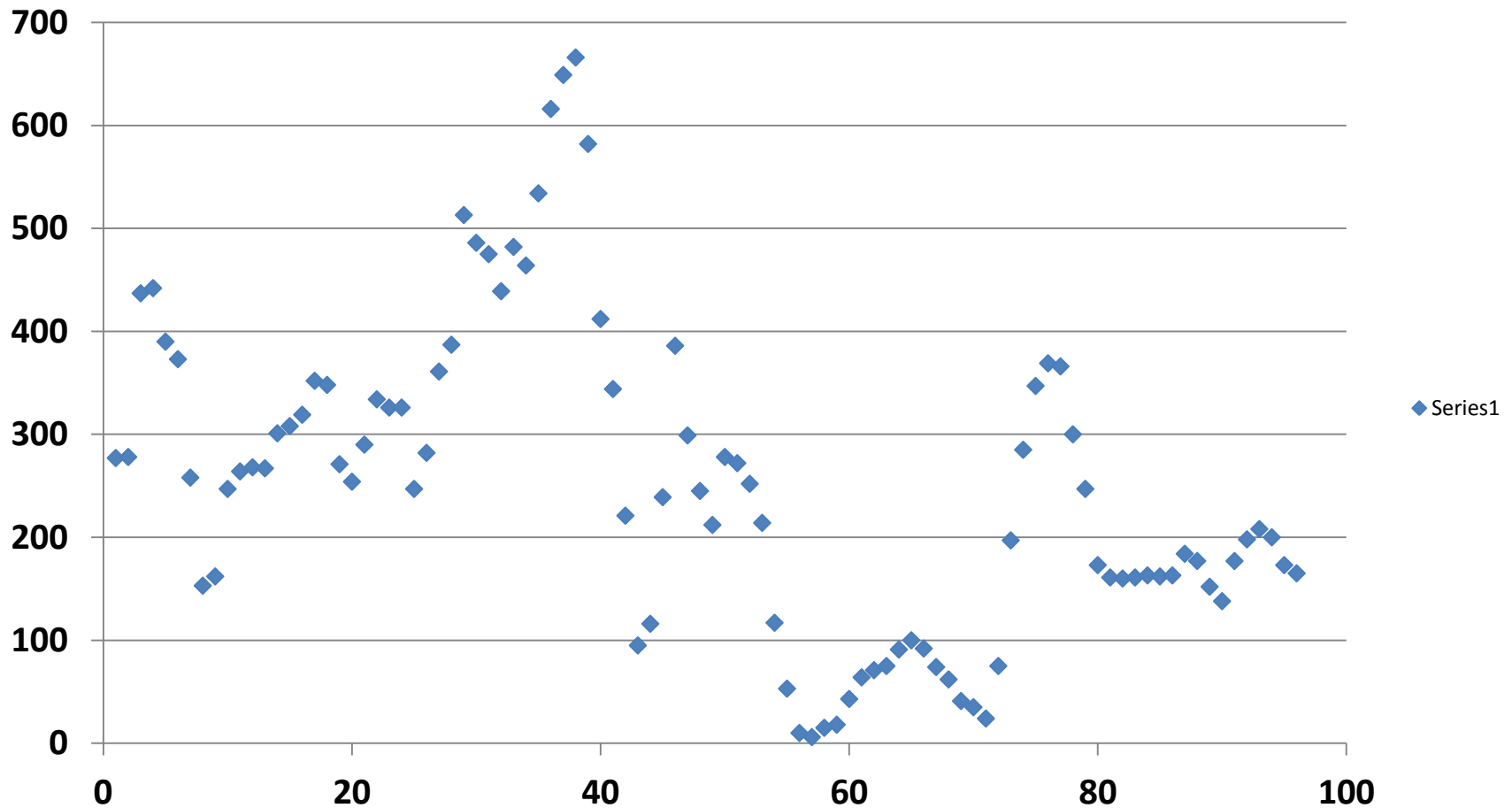
Matt Davison (Western)
Lindsay Anderson (Cornell),
Natasha Kirby (RBC)
3C Conference, Oct 28 2011
Fields Institute, Toronto.

The problem

- Renewables (Wind, Solar, small Hydro) are the cornerstone of green power initiatives both in Ontario and worldwide.

- Wind "penetration" has increased dramatically in recent years

- But wind and other renewables require expensive subsidies and do not result in dispatchable power.

Hourly Ontario wind production (MW):
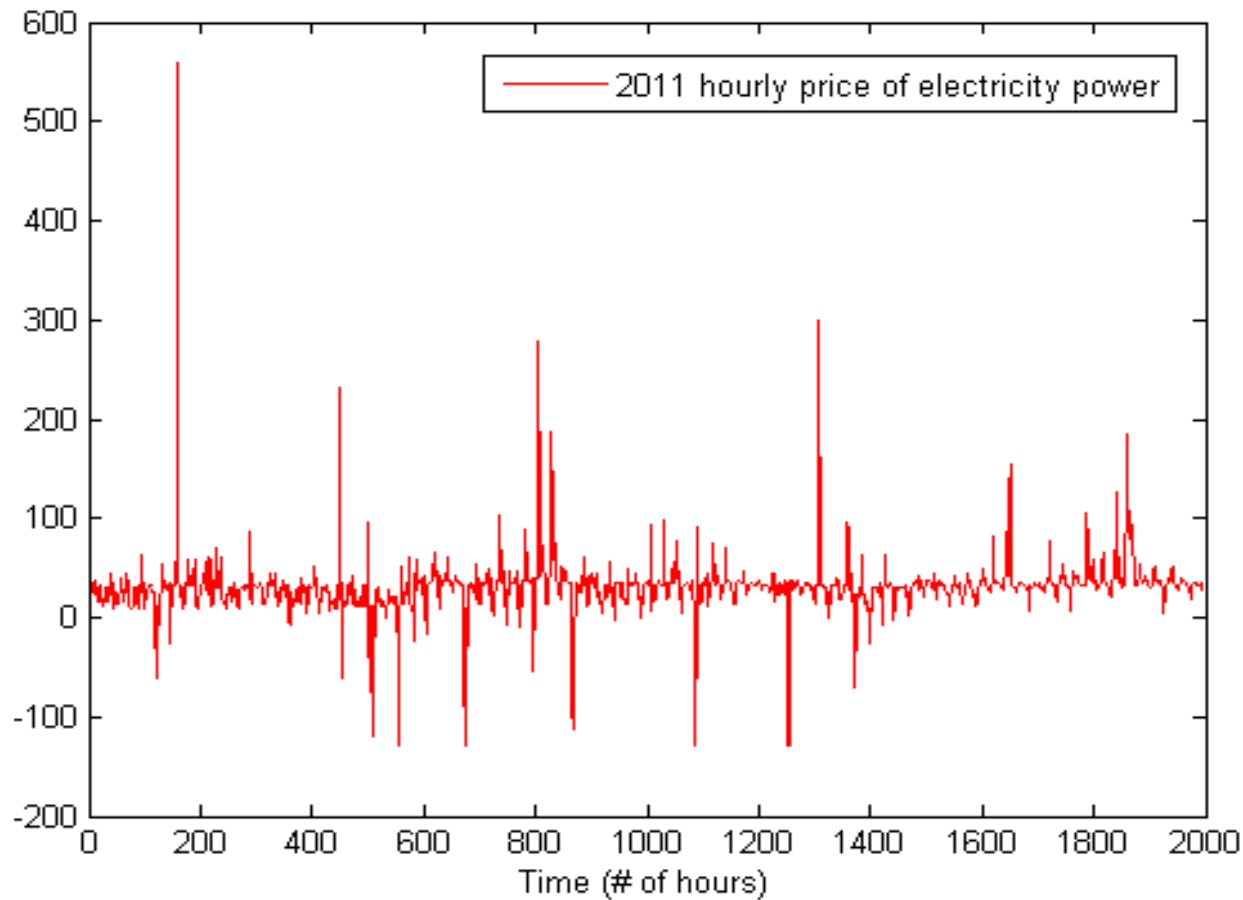May 5 – May 8 2011. Source: IESO, Melissa Mielkie

Some see impacts of this on markets

- Greater instability, even negative prices
- Wind in Ontario is given a fixed feed in Tariff and, as such, is treated as "must dispatch"
- So wind arriving in the middle of the night, in the fall, when demand is low can actually put a glut on the market (since nuclear baseload is hard to shift).
- Result is negative prices.   Validity of story uncertain, but negative prices are a fact:

Ontario Electricity Price, $/MWh

Source IESO, Ying Wang

Is the solution storage?

- Some see energy storage as the solution.

- This could smooth out uncertainties.

- (Old use for energy storage was to smooth predictable fluctuations in daily load).

But storage is expensive!

- Discuss energy storage: some quantitative examples. How much water do you need to store (and how high) to store $1000 worth of electricity?

- How many 1.5 volt AA batteries would you need? And how much would they cost? And how much would they weigh?
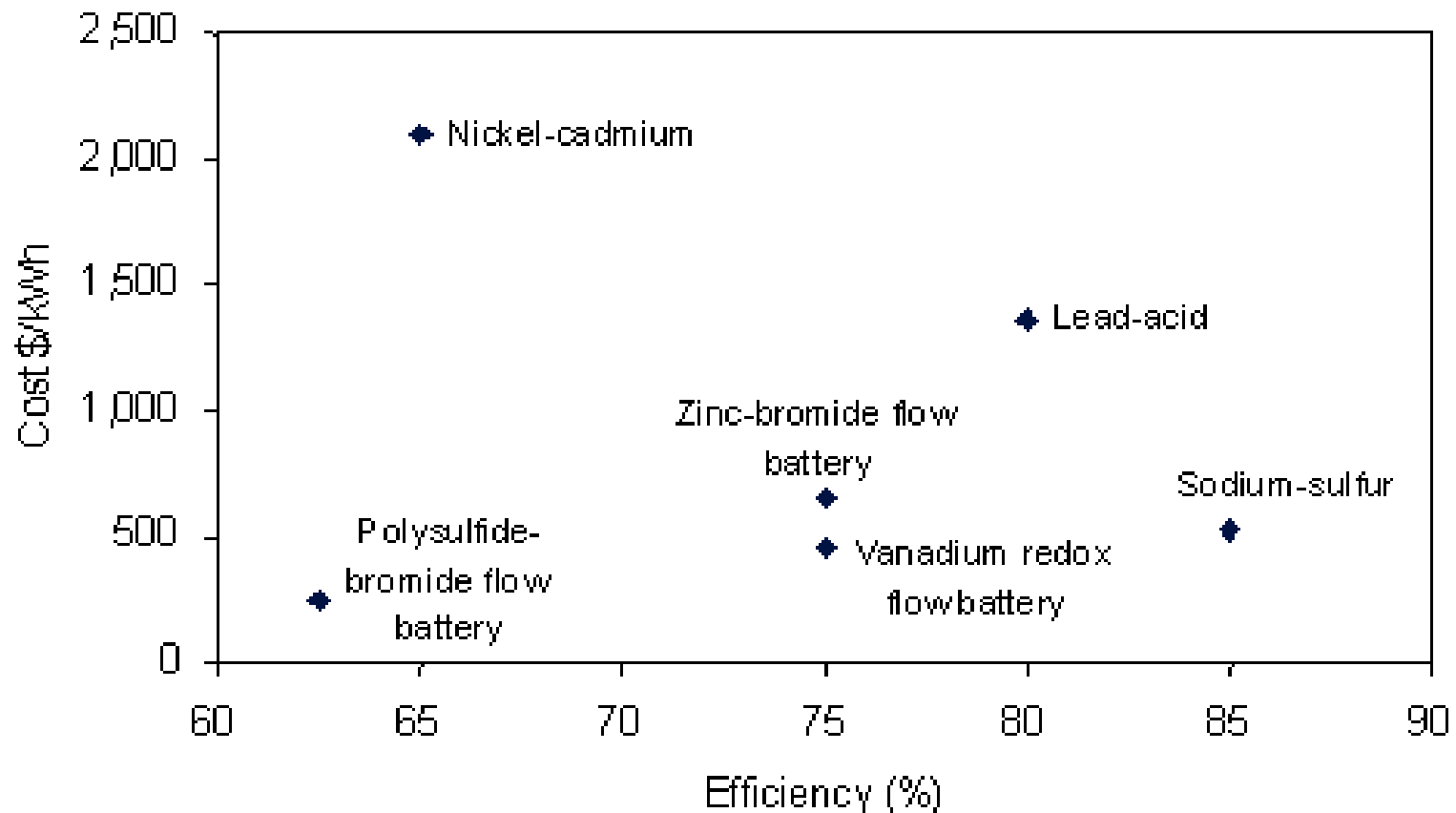
Pump storage

- 1MWh costs about $50 on the market.
  It is 3.6 billion Joules.

- If you raise a kg of water 100m you have about 1000 Joules of potential energy.

- So to get 3.6 billion J of potential energy you need to raise 3.6 million kg of water 100m.

- That's 3600 cubic metres, or a 60 m x 60m x 1 m wading pool hauled up a mountain.

- The hardware to do this will be expensive!

AA batteries

- A Nickel-Cadmium AA battery contains 1.2W-h of electricity, weighs about 30g and costs about $0.50.

- To store a Megawatt hour in AA batteries would take about 800,000 of them, for a total weight of 24,000 kg and cost about $400K.

- Maybe you'd be better off using them as a working fluid for a pump storage unit!

- The goal for AA cells is different; next slide shows cost of industrial size battery storage units.

Battery costs vs. efficiencies

(Source: The Future of Energy Storage, Global Business Insights)

Who pays for storage?

- In current Ontario setting, not the wind or solar producers

- The cost of uncertainty is another cost of running green markets.

- In fact, in Ontario if you did build a PSR facility you'd have to buy and sell at the open market price: No Feed In Tariff

My existing storage work

- I've worked a fair bit on Energy storage, with papers on Pump Storage (Thompson, MD, Rasmussen *Operations Research* 2004, Zhao & MD *Renewable Energy* 2009, *J. Hydrology* 2009), Natural Gas Storage (Thompson MD & Rasmussen *Naval Research Logistics* 2009) and Compressed Air Energy Storage (Carriveau, Ting, Konrad, MD, Simpson) *Renewable Energy Reviews*, under review)

A new tack

- All my work works on building realistic models of the physics and engineering of various facilities, getting some kind of stochastic model for prices or water inflows, turning it all into a big PDE or stochastic program, and generating numerical work after some numerical analysis.

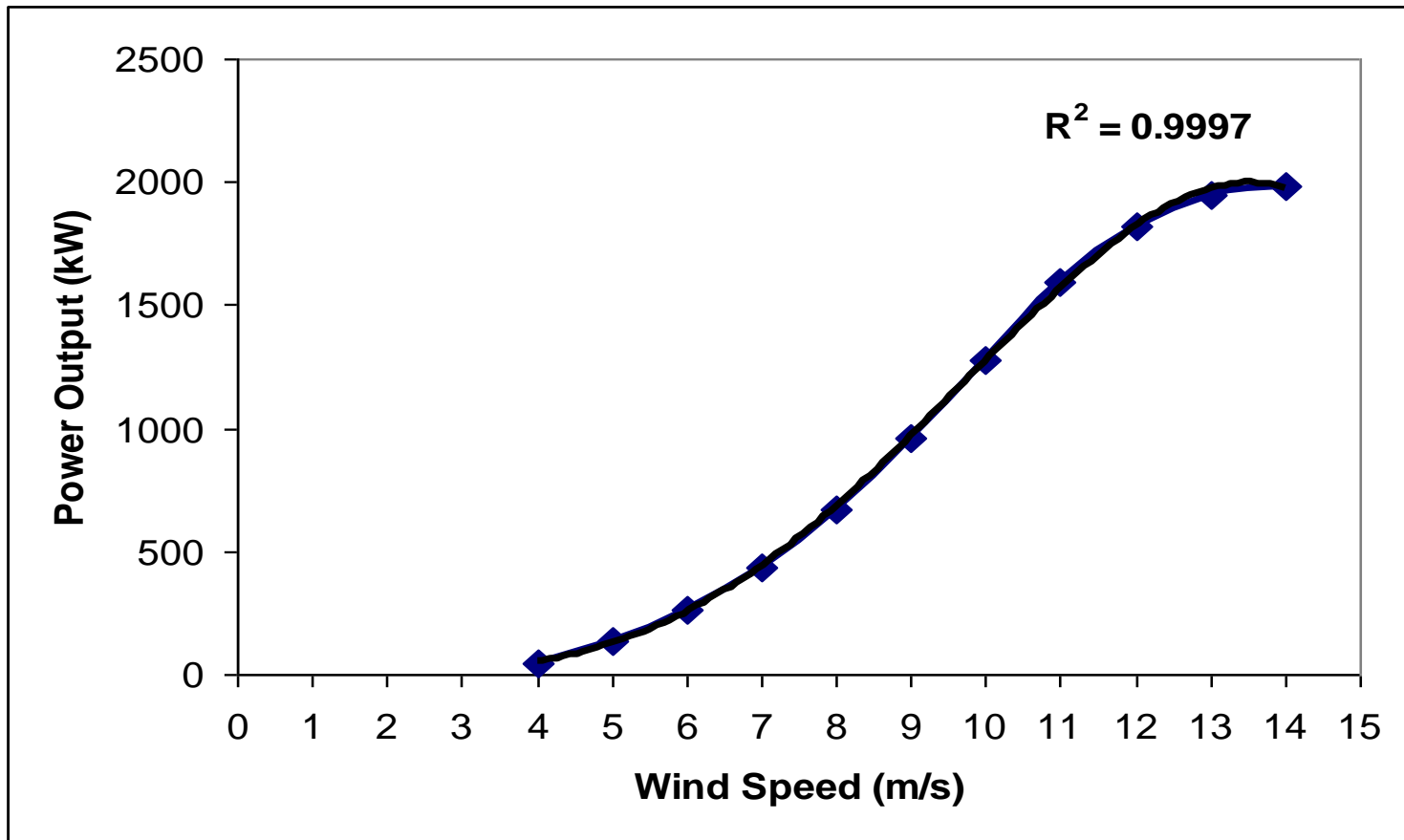- All this work ignores regulation entirely.

Simple model

- Today I present a very simple model of a storage facility with just 4 parameters: the value of a unit of power, the fractional loss in storing the power, the probability of generating the power, and the penalty resulting in bidding power into the market that isn't delivered.

- The model generates a nonlinear system of difference equations that can be solved in closed form!

- This closed form solution allows us to obtain many insights.

Wind "physics"

- A wind producer either produces M electricity if it is windy, otherwise none.

- Each hour it is windy with probability p and calm with probability 1-p, $0 \leq p \leq 1$.

- Bids for each hour must be made without information and can either be to deliver a unit of power or to deliver nothing.

Power curve

Model bidding

- The wind producer must decide in advance whether to offer power into the market.

- If power is offered and it is windy, producer gets $M.

- If power is not offered it cannot be sold, whether or not it is windy.

Penalties

- If the producer bids and it is not windy she must (without storage) pay a penalty of $-xM$

- The penalty satisfies $x \geq 0$

- Current Ontario market practice is for $x = 0$; current New York market practice has $x > 0$.

Storage physics

- The wind producer has access to a storage facility allowing them to store a single unit of wind energy. This storage can be filled or withdrawn in a single hour.

- Storage is "lossy" and we assess the cost of this loss at withdrawal. If a unit of energy is withdrawn it earns $(1-\gamma)M$, $0 \leq \gamma \leq 1$.

Storage contracts

- Storage facilities are leased for a period of N hours.

- At the end of the lease period they are returned to the owner.   A full facility gets a cash refund of $(1-\gamma)M$, and empty facility gets nothing.

V(F,k) and V(E,k)

- The value of a full storage facility, assuming optimal operation, with k periods left before the end of the lease is denoted by V(F,k)

- The value of an empty storage facility, assuming optimal operation, with k periods left before the end of the lease is V(E,k)

V(F,B,k) and V(F,N,k)

- With k periods remaining we must decide whether to bid in (offer) power or not.

- The value of a full facility with k periods remaining given that we bid in is V(F,B,k)

- If we don't offer power the value of the full facility with k periods remaining is V(F,N,k).

- Clearly V(F,k) = max[V(F,B,k),V(F,N,k)]

V(E,B,k) and V(E,N,k)

- The value of an empty facility with k periods remaining given that we bid in is V(E,B,k)

- If we don't offer power the value of the full facility with k periods remaining is V(E,N,k).

- V(E,k) = max[V(E,B,k),V(E,N,k)]

The recursion relation: empty

- We use dynamic programming to solve this.
- We've already 'turned around' time by describing everything in terms of time remaining.
- V(E,B,k) = p[M+V(E,k-1)] + (1-p)[-xM+V(E,k-1)]
- V(E,N,k) = pV(F,k-1) + (1-p)V(E,k-1)
- Since if you don't bid and it's empty, you might as well fill the facility to sell later.

The recursion relation: full

- V(F,N,k) = pV(F,k-1) + (1-p)V(F,k-1) = V(F,k-1).

- The V(F,B,k) case is a bit harder because if we bid and it's not windy we can choose whether to pay the penalty or empty the storage. Hence:

- V(F,B,k) = p[M + V(F,k-1)]

$$+(1-p)\max[-xM+V(F,k-1), (1-\gamma)M+V(E,k-1)]$$

A note on expectations

- It probably makes sense to optimize the expected value of the cash flows as done above since the procedure will be repeated many times.

- If, however, you want to add risk aversion via for instance a utility, that will only have the effect of distorting the probability, so replace p by q and the structure of the equations remains.

System 0

$V(F,B,k) = p[M+V(F,k-1)]+(1-p)\max[-xM+V(F,k-1), (1-\gamma)M+V(E,k-1)]$

$V(F,N,k) = V(F,k-1)$

$V(F,k) = \max[V(F,B,k),V(F,N,k)]$

$V(E,B,k) = p[M+V(E,k-1)] + (1-p)[-xM+V(E,k-1)]$

$V(E,N,k) = pV(F,k-1) + (1-p)V(E,k-1)$

$V(E,k) = \max[V(E,B,k),V(E,N,k)]$

$\quad V(F,0) = (1-\gamma)M$

$\quad V(E,0) = 0.$

Solving this

- Solution of this system requires at each time:
- **Optimal bidding rules**, when empty and when full, before we know if the wind will blow or not.

- The **optimal decision** about whether to pay the **penalty or empty** the storage in the full,bid,no wind case.

- Expressions for **V(F,k) and V(E,k).**

Analytic solution

- We find a complete analytic solution for this nonlinear system of difference equations.

- We could, of course, have coded this system without any difficulty (even in a spreadsheet!), but analytic solutions still are nice.

- I will now describe the solution and draw lots of insights about the wind storage problem from it.

Theorem 1: Solving for V(F,k)-V(E,k)

Theorem 1:

For the above system of difference equations,
V(F,k)-V(E,k) = (1- γ)M + min[pγM, (1-p^k)xM]

The following preliminary lemma is useful in this
proof:

Lemma 2:

Let $V(F,k) - V(E,k) = Mm(k) + (1-\gamma)M$. Then

$m(k) = m(k-1) + \min[x(1-p), \gamma p - pm(k-1)] - \min[p, x(1-p), (1-p)m(k-1)]$;

$m(0) = 0$.

Proof of Lemma 2:

- Subtract V(F,k)-V(E,k) in System 0, repeatedly using facts like max(a+b,a+c) = a + max(b,c) and max(-a,-b) = -min(a,b) and easy but tedious algebra to obtain the first order difference equation:

Proof of Theorem 1:

Theorem 1 <-> $m(k) = \min[\gamma p, \ x(1-p^k)]$ solves the Lemma 2 system:

$m(k) = m(k-1) + \min[x(1-p), \gamma p - pm(k-1)] - \min[p, x(1-p), (1-p)m(k-1)]$; $m(0) = 0$.

Proof: divide into 4 cases and use induction:
  i) $x \geq \gamma * \max(1, p/(1-p)$
  ii) $\gamma \leq x < \max(1, p/(1-p)$,
  iii) $\gamma p \leq x < \gamma$
  iv) $0 \leq x < \gamma p$

Discussion

- Note that V(F,k)-V(E,k) is nondecreasing in time remaining, just like an option.

- If x ≤ γp, V(F,k)-V(E,k) is always increasing with a limit of (1-γ+x)M,

- If x > γp, V(F,k)-V(E,k) increases until k = k* and then reaches a limit of [1-γ(1-p)]M at the finite time k*.

- k* is the largest integer satisfying

    k* < [ln(x-γp)-ln(x)]/ln(p)

Getting to the optimal control

Theorem 1 is the backbone allowing all the other results to come easily.

Corollary 3:$-xM + V(F,k) \leq (1-\gamma)M+V(E,k)$ for all k.

Proof: $-xM \leq (1-\gamma)M -[V(F,k)- V(E,k)]$

 $-xM \leq - \min[p\gamma M, (1-p^k)xM]$, (Thm 1) or

$x \geq \min[\gamma p, x(1-p^k)]$ which is clear.

Never optimal to pay penalty

- Corollary 3 implies that, if you bid and the wind doesn't blow, it is never optimal to pay the penalty but always better to empty the full storage. This is true even for tiny penalties, or for no penalties at all.

- It also says we can simplify System 0 a bit by replacing the expression for V(F,B,k) with:
V(F,B,k) = p[M+V(F,k-1)]+(1-p)[(1-γ)M +V(E,k-1)

You should always bid when full

Corollary 4:   V(F,B,k) >= V(F,N,k) for all k.

Proof: From the above slide and System 0:

V(F,B,k)-V(F,N,k) = p[M+V(F,k-1)]+(1-p)[(1-γ)M
    +V(E,k-1) − V(F,k-1)]
    = pM + (1-p)){(1-γ)M − [V(F,k-1) − V(E,k-1)]}

Using Theorem 1, = pM − (1-p)min[pγM,x(1-p^{k-1})M]

- (1-p)M*max[p/(1-p) - pγ,-x(1-p^{k-1})]
    = M*max[p(1-γ(1-p), -x(1-p) (1-p^{k-1}) >= 0,
    since γ*(1-p) < 1.

Bidding rules when empty

$V(E,B,k) - V(E,Nk) = p[M+V(E,k-1)] + (1-p)[-xM+V(E,k-1)] - \{pV(F,k-1) + (1-p)V(E,k-1)\}$

$= pM - p[V(F,k-1)-V(E,k-1)] - (1-p)xM$

$= [p-(1-p)x]M - p[(1-\gamma)M + \min\{\gamma p, x(1-p^{k-1})\}M]$

$= [\gamma p - (1-p)x]M - pM*\min\{\gamma p, x(1-p^{k-1})\}$

$= [\gamma p - (1-p)x]M + pM*\max\{-\gamma p, -x(1-p^{k-1})\}$

$= \max\{\gamma p - (1-p)x, \gamma p - x(1-p^k)\}M$. But, unless $k = 0$ in which case we can't bid anyway, $1-p \geq 1-p^k$,

Hence $V(E,B,k) - V(E,N,k) = [\gamma p - x(1-p^k)]M$

Empty bid rules: Large penalties

- $V(E,B,k) - V(E,N,k) = [\gamma p - x(1-p^k)]M$
- Large penalty: $x >= \gamma * \max[1, p/(1-p)]$
- Then, if $p < \frac{1}{2}$, $x(1-p^k) > x(1-p) > px > \gamma p$ and the expression is negative. If $p > \frac{1}{2}$ $x(1-p^k) > x(1-p) > \gamma p$ and the expression is still negative.
- so $V(E,B,k) - V(E,N,k) < 0$
- and so it's optimal not to bid.

Optimal control: large penalties

- If x ≥ γ*max[1,p/(1-p)] then the optimal control is to bid when full and not bid when empty. That way you never have to pay penalties and you refill the first time it's windy after a calm day.

- Note that sufficiently huge penalties are never collected!

- Here V(F,k)-V(E,k) = [1-γ(1-p)]M

Empty bid rules: Small penalties

- $V(E,B,k) - V(E,N,k) = [\gamma p - x(1-p^k)]M$
- Small penalty: $x \leq p\gamma$
- Then $x(1-p^k) < x \leq p\gamma$
- so $V(E,B,k) - V(E,N,k) > 0$
- and so it's always optimal to bid.
- Here $V(F,k)-V(E,k) = [1-\gamma p + x(1-p^k)]M$

Optimal control: small penalties

- With small penalties you always bid whether you are full or empty.   The effect of this is that if you start empty you never fill the storage, and if you start full you only use the storage once, to empty it.

- So the penalties are too small to encourage use of the storage, even though it looks like you are using the storage when it's full.

Empty bid rules: medium penalties

- $V(E,B,k) - V(E,N,k) = [\gamma p - x(1-p^k)]M$

- Medium penalty: $\gamma p < x < \gamma \max[1,p/(1-p)]$

- Here $\gamma p - x(1-p^k)$ is positive (when $k < k*$) or negative (when $k \geq k*$).

- Here $k*$ is the largest integer satisfying

-   $k* < \ln[1-\gamma p/x]/\ln(p)$

- So you don't bid (sufficiently far from maturity) and then bid (sufficiently close to maturity)

Optimal control: medium penalties

- The thinking here is that, with a sufficiently small amount of time left, you might be able to "get away" with bidding even when empty, in the expectation of never having to pay a penalty.

- $x(1-p^k)$ is the expected proportional penalty paid with k time steps remaining.

- Eventually it's better to play it safe and bid, with proportional loss of γ incurred with probability p.

- Hence we compare γp and $x(1-p^k)$.

Facility values: large penalties

- Here the equations are:
  $V(F,k) = V(F,B,k)$
  $= p[M+V(F,k-1)]+(1-p)[(1-\gamma)M+V(E,k-1)]$

- $V(E,k) = V(E,N,k) = pV(F,k-1) + (1-p)V(E,k-1)$

- So $V(E,k) = V(E,k-1) + p[V(F,k-1)-V(E,k-1)]$

- $V(E,0) = 0$ and $V(F,k)-V(E,k) = [1-\gamma(1-p)]M$, so

- **$V(E,k) = kp[1-\gamma(1-p)]M$ (k ≥ 0).**

- **$V(F,k) = (kp+1)[1-\gamma(1-p)]M$ (k ≥ 1).**

Facility Values: Small penalties

- Here V(F,k) = V(F,B,k) and V(E,k) = V(E,B,k) so the recursion relations are:
- V(F,k) = p[M+V(F,k-1)]+(1-p)[(1-γ)M+V(E,k-1)]
- V(E,k) = p[M+V(E,k-1)] + (1-p)[-xM+V(E,k-1)] or
- V(E,k) = V(E,k-1) + [p-x(1-p)]M
- Or **V(E,k) = k[p-x(1-p)]M**
- **V(F,k) = k[p-x(1-p)]M + (1-γ)M + x(1-p^k)M.**

Facility Values: Medium penalties

- When k < k* it's as if the penalties were small, so V(E,k) = k[p-x(1-p)]M, k < k*
- When k > k* the penalties are now large, so we can solve the large penalty difference equation with the "initial condition"
V(E,k*) = k*[p-x(1-p)]M
- V(E,k) = V(E,k-1) + p[V(F,k-1)-V(E,k-1)], where for k > k*, V(F,k-1)-V(E,k-1) = [1-γ(1-p)]M, so
- **V(E,k) = kpM –k*x(1-p)M –(k-k*) γ(1-p)M, k ≥ k*.**
- **k*: largest int satisfying k* < ln[1-γp/x]/ln(p)**

Impact on storage values

- So far the analysis has only told us what to do if we were given a storage facility.

- In this light it's not so surprising that we'd choose to empty a full facility rather than pay penalties.

- But what if we had to rent a storage facility? Would it be worth it?

- We need to compare with a turbine operated without a companion storage.

Compare with turbine with no storage

- Consider a turbine with no storage. W(k) is the value of this turbine with k periods remaining.
- W(k) follows the difference equation:
- W(k) = max[W(B,k), W(N,k)]
- W(B,k) = p[M + W(k-1)] + (1-p)[-xM + W(k-1)]
- W(N,K)= pW(k-1) + (1-p)W(k-1) = W(k-1)
- So W(k)=W(k-1)+max[p-x(1-p),0]*M; W(0) = 0.
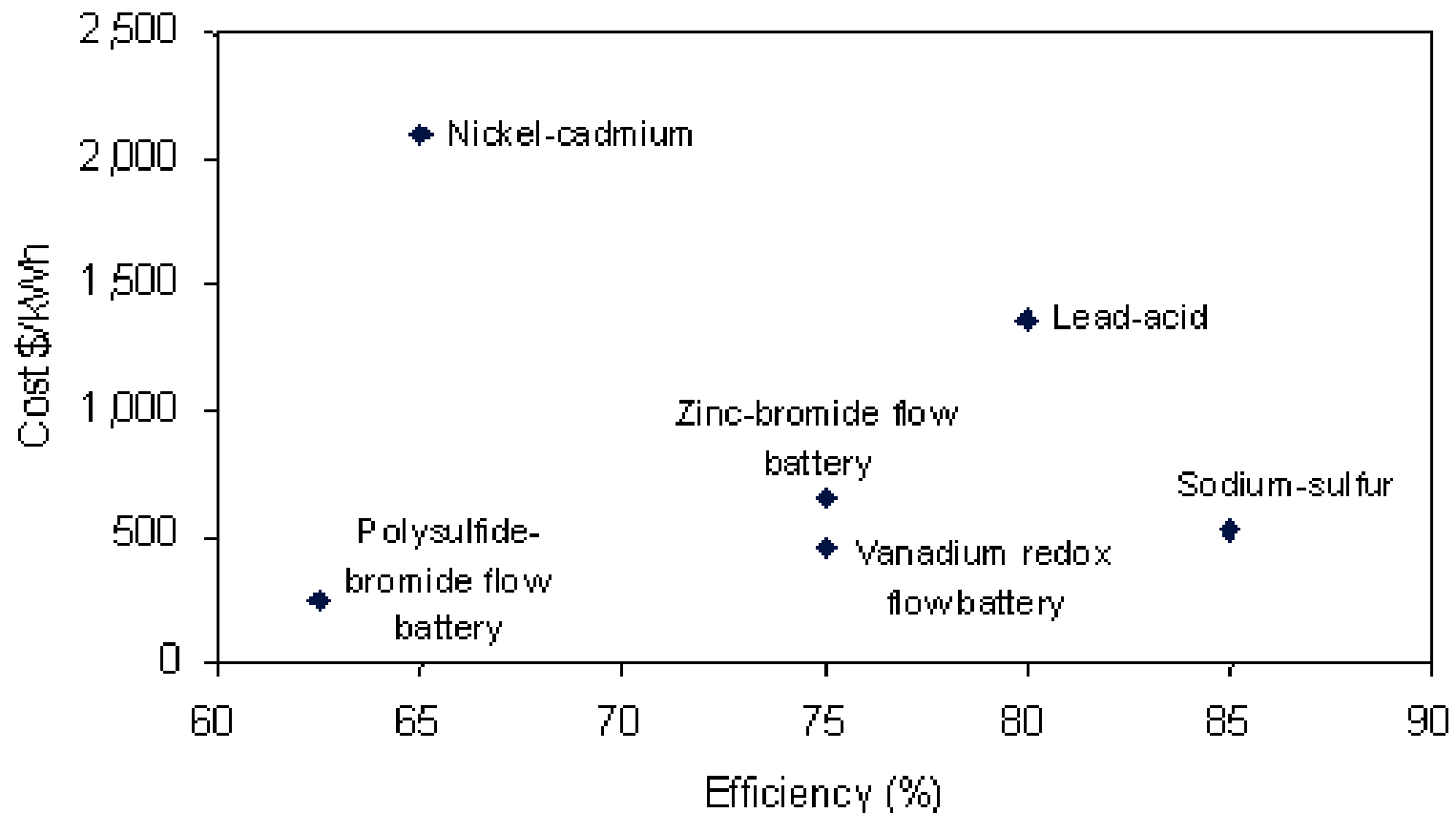- So **W(k) = kM*max[p-x(1-p),0].**

No storage wind turbine: controls

- If x < p/(1-p) the fines are small enough to make it worthwhile to operate, and you will always bid, and have W(k) = kM*[p-x(1-p)].

- If x > p/(1-p) the fines are large enough for the best policy be never to bid, with W(k) = 0.

- These are the correct "comparator" values for the combined wind– storage facility.

Added value of high penalty storage

- Here base case is to have no value from wind, so penalty is equivalent to a law requiring wind turbine operators to operate storage (or other backup) facility.

- The added value from the storage, for a N period facility, is $Np[1-\gamma(1-p)]M$

- Note this value doesn't depend on the value of x (once it's big enough).

- It says that the more wind the better, the longer the facility life the better, and the more efficient the facility the better.
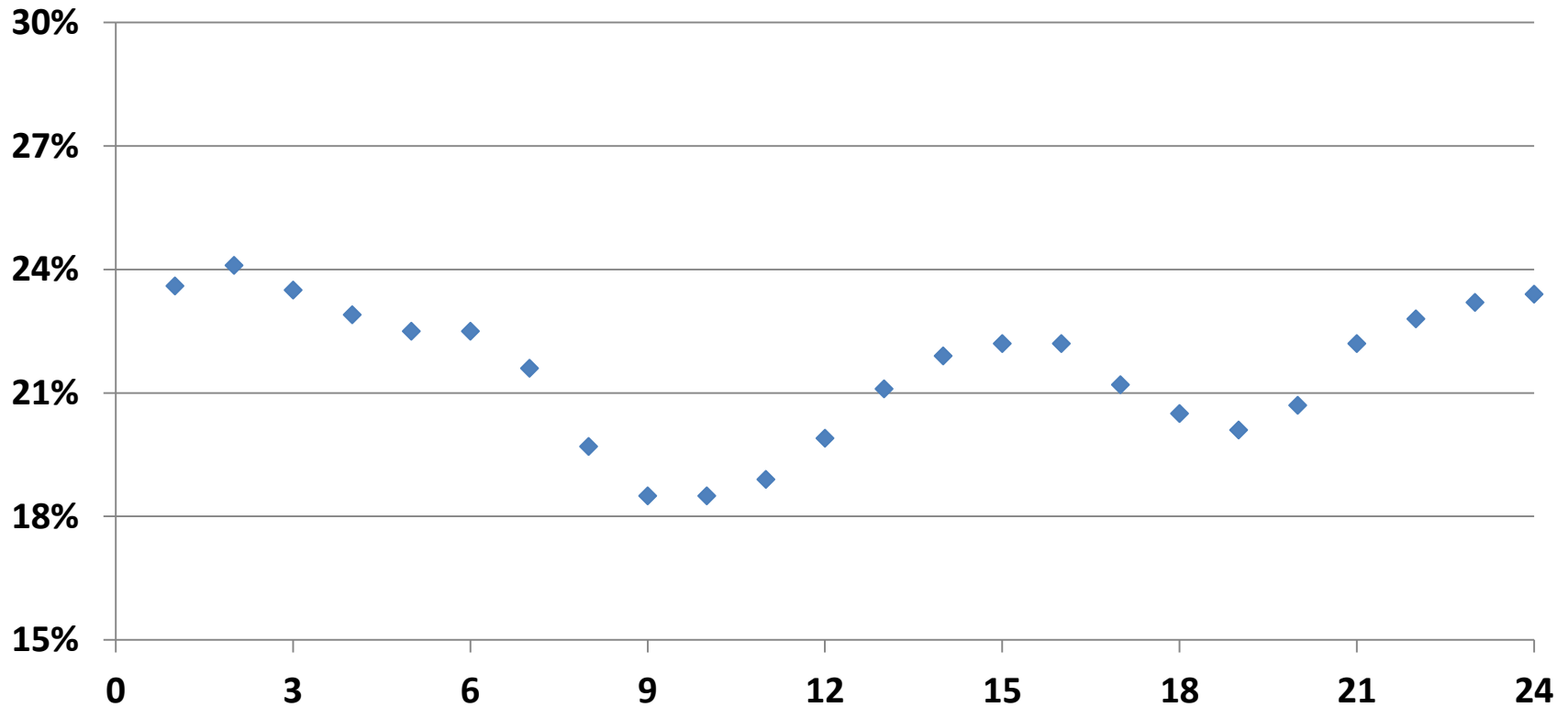
Battery costs vs. efficiencies

Estimating p

- We have access to the total production of wind in Ontario at each time and to the total availability of wind turbines at each time.

- If our model were correct for each turbine, we'd expect the long run average of this ratio to be the probability of full output.

- Next slide shows the data

- Yields (slightly conservative) estimate p = 20%

Estimating p (data)

Ontario Wind: Generated/Available
May 5 - Oct 11 2011. Source: IESO

Value: High penalty cost

- Take M = $140 (i.e. 1MW turbine).
- Take γ = 15% (Sodium-Sulfur battery)
- Take p = 20%.
- Take N = 8760 hours
- Then the value of the storage is about $215,000 per MWh (per year).
- Cost is about $500,000 but lasts for a number of years. So storage is "in the conversation".

Added value of low penalty storage

- Here $x < \gamma p$, so $x < p$ so $x < p/(1-p)$ and the base case is the "always run" no storage facility with value $W(k) = kM*[p-x(1-p)]$.

- Here also, though $V(E,k) = k[p-x(1-p)]M$, so the additional value of the storage is zero (since it doesn't change bidding behaviour).

- On the other hand, in this regime we'd expect the regulator to collect on average $N*(1-p)xM$ in penalties, which could be used to defray the costs of storage. With $N = 8760$, $p = 20\%$, $M = \$140$ and $x = \gamma p = 3\%$, this is about $\$30,000$.

Conclusions

- A simple model can be exactly solved and shows some interesting intuition.

- Of course this is way too unrealistic for reality – we need correlated wind speeds, storage with ability to store fractional units and seasonality, at the very least!

- Our key focus now is adding simple weather forecast models to this, but there wasn't time to share these preliminary results today.

- It's fun to see how much insight we can get without a lot of computing.