How to mitigate the risk of blowing up and the cost of being too cautious?

Endre Boros

RUTCOR, Rutgers University

Joint work with L. Fedzhora, N. Goldberg, P.B. Kantor, K. Saeger, P. Stroud and J. Word

Outline

What is RISK?

- *To preserve the distinction ... between the measurable uncertainty and the unmeasurable one we may use the term "risk" to designate the former and the term "uncertainty" for the latter.* (Frank Knight, 1921)
- *The concepts "yield" and "risk" appear frequently in financial writings. Usually if the term "yield" were replaced by "expected yield" or "expected return," and "risk" by "variance of return," little change of apparent meaning would result.* (Harry Markowitz, 1952)
- **DHS Risk Lexicon (2008):**
    - **Risk**: *potential for an unwanted outcome resulting from an incident, event, or occurrence, as determined by its likelihood and the associated consequences*
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- A procedure, call it a **sensor**, yielding a **score** of **riskiness** (say on a scale of 0-100).
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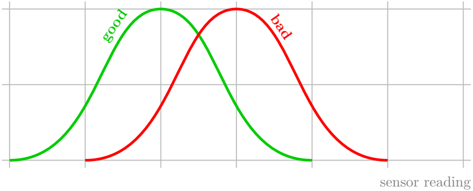

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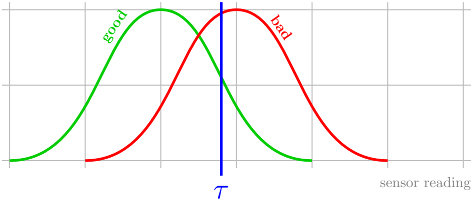

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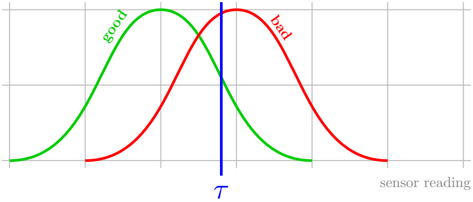

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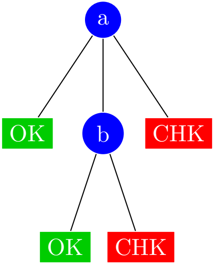

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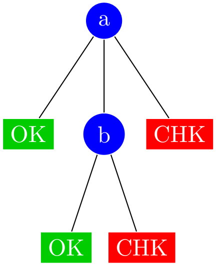

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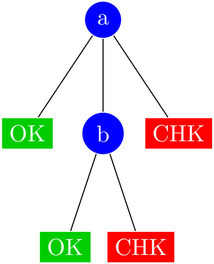

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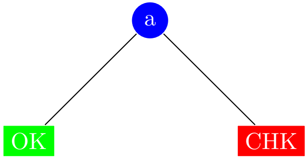
A small example involving two sensors



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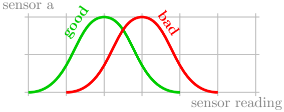

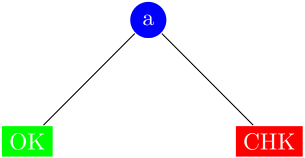

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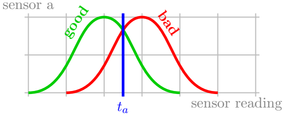

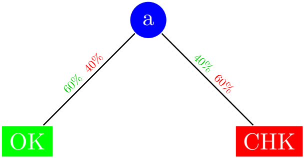

Inspection cost	$\mathbf{0.4C_{CHK}}$ $\mathbf{+C_a}$
Detection rate	**60%**

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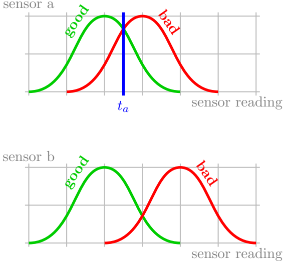

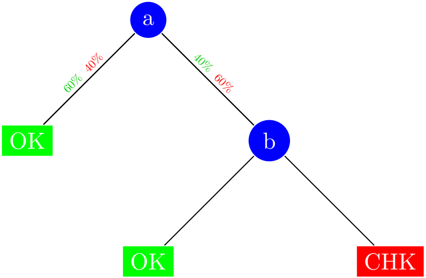

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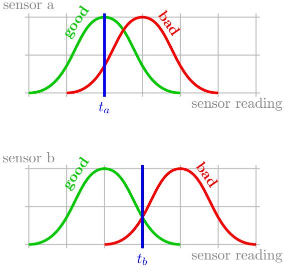

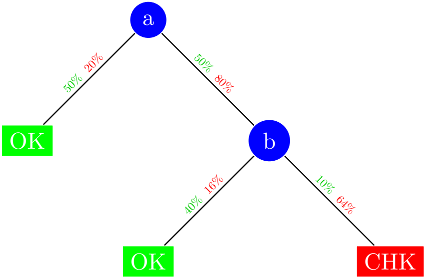

Inspection cost	$0.4C_{CHK}$ $+C_a$	**$0.1C_{CHK}$ $+C_a$ $+0.5C_b$**
Detection rate	60%	**64%**

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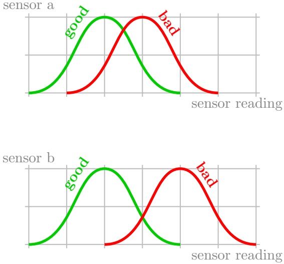

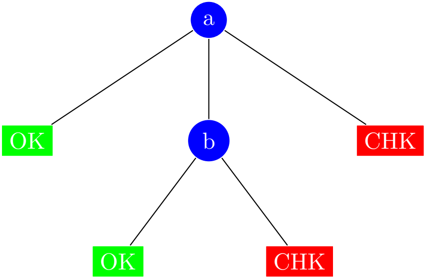

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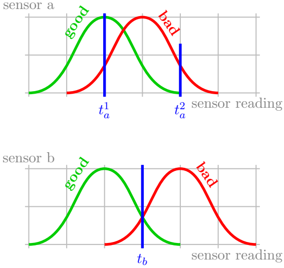

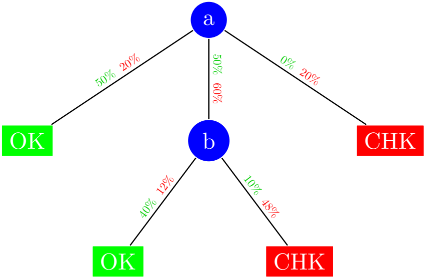

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Container Inspection

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- Finding ways to intercept illicit nuclear materials and weapons destined for the U.S. via the maritime transportation system is an exceedingly difficult task. Today, only a small percentage of containers arriving to U.S. ports are inspected.
- Inspection involves checking paperwork, using imaging sensors (X-rays, gamma-rays, etc.), and manual inspection.
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Maximize detection rate $\mathbf{\Delta(D, t)}$

- over all **decision trees D** and **threshold selections t**
- subject to **budget**, **capacity**, and **delay** constraints

A possible solution (Stroud and Saeger, 2003)

- Enumerate all possible (binary) **decision trees** and compute best possible **threshold selections** for each.

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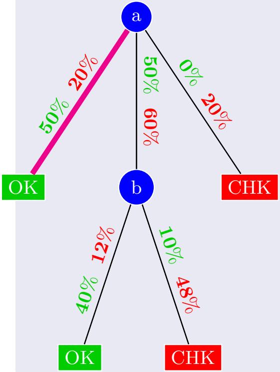

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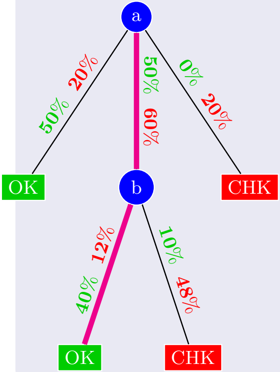

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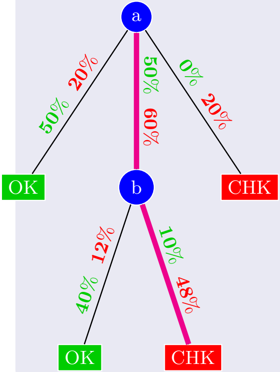

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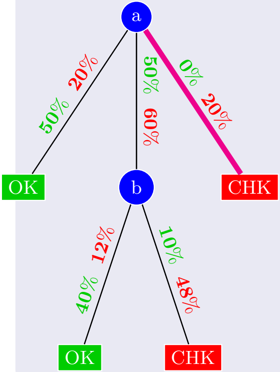

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- Let Ξ be the set of all possible histories, and y_η denote the *fraction of good containers* we plan to test along history η (for all $\eta \in \Xi$).
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Polyhedron of decision Trees

Let $\mathbf{y} = (y_\eta | \eta \in \boldsymbol{\Xi})$, and consider the polyhedron

$$P = \left\{ y \;\middle|\; \begin{array}{l} \displaystyle\sum_{\substack{\eta \in \boldsymbol{\Xi} \\ (\nu,(s,j)) \in \boldsymbol{\Pi}(\eta)}} y_\eta = \gamma(s,j) \sum_{\substack{\eta \in \boldsymbol{\Xi} \\ (\nu,(s,*)) \in \boldsymbol{\Pi}(\eta)}} y_\eta \quad \forall (\nu,(s,j)) \in \boldsymbol{\Pi} \\ \displaystyle\sum_{\eta \in \boldsymbol{\Xi}} y_\pi = 1 \\ y_\eta \geq 0 \quad \forall \eta \in \boldsymbol{\Xi} \end{array} \right\}$$

Theorem

Vertices of P correspond to decision trees; $y \in P$ correspond to mixed strategies.

Polyhedron of decision Trees

Let $\mathbf{y} = (y_\eta | \eta \in \boldsymbol{\Xi})$, and consider the polyhedron

$$P = \left\{ y \;\middle|\; \begin{array}{l} \sum_{\substack{\eta \in \boldsymbol{\Xi} \\ (\nu,(s,j)) \in \boldsymbol{\Pi}(\eta)}} y_\eta = \gamma(s,j) \sum_{\substack{\eta \in \boldsymbol{\Xi} \\ (\nu,(s,*)) \in \boldsymbol{\Pi}(\eta)}} y_\eta \quad \forall(\nu,(s,j)) \in \boldsymbol{\Pi} \\ \sum_{\eta \in \boldsymbol{\Xi}} y_\pi = 1 \\ y_\eta \geq 0 \quad \forall \eta \in \boldsymbol{\Xi} \end{array} \right\}$$

Theorem

Vertices of P correspond to decision trees; $y \in P$ correspond to mixed strategies.

Large Scale LP Formulation

$$\sum_{\eta \in \boldsymbol{\Xi} : \mathbf{CHK} \in \eta} y_\eta \frac{\beta(\eta)}{\gamma(\eta)} \longrightarrow \max \qquad \text{(Detection Rate)}$$

$$\sum_{\eta \in \boldsymbol{\Xi}} y_\eta C(\eta) \leq \mathbf{B} \qquad \text{(Budget Limit)}$$

$$\ldots$$

$$y \in P$$

- Tractable model up to 12 sensors.
- Detection rate – unit inspection cost ROC curve can be tabulated.
- Effects of capacity and time delay limitations can be analyzed.
- Benefits of new sensor technologies can be evaluated.

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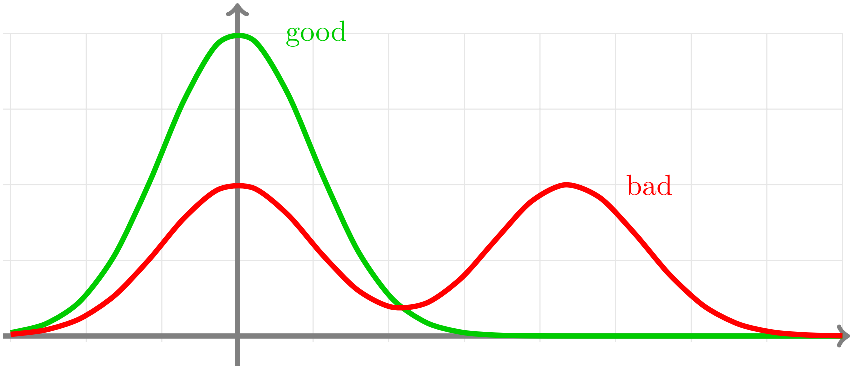
Outline

Experiments with 4 sensors (Stroud and Saeger, 2003)

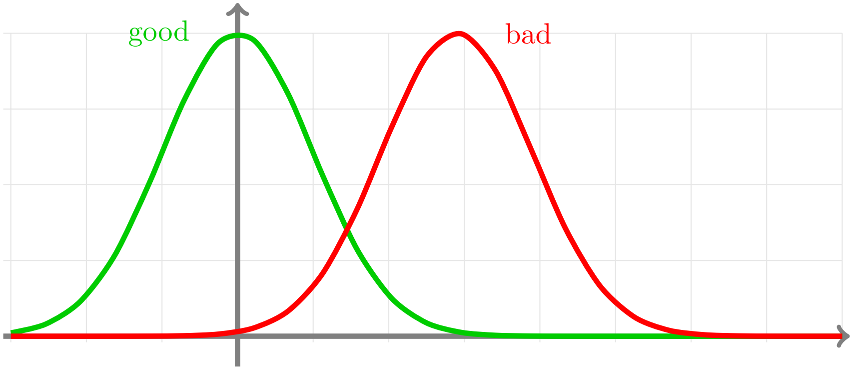


Distributions of sensor readings of **sensor 1**. The cost of inspection of a container by this sensor is $\mathbf{C_1 = \$0.32}$.

Experiments with 4 sensors (Stroud and Saeger, 2003)

Distributions of sensor readings of **sensor 2**. The cost of inspection of a container by this sensor is $\mathbf{C_2 = \$0.92}$.

Experiments with 4 sensors (Stroud and Saeger, 2003)

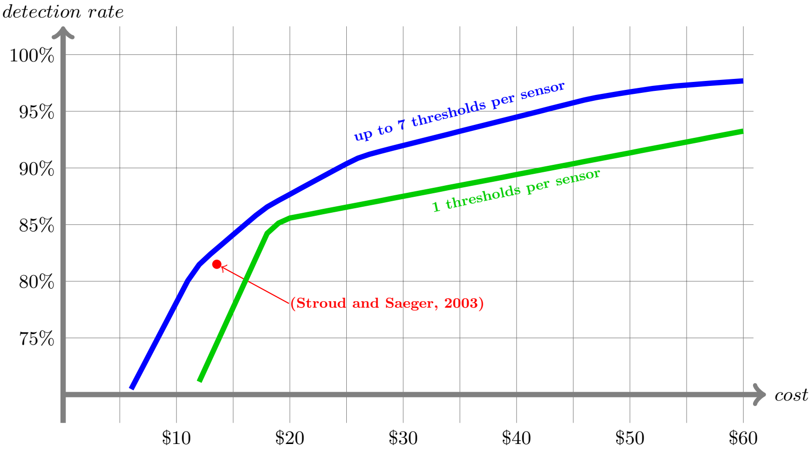


Distributions of sensor readings of **sensor 3**. The cost of inspection of a container by this sensor is $\mathbf{C_3 = \$57}$.

Experiments with 4 sensors (Stroud and Saeger, 2003)

Distributions of sensor readings of **sensor 4**. The cost of inspection of a container by this sensor is $\mathbf{C_4 = \$176}$.

$C = (0.32, 0.92, 57, 176, 600)$



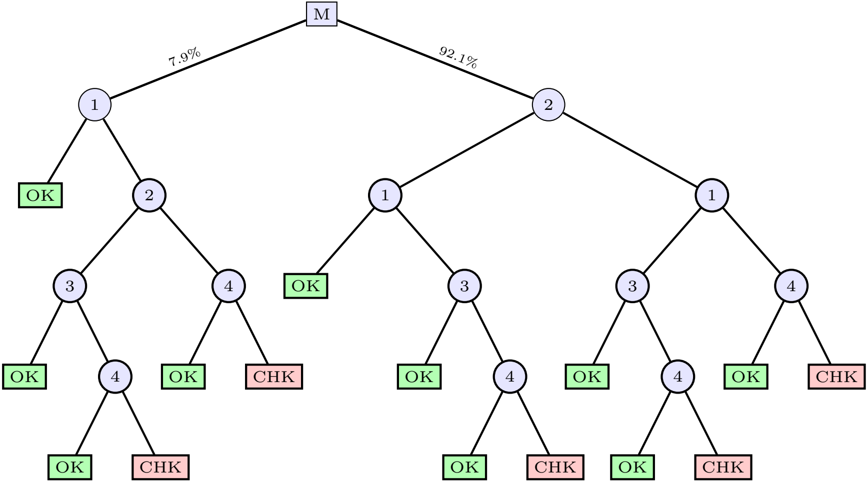


Figure: Optimal decision tree with one threshold, that is a binary partition for each sensor. It is a mixed strategy, since 92.1% of the containers are inspected by a different subtree than the other 7.9%.

Publications

- Boros, E., Fedzhora, P.B., Kantor, P.B., Saeger, K., and Stroud, P. Large Scale LP Model for Finding Optimal Container Inspection Strategies. *Naval Research Logistics Quarterly*, Vol. **56** (5), 404-420, 2009.
- Kantor, P. and Boros, E. Deceptive Detection Methods for Effective Security with Inadequate Budgets: The Testing Power Index. *Risk Analysis*. **30** (4), 663-673, 2010.
- Boros, E., Goldberg, N., Kantor, P.B., Word, J. Optimal sequential inspection policies. *Annals of Operations Research* **187**(1), 89-119, 2011.