

Modeling Mode-Locked Lasers in the Few Femtosecond Regime

Coherent Structures as Attractors

J. N. Kutz, **Mode-locked soliton lasers**, SIAM Review (December 2006)

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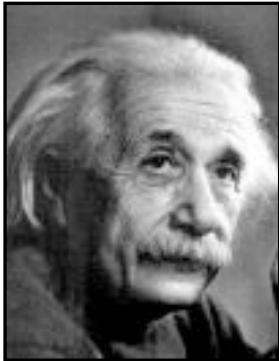
Steven Cundiff
Physics, JILA

Experiment

Laser theory (Kutz) group at UW



The Maser, Laser and Nonlinear Optics



Postulated by Satyendra Bose (1924)

- Predicted **condensed** state for photons

Extended by Albert Einstein

- Predicted **condensed** state for matter

MASER: first coherent EM radiation (1953)

PHYSICAL REVIEW

VOLUME 99, NUMBER 4

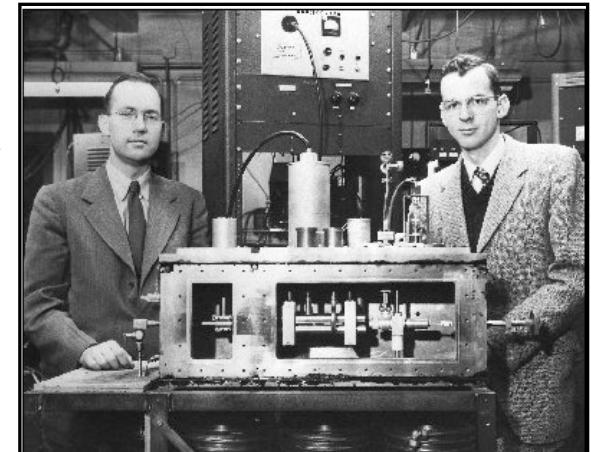
AUGUST 15, 1955

The Maser—New Type of Microwave Amplifier, Frequency Standard, and Spectrometer*†

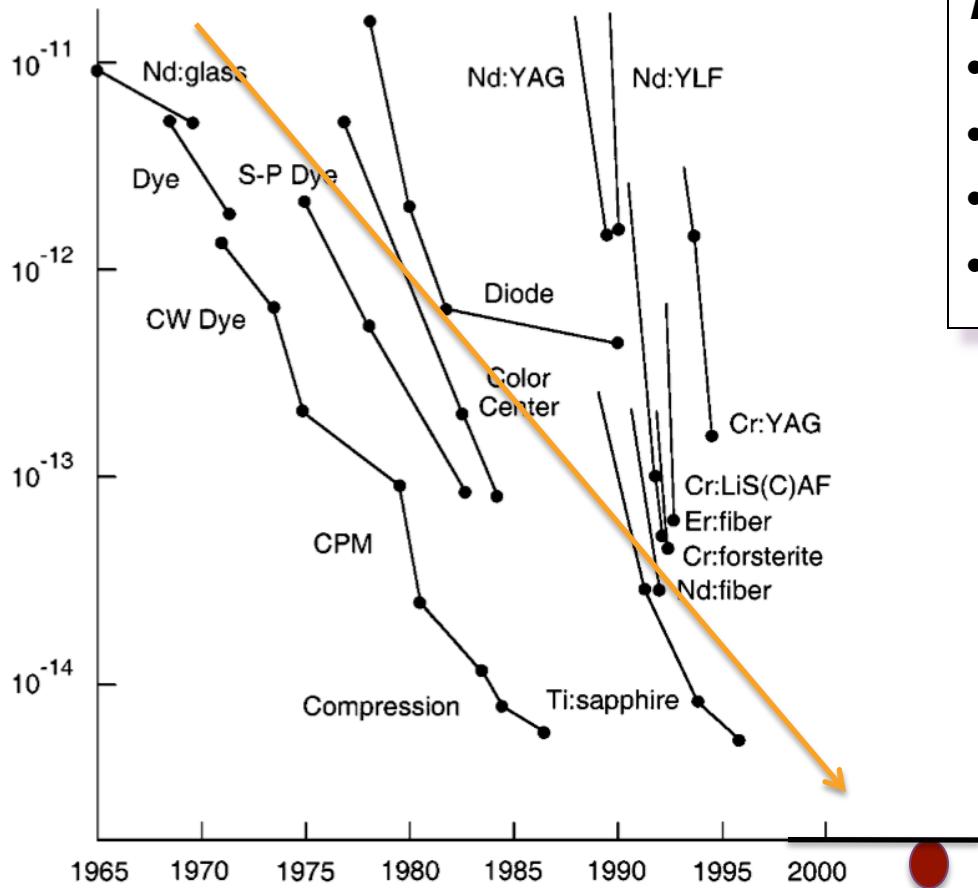
J. P. GORDON,‡ H. J. ZEIGER,§ AND C. H. TOWNES
Columbia University, New York, New York
(Received May 4, 1955)



Maiman (Hughes) ruby laser (May 16, 1960)



Pulsed Lasers



Dominant Physical Effects

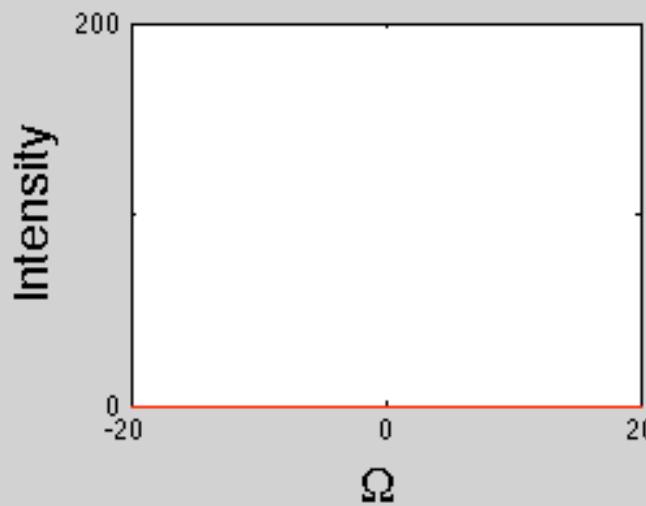
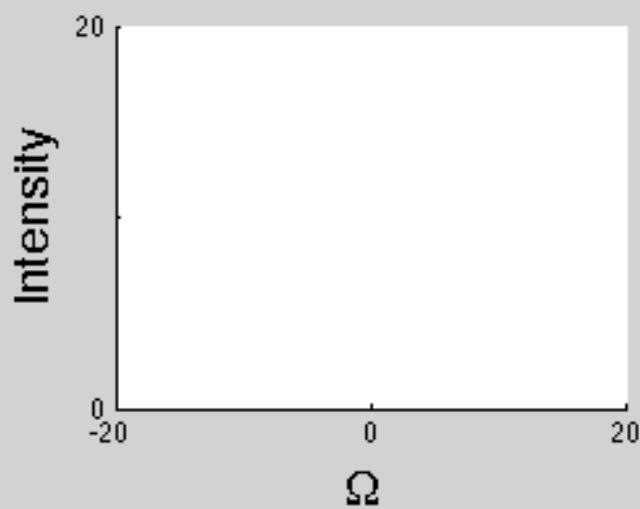
- chromatic dispersion
- self-phase modulation
- attenuation
- gain (bandwidth limited)

Mode-locking

- Physical effects + periodic intensity dependent perturbation
- Pulse stream integrity (cheap, compact, reliable)

Attosecond Physics

Mode-Locking Dynamics



Theoretical Framework

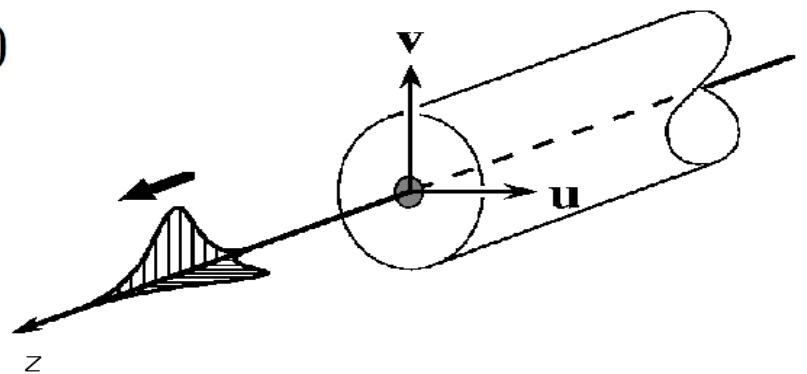
- Qualitative Modeling
 - Haus Master Equation
- Quantitative Modeling
 - Modeling Physical Laser Cavities



The Nonlinear Schrodinger Equation

Nonlinear wave propagation [Gross-Pitaevskii 1950s]

$$i \frac{\partial Q}{\partial Z} + \frac{1}{2} \frac{\partial^2 Q}{\partial T^2} + |Q|^2 Q = 0$$



1972 - Zakharov & Shabat
soliton solutions



1973 - Hasegawa & Tappert
theoretically proposed

$$n(\omega, E) = n_0(\omega) + n_2(\omega)|E|^2$$



1980 - Mollenauer
experimentally demonstrated

Haus Master Mode-Locking Model

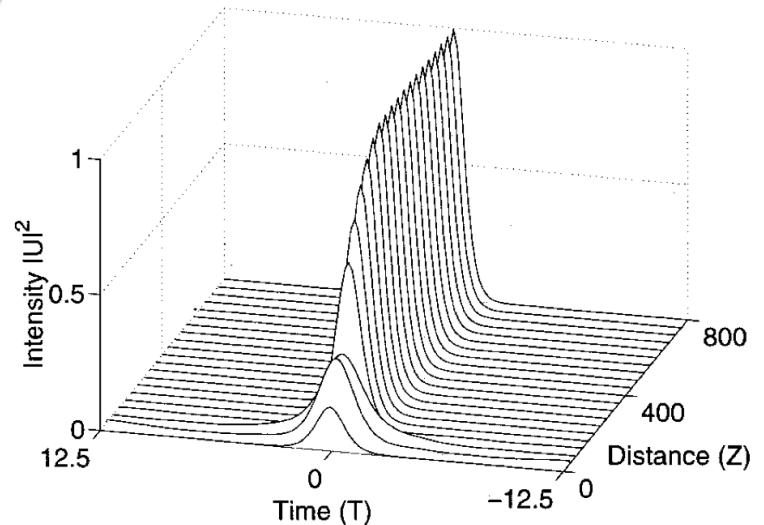


Hermann Haus, MIT

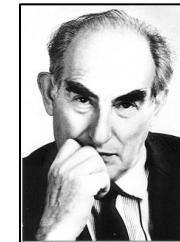
$$g(Z) = \frac{2g_0}{1 + \|Q\|^2/e_0}$$

Intensity-dependent attenuation

$$i \frac{Q}{Z} + \frac{1}{2} \frac{\partial^2 Q}{T^2} + \|Q\|^2 Q - i \circ \|Q\|^2 Q + i \cdot Q \circ i \frac{\partial g(Z)}{\partial Z} + \frac{\beta}{T^2} \|Q\|^2 = 0$$



- heuristic model: beta is modelocking mechanism
- pulse solutions: $Q(Z, T) = \eta \operatorname{sech}(\omega T)^{1+iA} \exp(i\theta Z)$
- physical connection: quantitative match not possible



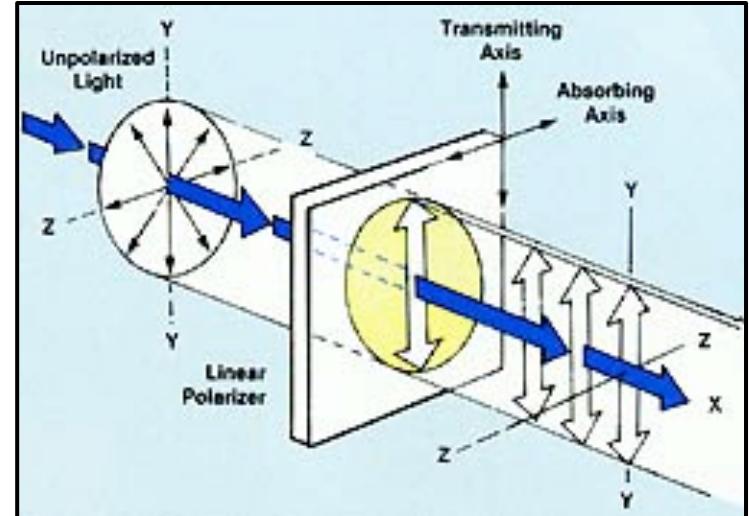
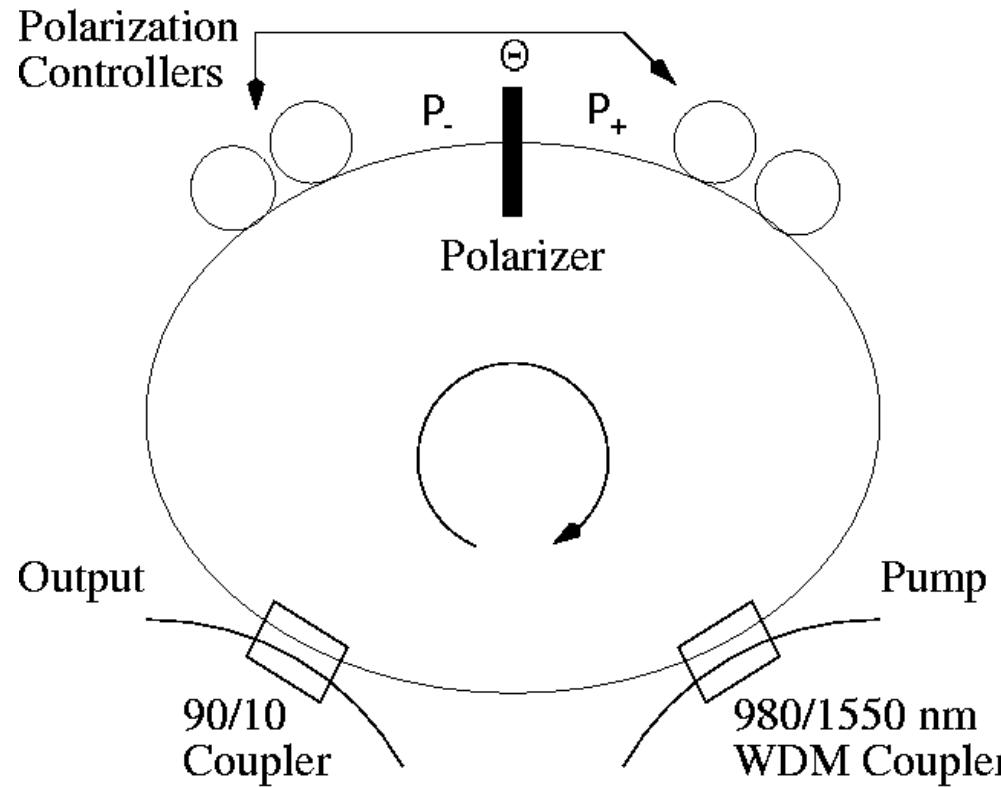
*V. Ginzburg
Nobel 2003*



*L. Landau
Nobel 1962*

Quantitative Models

Passive Polarizer



Ding & Kutz, JOSA B (July 2009)

- Nonlinear rotation+polarizer = mode-locking

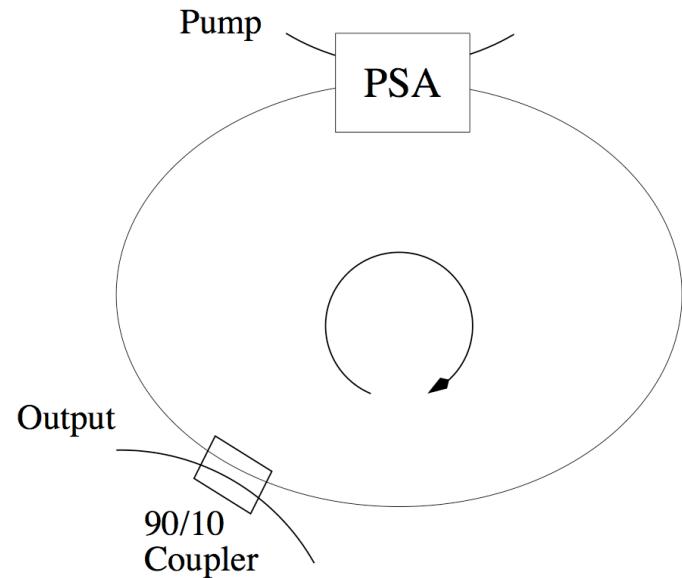
- Polarizer action:

$$P_+ = \tan^{-1} [\alpha \tan(P_- - \Theta)] + \Theta$$

Phase-Sensitive Amplification

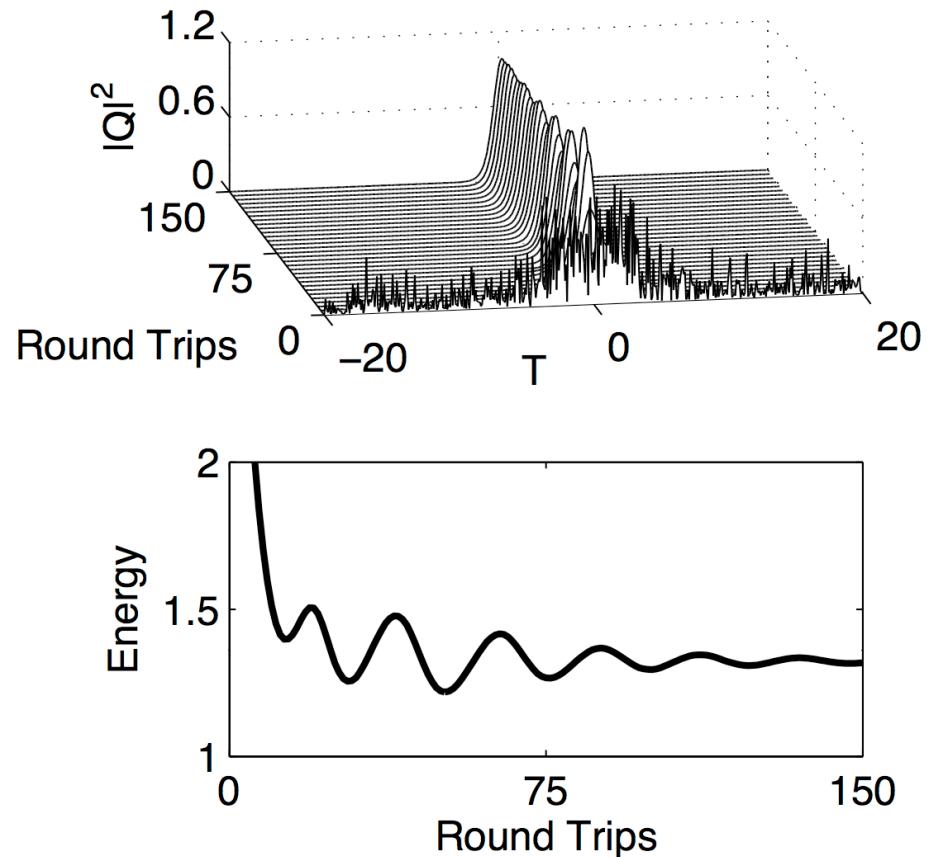
Soliton phase rotation: intensity-dependent

$$Q(Z, T) = \eta \operatorname{sech}(\eta T) \exp(i\eta^2 Z/2)$$

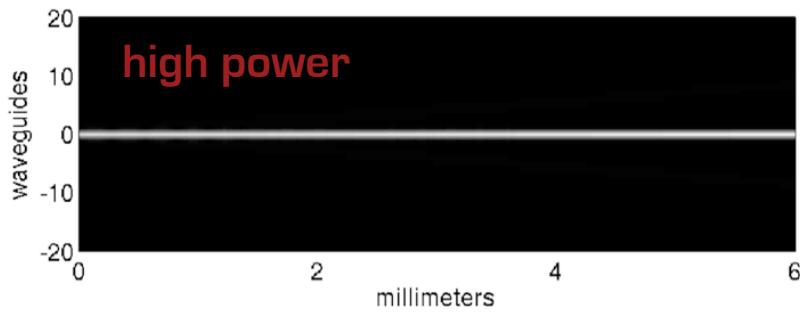
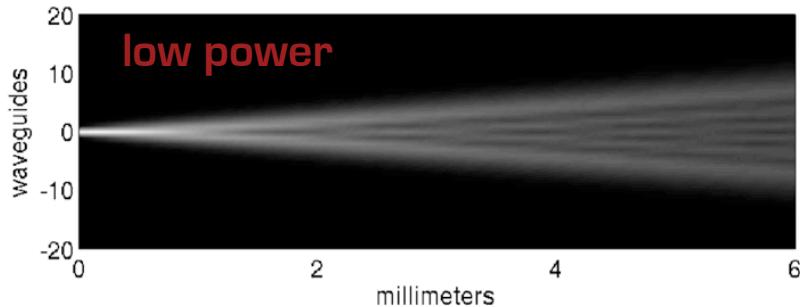


$$\eta = \sqrt{\frac{2\Delta\Omega_p}{Z_R}}$$

In-phase intensity constraint

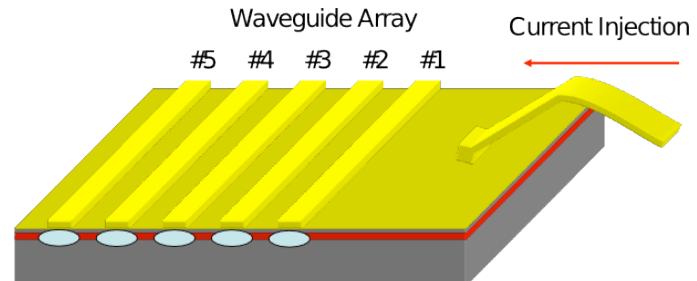
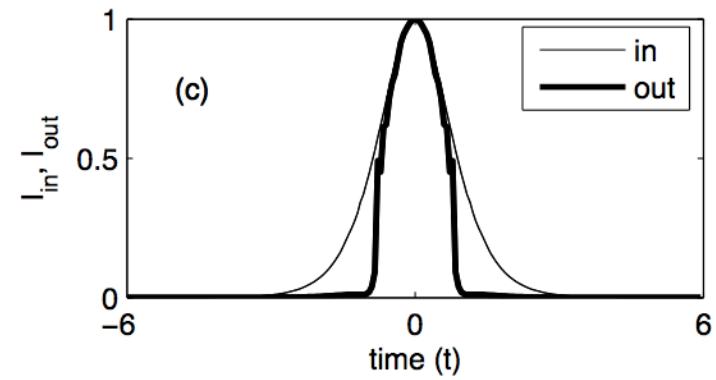
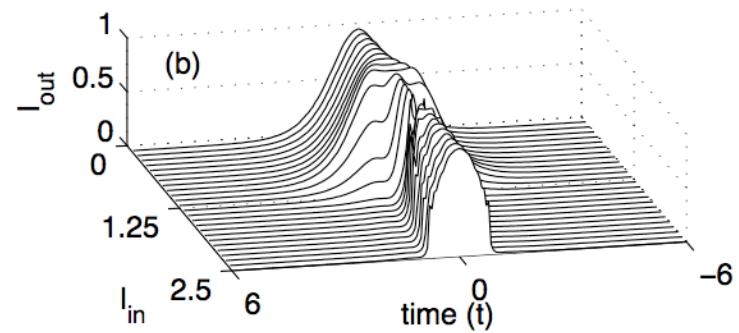


Waveguide Arrays

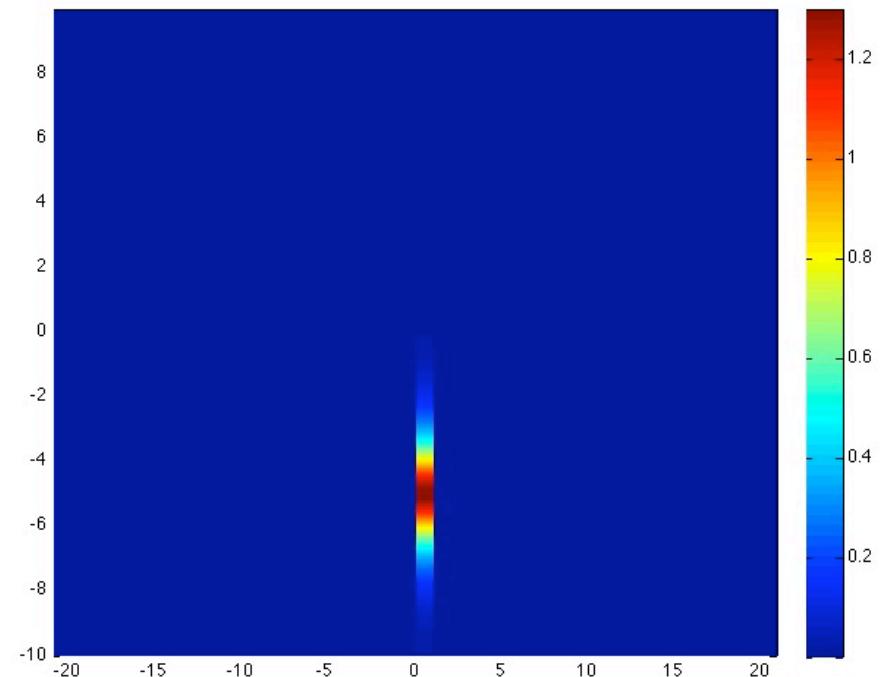
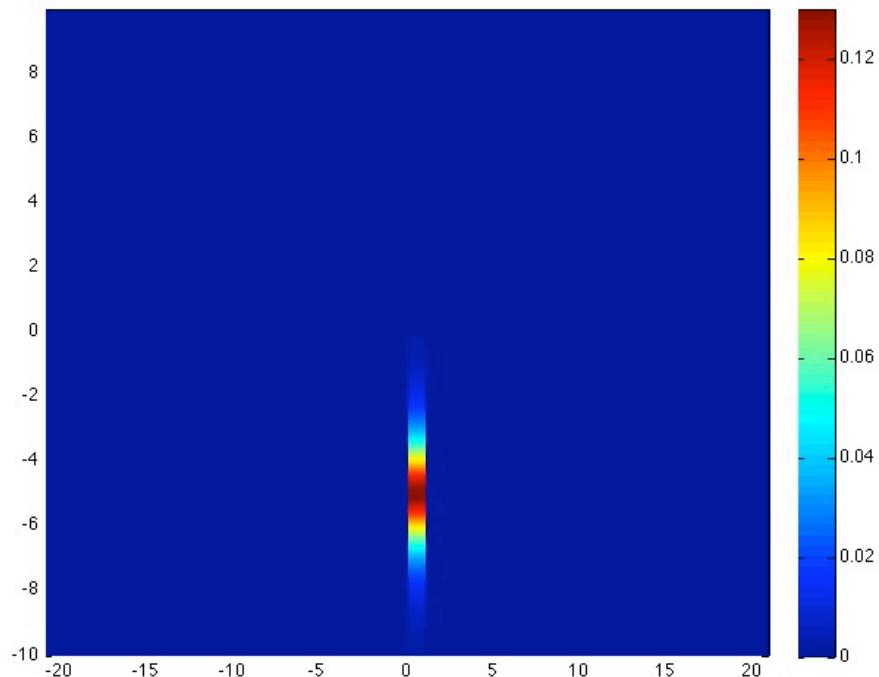


$$i \frac{dA_n}{d\xi} + C(A_{n-1} + A_{n+1}) + \beta |A_n|^2 A_n = 0$$

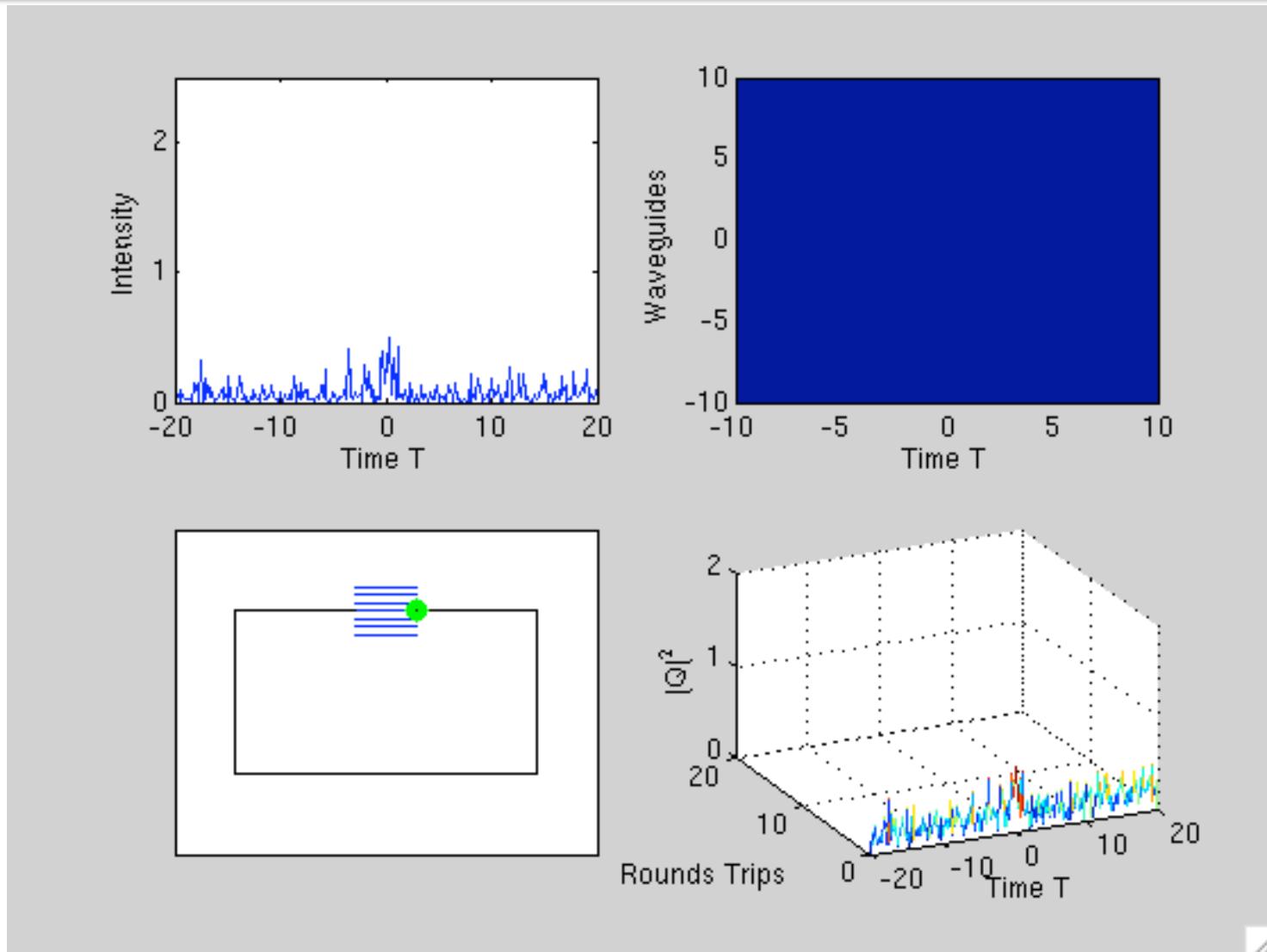
Christodoulides & Joseph, Opt. Lett. (1988)



Waveguide Arrays



Waveguide Array Mode-locking



Average Cavity Dynamics

waveguide array

$$i\frac{\partial A_0}{\partial Z} + \frac{1}{2}\frac{\partial^2 A_0}{\partial T^2} + \beta_0|A_0|^2 A_0 + CA_1 + i\gamma_0 A_0 - ig(Z)\left(1 + \tau\frac{\partial^2}{\partial T^2}\right)A_0 = 0$$

$$i\frac{\partial A_1}{\partial Z} + C(A_2 + A_0) + i\gamma_1 A_1 = 0$$

$$i\frac{\partial A_2}{\partial Z} + CA_1 + i\gamma_2 A_2 = 0$$

passive polarizer [cubic-quintic Ginzburg-Landau]

$$\begin{aligned} q_z = & -i\frac{D}{2}q_{TT} + i|q|^2q + G(z)(1 + \tau\partial_T^2)q - \Gamma q \\ & + id_2|q|^2q + id_3|q|^4q + (c_1 + c_2|q|^2 + c_3|q|^4)q \end{aligned}$$

phase-sensitive amplifier [Swift-Hohenberg equation]

$$u_z + \frac{1}{4}(\partial_t^2 - 1)^2 u - \gamma(z)u - u^3 + u^5 + 3\beta u(u_t)^2 + (\beta + 1)u^2 u_{tt} = 0$$

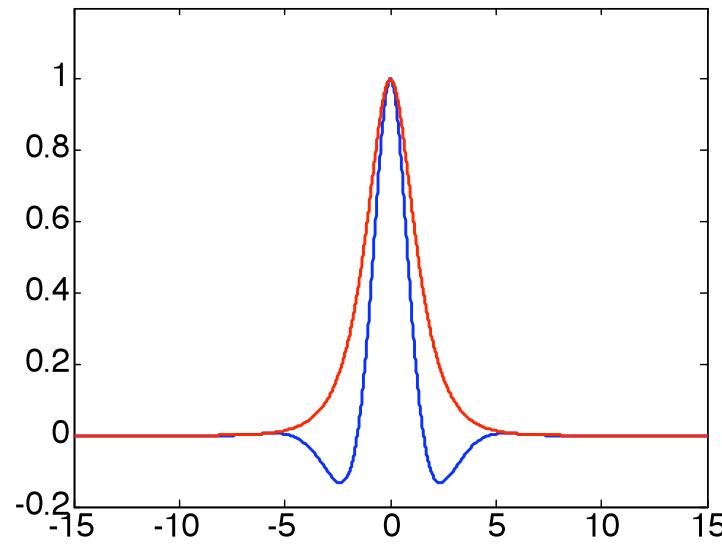
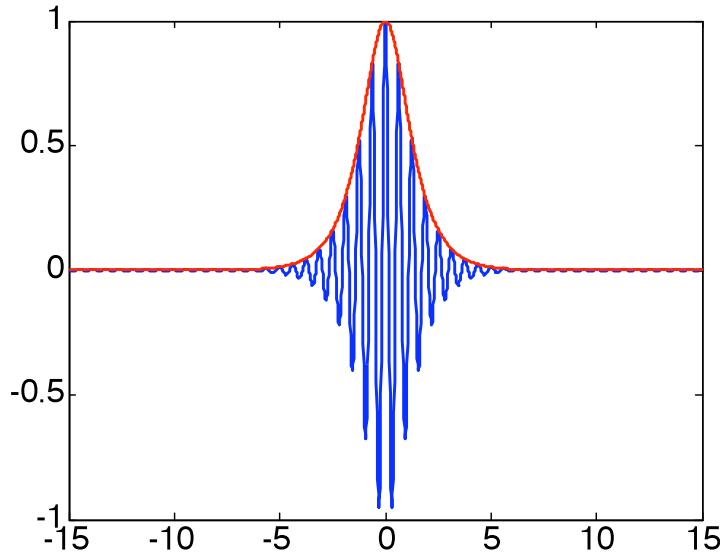
Modeling Ultra-Short Mode-Locking Dynamics

Beyond the center frequency expansion



Limit of NLS Based Models

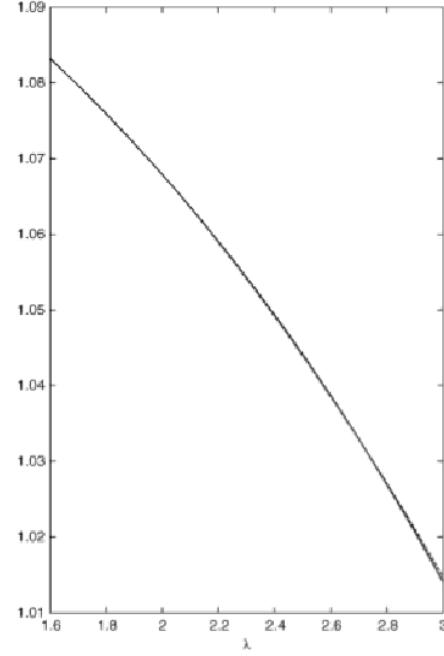
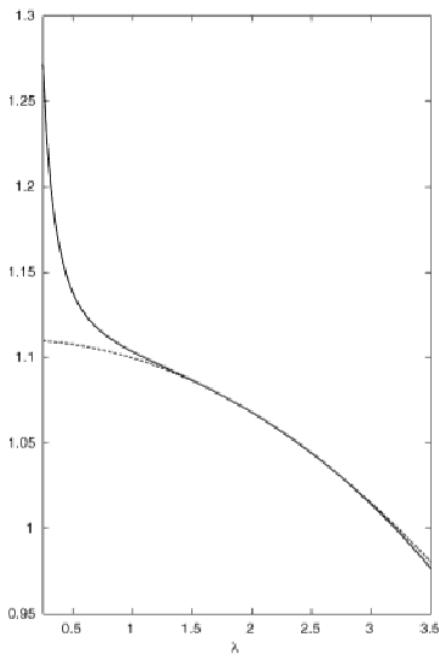
NLS relies on slow envelope approximation, which becomes invalid for few cycle pulses



- Sh Amiranashvili, A.G. Vladimirov, U. Bandelow, "Solitary wave solutions for few cycle optical pulses",
Phys. Rev. A. **77**, 063281 (2008)
- T. Shaefer, C.E. Wayne, "Propagation of ultra-short optical pulses in cubic nonlinear media," Physica D
196, 90- 105 (2004)
- Chung, Jones, Shaafer, Wayne, "Ultra-short pulses in linear and nonlinear media", Nonlinearity **18**,
1351 – 1374 (2005)
- M. Kolesik, J.V. Moloney, and M. Mlejnek, "Unidirectional Optical Pulse Propagation Equation,"
Physical Review Letters 89, 283902 (2002)

A Different Asymptotics

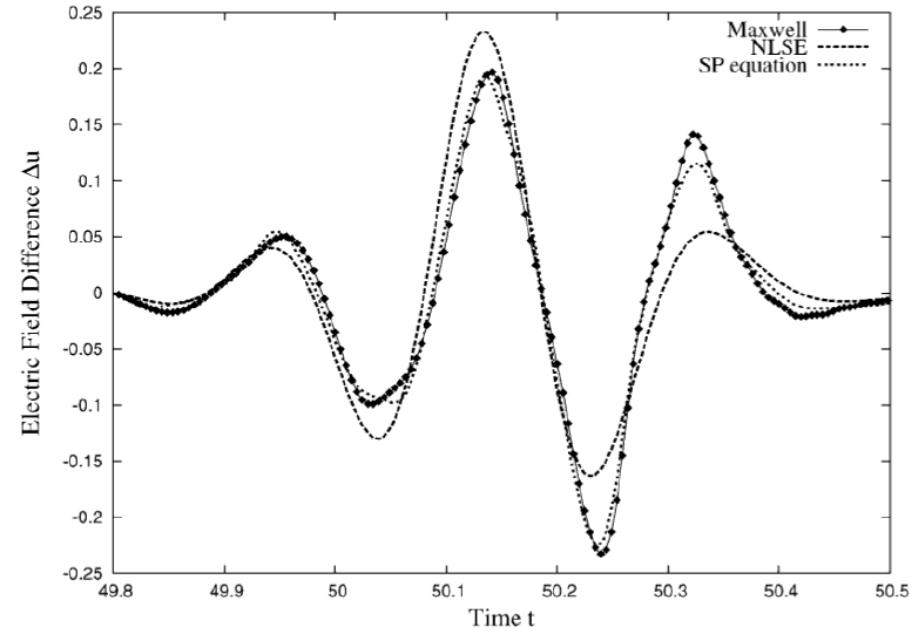
Express linear susceptibility as polynomial in wavelength



$$\phi = \frac{t - x}{\varepsilon}, x_n = \varepsilon^n x,$$

$$u(x, t) = \varepsilon A_0(\phi, x_1, x_2, \dots) + \varepsilon^2 A_1(\phi, x_1, x_2, \dots) + \dots$$

multiple scales



$$-2 \frac{\partial^2 A_0}{\partial x_1 \partial \phi} = \frac{1}{c_2^2} A_0 + \chi^{(3)} \frac{\partial^2}{\partial \phi^2} \left(A_0^3 \right)$$

The Short Pulse Equation

SPE

$$u_{xt} - u - \frac{1}{6} (u^3)_{xx} = 0$$

Exact solutions from Sine-Gordon breathers

$$u = 4mn \frac{\sin\psi \sinh\phi + n \cos\psi \cosh\phi}{m^2 \sin^2\psi + n^2 \cosh^2\phi},$$

$$x = y + 2mn \frac{m \sin 2\psi - n \sinh 2\phi}{m^2 \sin^2\psi + n^2 \cosh^2\phi}$$

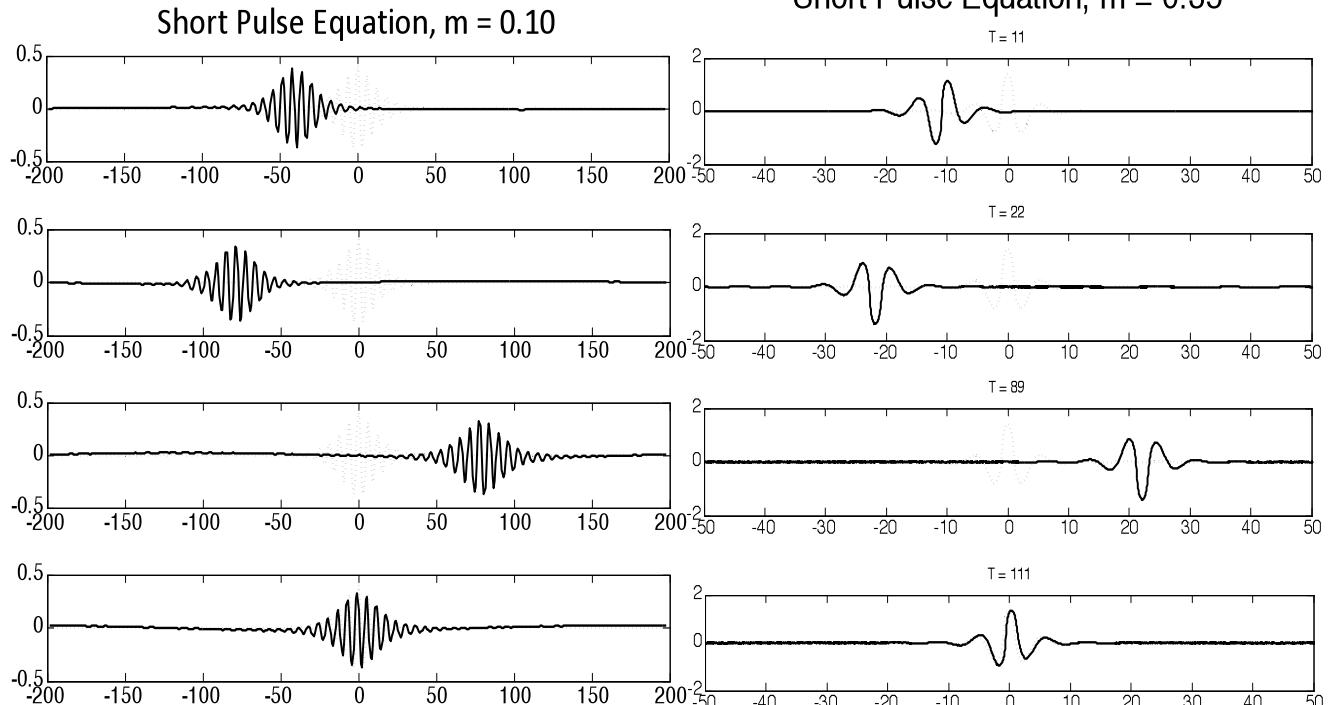
Short Pulse Equation, $m = 0.35$

$$\phi = m(y + t),$$

$$\psi = n(y - t),$$

$$n = \sqrt{1 - m^2},$$

$$0 \leq m \leq \sin(\pi/8) \approx 0.3827$$



- parameter m describes “shortness”
- $m \rightarrow 0.383$ is ultra-short limit
- $m \rightarrow 0$ is the NLS limit.
 $u \approx 4m \cos(x - t) \operatorname{sech}(m(x + t))$

Master Mode-Locking Theory

$$iu_z + \frac{1}{2}u_{tt} + |u|^2 u = i \left(g(z) \left(1 + \tau \frac{\partial^2}{\partial t^2} \right) - \gamma \right) u + i\beta |u|^2 u$$

$$g(z) = \frac{2g_0}{1 + \|u\|^2}$$

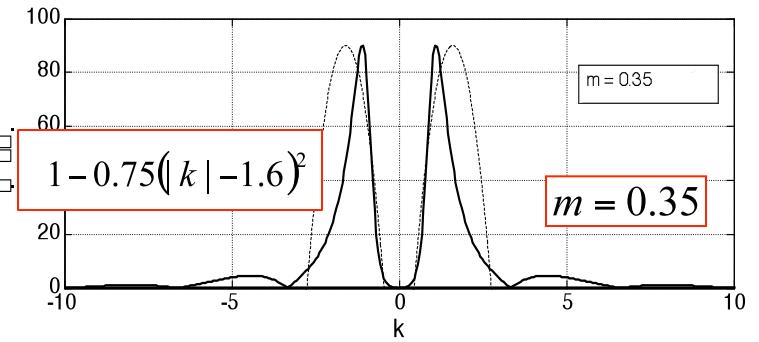
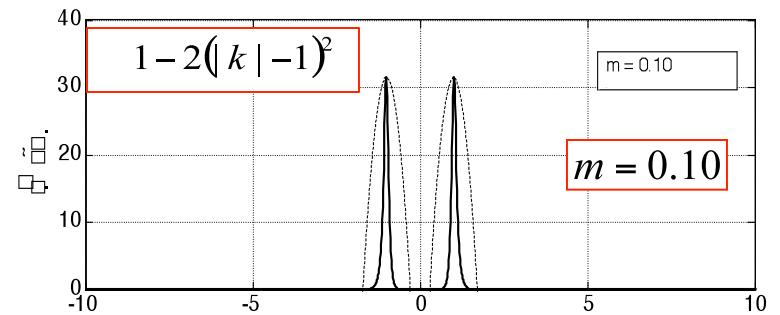
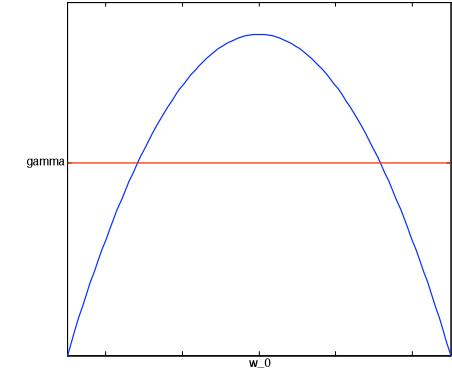
SPE is not an envelope equation!

Want symmetric gain windows centered around laser frequency

$$u_{xt} - u - \frac{1}{6}(u^3)_{xx} = \left(g(t) \mathcal{F}^{-1} [P(k) \hat{U}] - \gamma u + \beta u^3 \right)_x$$

SPMMLE

$$u_{xt} - u - \frac{1}{6}(u^3)_{xx} = \left(g(t)(\tau_2 u_{xx} + \tau_4 u_{xxxx}) - \gamma u + \beta u^3 \right)_x$$

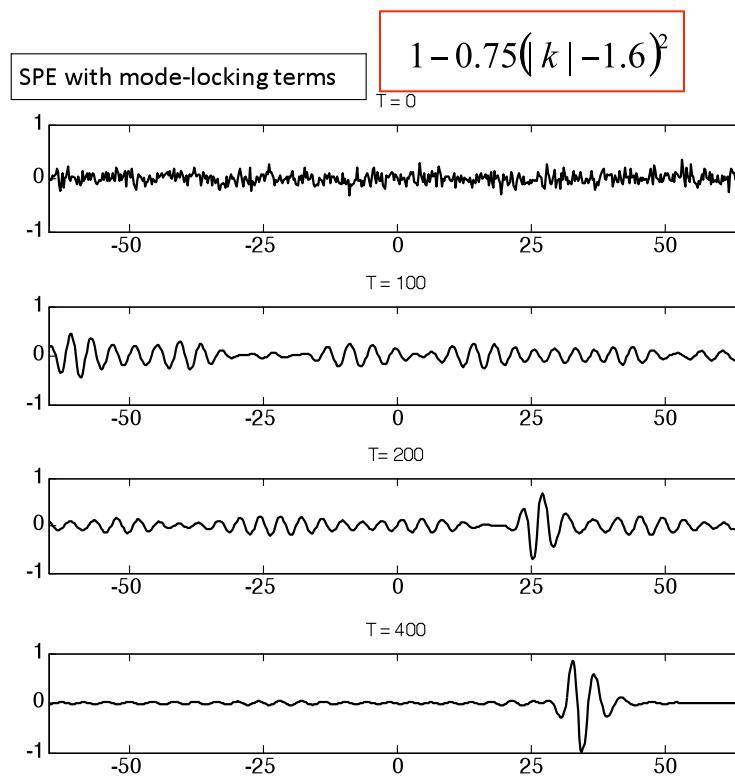
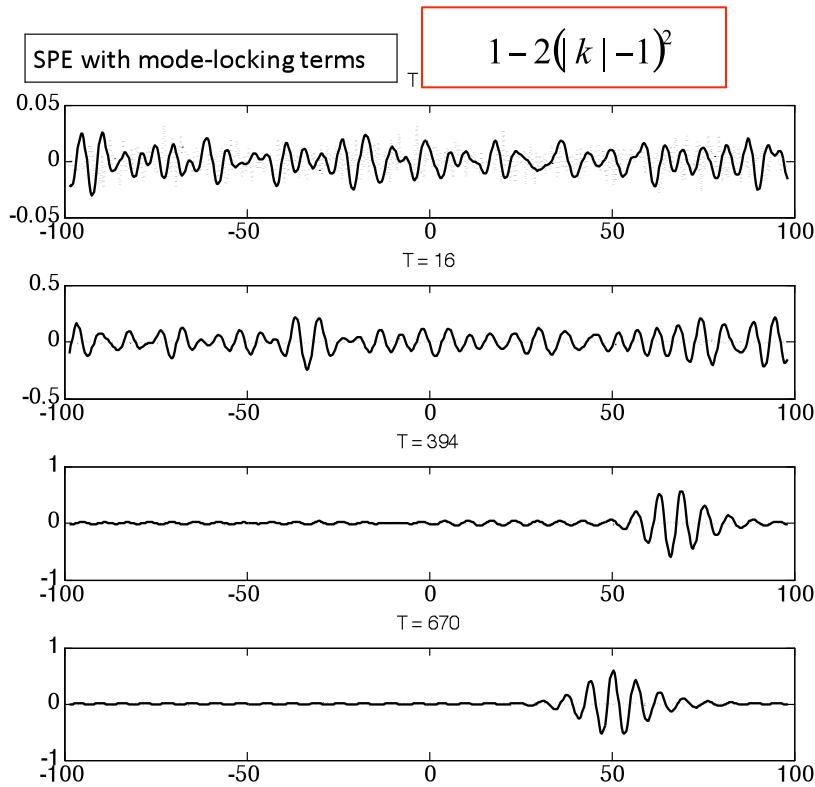


Stable 2-5 fs Mode-Locking in SPMML

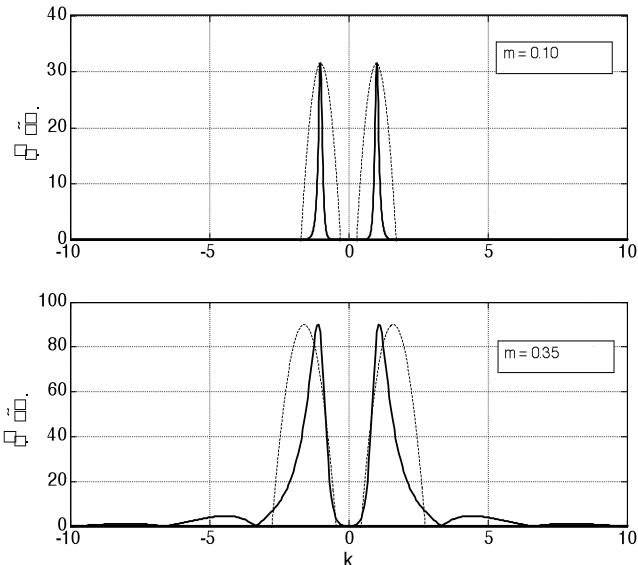
$$u_{xt} - u - \frac{1}{6}(u^3)_{xx} = \left(g(t) \mathcal{F}^{-1} [P(k)\hat{U}] - \gamma u + \beta u^3 \right)_x$$

$$\boxed{g(z) = \frac{2g_0}{1 + \|u\|^2}}$$

$$\boxed{P(k) = 1 - a(|k| - b)^2}$$

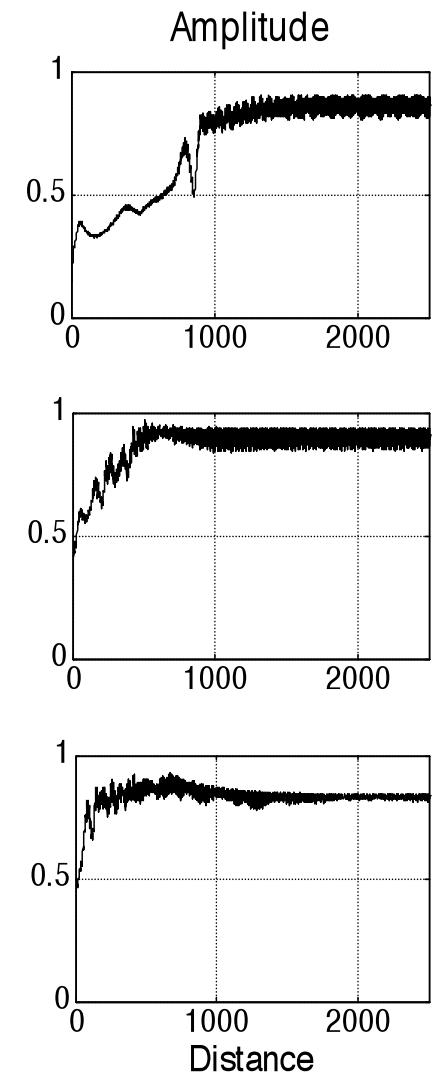
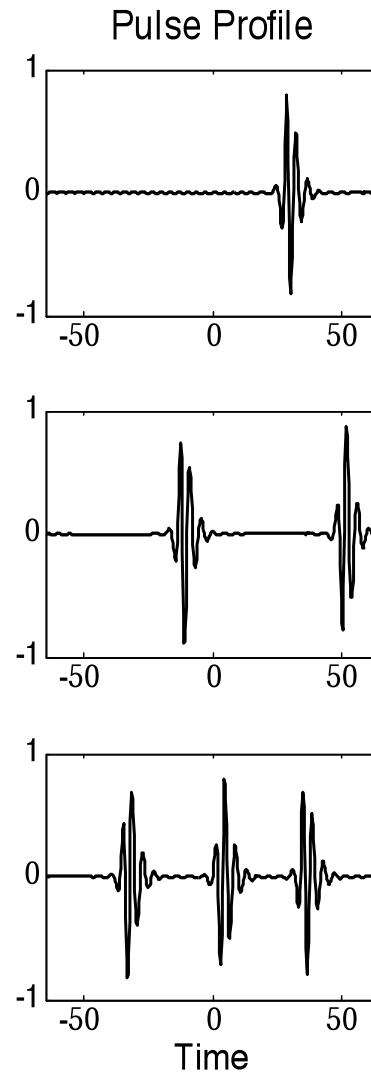


Multi-pulsing instability

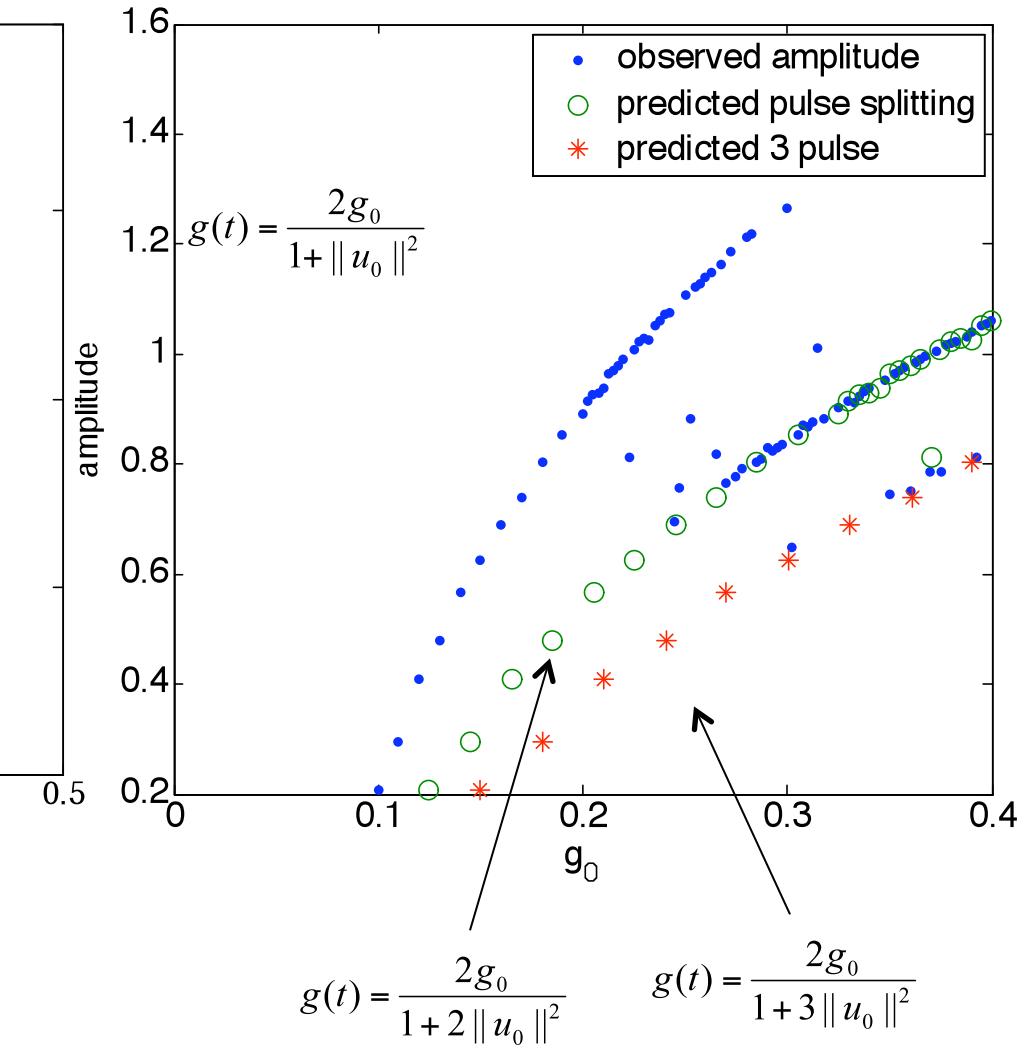
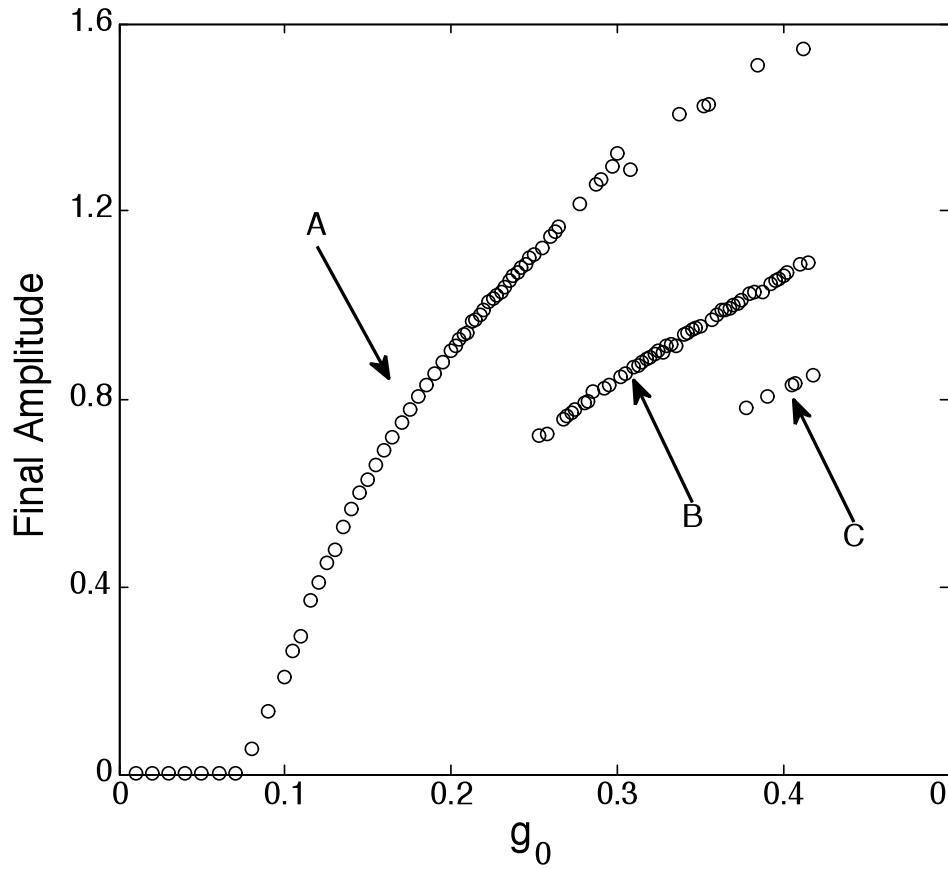


$$g(t) = \frac{2g_0}{1 + \|u\|^2}$$

- high gains give taller narrower pulses
- require more bandwidth
- sharper pulses approach bandwidth limit



Multi-pulsing



Summary & Conclusions

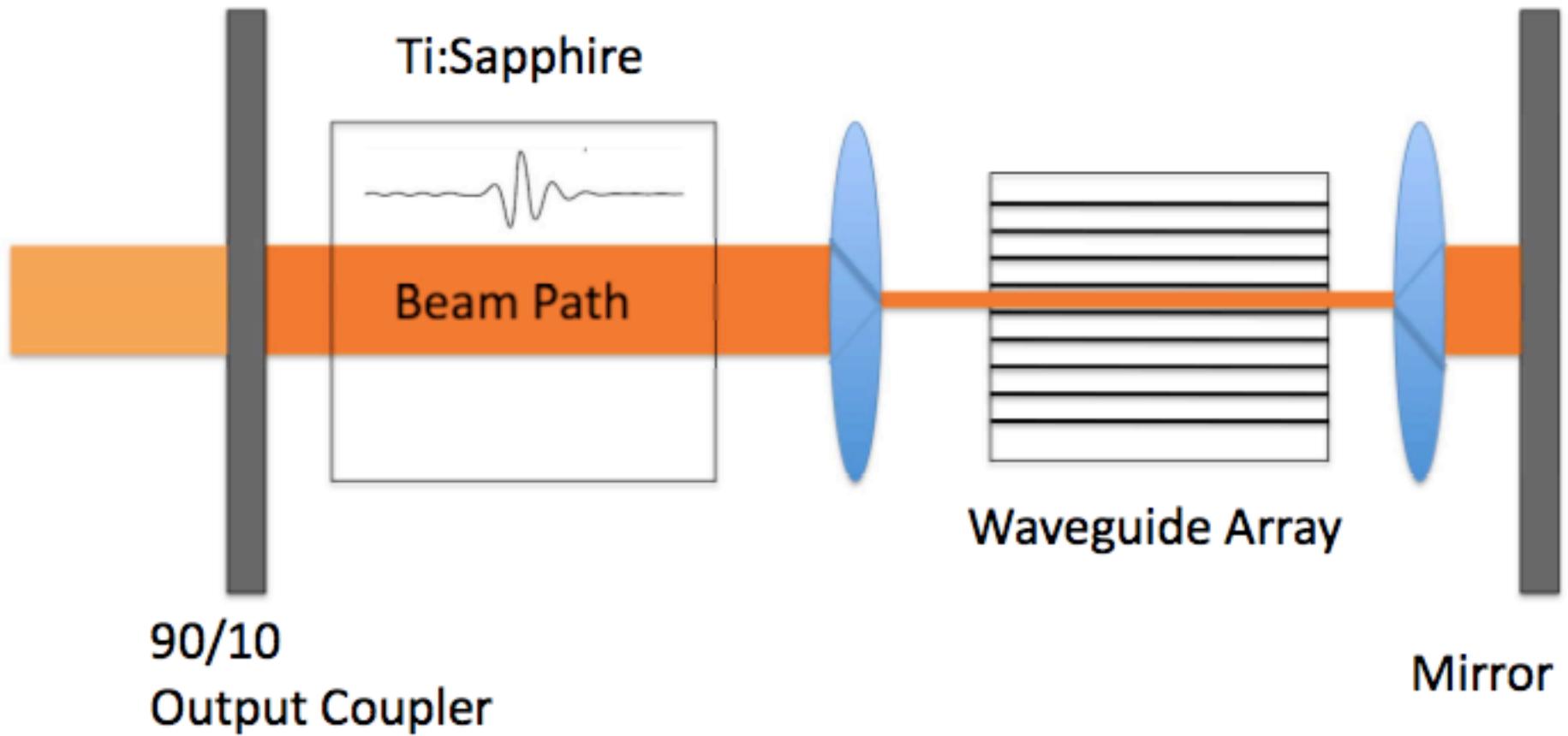
Ultra-Short

- Need theory beyond center-frequency expansion
- Excellent treatment of full dispersion from 800-3500nm
- Retains entire phase information and evolution
- Nonlinearity still needs a more advanced treatment

$$u_{xt} - u - \frac{1}{6} \left(u^3 \right)_{xx} = \left(g(t) \left(\tau_2 u_{xx} + \tau_4 u_{xxxx} \right) - \gamma u + \beta u^3 \right)_x$$

Quantitative Models

Waveguide Array Mode-Locking



Governing Equations

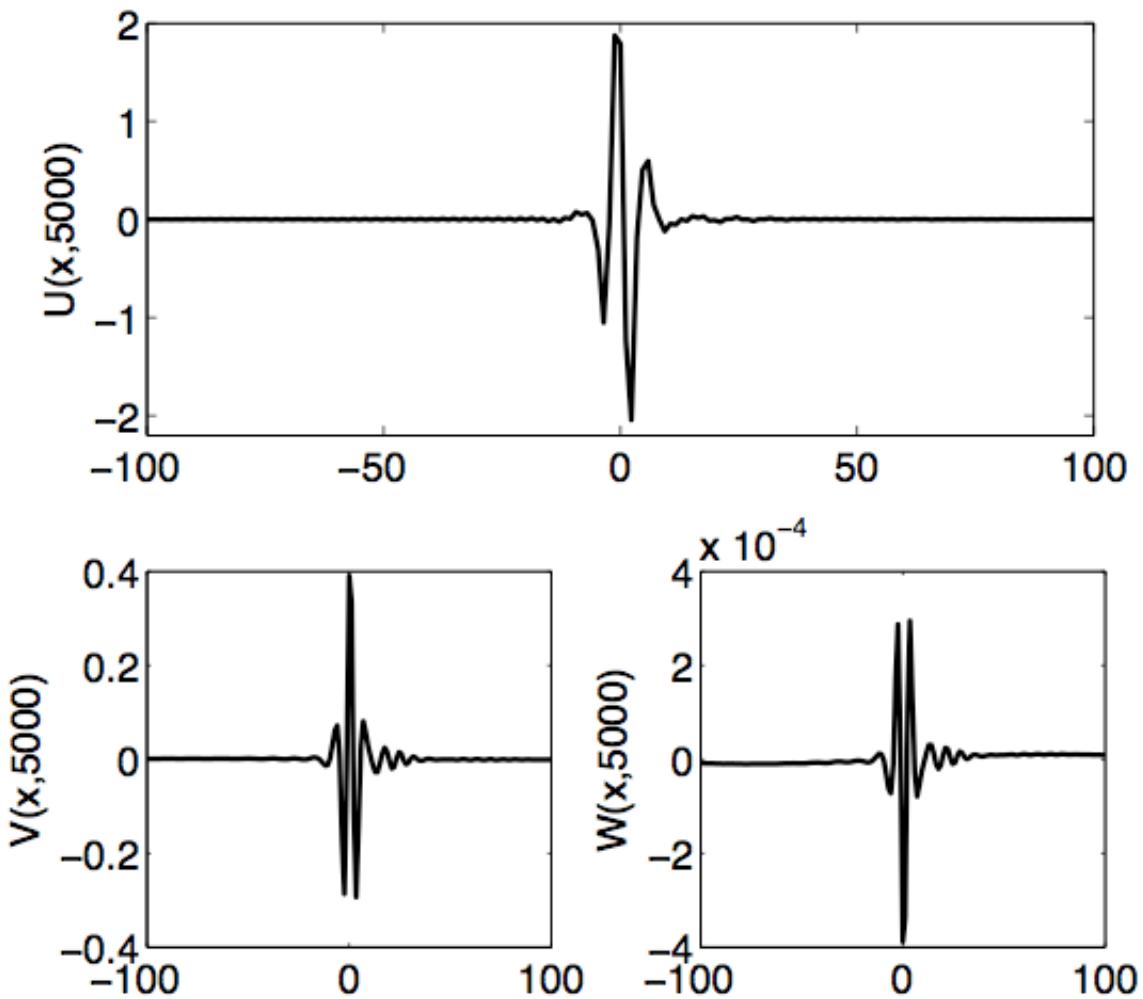
$$U_{xt} - U - \frac{1}{6}(U^3)_{xx} = [g(t)(\tau_2 U_{xx} + \tau_4 U_{xxxx}) - \gamma_0 U - cV]_x$$

$$V_{xt} - V = [-\gamma_1 V + c(U + W)]_x$$

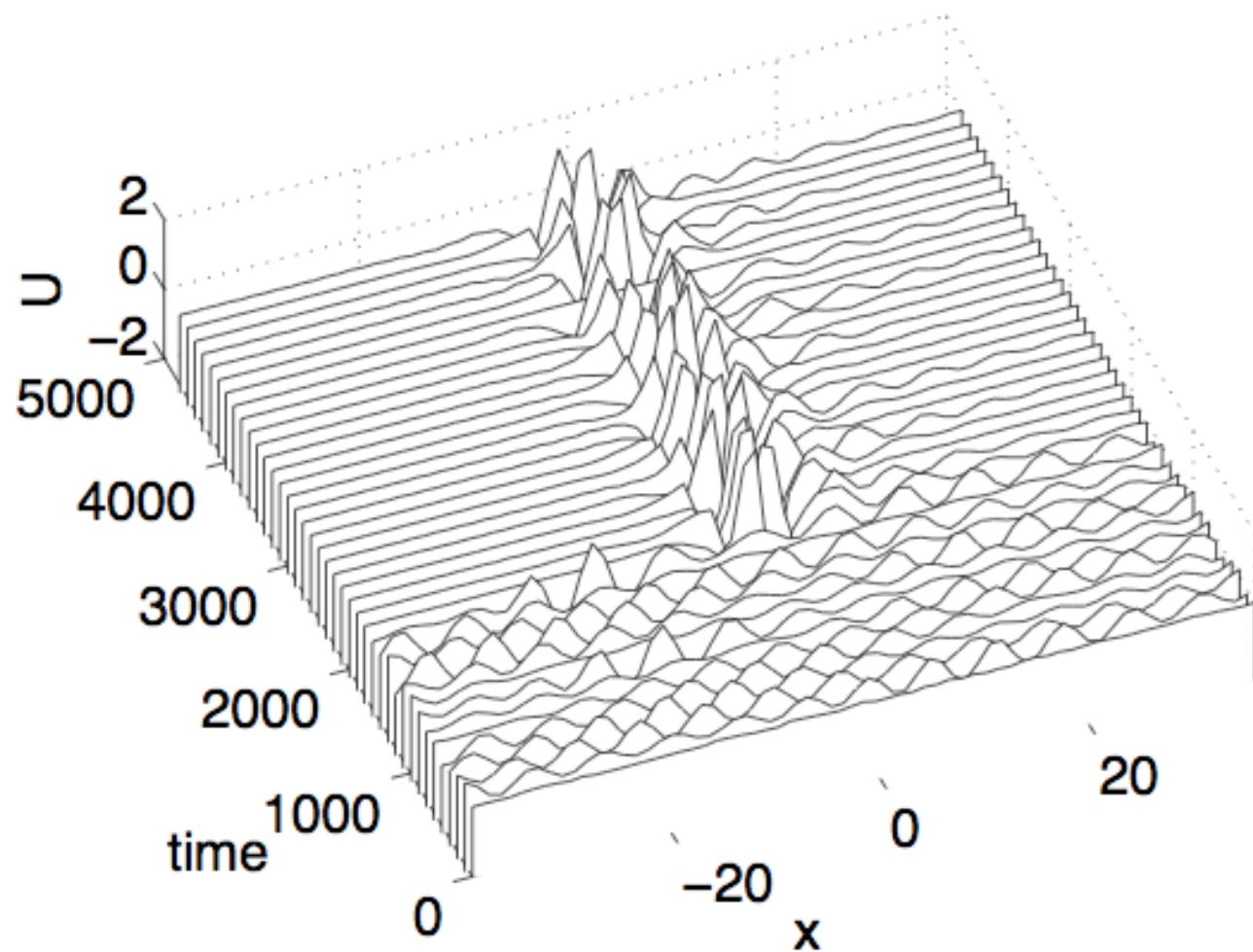
$$W_{xt} - W = [-\gamma_2 W - cV]_x$$

$$g(t) = \frac{2g_0}{1 + \|U\|^2}$$

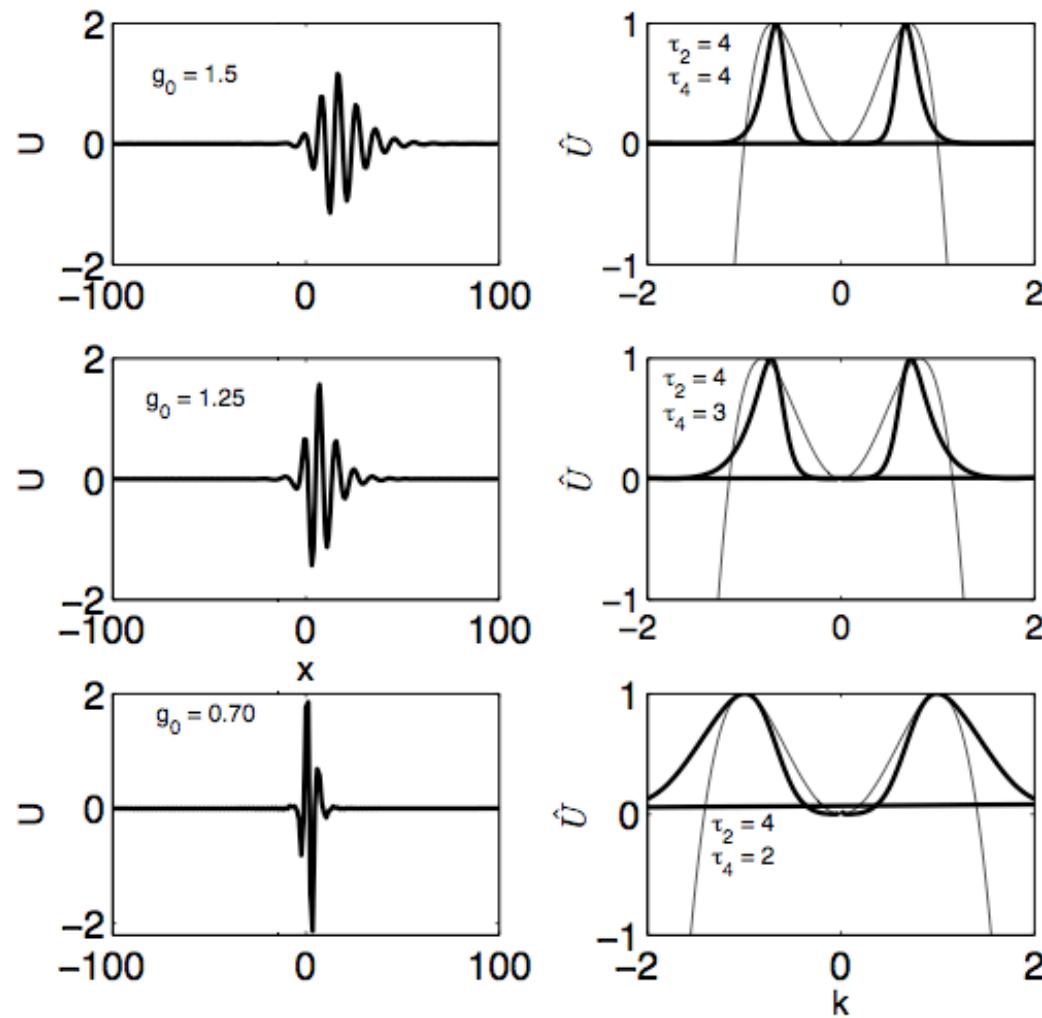
Mode-Locking Dynamics



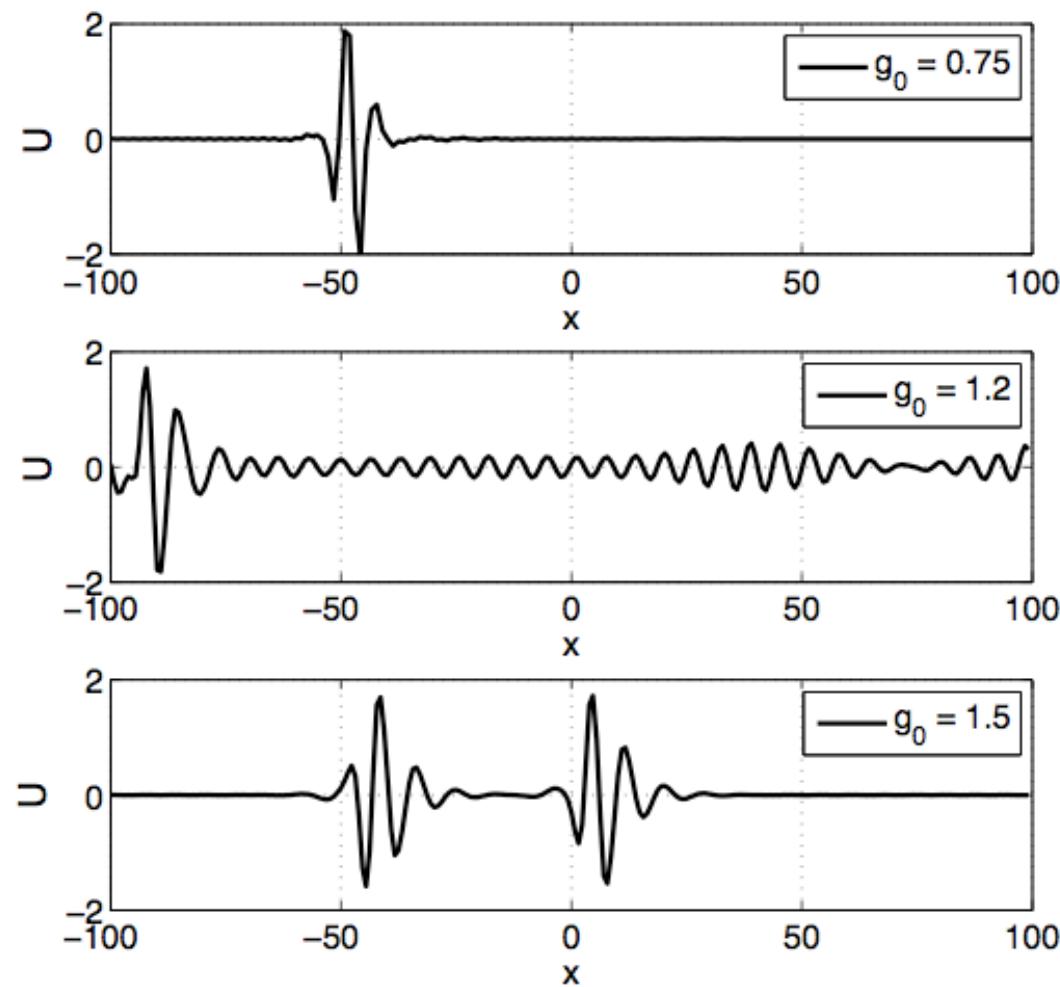
Spatio-Temporal Dynamics



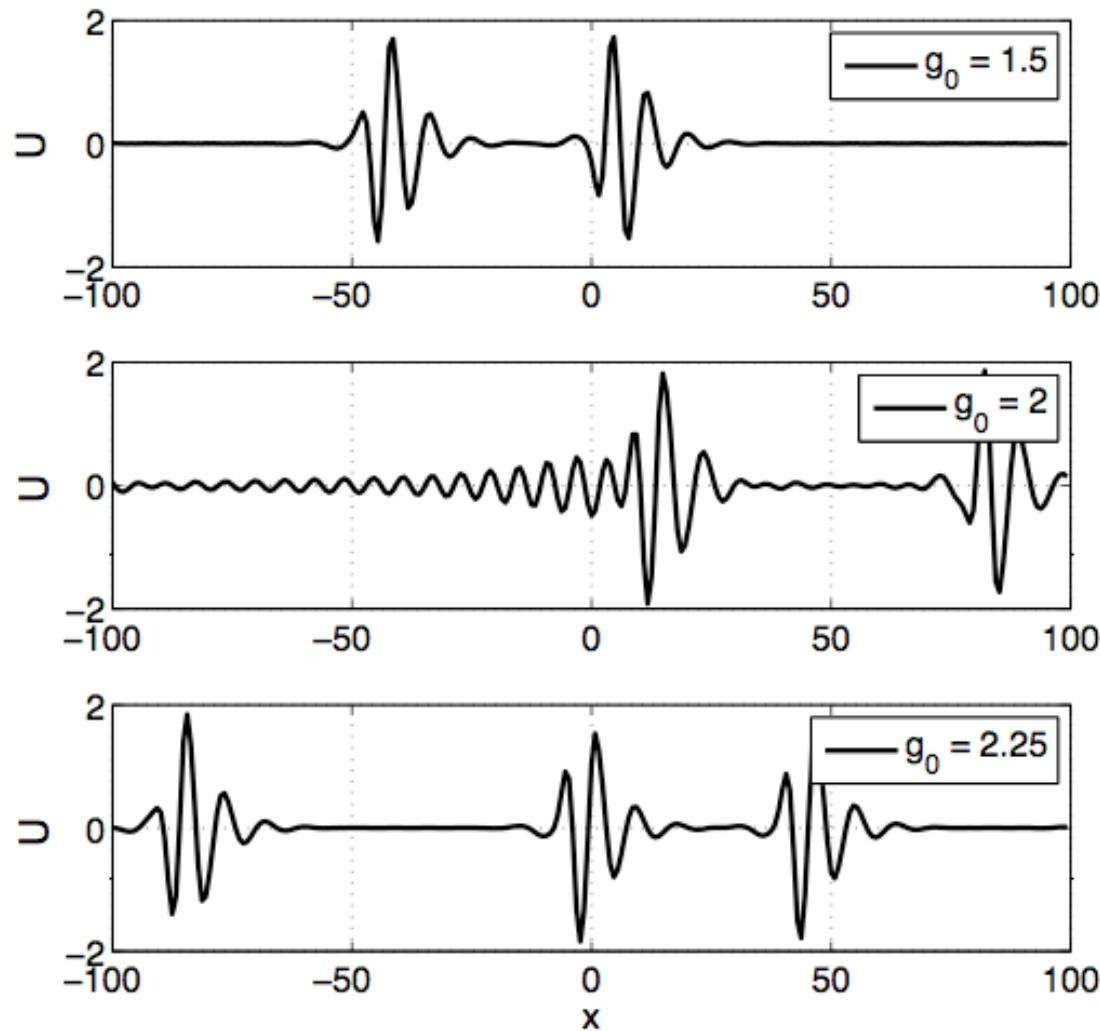
Bandwidth Engineering



Multi-Pulsing Instability



Multi-Pulsing Instability



Short-Pulse Mode-Locking

Mode-Locking in the Few Femtosecond Regime Using Waveguide Arrays and the Coupled Short-Pulse Equations

E. Farnum and J. Nathan Kutz

To Appear: IEEE J. Selected Topics in Quantum Electronics (2011)



APPLIED MATHEMATICS
UNIVERSITY of WASHINGTON

Short Comings

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left[\int_{-\infty}^t \chi^{(1)}(t-\tau) E(\tau, x) d\tau + \chi^{(3)} E^3 \right]$$

What about time-frequency response of nonlinearity?

Center-Frequency (NLS) Description

$$\begin{aligned} iQ_Z - \frac{k''}{2}Q_{TT} + \sum_{n=3}^{\infty} \beta_n \partial_T^{(n)} Q \\ + \left(\alpha + i\alpha_1 \frac{\partial}{\partial t} \right) \left(Q(Z, T) \int_0^\infty R(\xi) |Q(Z, T - \xi)|^2 d\xi \right) = 0 \end{aligned}$$

Super-continuum generation model (Dudley et al)

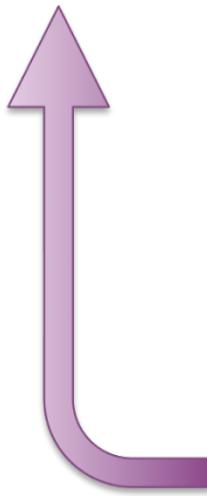
Short-Pulse Equivalent

$$E_{ZT} - E - \frac{1}{6} \left(E \int_0^\infty R(\xi) E^2(Z, T - \xi) d\xi \right)_{TT} = 0$$

Correct Short-Pulse Evolution?

Chi^2 vs omega response

It is concluded that the currently available data are insufficient and should be augmented to provide better guidance for experimental work. As a result, very little can be definitively said about $\bar{\chi}^{(3)}(\omega)$.



- D. Milam, “Review and assessment of measured values of the nonlinear refractive-index coefficient of fused silica,” App. Opt. **37**, 546-550 (1998).
- S. Santran, L. Canioni, L. Sarger, T. Cardinal and E. Fargin, “Precise and absolute measurements of the complex third-order optical susceptibility,” J. Opt. Soc. Am. B **21**, 2180-2190 (2004).
- M. Samoc, “Third-order nonlinear optical materials: practical issues and theoretical challenges,” J. Mol. Model., (2010) DOI 10.1007/s00894-010-0856-8.

Summary & Conclusions

High-Energy

- Need to suppress multi-pulsing
- New models needed
- Engineering of nonlinear losses
- Dimensionality for energy increase (light bullets)

Ultra-Short

- Need theory beyond center-frequency expansion
- Nonlinearity still needs a more advanced treatment

