Strong Collapse Turbulence in Quintic Nonlinear Schrödinger Equation

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Short pulse

- Ultrashort pulse of light: Picosecond, femtosecond (10^{-15} second), attosecond (10^{-18} second) pulses.
- Rogue waves.

- Main assumptions of cubic NLSE: (a) The response of material attains a quasi-steady state, (b) Pulse width is large in comparison to the oscillations of carrier frequency.
- HNLSE
- SPE
- Valid model for rogue waves?

- Ultrashort time duration, ultra-broad spectral bandwidth, high-average power.
- Applications: Frequency metrology, Terahertz generation and detection, optical coherence tomography, etc.

Quintic NLS

$$\boxed{i\psi_t + \square^2\psi + \alpha|\psi|^2\psi + \beta|\psi|^4\psi = 0} \qquad \text{(NLS)}$$

- Bose-Einstein condensate, NLS type systems near the transition from supercritical to subcritical bifurcations, pattern formation (QGL), dissipative solitons in lasers.
- 1D CNLS is integrable with global existence of all solutions.
- $\beta>0$, $D\geq 1$ can develop a finite time singularity accompanied by dramatic contraction of the solution => wave collapse, collapse.

Regularized QNLS

- 1D Regularized QNLS (complex quintic GL)

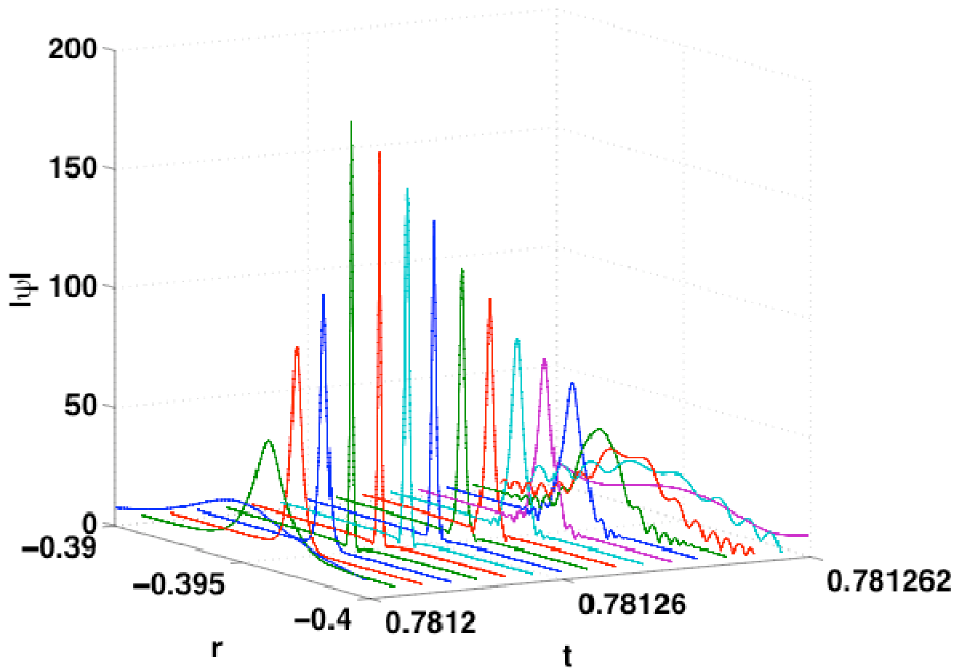
$$\boxed{i\psi_t + (1 - ia\varepsilon)\partial_x^2\psi + (1 + ic\varepsilon)|\psi|^4\psi = i\varepsilon\phi}$$

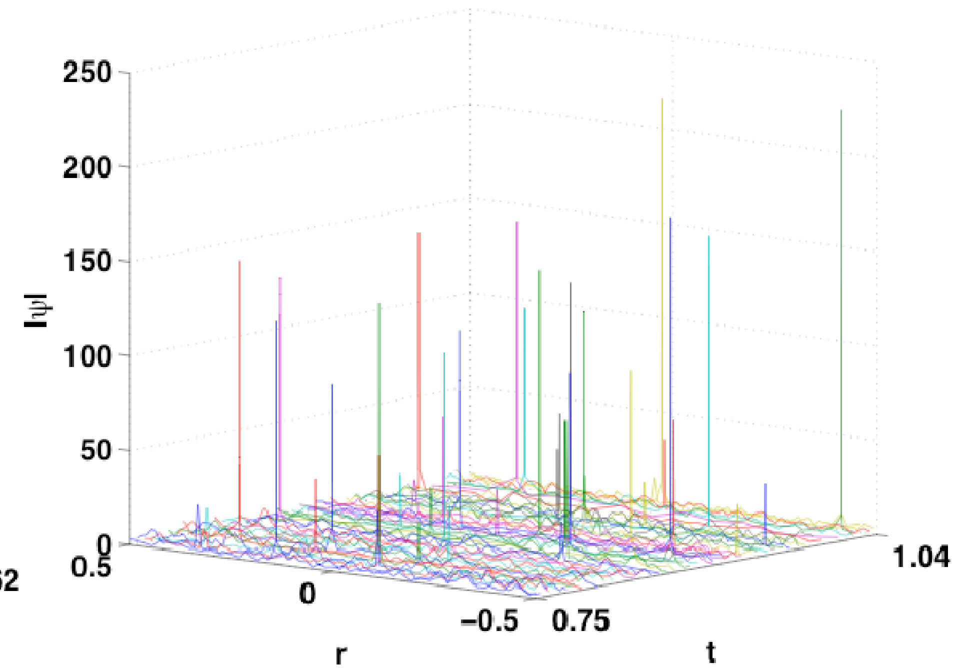
- is a small parameter.
- a ~ 1: Linear dissipation (angular-dependent loss or optical filtering).
- c ~ 1: Nonlinear dissipation (three-photon absorption in optics or four-body collisions).
- : general forcing.

e.g.,

Regularized QNLS

- Forcing => pumping of energy into the system.
- Formation of multiple collapse events randomly distributed in space and time.

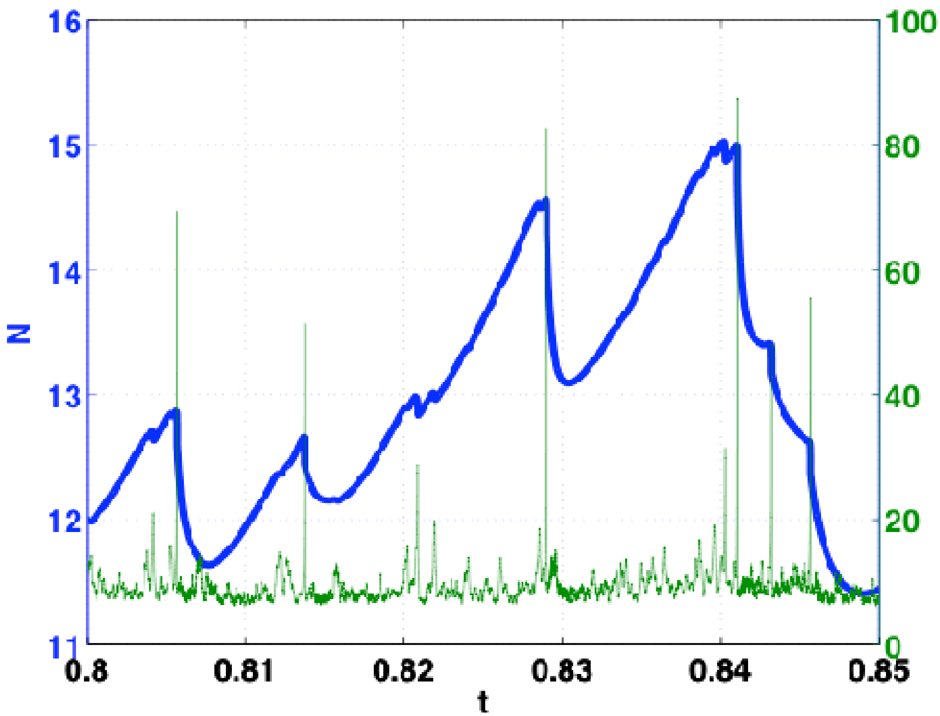


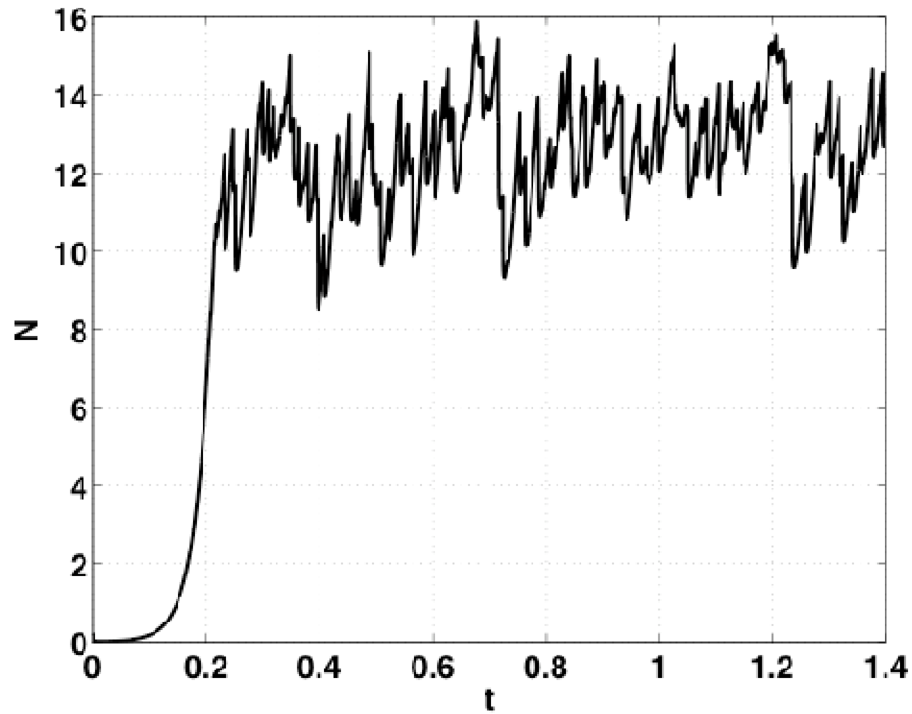


Regularized QNLS

- After the initial transient, statistical steady state (state of developed turbulence) is achieved.
- Forcing in average is compensated by dissipation.

(Number of particles, optical power, energy, wave action)





Self-similar form

- For $\varepsilon=0$, the collapsing solution of RQNLS has the self-similar form.

- For small $L<<1$,

$$V($$

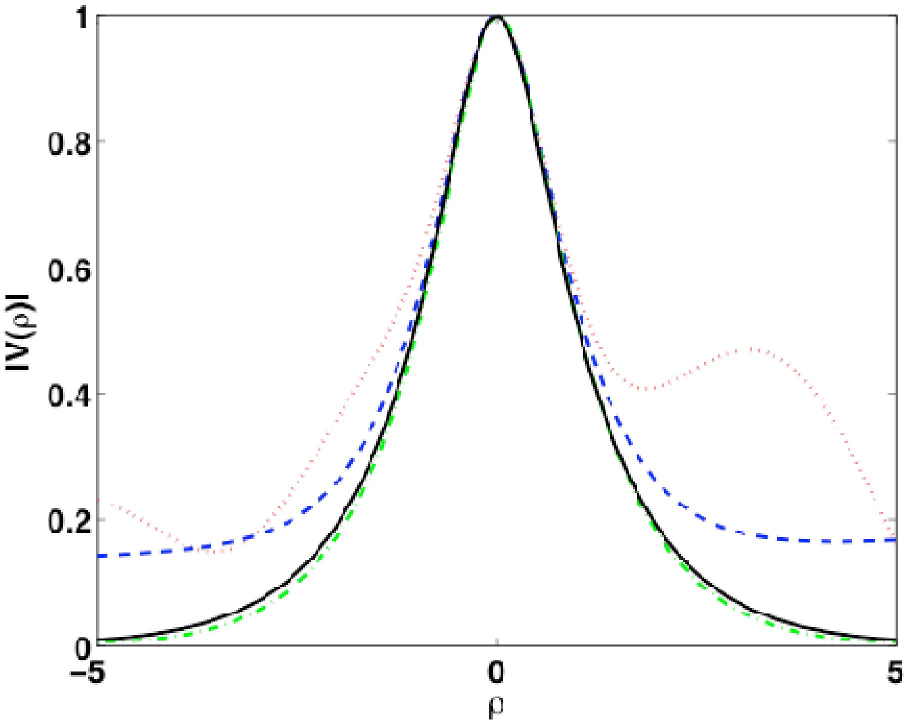
Self-similar form

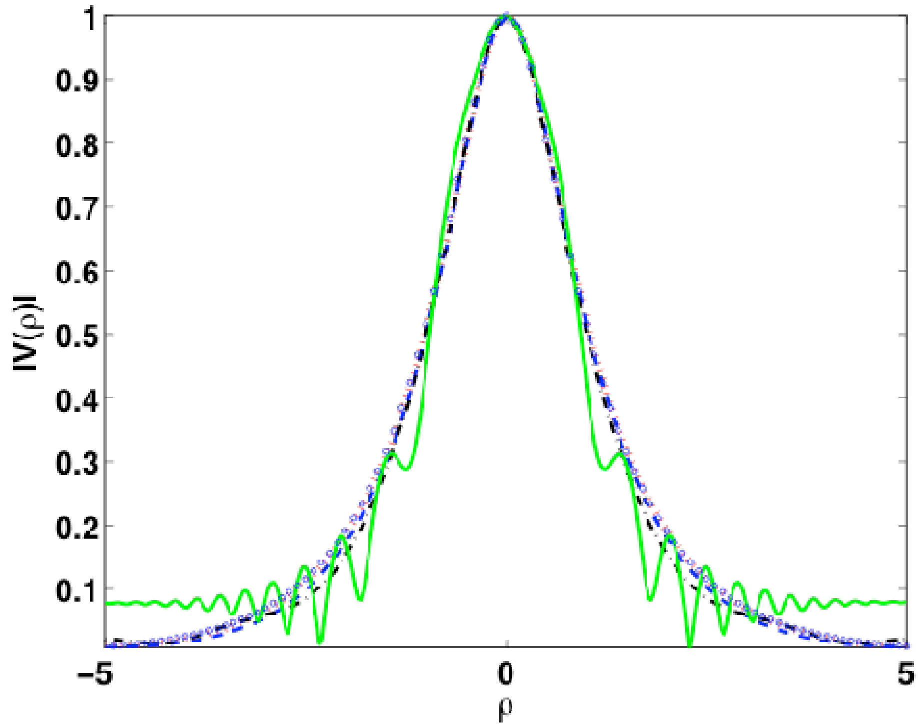
- Hamiltonian vanishes at the ground state soliton solution.
- The number of particles at the ground state,

- $N<N_c$; Collapse is impossible.
- As $L(t)$ decreases with $t \rightarrow t_0$ (time at which a singularity develops), $V(\rho) \longrightarrow R(\rho)$ for $\rho < 1$. => $N_{collapse} \longrightarrow N_c$
- The collapse of QNLS is strong.

Self-similar form

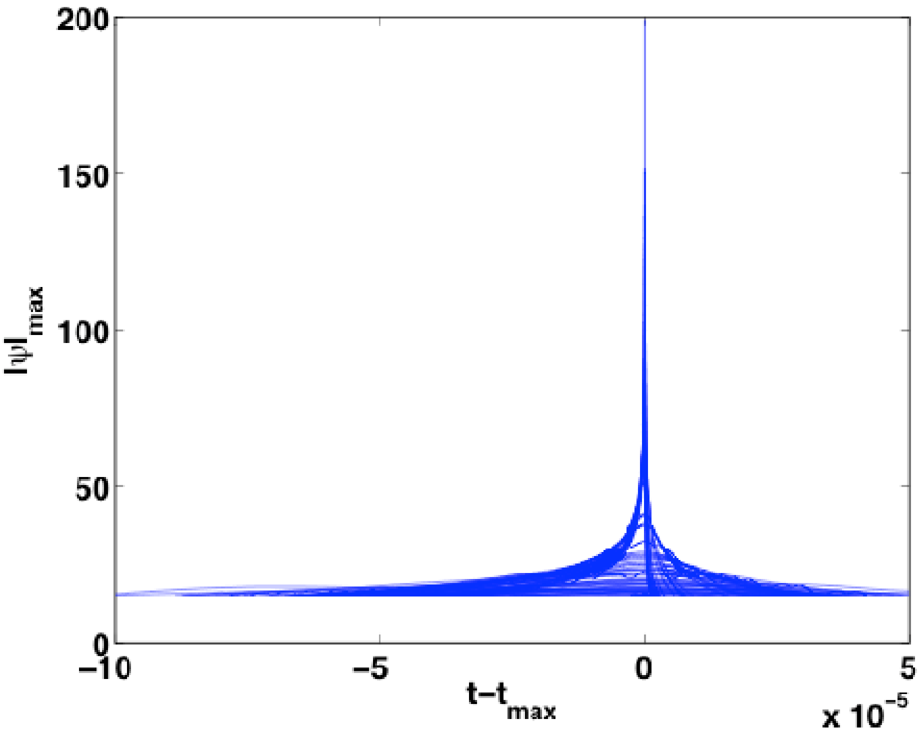
- RQNLS with $\varepsilon>0$ does not allow singular collapses.
- Collapse amplitude goes through a maximum at some $t=t_{max}$.

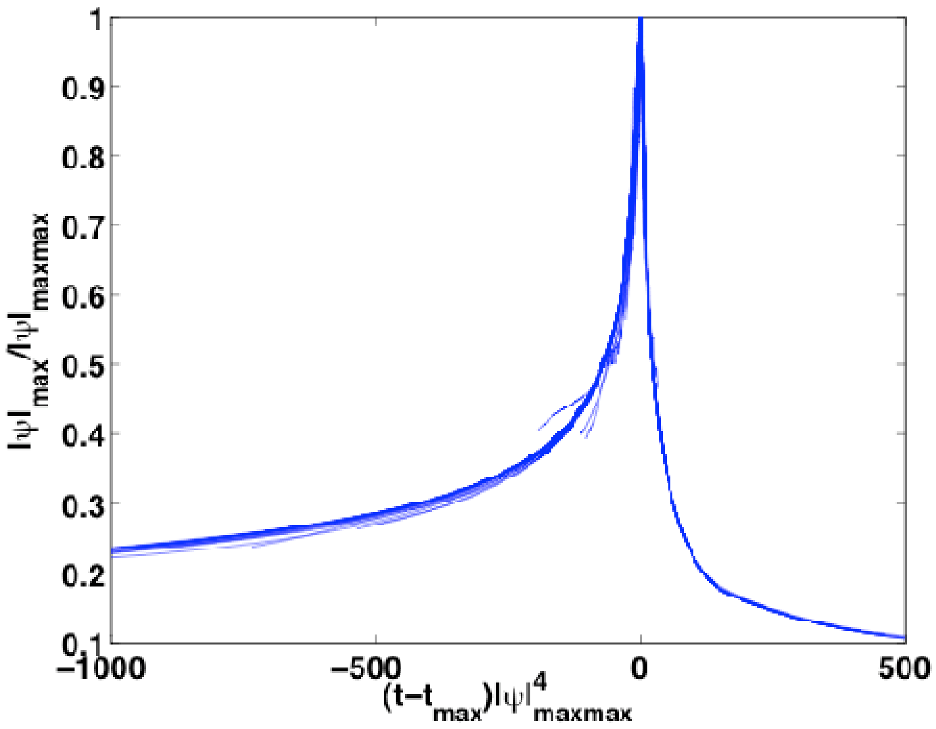




Strong collapse

- $\psi_{max}(t)$ in rescaled units shows a universal behavior.
- All collapse events in RQNLS are identical upto rescaling.



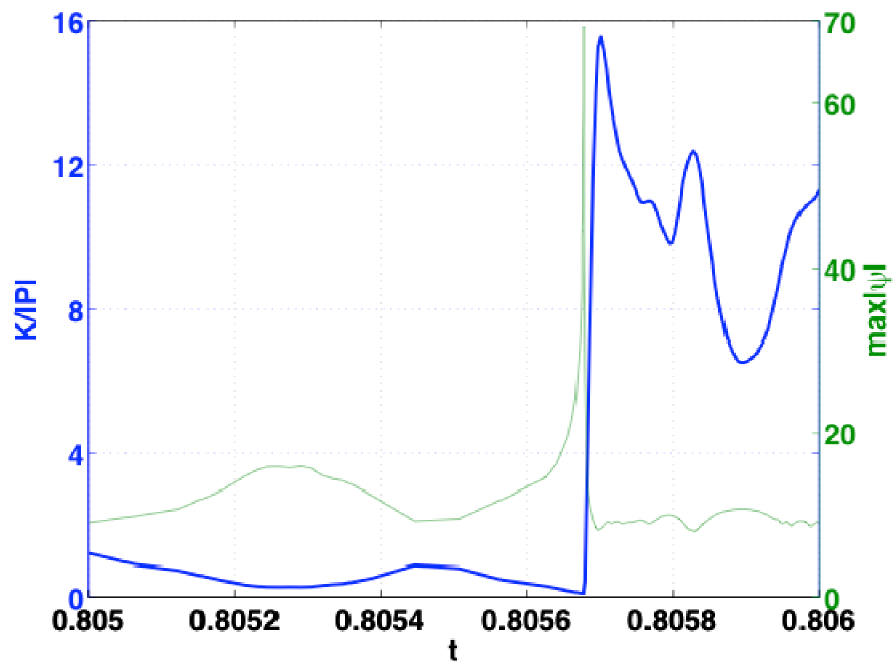


Strong collapse

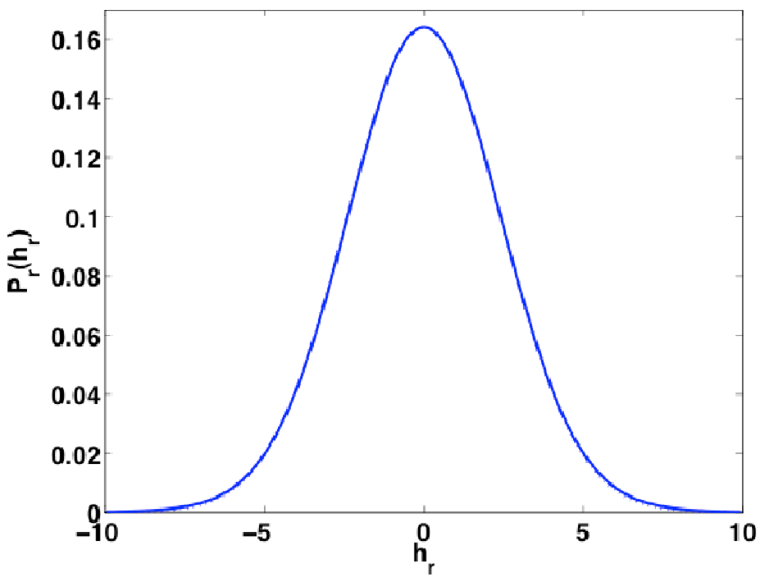
- Growth phase: Kinetic energy and potential energy nearly cancel each other inside the collapsing region.
- Decay phase: Kinetic energy dominates over potential energy.
- Superposition of many of outgoing (almost linear) waves forms a nearly random Gaussian field => seeds new collapse events.

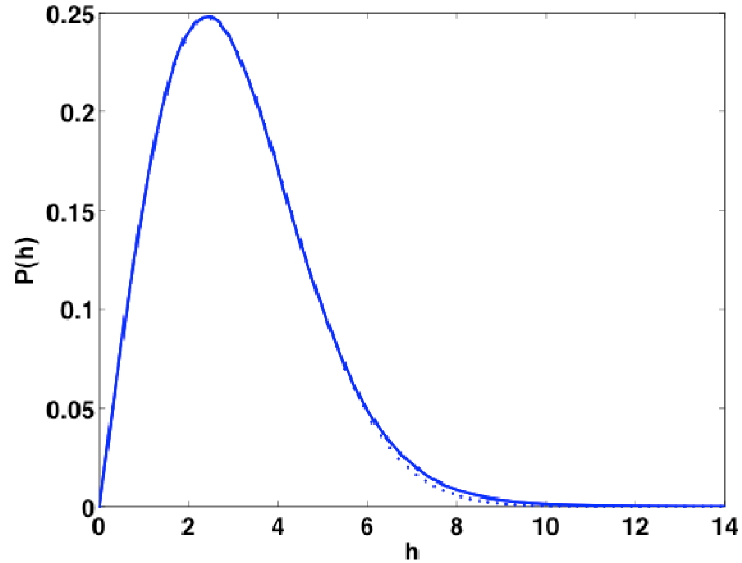
K=

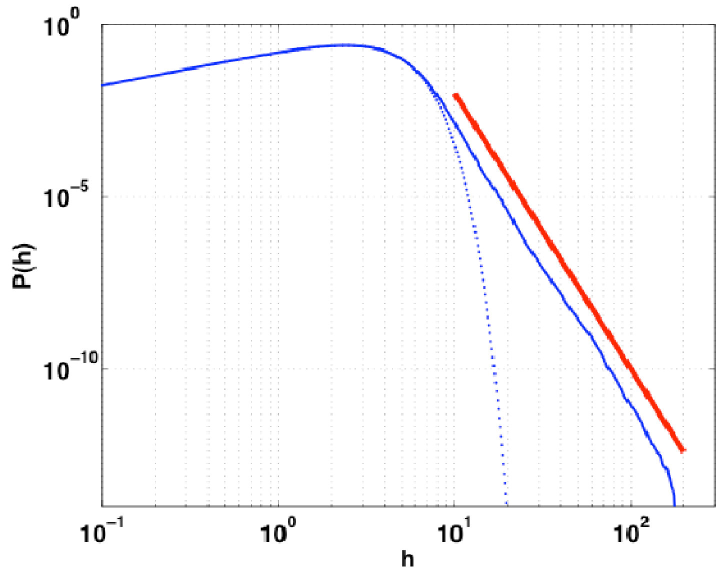
P=



Strong collapse

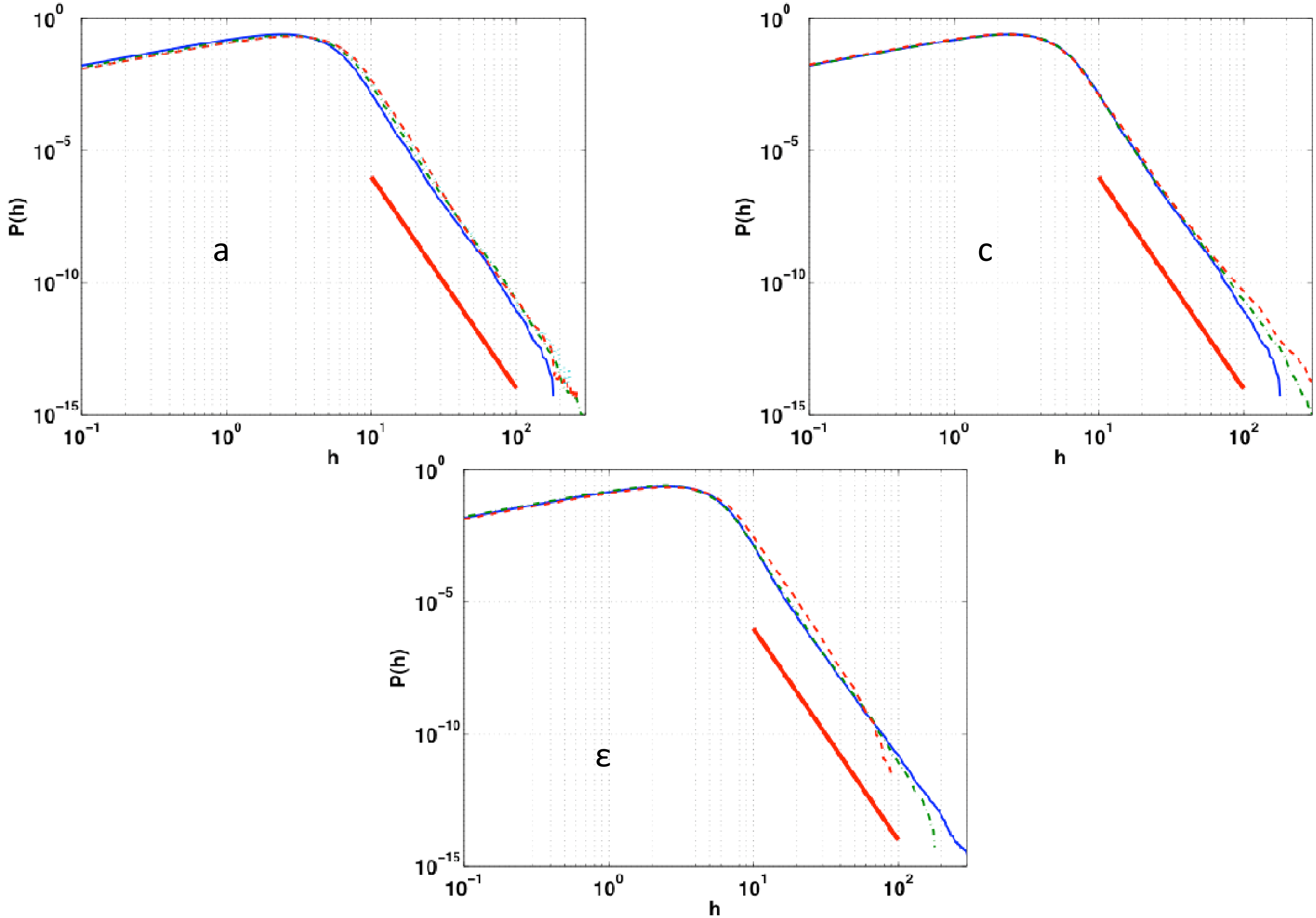






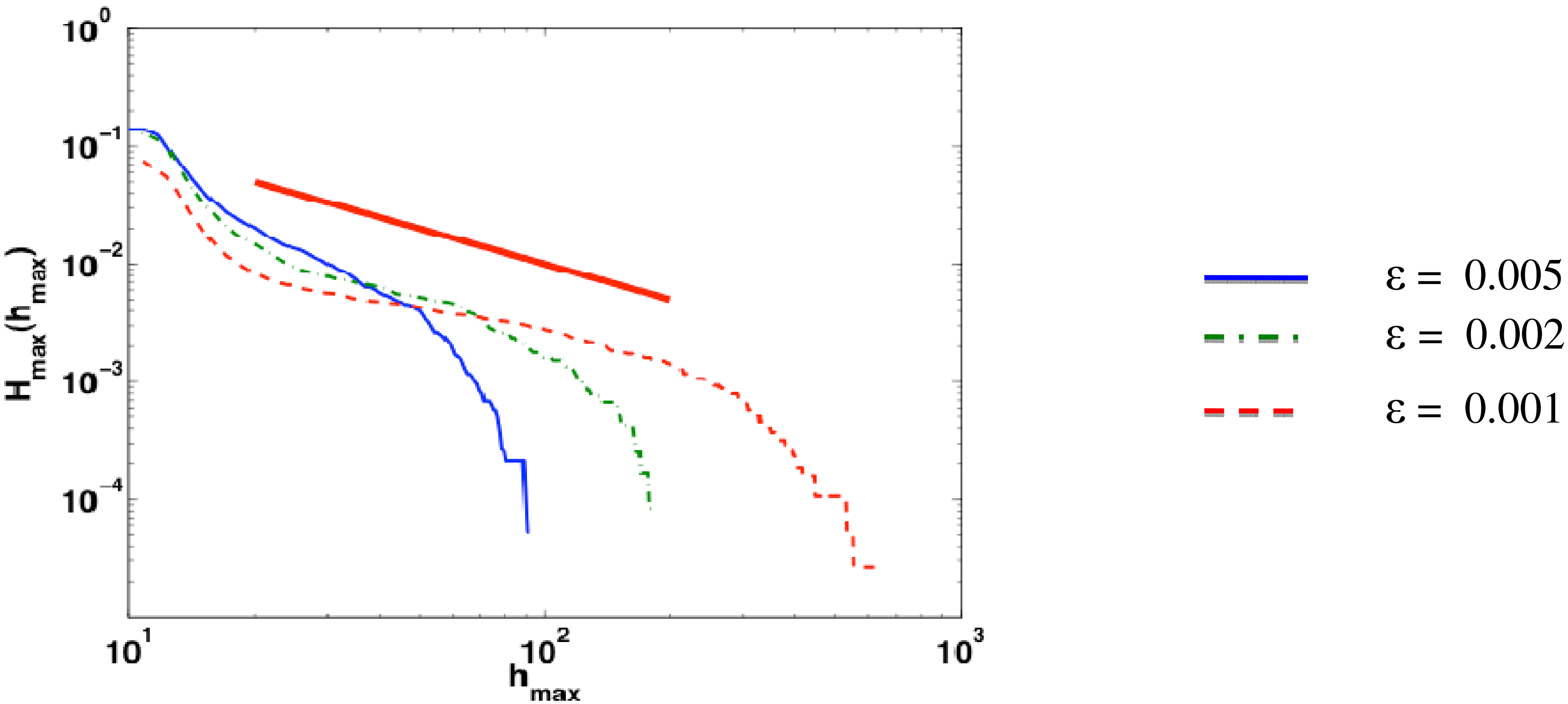
- Power-like tail ~ h^{-8}.
- Independent of type of forcing.
- Intermittency of optical turbulence.

Strong collapse



Strong collapse

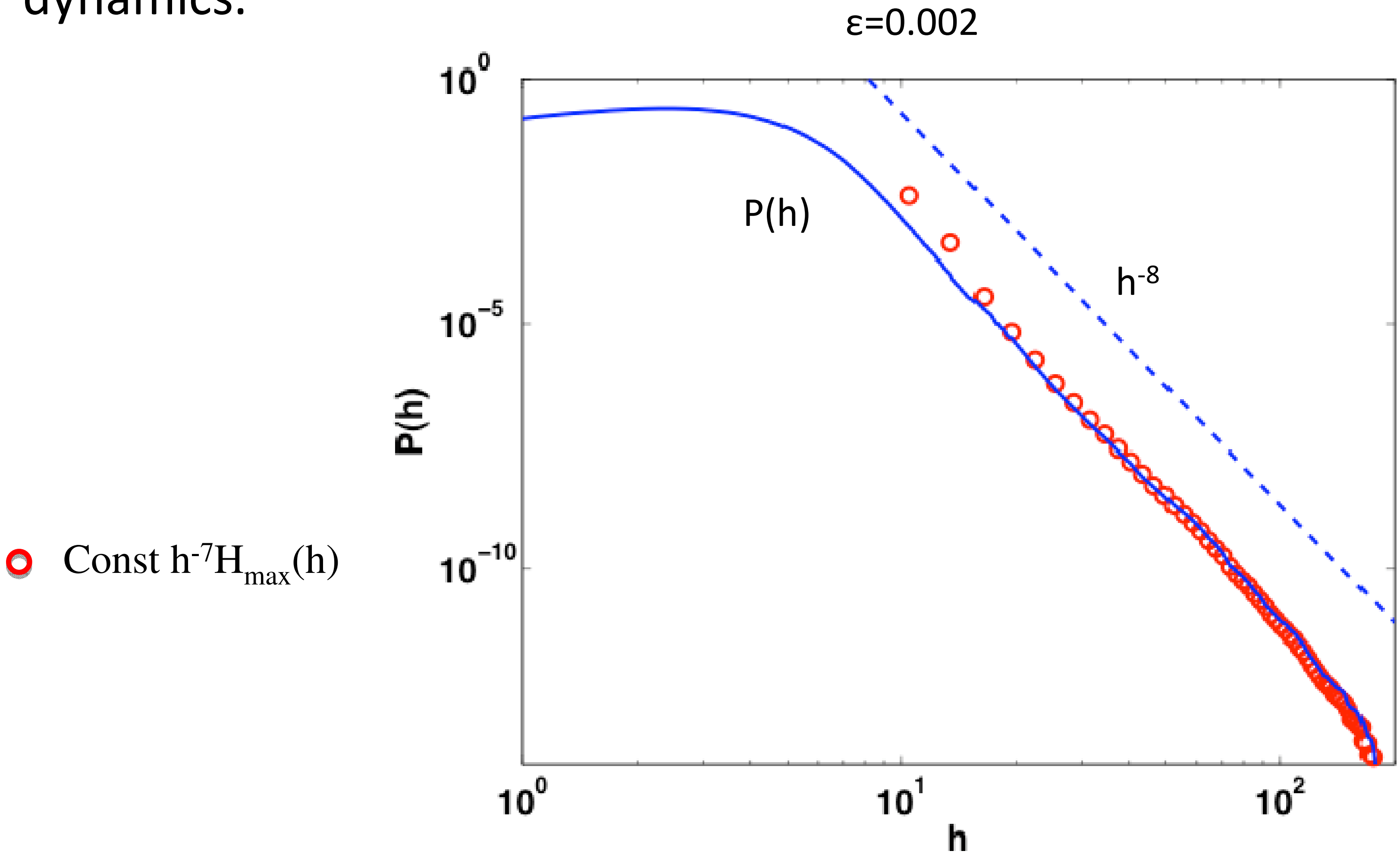
- $H_{max}(h)$: Probability of collapses that have maximum heights > h.
- $P(h) = \text{Const } h^{-7} H_{max}(h)$
- Power-like dependence h^{-1}?
- Conjecture first made in [IT92] appears to be wrong.



[IT92] H. Iwasaki and S. Toh, Progress of Theoretical Physics 87,1992.

Strong collapse

- Intermittency of optical turbulence of RQNLS is solely due to collapse dynamics.



Numerical scheme

- A version of Fourth order Split-step scheme [Yoshida].

$$C_1 = 1/(2(2-2^{1/3})),\ C_2 = (1-2^{1/3})/(2(2-2^{1/3})),\ C_3 = C_2,\ C_4 = C_1,$$
$$d_1 = 1/(2-2^{1/3}),\ d_2 = -2^{1/3}/(2-2^{1/3}),\ d_3 = d_1$$

- Generalization of the fourth order symplectic integration to non-Hamiltonian systems.
- Adaptive change of Δx.
- $\Delta t = q\Delta x^2/\pi$, $q<1/2$.

Conclusion

- In the statistical steady-state, the dynamical balance is achieved between forcing and both linear and nonlinear dissipation.
- RQNLS has multiple collapse events randomly distributed in space and time.
- PDF of amplitude fluctuations has strongly non-Gaussian tail with power-like behavior =>intermittency of strong collapse-dominated turbulence.
- Cumulative probability $H_{max}(h)$, (probability of the maximum amplitude of collapse that exceeds h) is not universal and depends on the parameter of RQNLS.
- For some range, $H_{max}(h)$ can be roughly estimated as h^{-1}, but only an intermediate asymptotic at best.
- Analytical form of $H_{max}(h)$ from the parameters of RQNLS ?

Strong Collapse