

Mental Packing and Unpacking in Mathematics

Mark Turner

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Fields Institute for Research
in Mathematical Sciences

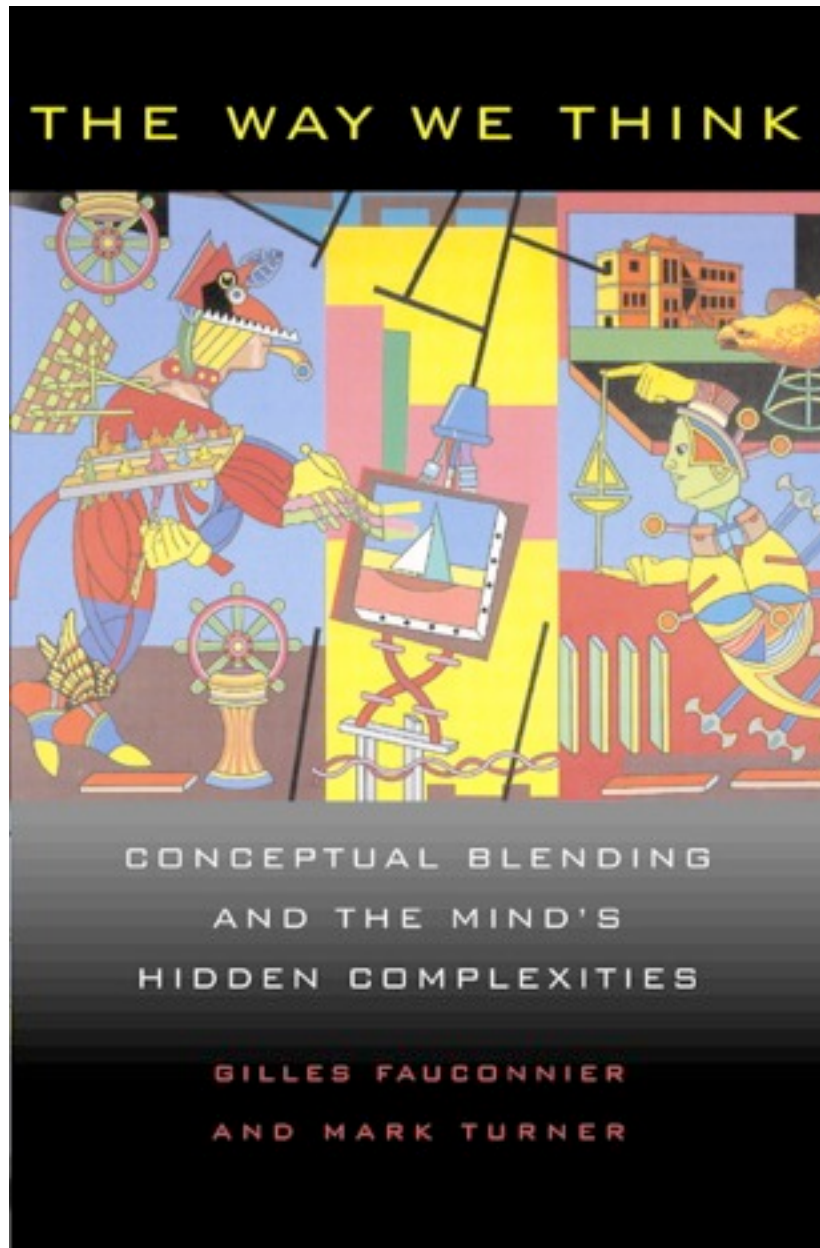
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Department of Cognitive Science



Packing and Unpacking

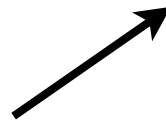
in Conceptual Integration Networks



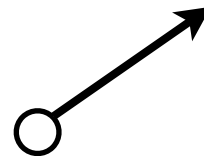
*The Way We Think:
Conceptual Blending and
the Mind's Hidden
Complexities*

Gilles Fauconnier & Mark
Turner, Basic Books, 2002

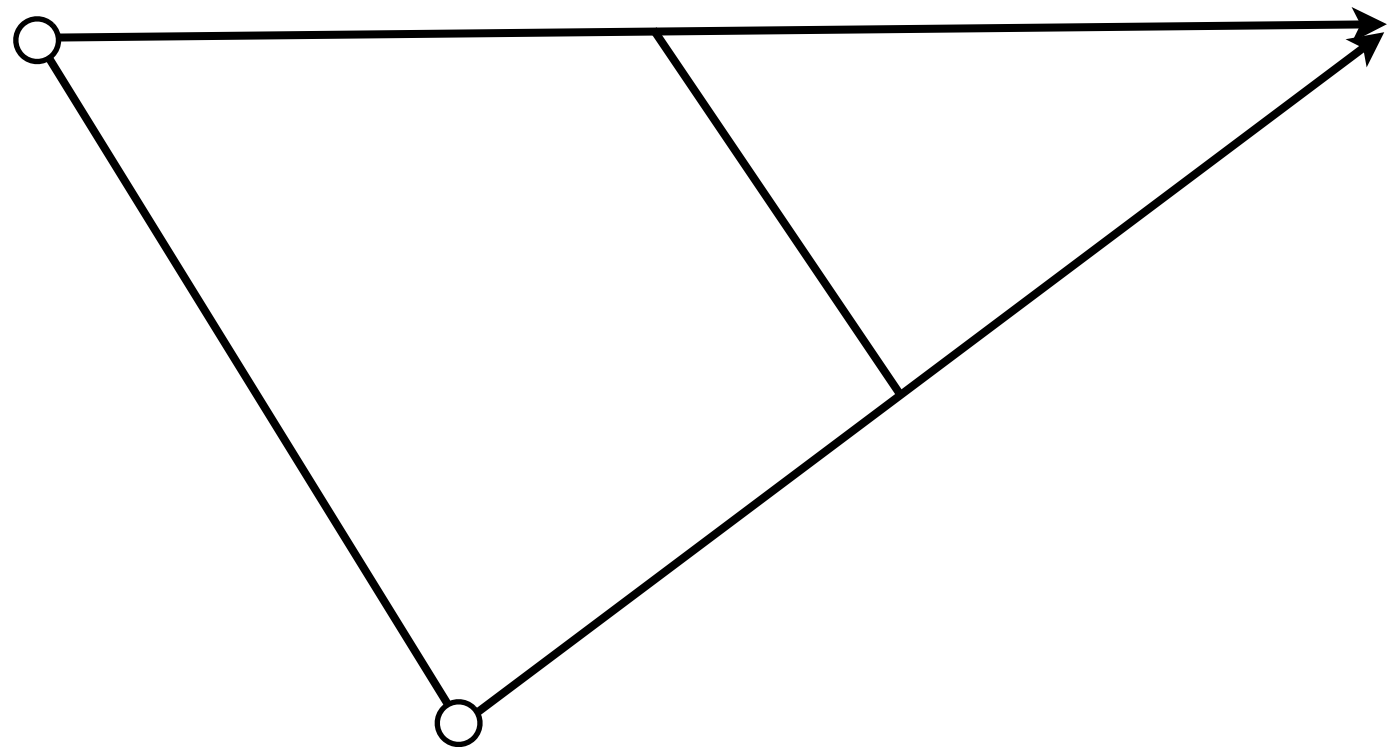
blending.stanford.edu



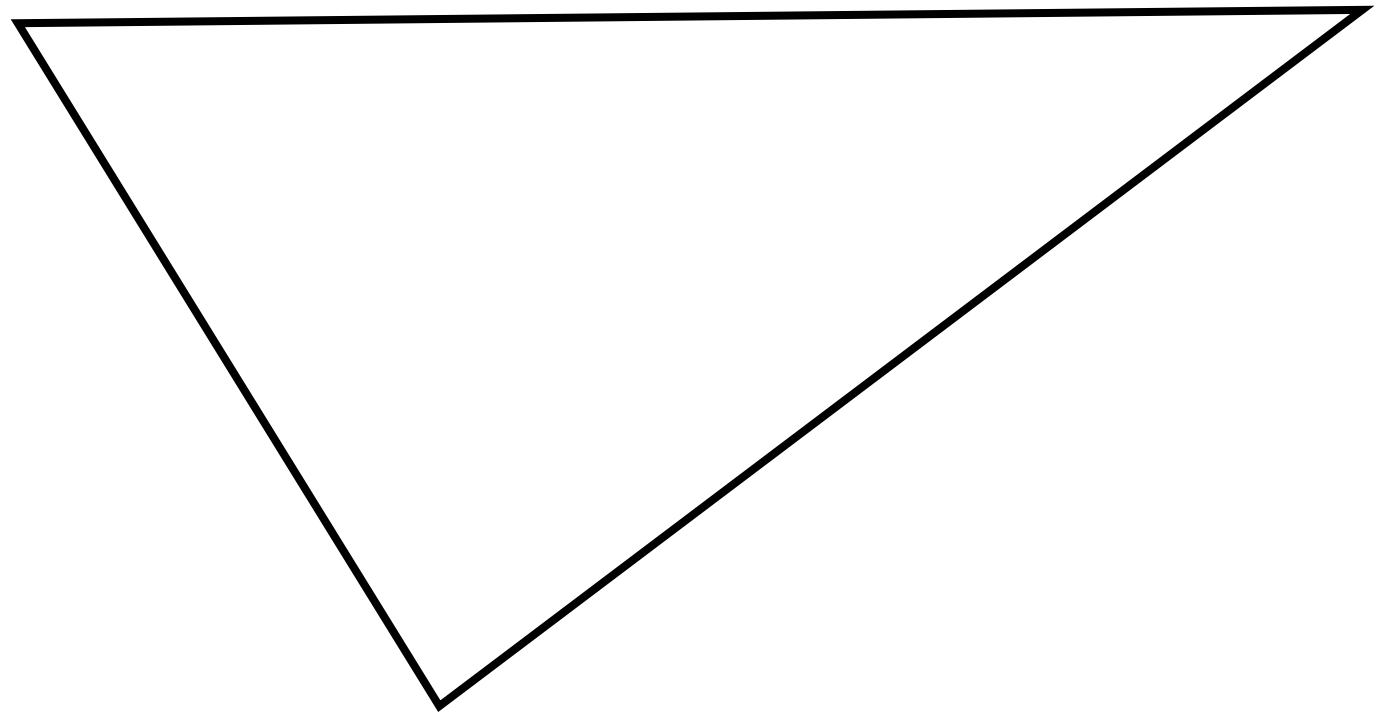
Constant Bearing, Decreasing Range



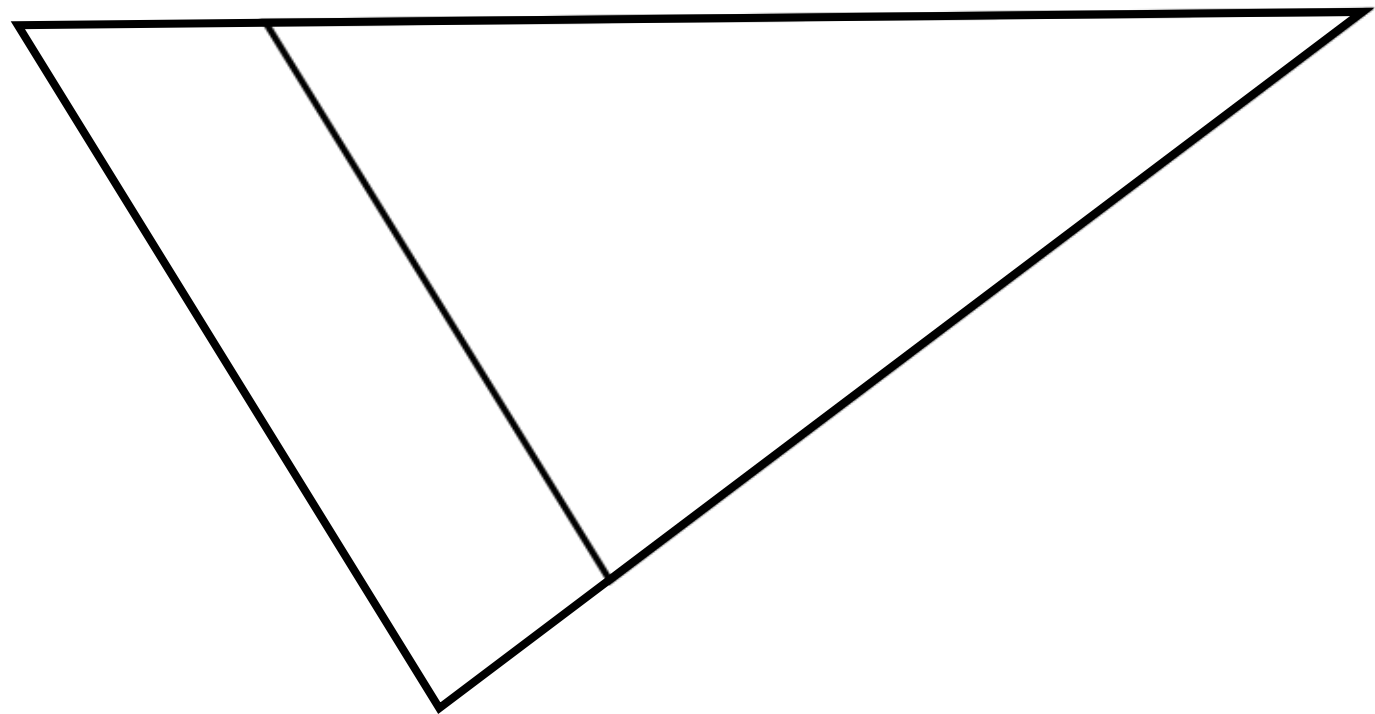
Constant Bearing, Decreasing Range



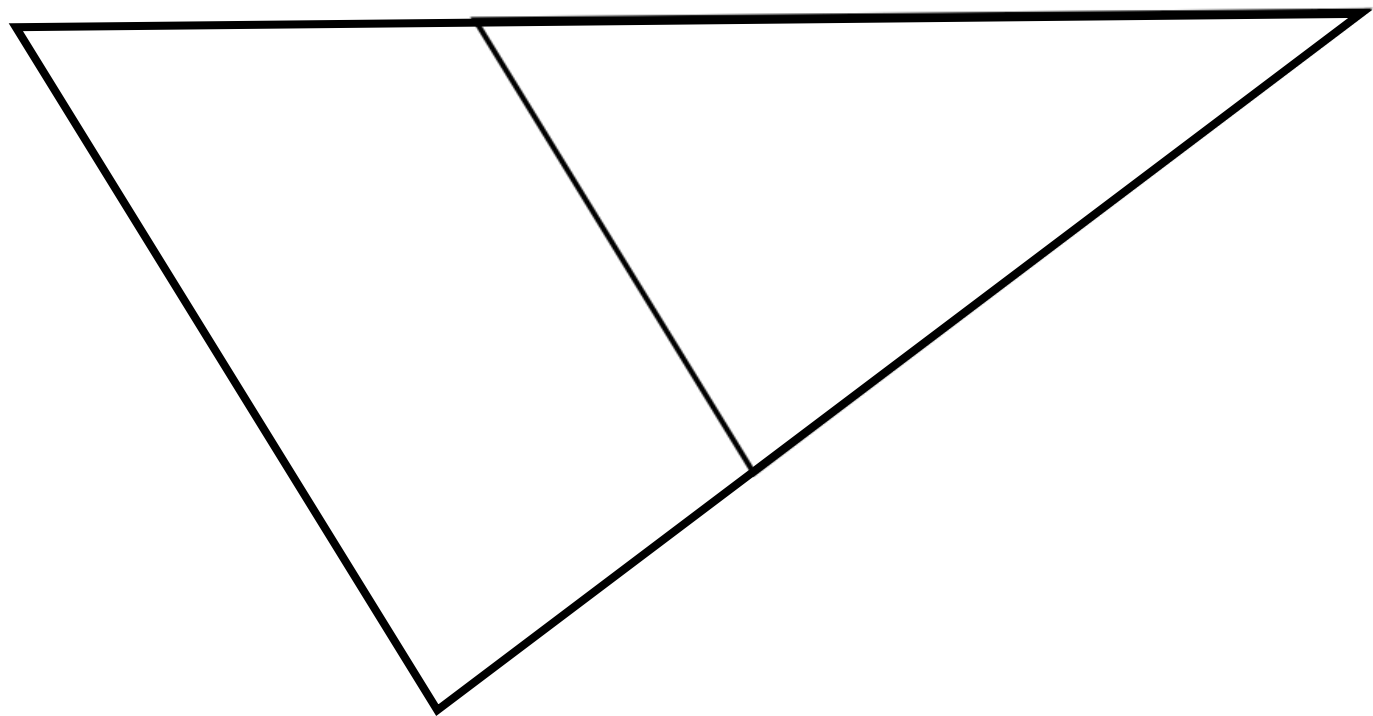
Constant Bearing, Decreasing Range



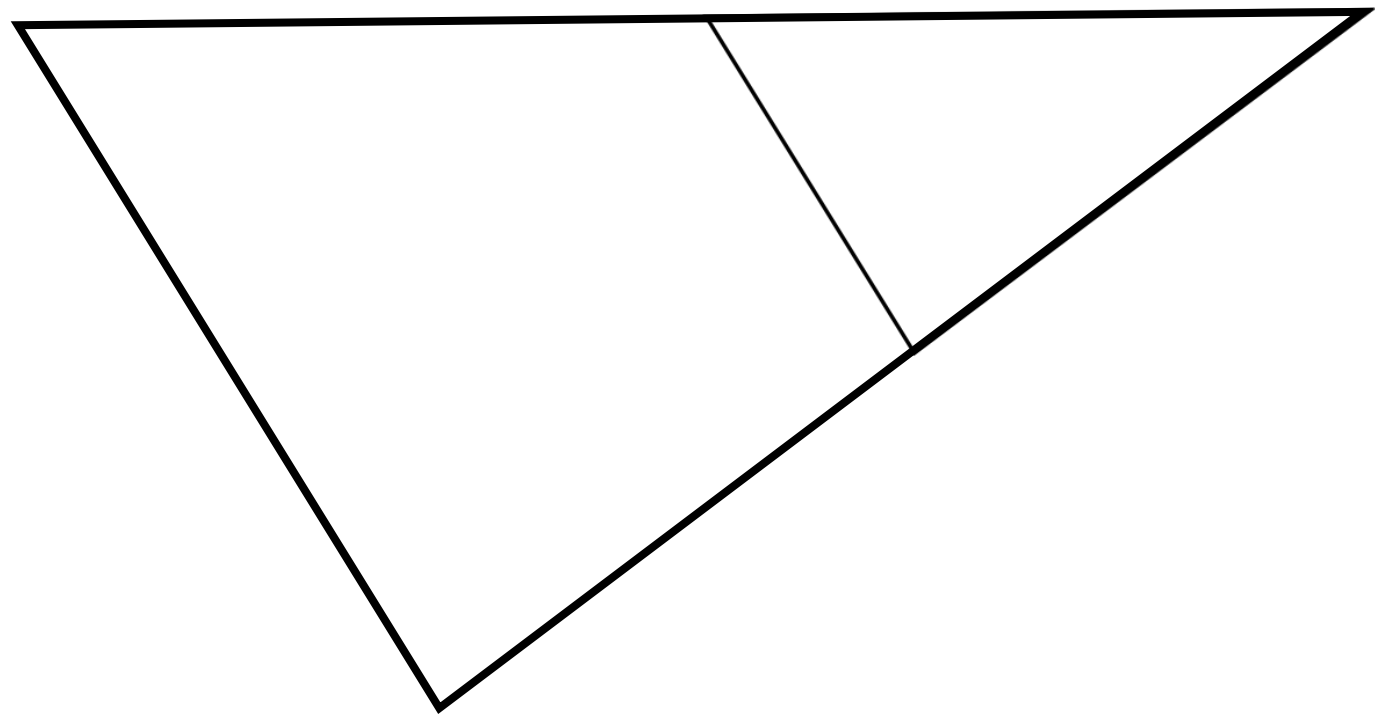
Constant Bearing, Decreasing Range



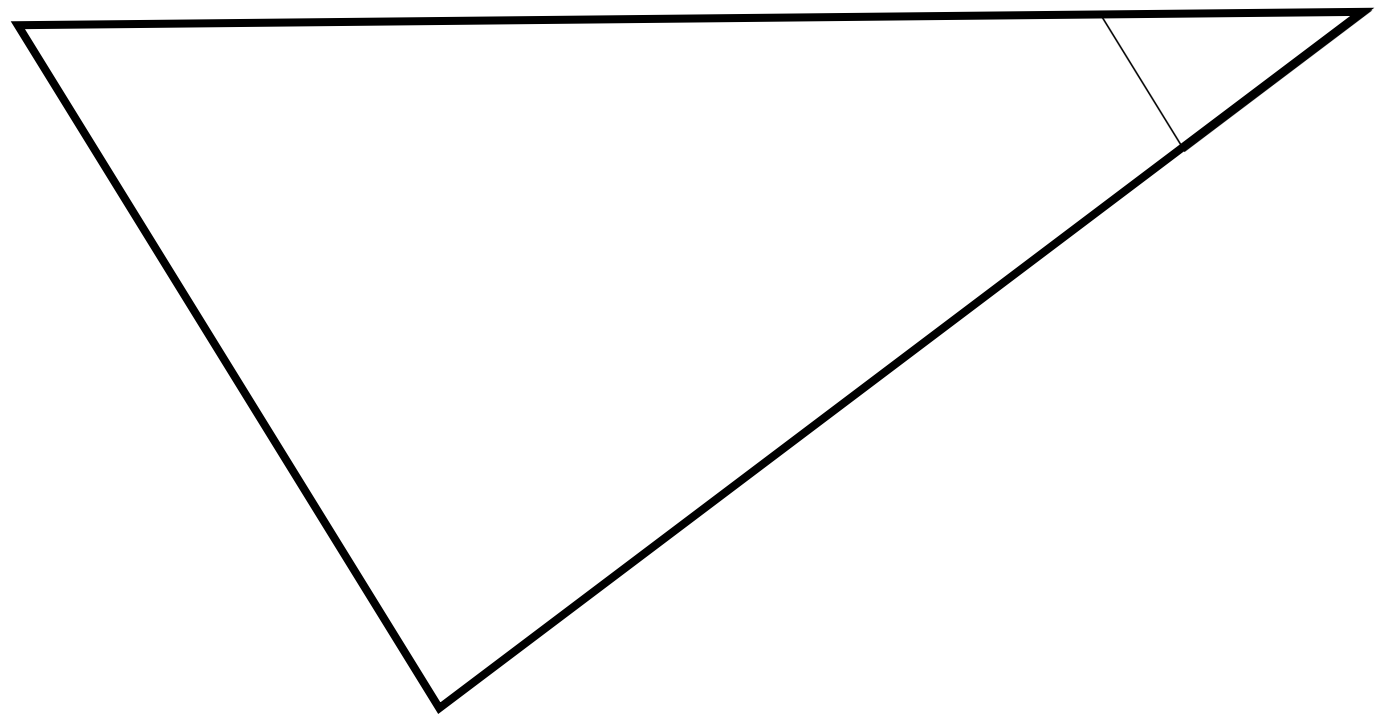
Constant Bearing, Decreasing Range



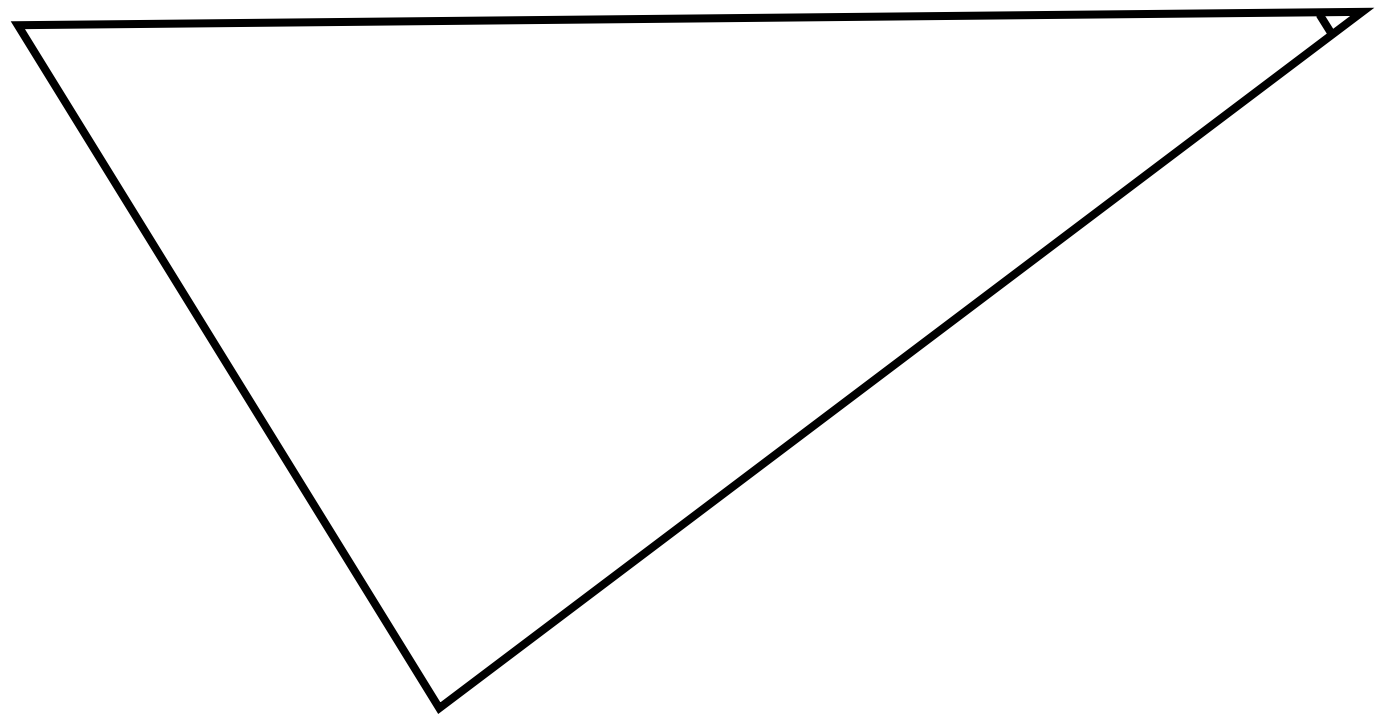
Constant Bearing, Decreasing Range



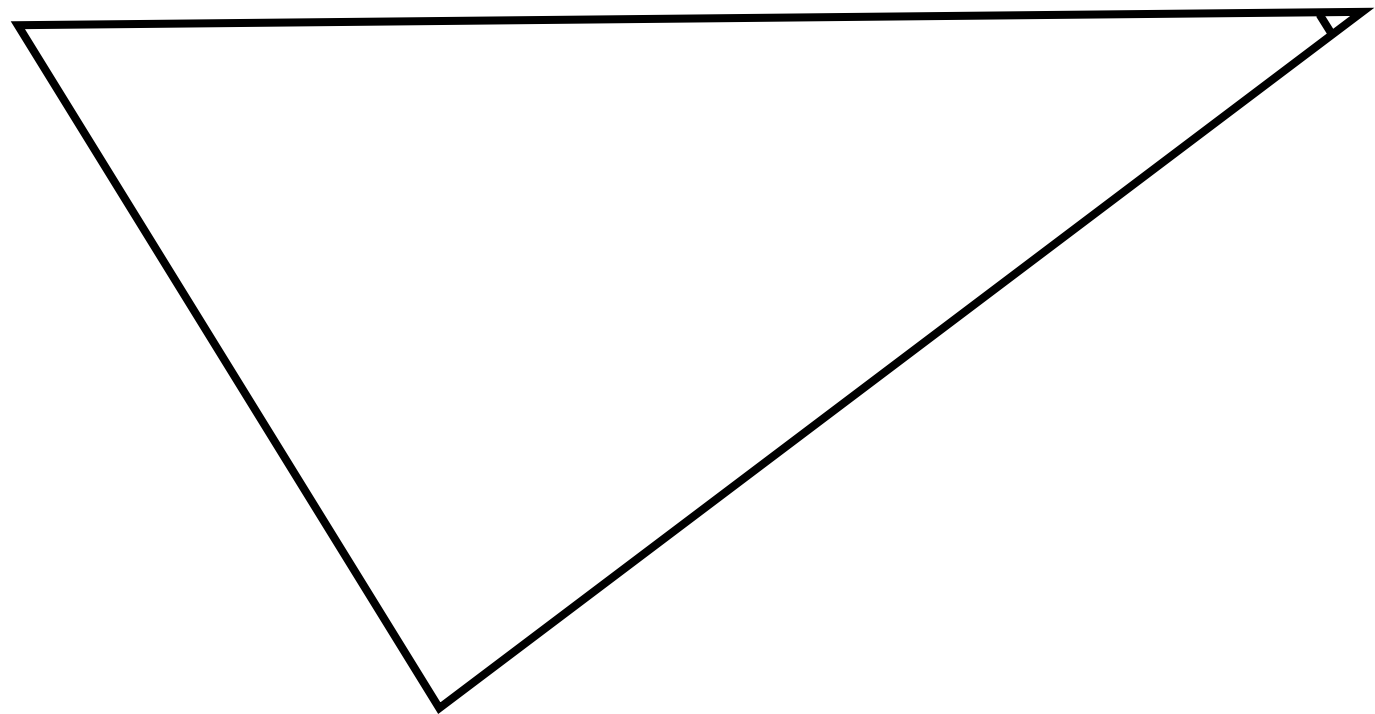
Constant Bearing, Decreasing Range



Constant Bearing, Decreasing Range



Constant Bearing, Decreasing Range



Constant Bearing, Decreasing Range



Constant Bearing, Decreasing Range

Emergent Meaning and Home Turf in the Blended Space


Constant Bearing, Decreasing Range

Emergent Meaning and Home Turf in the Blended Space


Constant Bearing, Decreasing Range


The Shrinking Triangle

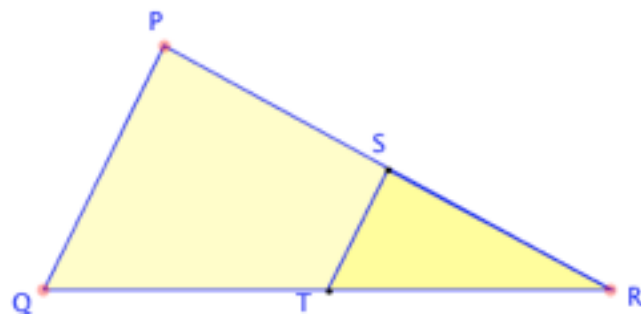
The Shrinking Triangle

 **There's no triangle to begin with. Imagining a single triangle on the water is already an amazing compression beyond the conceptual powers of other species. But once we have achieved the packed identity of a single triangle on the water . . .**

The Shrinking Triangle


 There's no triangle to begin with. Imagining a single triangle on the water is already an amazing compression beyond the conceptual powers of other species. But once we have achieved the packed identity of a single triangle on the water . . .

 Notice that even that *is not a triangle*. The sailor does not know any of the lengths of this triangle. Therefore, what he knows is *not a triangle*, but rather a set of constraints on the relevant (but unknown) triangle. There is an uncountable infinity of triangles fitting those constraints, but they are all packed into one.





The Shrinking Triangle

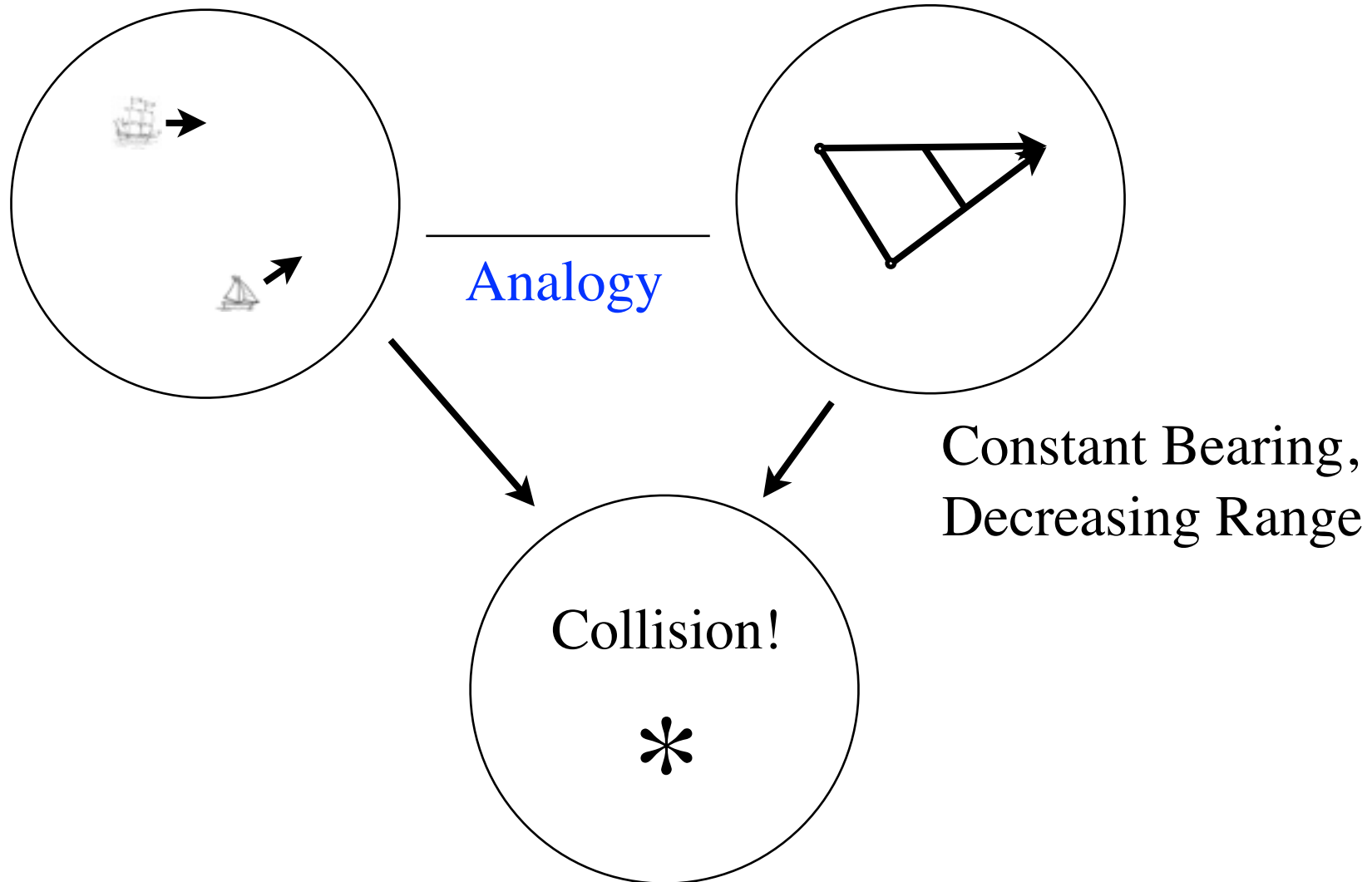
The Shrinking Triangle

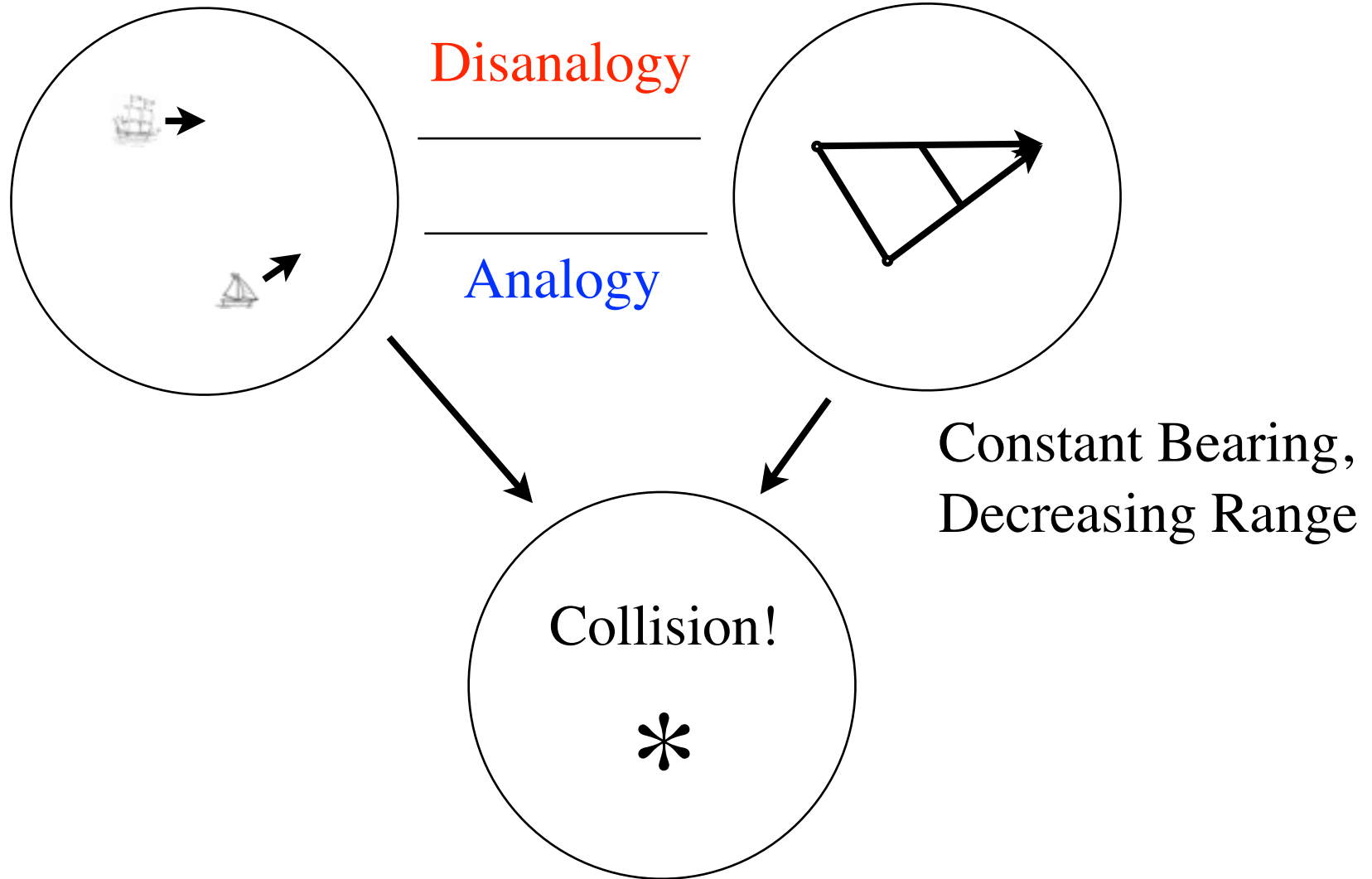
 **The next compression, and quite different from the compression of all the possible static triangles into one static triangle: Over time, there will be an uncountable infinity of such similar triangles. But each of them is static. There is no shrinking. The shrinking is an emergent phenomenon in the blend.**

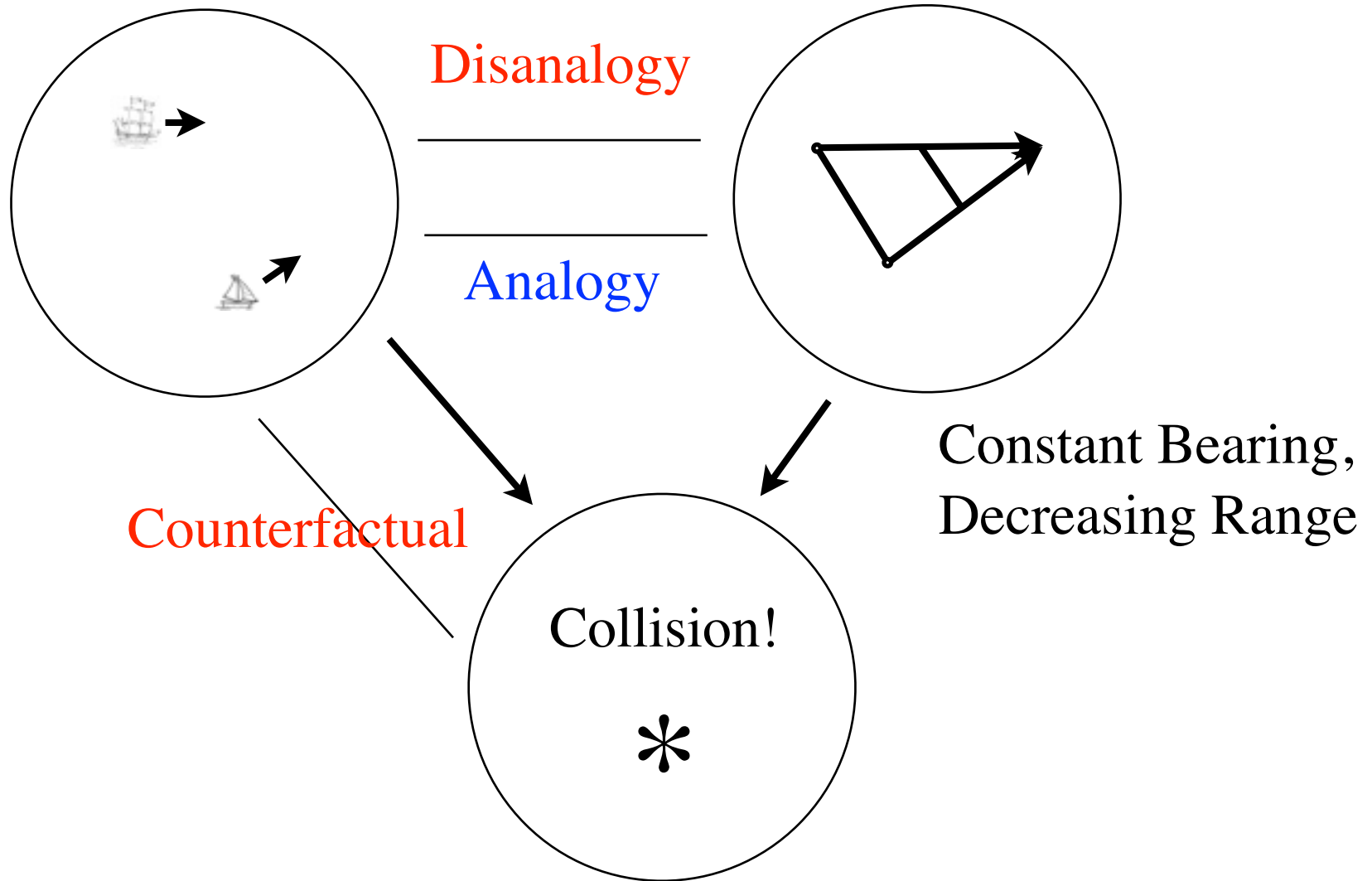
The Shrinking Triangle

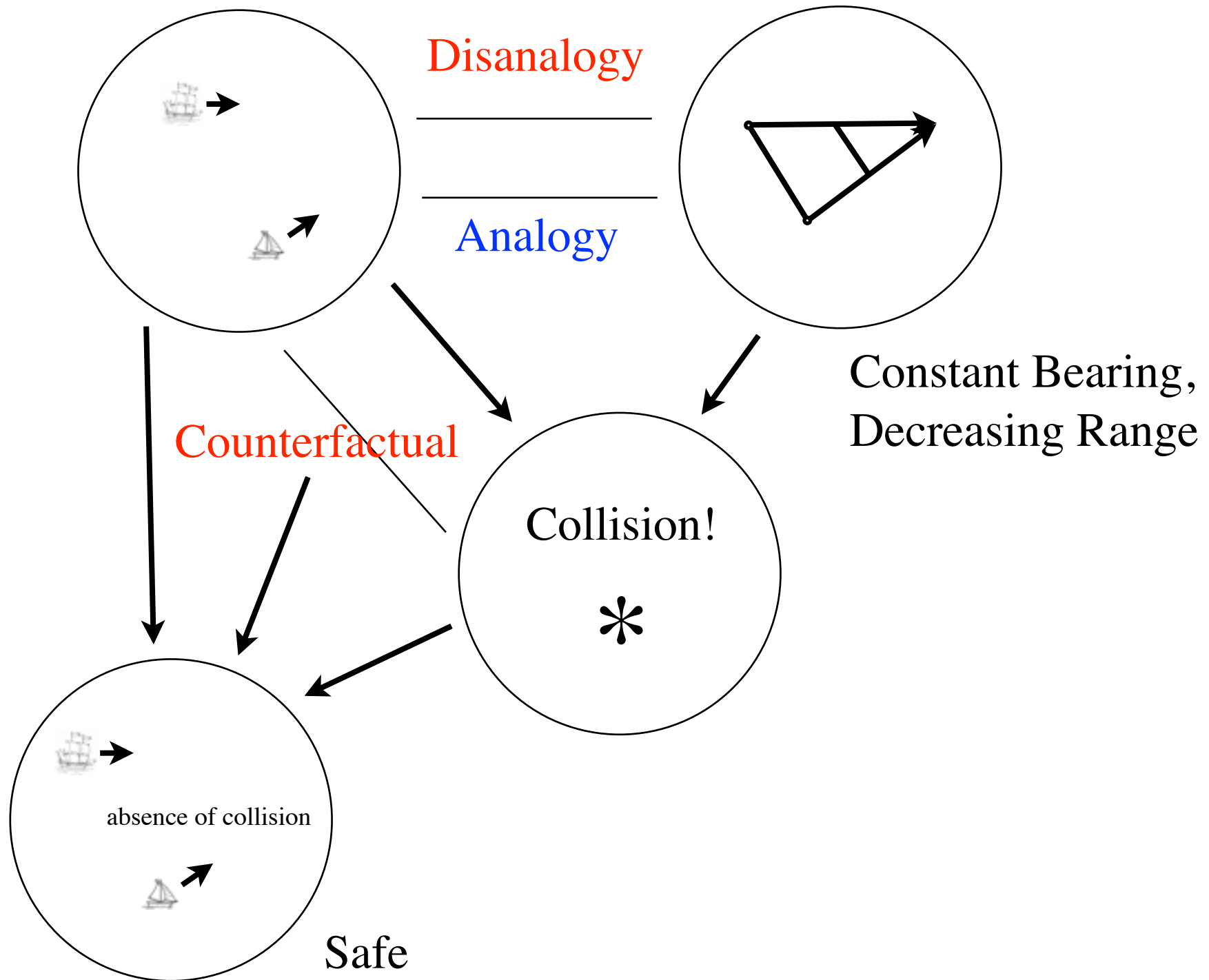
 The next compression, and quite different from the compression of all the possible static triangles into one static triangle: Over time, there will be an uncountable infinity of such similar triangles. But each of them is static. There is no shrinking. The shrinking is an emergent phenomenon in the blend.

 It is not even given that there is a triangle that results in a collision. Therefore, the entire conceptual integration network for the collision triangle (already a compression of an uncountable infinity of possibilities) is held as potentially counterfactual to another conceptual integration network in which the the two lines representing journeys either do not intersect or their point of intersection is not a simultaneous location for the boats.

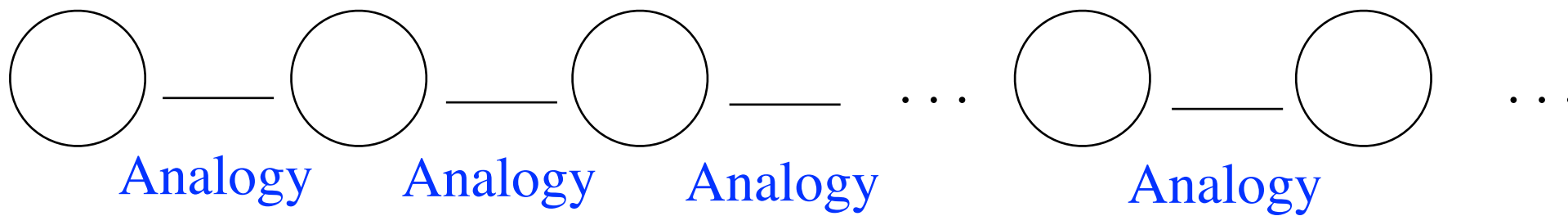






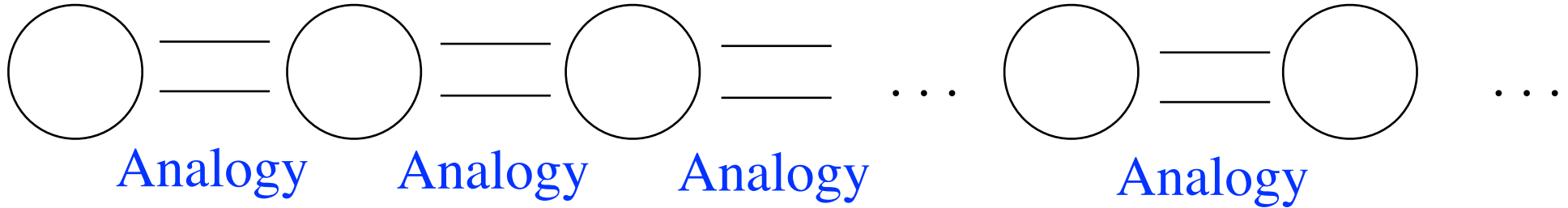


The Shrinking Triangle is the result of a very common Generic Compression Template



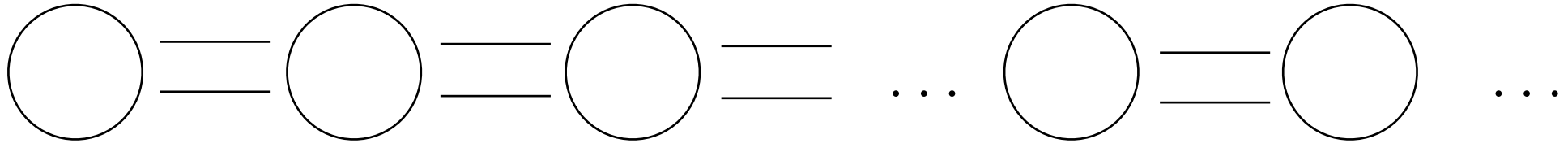
Disanalogy Disanalogy Disanalogy

Disanalogy



Disanalogy Disanalogy Disanalogy

Disanalogy

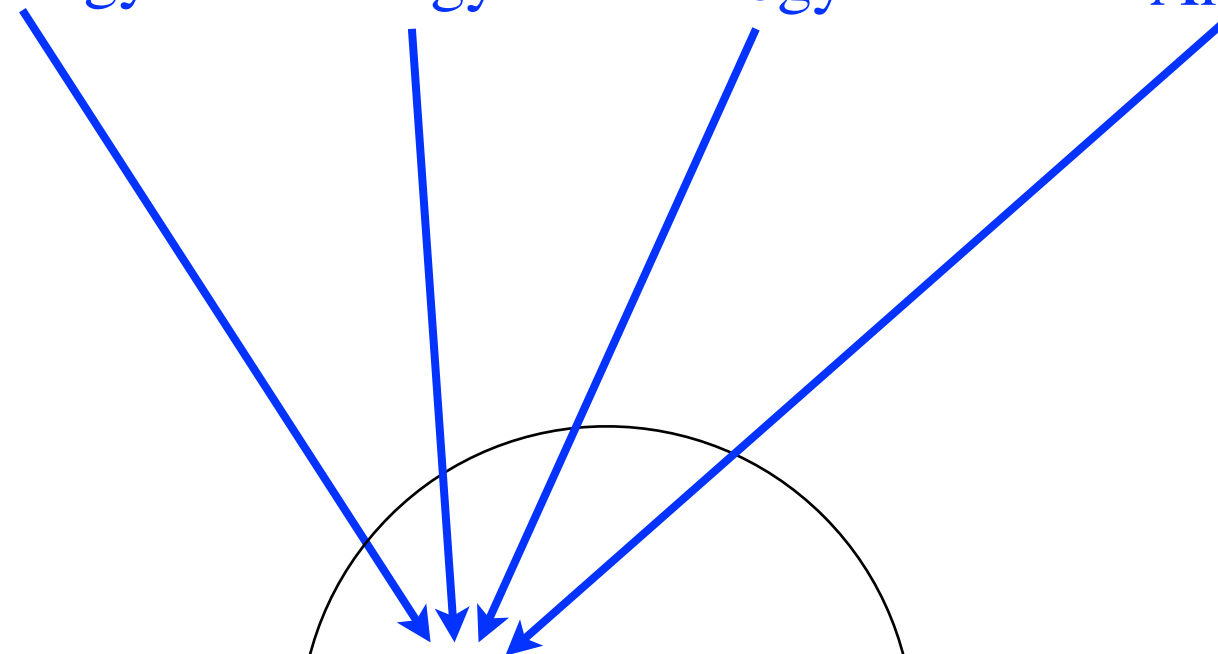
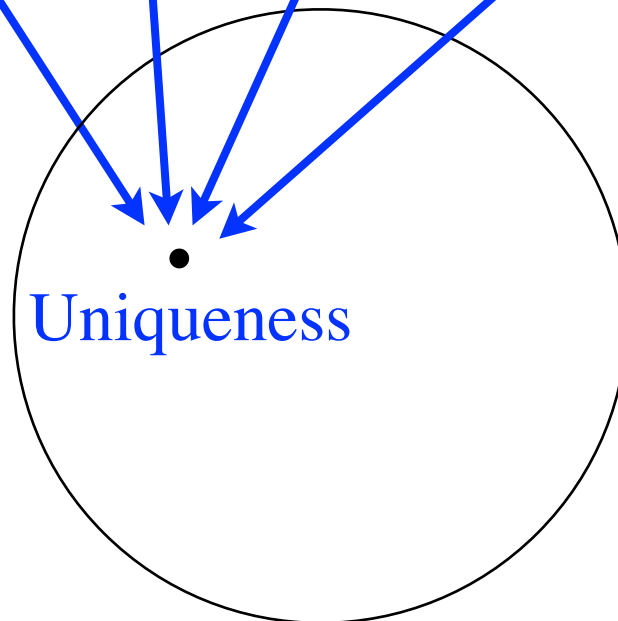


Analogy

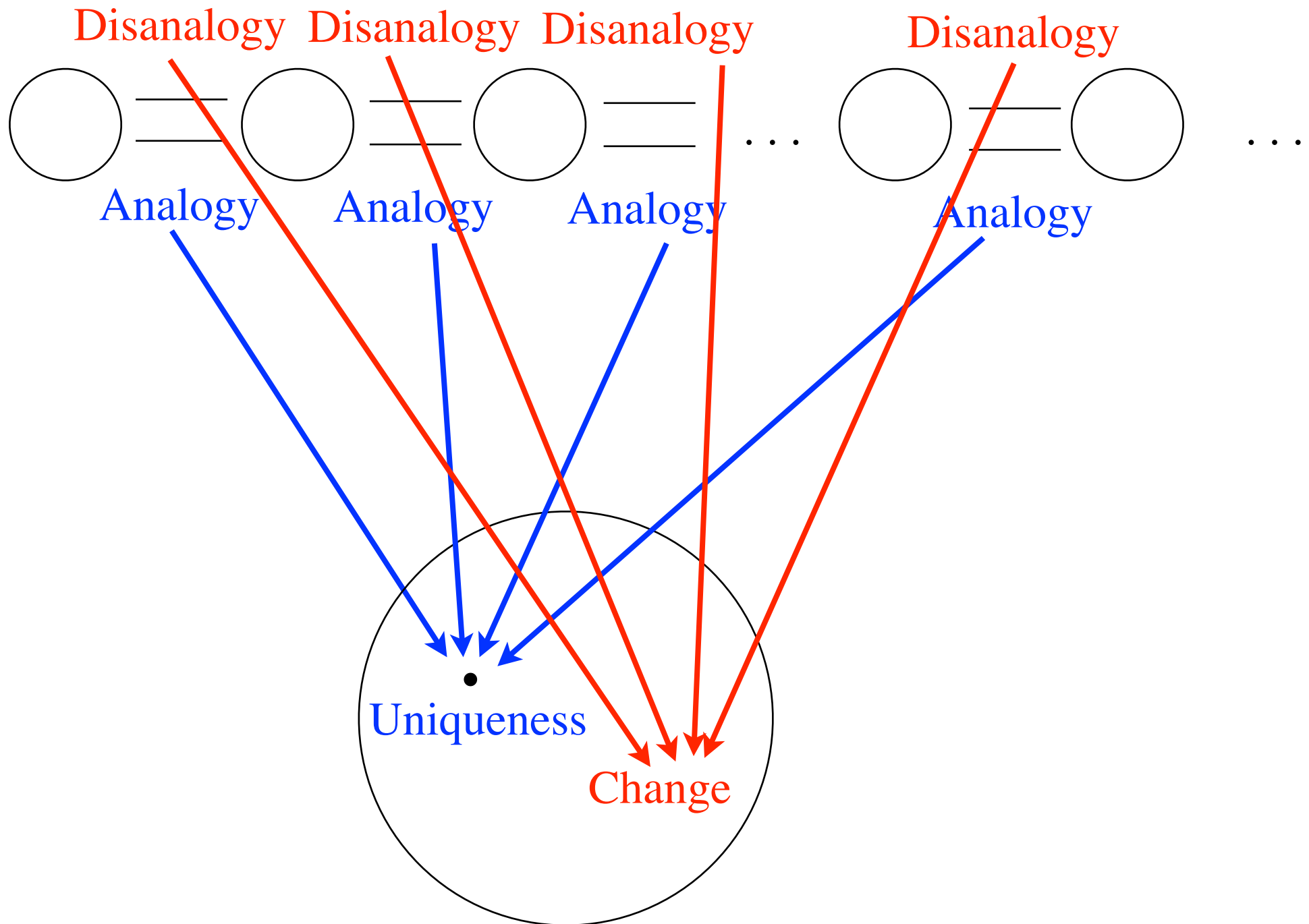
Analogy

Analogy

Analogy



Uniqueness



Compression and Decompression of Identity

The Mindful Pronghorn.

A general strategy available in building integration networks: *a vital relation between spaces is compressed into structure inside the blend.*

With the Arts
And Entertainment

Science Times

TUESDAY, DECEMBER 24, 1996

The New York Times

C1



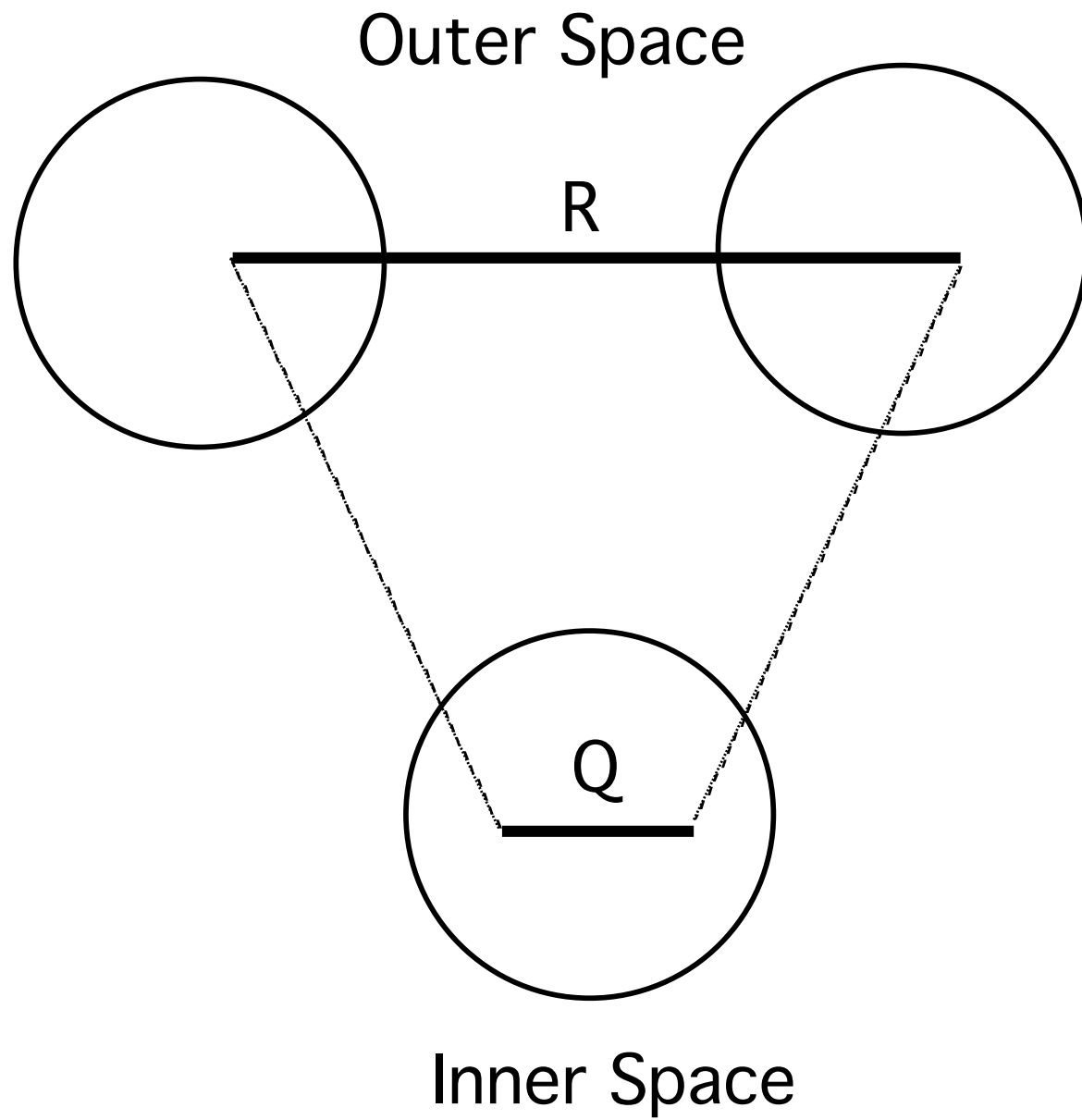
Pantheon of Ghosts

At various times, three extinct species, the giant short-faced bear, *Arctodus simus*, the long-legged hyena, *Chasmaporthetes ossifragus*, and the North American cheetah, *Miracinonyx trumani*, swiftly pursued both the North American pronghorn and its direct ancestors.

Pronghorn's Speed May Be Legacy of Past Predators

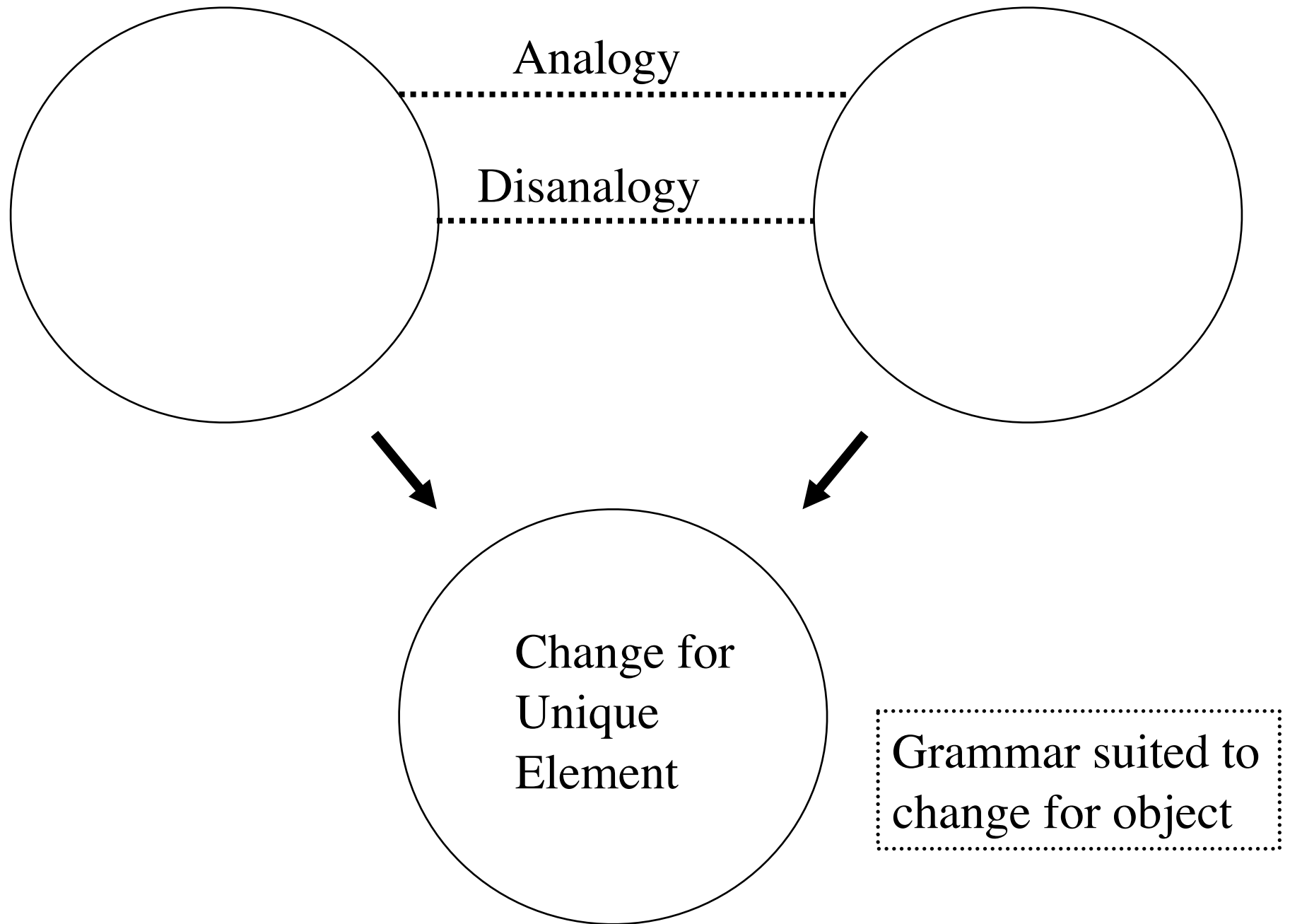
The Mindful Pronghorn

The front page of the science section of the *New York Times* for Tuesday, 24 December 1996 carried a story with the headline "Ghosts of Predators Past." It was illustrated by a large photograph of a small American pronghorn chased by pen-and-ink prehistoric cheetahs and long-legged dogs. The American pronghorn is much faster than any of its modern predators. Why would evolution select for this costly speed when it brings no additional reproductive benefit? The scientists propose that "the pronghorn runs as fast as it does because it is being chased by ghosts—the ghosts of predators past. . . . As researchers begin to look, such ghosts appear to be ever more in evidence, with studies of other species showing that even when predators have been gone for hundreds of thousands of years, their prey may not have forgotten them."



Outer-space → Inner-space Compressions

Blending compresses "outer-space" mappings into inner-space human-scale elements that can then be expressed through existing basic human-scale grammatical constructions.



Identity & Change from Analogy & Disanalogy

“In a leap year, we add a day to February.”

“*Kick the habit*”

“*Your agent* is waiting for you.”

“The customer is always right.”

“His girlfriend gets younger every year.”

“Make this problem go away.”

“Get your tennis serve back.”

“Your French has disappeared.”

Identity & Change from Analogy & Disanalogy

“Make this envelope disappear”

—Fauconnier

The Fix Triangle

Ed Hutchins

The Fix Triangle



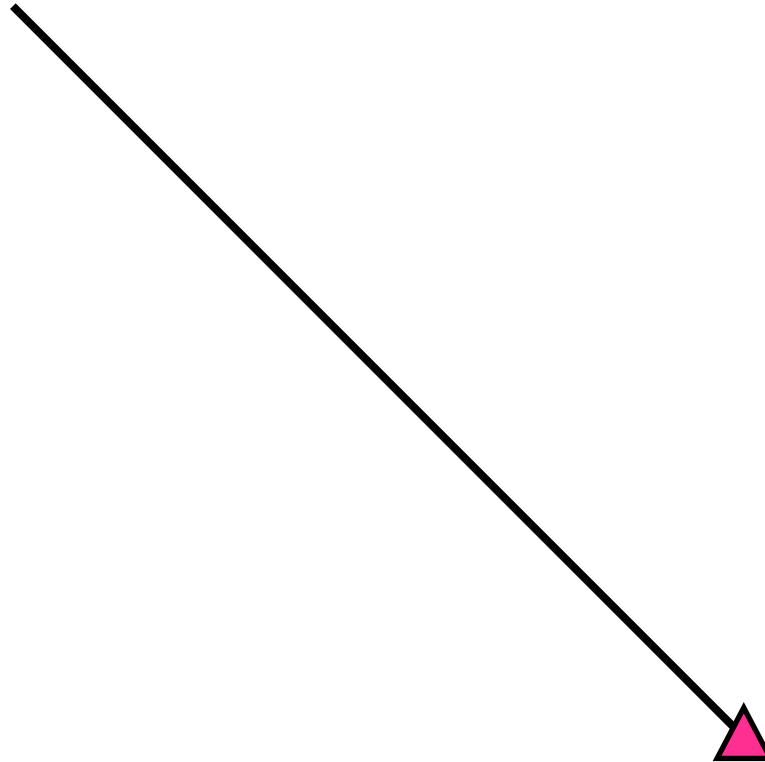
Ed Hutchins

The Fix Triangle



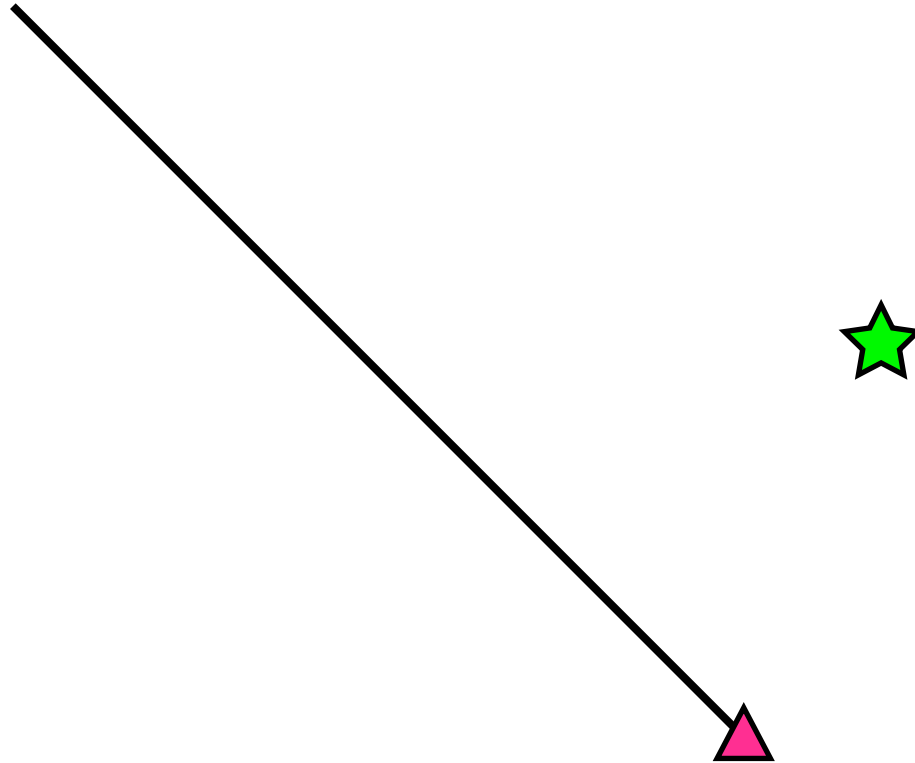
Ed Hutchins

The Fix Triangle



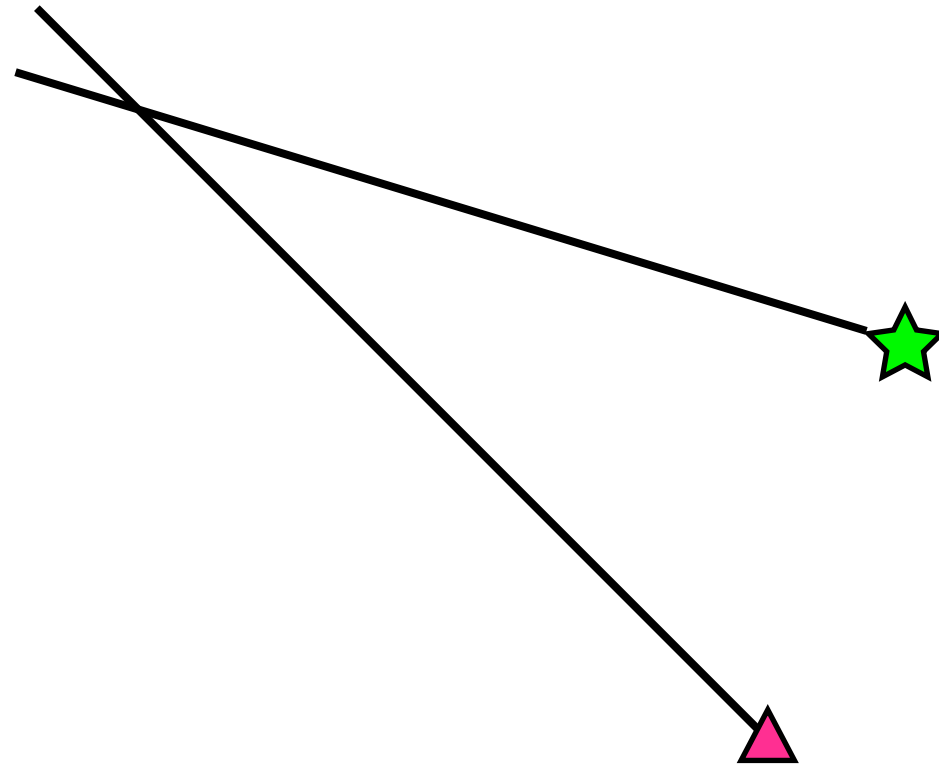
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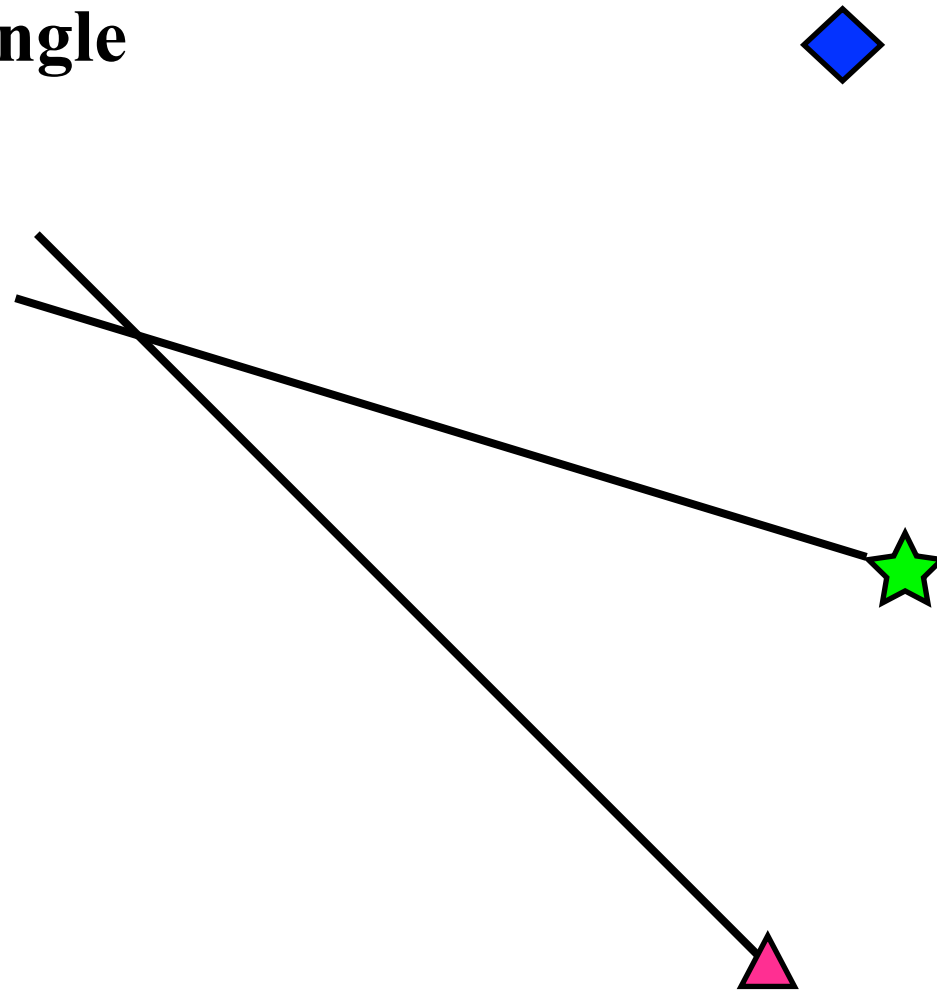
Ed Hutchins

The Fix Triangle



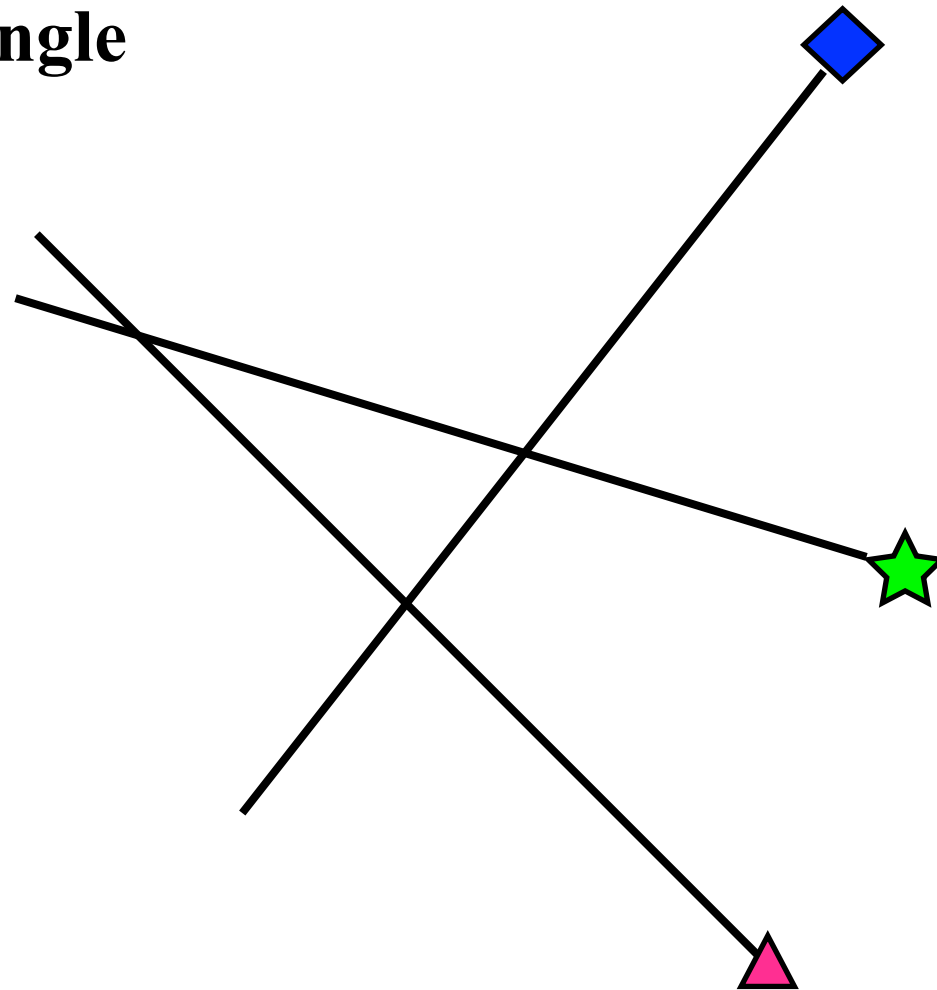
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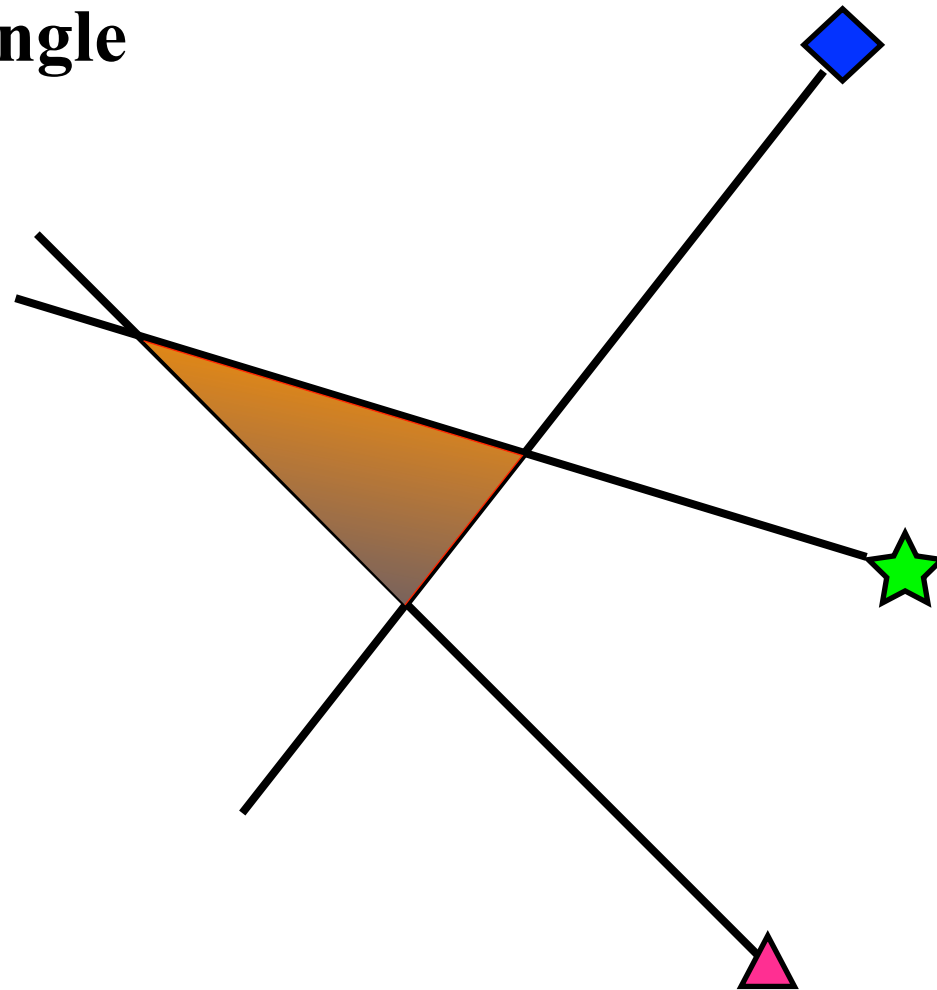
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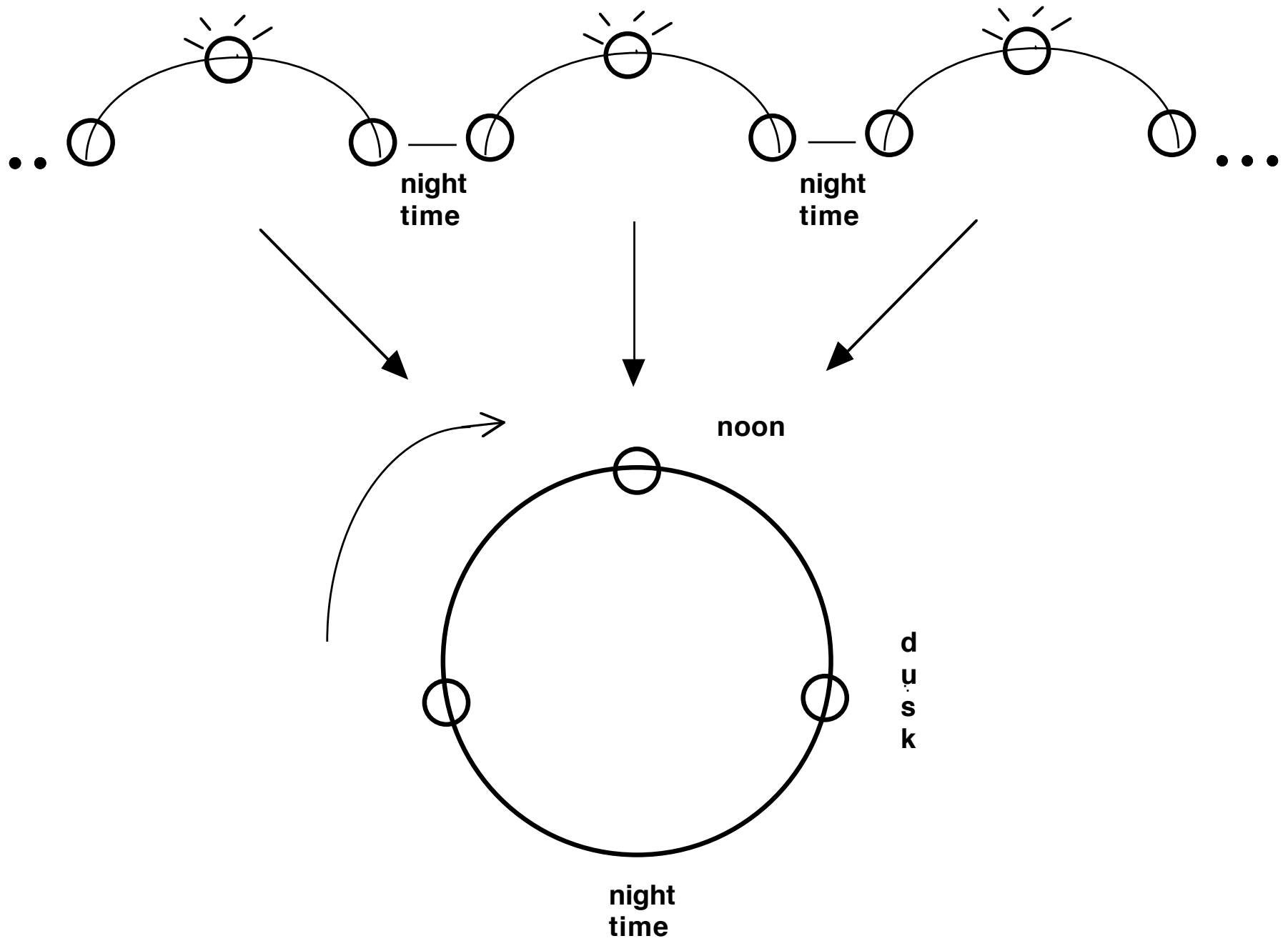


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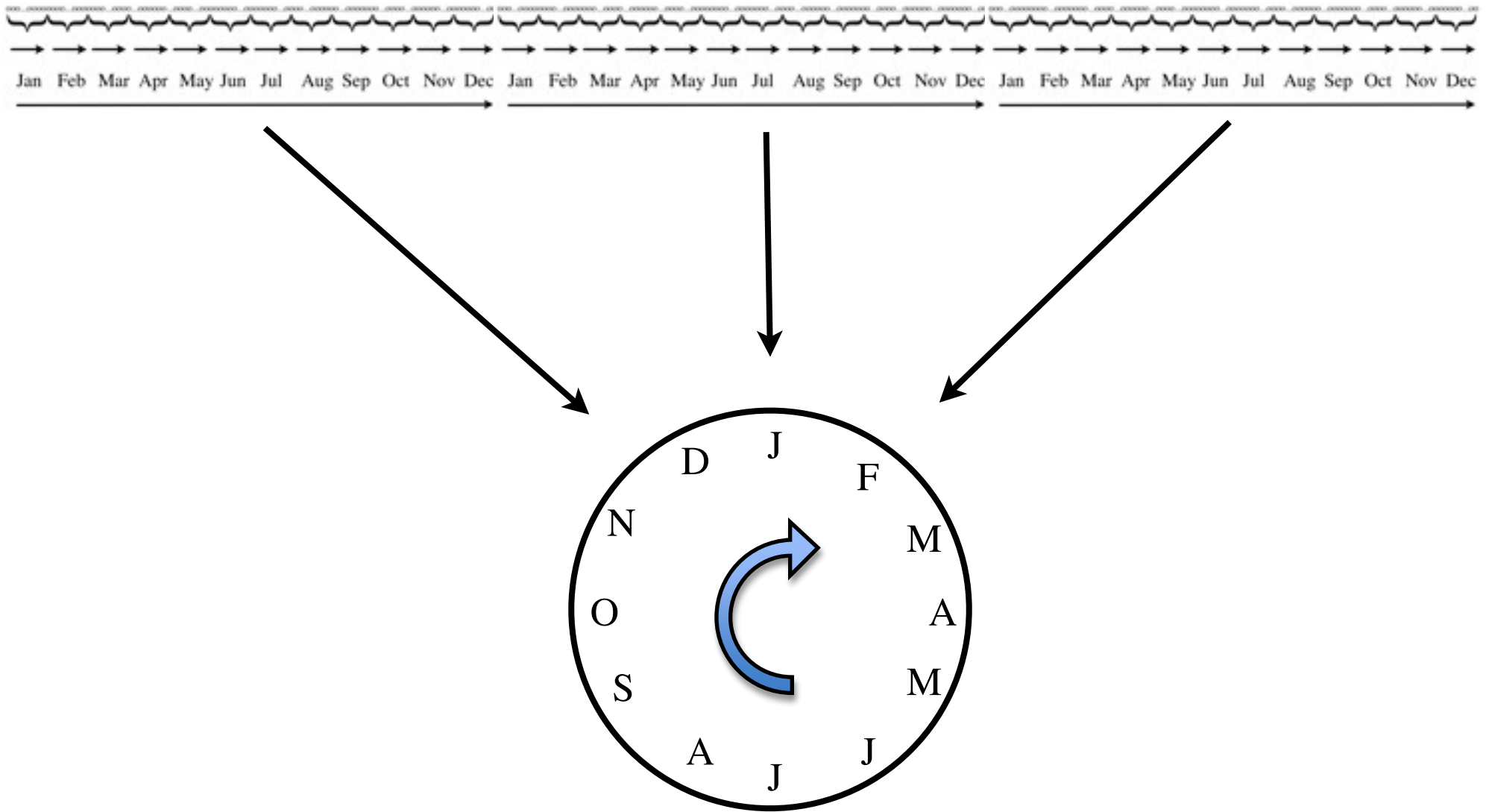
• Time & Space

Packing and Unpacking

In a leap year, we add a day to February. It seems perfectly normal, uncreative. Every four years, we must add one day to the year. But what is a day? What is a year? What is February? In your experience, there is just one day and then another day and then another day and then another day and then another day and then another day and then another day and it never ends. There is no *February* out there in the world and there is no *year* out there in the world, either. Of course, there is one day and another day and another day and another day. We blend and pack these into a conception of a cyclic day, which makes it possible to say “Soon it will be morning *again*,” “this park closes at *dusk*,” and so on.








The Cyclic Year
("Soon it will be January *again*, . . .")

Neurobiological substrate?


Neurobiological substrate?


 **Hypertrophy and replication of domain-specific integrations, e.g. chase play.**

 **There are other restricted-domain abilities that look as if they involve integration, such as chase play, a kind of simulation of aggression, which evidently is common throughout the mammalian world for species involved in predation. During chase play, parent and offspring simultaneously activate motor patterns, attention patterns, and motivational structures that belong to two clashing domains, such as parent-offspring and predator-prey. Maybe the neural circuitry subtending binding, synaesthesia, or special-purpose blending of the sort we find in chase play got the ball rolling in the run-up to full cognitive modernity.**

Neurobiological substrate?

Neurobiological substrate?

 **Integration, binding. Damasio & broker neurons. Edelman and Neural Darwinism over neuronal group patterns. Time-locking. Neurons that fire together wire together.**

 **Antonio Damasio, in *Descartes' Error*, puts forward the notion of “broker neurons,” which might connect up what we think of as disparate neuronal groups subtending clashing conceptual arrays.**

Neurobiological substrate?

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Integration, binding. Damasio & broker neurons. Edelman and Neural Darwinism over neuronal group patterns. Time-locking. Neurons that fire together wire together.




Conceptual integration could be a hypertrophy of perceptual integration: the neural mechanisms of perceptual integration might have been recruited and expanded by biological evolution, resulting in a computational ability that made double-scope blending possible. Perceptual integration, called the “binding problem,” is perhaps the major open scientific question in neuroscience.

Neurobiological substrate?

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 **Integration, binding. Damasio & broker neurons. Edelman and Neural Darwinism over neuronal group patterns. Time-locking. Neurons that fire together wire together.**

 **A worthy ambition, but it runs up against . . .**


Neurobiological substrate?

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Neurobiological substrate?

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 **A worthy ambition, but it runs up against . . .**

 **The 3,000-fold problem**, which is the 800-pound gorilla in human evolution:

 **Neural binding has been here at least since mammals, perhaps since arthropods, so a minimum of 150 million years, while human-level conceptual integration has been here perhaps 50 thousand years. Binding as we know it has been here at least 3,000 times longer than human-level blending. Accordingly, by itself, it offers no explanation, although it is hard to imagine that it is not in some way indispensable to blending & packing.**

COGNITIVE SCIENCE

A Multidisciplinary Journal



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The AHA! Experience: Creativity Through Emergent Binding in Neural Networks

Paul Thagard, Terrence C. Stewart

Department of Psychology, University of Waterloo

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The AHA! Experience: Creativity Through Emergent Binding in Neural Networks

Cognitive Science 35 (2011) 1–33

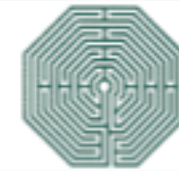


“Our account of creativity as based on representation combination is similar to the idea of blending (conceptual integration) developed by Fauconnier and Turner (2002), which is modeled computationally by Pereira (2007). Our account differs in providing a neural mechanism for combining multimodal representations, including emotional reactions.” —page 25.

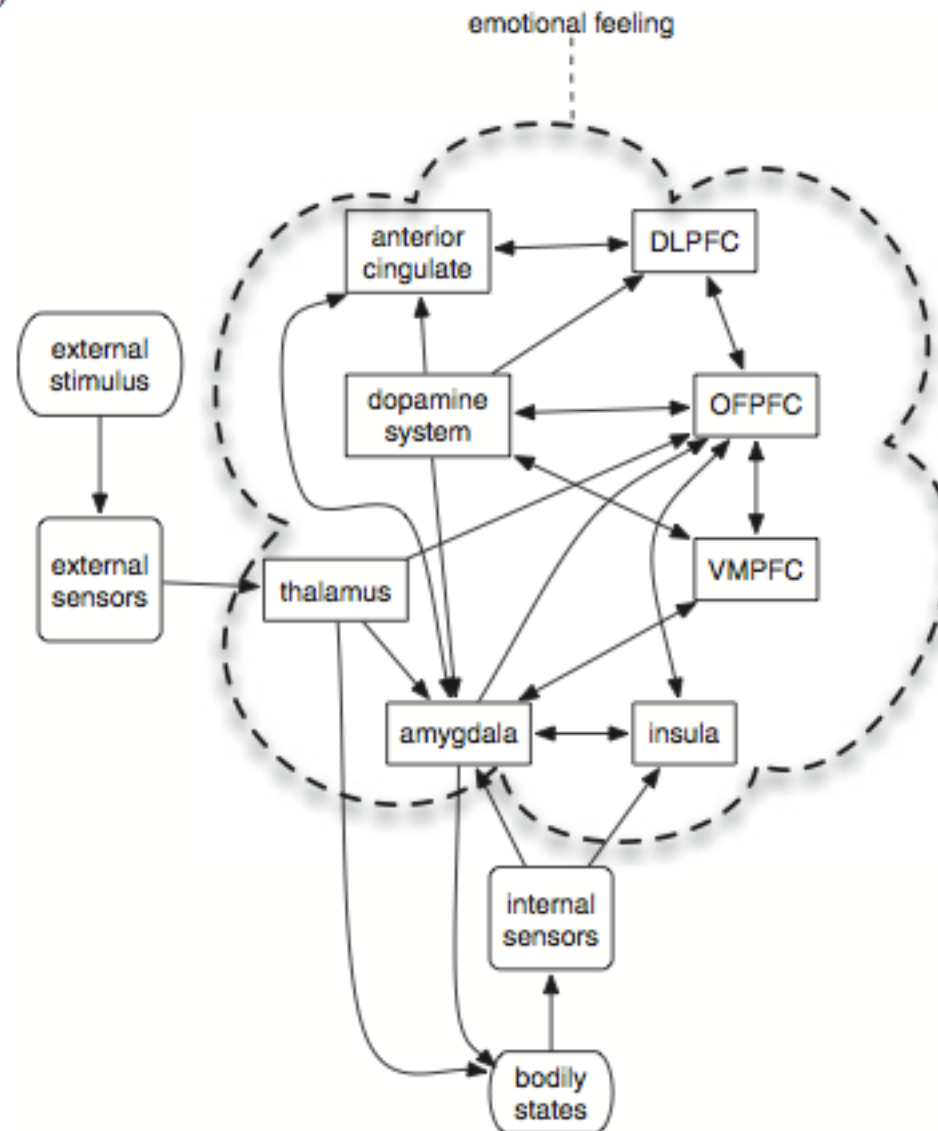
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Cognitive Science 35 (2011)





Convolution of Neural Patterns:



“Now that we have specified how neurons can represent vectors, we can organize neurons to perform the convolution operation (Eliasmith, 2004). **We need a neural model where we have two groups of neurons representing the input patterns** (using the representation scheme above), and **these neurons must be connected to a third group of neurons which will be driven to represent the convolution of the two original patterns.**” —page 15.

COGNITIVE SCIENCE

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Cognitive Science 35 (2011) 1–33





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A worthy ambition, and interesting, but . . .



-  A worthy ambition, and interesting, but . . .
-  It's still fundamentally **computational** rather than **neural**, and its “neurally-inspired” computational mechanisms are more **allegories** of neurobiology than neurobiology.



-  A worthy ambition, and interesting, but . . .
-  It's still fundamentally **computational** rather than **neural**, and its “neurally-inspired” computational mechanisms are more **allegories** of neurobiology than neurobiology.
-  It offers no answer to the **3,000-fold** problem, namely, the fact that neural binding as we know it and model it has been here at least 3,000 times as long as human-level blending.

Neurobiological substrate?



Synesthesia.



Synaesthesia, or more generally cross-wiring, could provide neurobiology useful for blending. V. S. Ramachandran, Edward Hubbard, and others have worked on the neurobiology of synaesthesia and considered its contribution to conceptual integrations involving metaphoric links. Stephen Mithen, in *The Prehistory of the Mind*, has also considered cross-wiring. Synaesthesia is a kind of neural binding in restricted domains. Perhaps it could have evolved into an ability that is not restricted to particular conceptual domains.

Neurobiological substrate?



Mirror and canonical neurons.



What is now referred to as the “mirror-neuron craze” has naturally led to notions that mirror neurons (and maybe canonical neurons) subtend blends of self and other.

Lascaux Bull



Chauvet



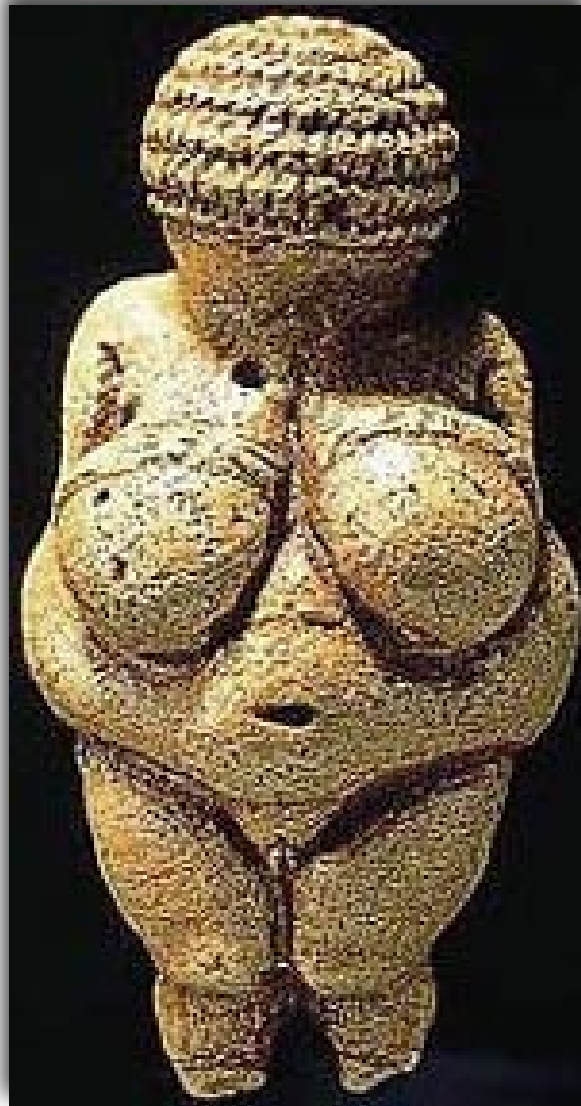
Chauvet



Chauvet



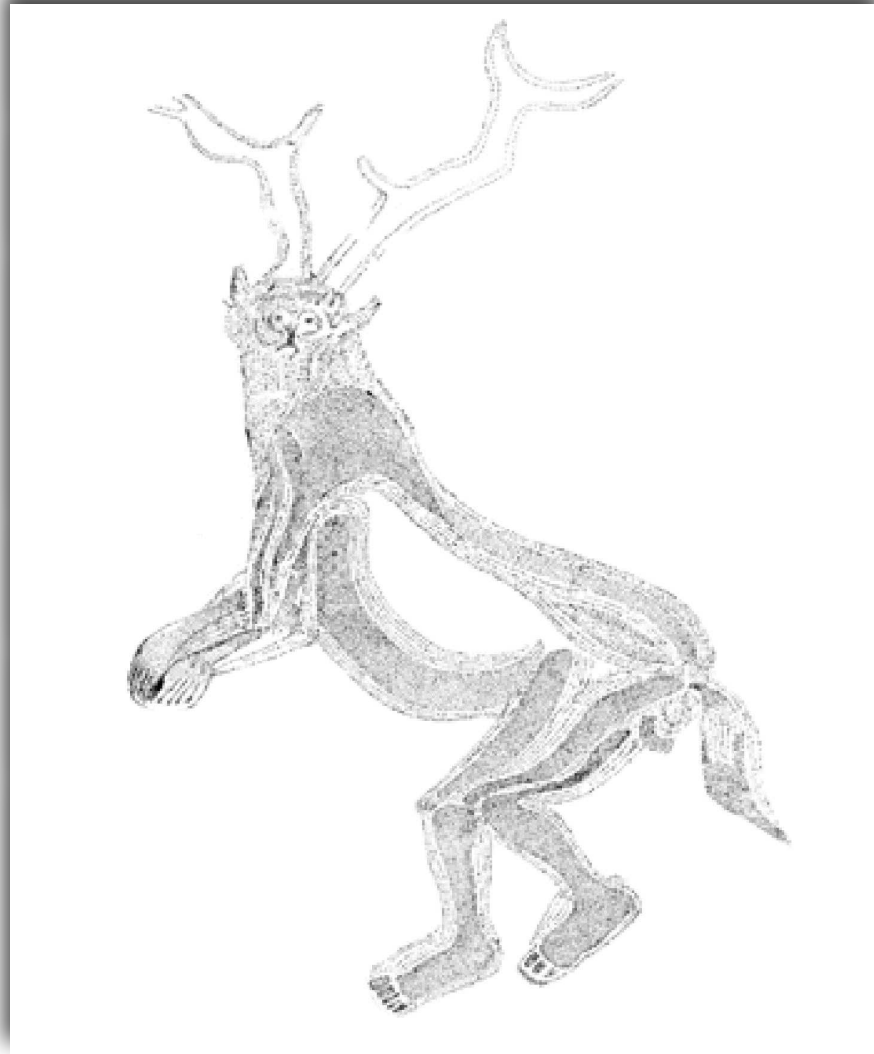
Willendorf

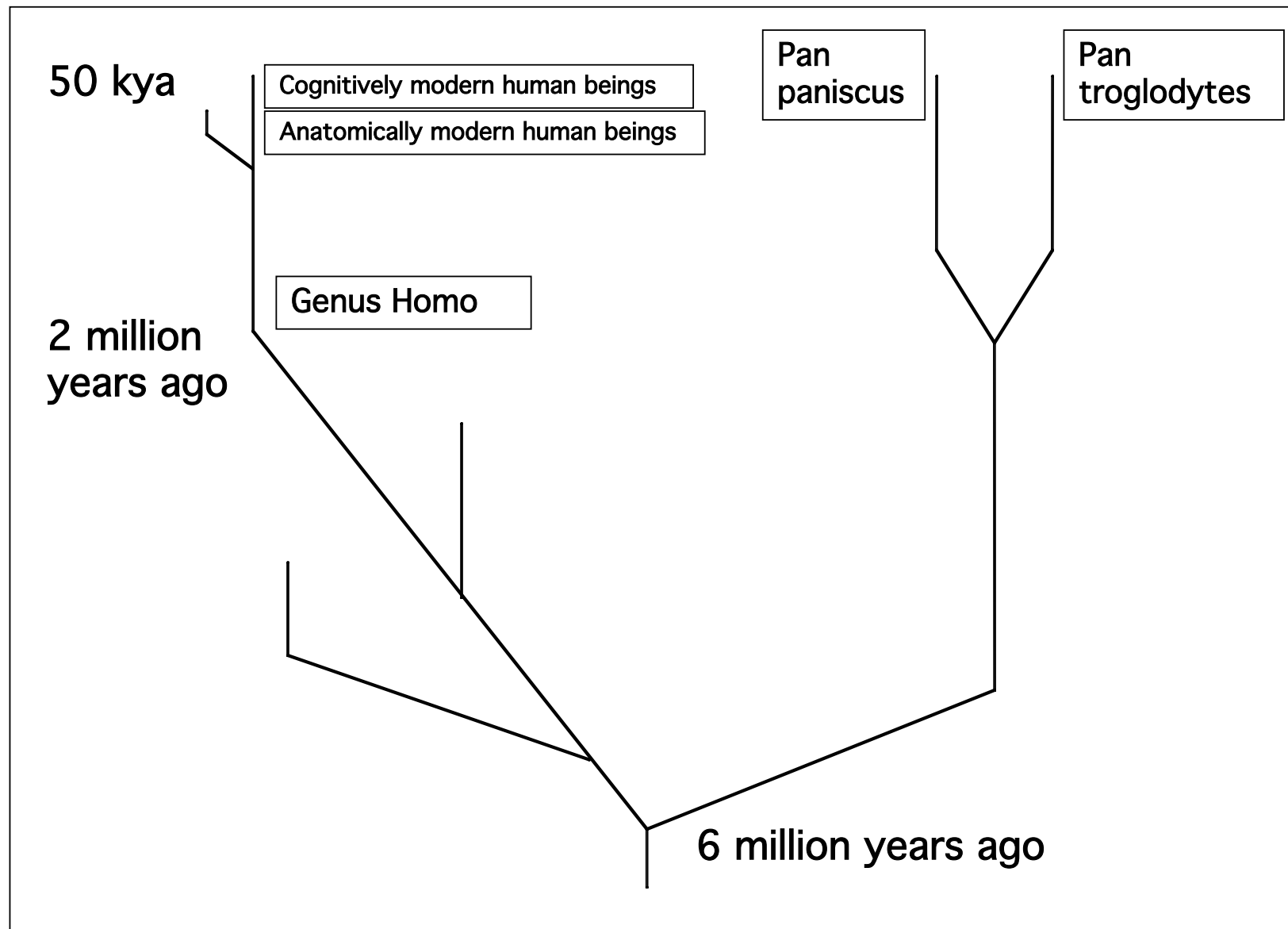


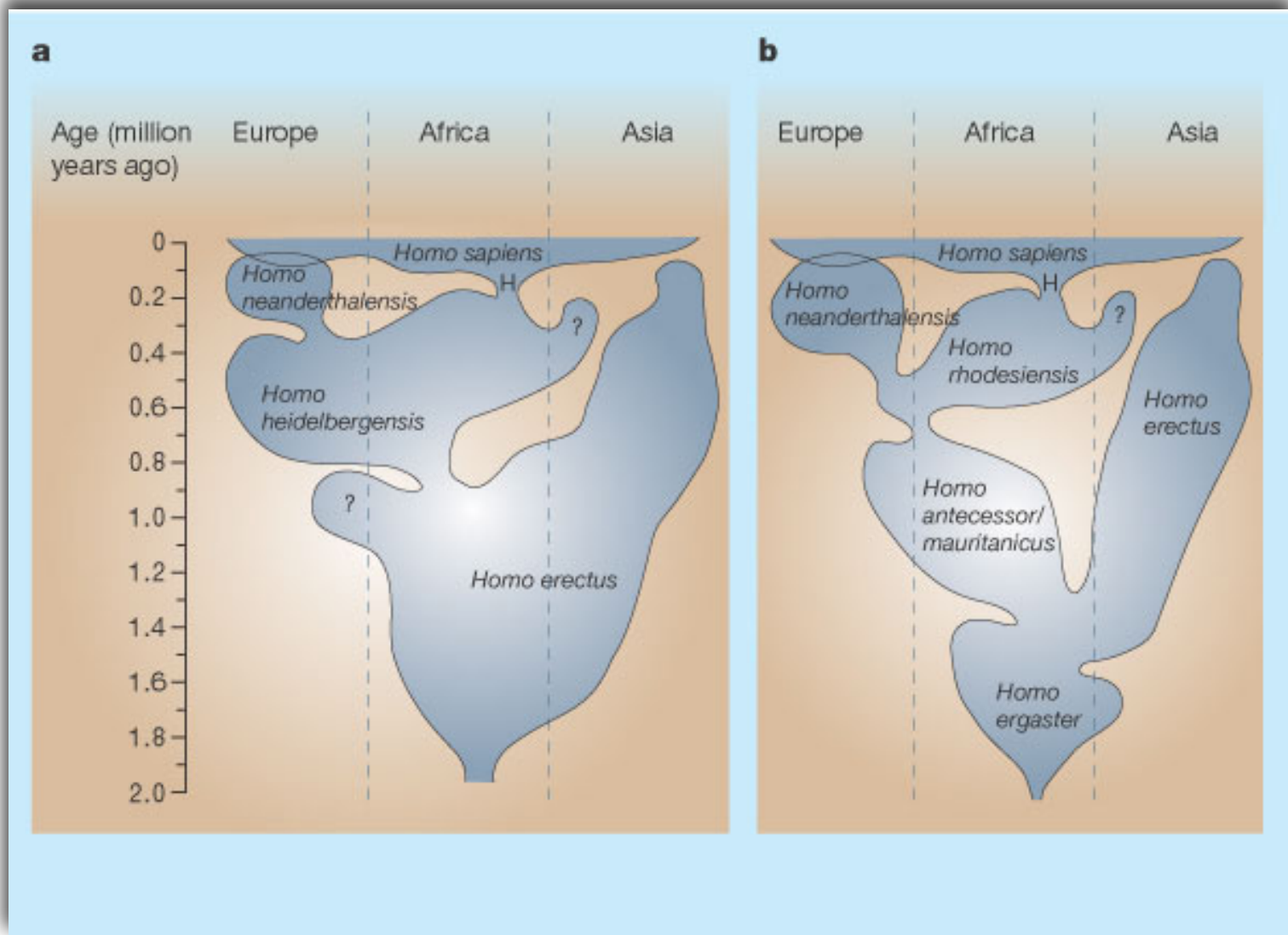
Ivory beads sewn into cloth, Burial at Sungir



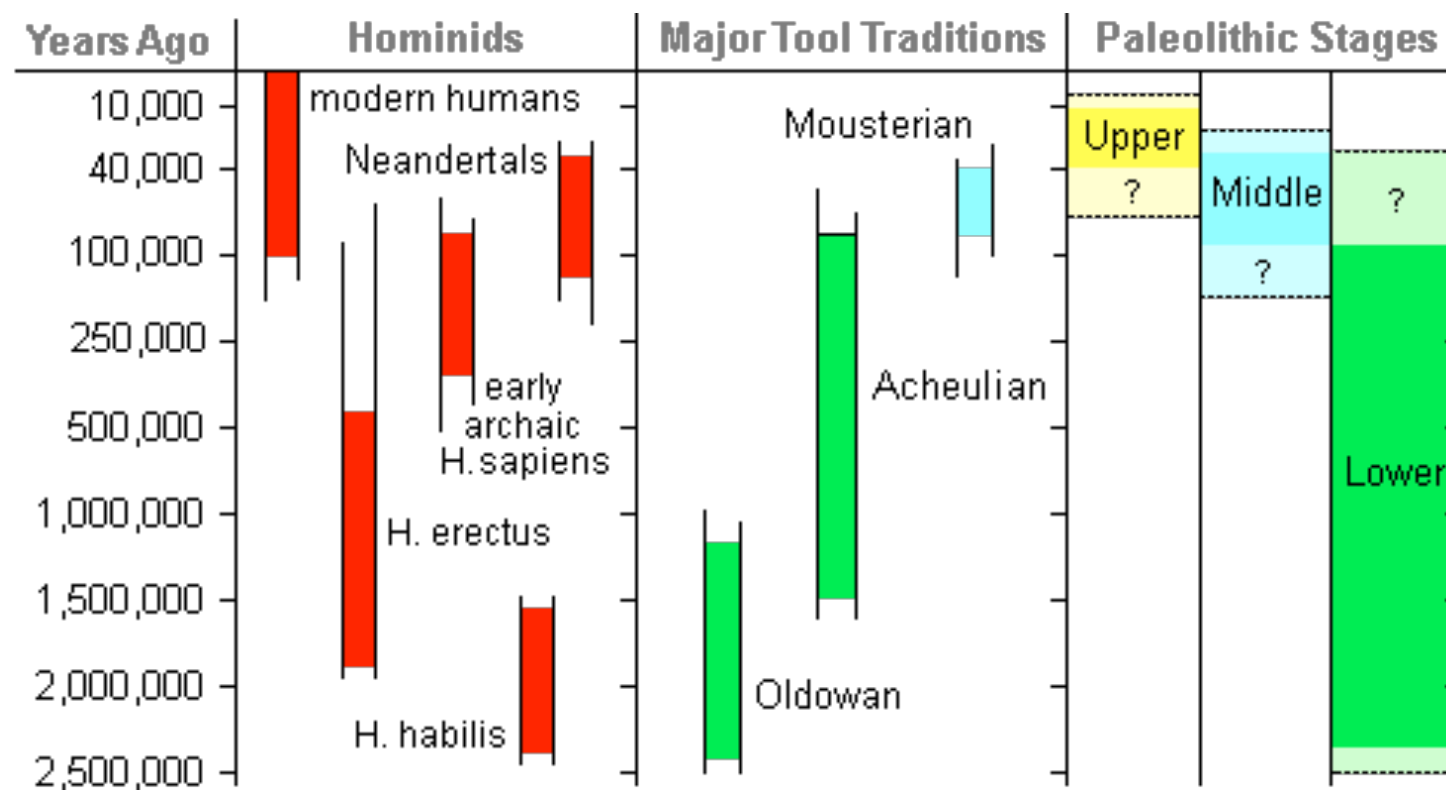
From “The Sanctuary” at Trois-Frères

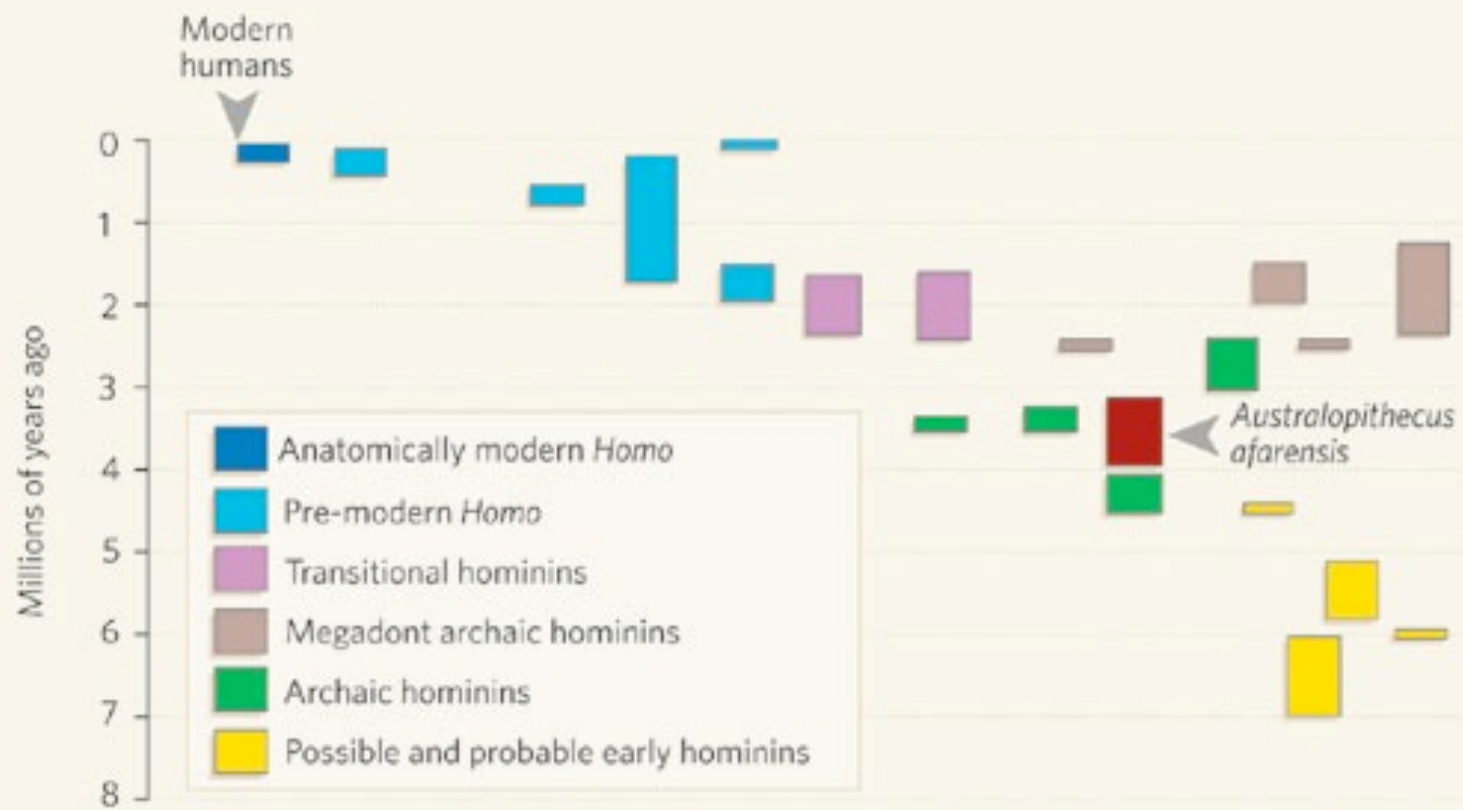


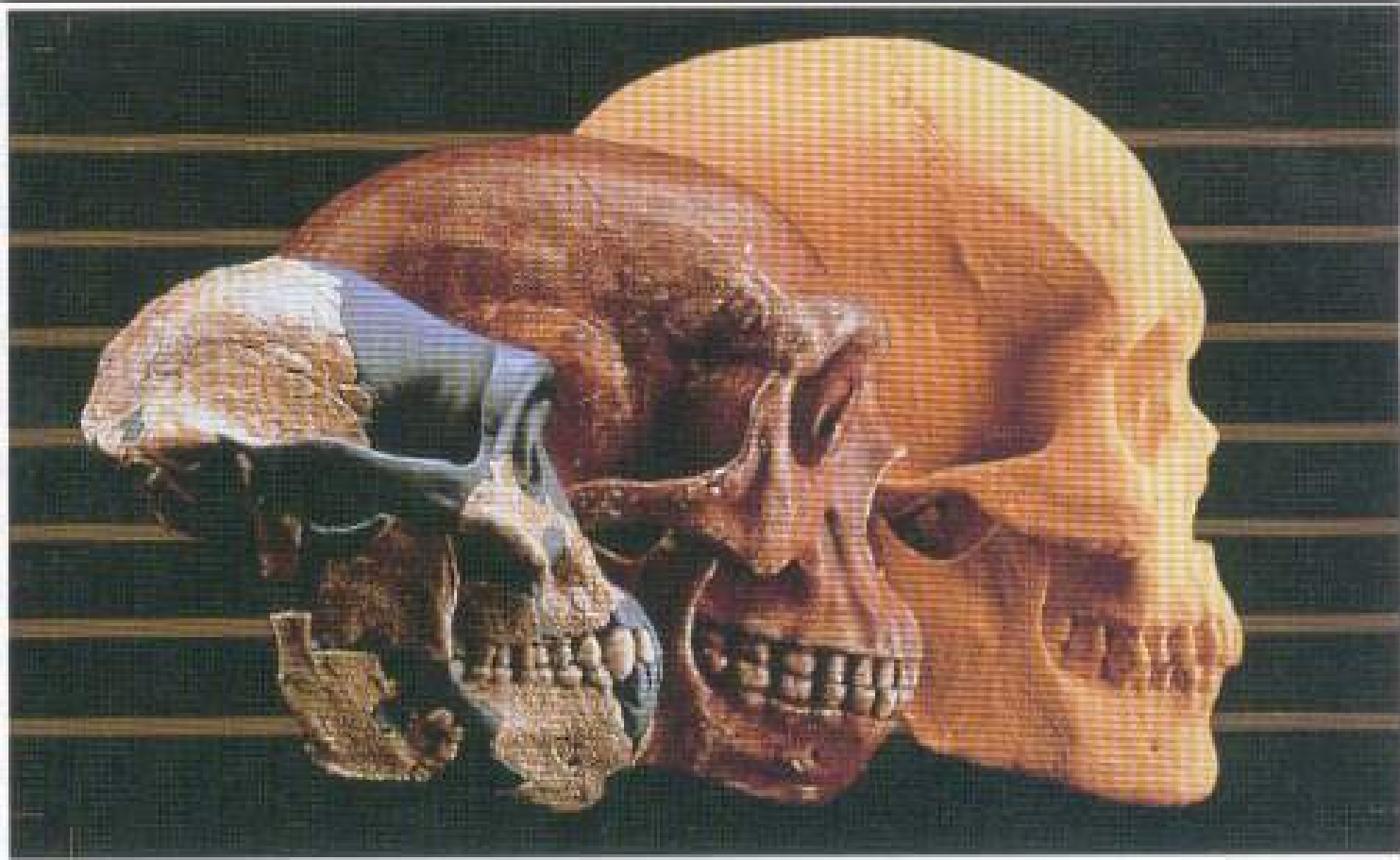




Stringer, Chris. 2003. "Human evolution: Out of Ethiopia." *Nature* 423, 692 - 695.







K. O'Farrell/Concepts

Separation through biological evolution



Gorilla gorilla beringei - Eastern gorilla species



Gorilla gorilla gorilla - Western gorilla species



Homo sapiens - Human beings (Homo sapiens sapiens)



Pan paniscus - Bonobo, or pygmy chimpanzee



Pan troglodytes - Chimpanzee



Pongo pygmaeus - Orangutan

Lemur catta catta

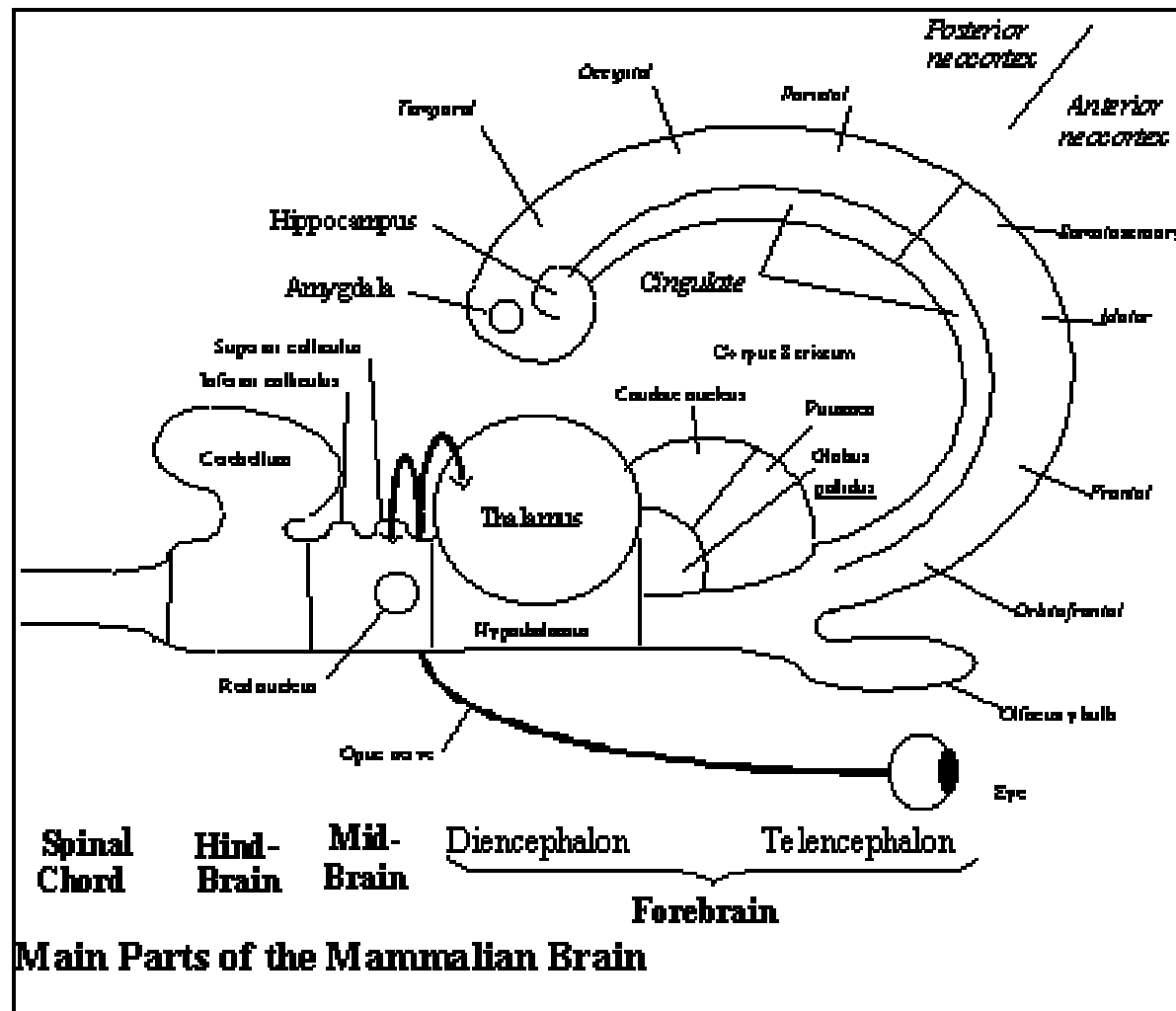
Homo sapiens sapiens

Kingdom: Animalia
Phylum: Chordata
Class: Mammalia
Order: Primates
Family: Lemnridae
Genus Lemur
Species: Lemur catta

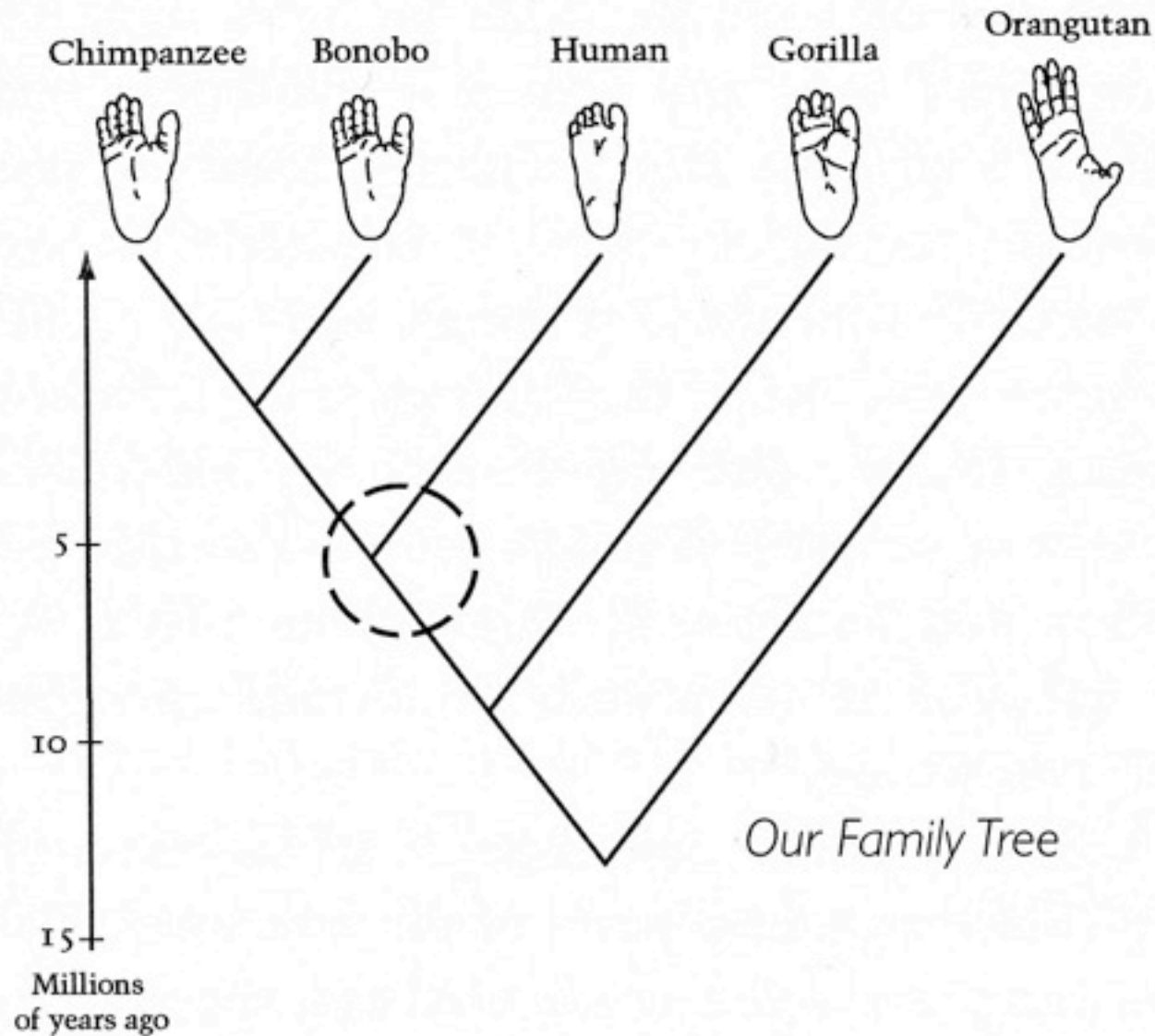
Kingdom: Animalia
Phylum: Chordata
Class: Mammalia
Order: Primates
Family: Hominidae
Genus Homo
Species: Homo sapiens







Great ape lineage



The human



160,000-year-old *Homo sapiens*, Ethiopia





Vesalius 1542

Did Working Memory Spark Creative Culture?



e.g., Alan Baddeley and Graham Hitch

9 APRIL 2010 VOL 328

SCIENCE

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“A provocative model suggests that a shift in what and how we remember may have been **key to the evolution of human cognition.**”



COLORADO SPRINGS, COLORADO—About 32,000 years ago, a prehistoric artist carved a special statuette from a mammoth tusk. Holding the abstract concepts of “human” and “animal” in his or her mind, the artist created an imaginary beast with the body of a human and the head of a lion. Archaeologists found the 28-centimeter-tall figurine in hundreds of pieces in the back of Germany’s Hohlenstein-Stadel cave in 1939, and after World War II, they put the fragments back together, reconstructing the ancient artwork.



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“A provocative model suggests that a shift in what and how we remember may have been **key to the evolution of human cognition.**”

Today, archaeologists hail the “Lion Man” as one of the earliest unambiguous examples of artistic expression, a hallmark of modern human behavior. The figurine “has acquired an iconic status for modern archaeologists as profound as it must have been for the original artisan,” wrote Thomas Wynn and Frederick Coolidge, both of the University of Colorado, Colorado Springs, in a paper last year.



“A provocative model suggests that a shift in what and how we remember may have been **key to the evolution of human cognition.**”

Wynn and Coolidge argue that the figurine's creation—as well as its subsequent reconstruction by archaeologists—is an excellent example of something unique to our species: an **enhanced capacity to hold and manipulate information** in one's conscious attention while carrying out specific tasks, an ability psychologists call working memory.



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2. Wynn, T., 2002. Archaeology and cognitive evolution. Behavioral and Brain Sciences 25, 389–403.
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Packing & Unpacking




Packing & Unpacking

 **We need a theory of packing & unpacking independent of a hypothesis about the expansion of working memory.**

Packing & Unpacking

-  **We need a theory of packing & unpacking independent of a hypothesis about the expansion of working memory.**
-  **But we really need one if the hypothesis of the expansion of working memory is right.**

Packing & Unpacking

-  **We need a theory of packing & unpacking independent of a hypothesis about the expansion of working memory.**
-  **But we really need one if the hypothesis of the expansion of working memory is right.**
-  **Because the expansion of working memory is useful only if we can pack, carry, & unpack.**

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Similar Co-Evolution for Blending & Human Memory

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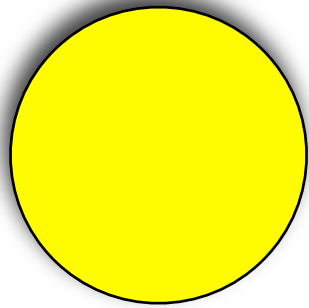
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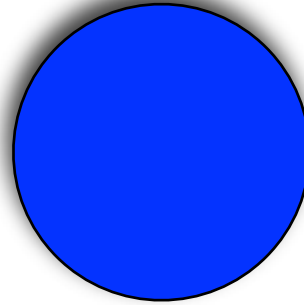
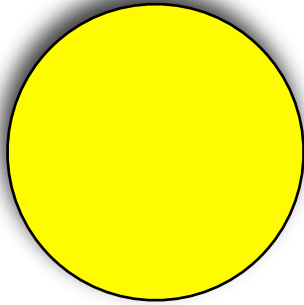
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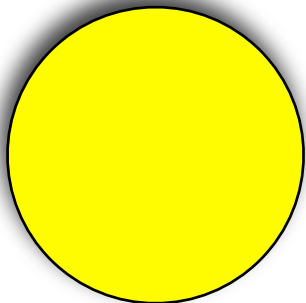
What Memory Is For

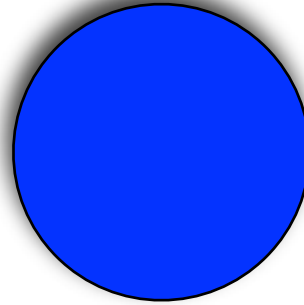
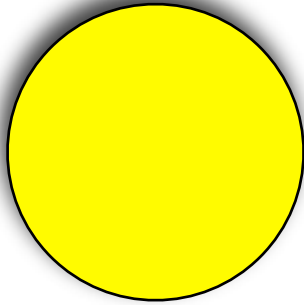
As Arthur Glenberg (1997) writes in “What Memory Is For,”

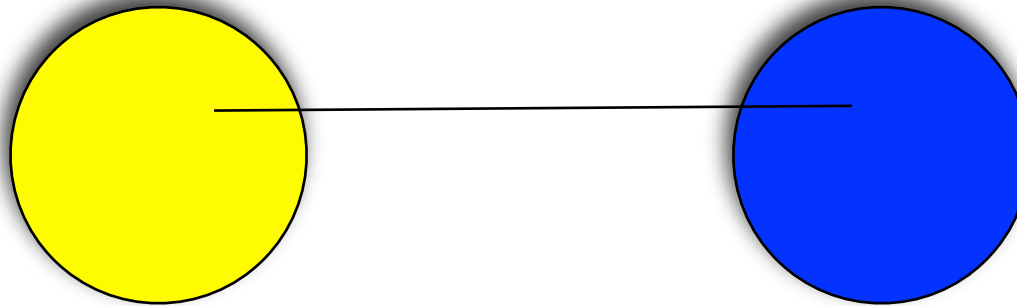
To avoid hallucination, conceptualization would normally be driven by the environment, and patterns of action from memory would play a supporting, but automatic, role. (Glenberg 1997, 1)

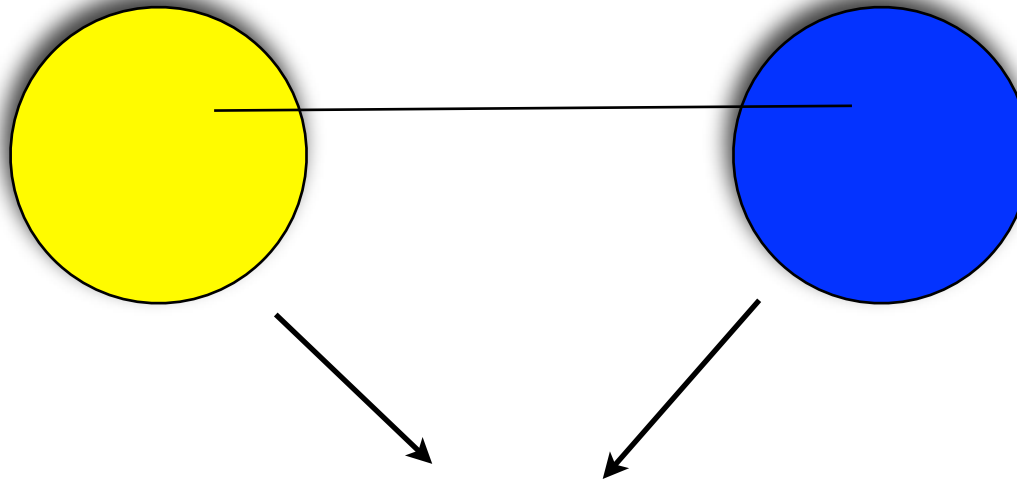
But as Glenberg astutely observes, for human beings, it is often the case that memory takes the upper hand in conceptualizing the narrative one is inhabiting:

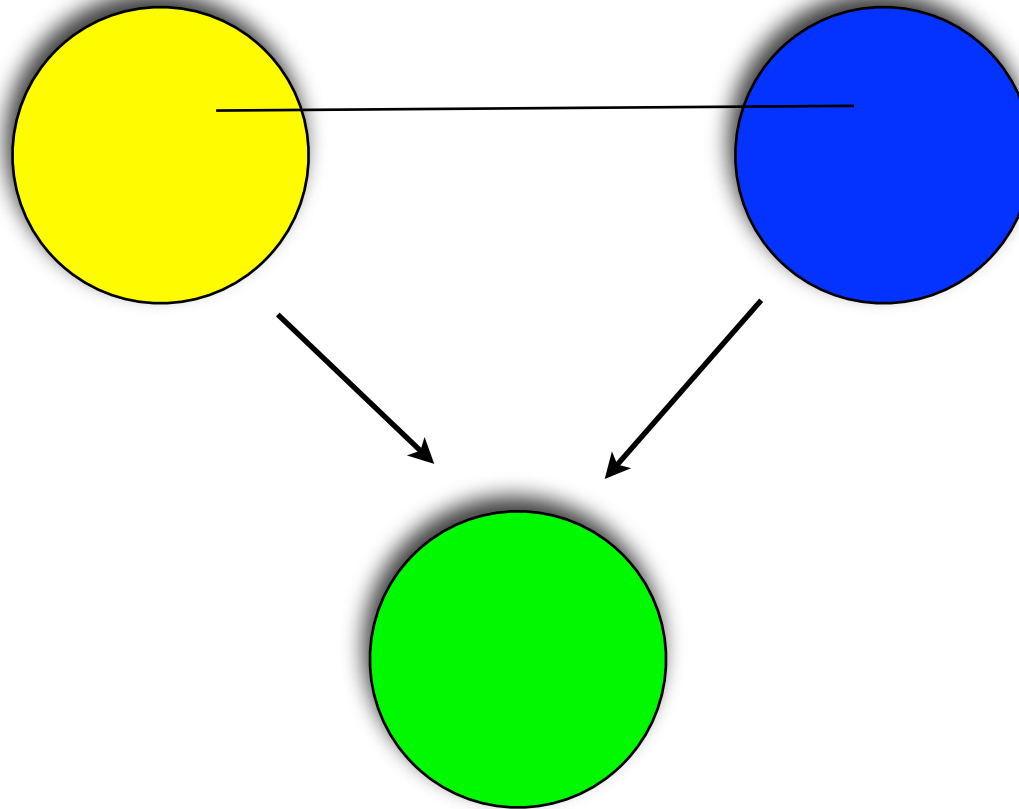
A significant human skill is learning to suppress the overriding contribution of the environment to conceptualization, thereby allowing memory to guide conceptualization. (Glenberg 1997, 1)

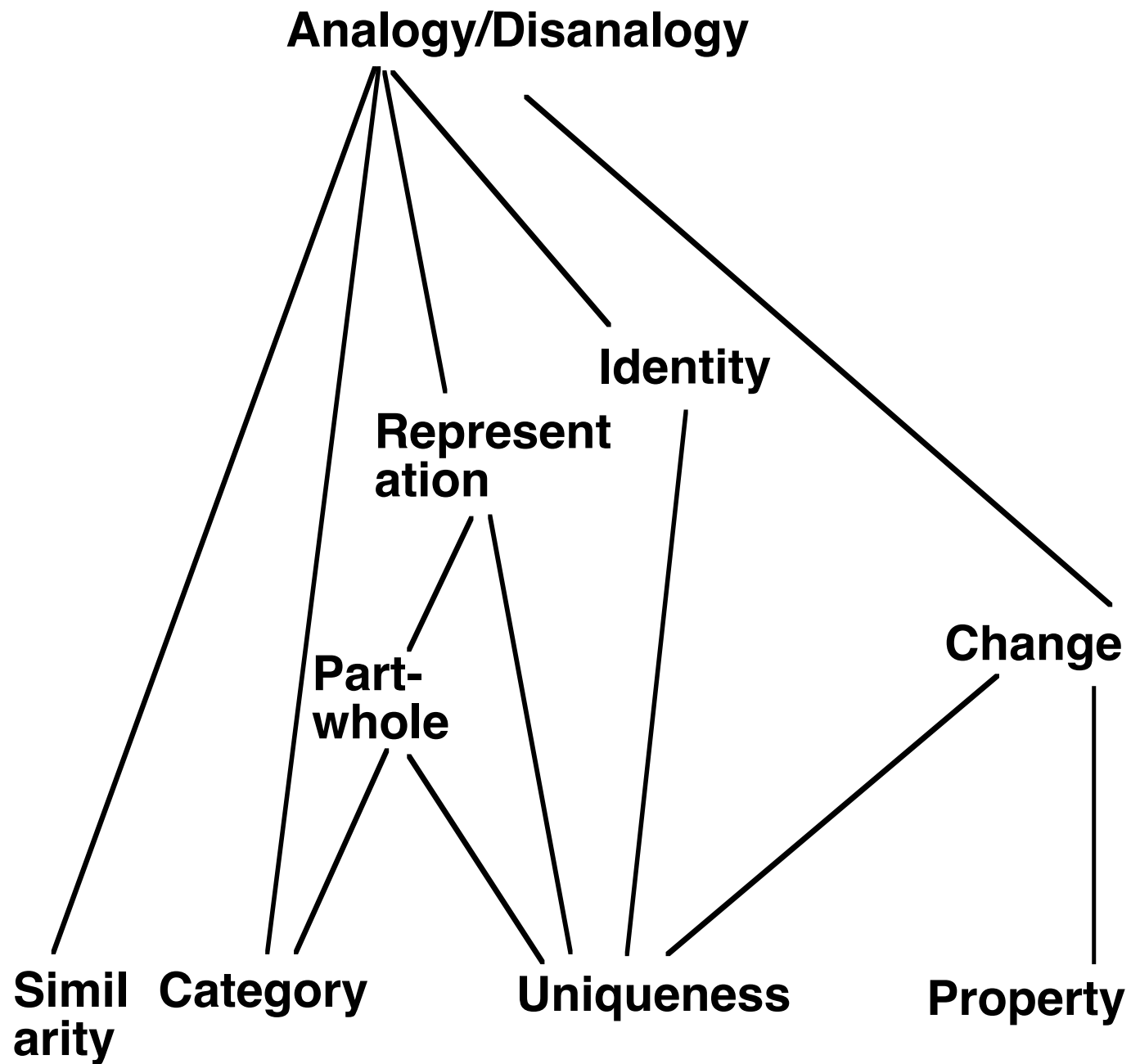


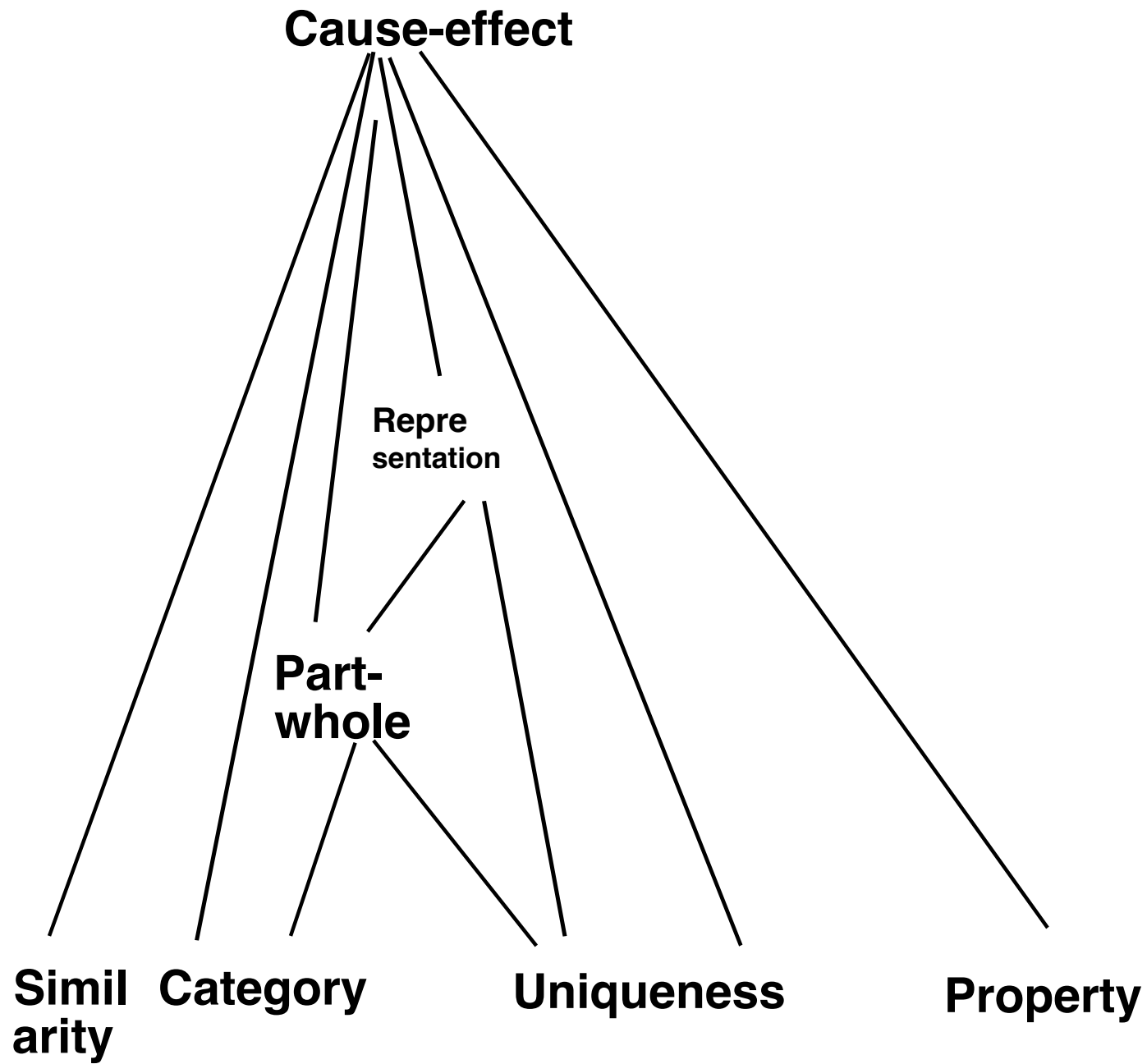












Packing & Unpacking in Mathematics

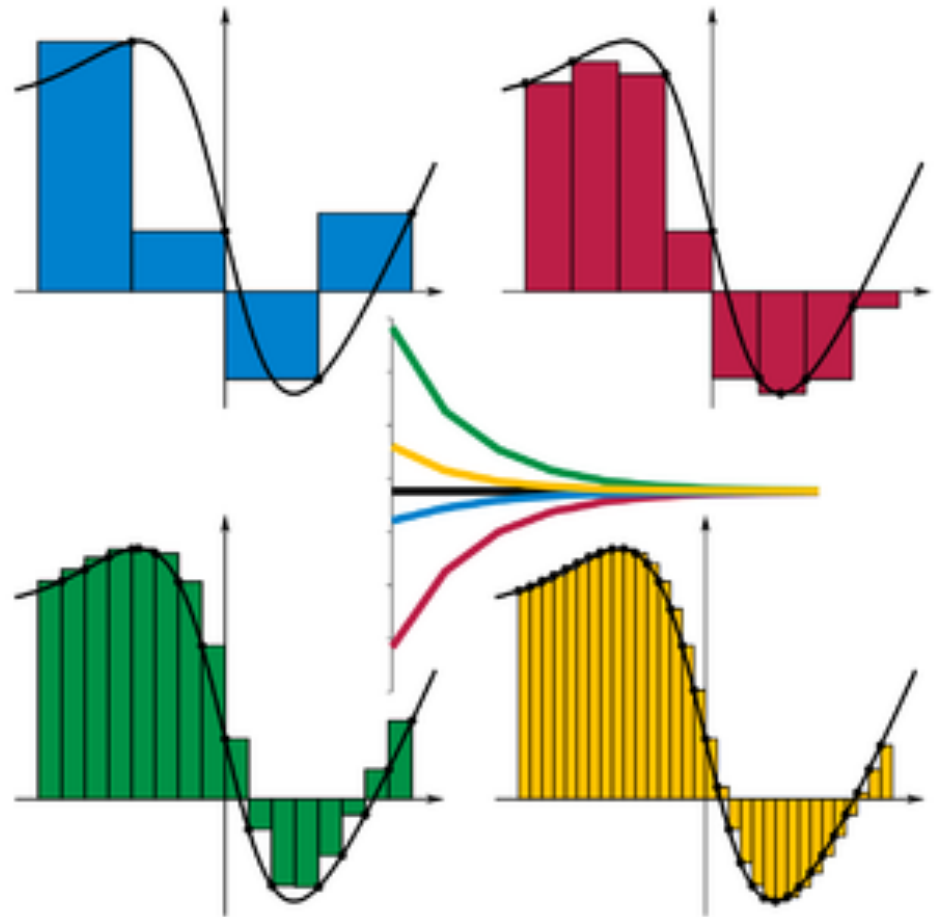
Riemann sum

$$S = \sum_a^b f(x_i)(x_i - x_{i-1})$$

Four of the Riemann summation **methods** for approximating the area under curves. **Right** and **left** methods make the approximation using the right and left endpoints of each subinterval, respectively.

Maximum and **minimum** methods make the approximation using the largest and smallest endpoint values of each subinterval, respectively.

The values of the sums converge as the subintervals halve from top-left to bottom-right.



— Wikipedia

Riemann integral

Let f be a non-negative **real**-valued function of the interval $[a,b]$, and let $S = \{(x,y) \mid 0 < y < f(x)\}$ be the region of the plane under the graph of the function f and above the interval $[a,b]$ (see the figure on the top right). We are interested in measuring the area of S . Once we have measured it, we will denote the area by:

$$\int_a^b f(x) \, dx$$

The basic idea of the Riemann integral is to use very simple approximations for the area of S . By taking better and better approximations, we can say that "in the limit" we get exactly the area of S under the curve.

Note that where f can be both positive and negative, the integral corresponds to *signed area* under the graph of f ; that is, the area above the x -axis minus the area below the x -axis. — Wikipedia

Philo of Alexandria, c. 20 BCE—40 CE

How, then, was it likely that the mind of man being so small, contained in such small bulks as a brain or a heart, should have room for all the vastness of sky and universe, had it not been an inseparable portion of that divine and blessed soul? For no part of that which is divine cuts itself off and becomes separate, but does but extend itself. The mind, then, having obtained a share of the perfection which is in the whole, when it conceives of the universe, reaches out as widely as the bounds of the whole, and undergoes no severance; for its force is expansive.

—Philo of Alexandria,

That the Worse Is Wont to Attack the Better (Quod Deterius Potiori Insidiari Soleat), 90.

Mental Packing and Unpacking in Mathematics

Mark Turner

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