The Representation of Mathematics in the Media

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Introduction

The purpose of this essay is to discuss the representation of mathematics in the print and non-print media as a semiotic phenomenon by illustrating how it is incorporated into these forms of mass communication. It will discuss the following points:

(1) Definition of key terms and concepts;

(2) The selective exemplification of mathematics in popular cultural print and non-print media manifestations.

Of necessity, this paper must be judicious in the scope of materials presented. In general, it will not discuss sporadic media allusions to mathematics, that is, where such references are incomplete, faulty, or lacking adequate identification of specific mathematical principles and concepts. Rather, it will consider specific exemplars in which mathematics is an integral part of the work, and accurate in the depiction of how mathematics functions.

Definitions

In this section, the following terms will be discussed:

(1) Mathematics

(2) Media

(3) Semiotics

(4) Popular culture

Mathematics

The American Heritage Dictionary of the English Language (Morris 1979: 806) defines "mathematics" as

"[t]he study of number, form, arrangement, and associated relationships, using rigorously defined linear, numerical and operational symbols ."

Devlin (2000: 5) poses the question “What is mathematics?” His simple response is that it is the “*science of patterns*” (Devlin 2000: 7). Devlin (2000: 8) goes on state that:

> … the patterns studied by the mathematician can be either real or imagined, visual, or mental, static or dynamic, qualitative or quantitative, utilitarian or recreational. They arise from the world around us, from the depths of space and time, and from the workings of the human mind. Different kinds of patterns give rise to different branches of mathematics. For example, number theory studies (and arithmetic uses) the patterns of number and counting; geometry studies the patterns of shape; calculus allows us to handle patterns of motion; logic studies patterns of reasoning; probability theory deals with patterns of chance; topology studies patterns of closeness and position.

Media

Danesi (2009: 192) describes “media” as:

> … any means of transmitting information … the various forms, devices, and systems that make up mass communications considered as a whole, including newspapers, magazines, radio stations, television channels, and Web sites. Before alphabetic writing, the media for communicating information were oral-auditory and pictographic. Writing facilitated the storage of printed texts. Later technology made such texts available to masses of people.

Semiotics

Danesi and Perron (1999: 73; see also Deely 1980: 88-91, 2010: 81-91; Petrilli and Ponzio 2005: 6-10) offer a clear and succinct description of the Peircean "sign" by noting that it consists of three main elements:

> Peirce called the perceivable part of the sign a *respresentamen* (literally 'something that does the representing') and the concept that it encodes the object (literally 'something cast outside for observation'). He then termed the meaning that someone gets from the sign the *interpretant*. This is itself a sign (or more accurately a signified in Saussurean terms) in that it entails knowing what a sign means (stands for) in personal, social and context-specific ways.

Figure 1 represents the tripartite elements of a sign and their interrelationships.

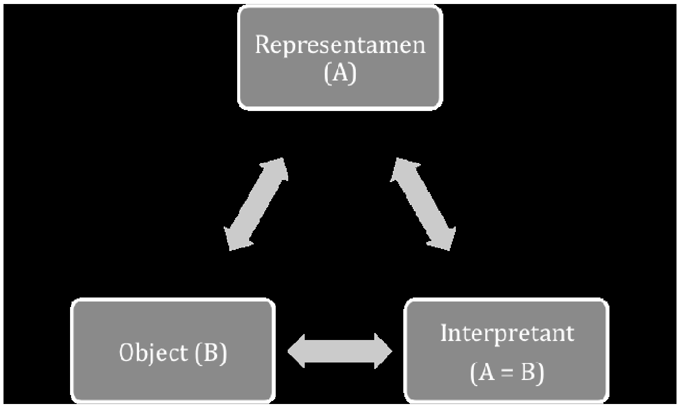


Figure 1. The Peircean Sign and Its Tripartite Components (Based on Danesi 2004b: 26)

A simple example of a mathematical sign involves the notion of numbers as signs. In this respect, Danesi (2008b: 14; See also Dunham 1990, 1994) notes:

> Numerals are signs that encode number concepts. They are so intrinsic to mathematical method that mathematicians rarely differentiate between the two in practice. This is akin to how we rarely make a distinction between words and the concepts they represent. We have become habituated since childhood to perceive all signs and their meanings as *Gestalts* (signifying wholes).
>
> The most basic are the counting numbers, known also as the *integers*, the *natural*, *cardinal*, or the *whole* numbers. These imply holistic quantitative concepts (rather than the ones designating partiality), and are thus similar to the concepts encoded in mass nouns (*rice*, *information*, etc.). This is why the sum or product of whole numbers always produces another whole number.

Danesi (2008b: 16) goes on to observe that numerals represent a code. Moreover, he (Danesi 2008b: 20-21) employs the line metaphor (Lakoff and Núñez 2000: 48, 424; See also Fauconnier 1997; Fauconnier and Turner 1998; Turner and Fauconnier 1995) “Numbers Are Points on a Line” in his discussion of negative and positive numbers together with a line to illustrate this mathematical concept (Figure 2).

______|_____|_____|_____|____|_____|_____|_____|____|______

-4 -3 -2 -1 **0** 1 2 3 4

Figure 2. Numbers are Points along a Line.

With respect to semiotics and its relationship with mathematics, Danesi (2008b: 9-10) states:

The historical link between semiotics and mathematics is an obvious one. In constructing appropriate signs (numerals, plus and minus signs, variables in algebra, equations, etc.) to represent numerical and spatial concepts, mathematicians have always been engaging in semiotic thinking. Since the late 1980s, this intrinsic link has caught the attention of math educators, as witnessed by the number of studies, special issues of journals, monograph series, websites, and a host of books dealing with the semiotics-mathematics interface, either directly or indirectly….

In his introductory chapter on problem-solving Danesi (2008b: 37) states that:

… math competence is related to semiotic competence–the ability to understand sign structures and how they represent things (iconically, indexically, symbolically, etc.). Semiotic pedagogy involves imparting this ability through techniques derived from semiotic practices and concepts (opposition, concretization, annotation, generalization, contextualization, analogies to real life, etc.).

Popular Culture

Kroeber and Kluckhohn (1952: 66) define culture as follows:

Culture consists of patterns of and for behavior acquired and transmitted by symbols, constituting the distinctive achievements of human groups, including their embodiments in artifacts, the essential core of culture consists of traditional [= historically derived and selected] ideas and especially their attached values.

It is possible to speak of a mathematics culture in the sense that its practitioners employ a common set of symbols (digits, integers, numbers, numeral, operations, and so forth), which constitutes a professional language, or jargon, that one must acquire over time as is the case with first language acquisition. Moreover, mathematicians communicate via email, telephone, the Internet, research publications, and professional meetings to form a mathematical community.

While there is no discrete dictionary entry for the phrase "popular culture", Morris (1979: 321) provides the following definition of "culture", namely, "[t]he totality of socially transmitted behavioral patterns, arts, beliefs, institutions, and all other products of human work and thought characteristic of a community or population." "Popular", of course" comes from the Latin "*popularis*", which means "[o]riginating among the people" (Morris 1979: 1020).

Mathematical Problems

Problem-solving activities are a common and effective strategy for teaching the various domains of mathematics. In his brief but cogent discussion of the notion of a mathematical problem, Danesi (2008b: 2) states that a problem is "… a type of question, consisting of information or a set or conditions designed to elicit a particular answer." In particular, he (Danesi 2008b: 2-3) notes that:

> Since antiquity, mathematicians have conceived of two basic types of problems–one in which the solution can be easily envisaged, and one in which it cannot. In English, the former is called a *problem* proper and the latter a *puzzle*. Generally speaking, the problem provides all the required information to reach a solution directly; the puzzle, on the other hand, provides information that appears to be incomplete or else that conceals a twist or clever trap, thus making it much more difficult to reach a solution. Both types are used typically in classroom pedagogy.

Subsequently, Danesi (2002: 30) speaks of problem solving as insight thinking, that is, "…as an admixture of the imagination and memory that leads us literally to see the pattern or twist that a puzzle conceals." Charles Sanders Peirce (1832-1898) refers to this as abductive thinking, or abduction, that is, a problem-solver is able to utilize a "hunch" on what is involved in the puzzle or problem (Ayim 2010: 1-2).

Mathematics in the Media

In his prefatory statement to *Problem-Solving in Mathematics: A Semiotic Perspective for Educators and Teachers*, Danesi (2008b: vii) points out that:

> Skill in math is becoming more and more of a practical necessity in today's information-based society. As journalist Thomas Friedman (2007: 300) has aptly put it, the reason is that the 'the world is moving into a new age of numbers' in which 'partnerships between mathematicians and computer scientists are bulling into whole new domains of business and imposing efficiencies in math.' Even the contemporary arts and entertainment worlds have taken notice. Popular television shows, such as the crime drama *Numbers*, and movies such as *A Hill on the Dark Side of the Moon*, *Good Will Hunting*, *Pi*, *Conceiving Ada*, and *Proof* (to mention just a few) are built around, or involve, mathematics as a basic element in their narrative makeup.

Danesi's observations about the ever-increasing role of mathematics in the media and popular culture are well taken. He (Danesi 2008b: vii) further notes that "math phobia" is a relatively new phenomenon. In fact, during the past millennia, people have engaged in numbers and mathematical problems as a source of pleasure and delight. As Danesi (2008b: vii) asserts there has always been a "math recreation industry", which goes back to ancient times. Current evidence of recreational mathematics is to be found in the plethora of materials including popular magazines devoted to puzzles and problem-solving featured in *Games Magazine*, as well as in most daily newspapers (Sudoku®, KenKen®). In the nineteenth century, Sam Loyd (1841-1911) was the best-known individual in the field of recreational mathematics. In fact, he his life and work have been chronicled because of his fame in the area of recreational mathematics (Gardner 1959, Loyd 2007, Slocum and Sonneveld, 2006).

As noted previously, this study examines selectively the use of "mathematics", broadly defined, in the media. It will examine issues related to this symbolic form of communication. Mathematics has a wide variety of media manifestations in print and non-print format.

Print Media

The print media take various forms, namely, books, newspapers, and magazines, although other less permanent forms may occur such as pamphlets, signboards, pieces of paper used as handouts or affixed to walls, and even *graffiti* on walls and other surfaces. In what follows, the representation of mathematics in the print media (books, newspapers, magazines) will address selectively the following domains: (1) creative literature; and (2) recreational mathematical problem-solving, and its occasional tangible manifestations.

Creative Literature

In his essay on mathematics in creative literature, or what Ian Stewart (Abbott 2002: 33) designates "mathematics fiction", Tony Mann (2010: 58) specifically omits from his discussion what he labels the two best known examples of this genre: Lewis Carroll's (Charles Lutwidge Dodgson, 1832-1898) *Alice in Wonderland* and its sequel *Through the Looking-Glass* as well Edwin A. Abbott's (1838-1926) *Flatland: A Romance of Many Dimensions* in order to discuss lesser known exemplars. Mann (2010: 60) proposes a classification of mathematics in fiction as follows:

- Fiction by mathematicians
- Fiction using mathematical structures
- Fiction expounding mathematics
- Other [to include fiction by Jorge Luis Borges (1899-1986)]
- Fiction about real mathematicians
- Fiction about doing mathematics
- Fictions about mathematical ideas
- Fictions with mathematicians as characters

Mann's essay provides an ample listing of mathematical fiction. However, the rubric "Fiction using mathematical structures" and "Fiction expounding mathematics" are the two categories that are most relevant to this study. For the purposes of this paper, the discussion will focus on the two works that Mann elected not to discuss, that is, Carroll's (2000) *Alice in Wonderland* and *Through the Looking-Glass*, and Abbott's (2002) *Flatland: A Romance of Many Dimensions* as exemplars, par excellence, of mathematics fiction. Moreover, several other related words will be noted. Nevertheless, Mann's (2010) discussion and analysis of other mathematical fiction merit the reader's attention.

Alice's Adventures in Wonderland & *Through the Looking-Glass*

At least twenty-two film and television versions of *Alice's Adventures in Wonderland* exist. These include US, English, Japanese, Russian and Australian versions in conventional film, animation, and stop action formats. The most recent version is the 2010 Walt Disney Pictures version with its computer-animated live action format. It is, in fact, a sequel to the original Lewis Carroll novel of 1865. In addition to the film adaptations, numerous published versions are available in translation in many languages worldwide. Furthermore, at least eight comic book renderings of the works are available. Moreover, there have been live performances as early as 1886 in the Prince of Wales Theatre in London. Numerous performances of this classic have occurred since then including one scheduled for Broadway in April 2011. The influence of this work in popular culture is significant. A few selected examples suffice to illustrate this, namely, art (Salvador Dalí, 1904-1989), computer and video games (*Alice in Wonderland*, 2000, Nintendo), literary restatements (*Wonderland Revisited and the Games Alice Played There*, Sheppard 2009; 1951-), classical music (*Through the Looking Glass*, 2009, opera by Alan John, 1958-), popular music (Jefferson Airplane [1965-1974], *White Rabbit*, 1967), radio (presentation by the Lewis Carroll Society of Canada), science and technology (*Mirrors in Mind*, Gregory 1997, Richard Gregory 1923-2010 with a discussion of the science of mirror images in Lewis Carroll's work), and tourist attractions (Walt Disney Parks and Resorts).

Mathematician Martin Gardner (1914-2010), a specialist in recreational mathematics, produced the annotated Alice (Carroll 2000), the definitive edition of *Alice's Adventures in Wonderland* &*Through the Looking-Glass* (Carroll 2000) authored by Carroll (pseudonym for Charles Lutwidge Dodgson, 1832-1898), a mathematician and author of children's books. Martin Gardner's annotations provide information and insight into this children's fantasy book by revealing some of its mathematical allusions. It should be noted that Gardner comments on other aspects of these two works, namely, the nineteenth-century English cultural allusions. What is of interest here are some selected mathematical references that shed light on the mathematics of that era.

In his annotated edition of *Alice's Adventures in Wonderland & Through the Looking Glass* (Carroll 2000), renowned recreational mathematician Martin Gardner (1914-2010) provides insightful and detailed marginal annotations concerning the mathematical allusions in this children's book.

Keith Devlin (2010) has also written about the hidden math in Lewis Carroll's masterpiece. In his discussion of Melanie Bayley's (2009; cf. Pycior 1984) essay on the mathematics in Carroll's work, Devlin (2010) points out that during the second half of the nineteenth century the field of mathematics was undergoing paradigm shifts in several of its sub-disciplines, which caused it to be more abstract in its formulation. Devlin (2010) cites, for example, non-Euclidean geometries, abstract, or symbolic, algebra, imaginary numbers, to name but a few of the groundbreaking developments. Devlin further points out that Charles Lutwidge Dodgson was a tutor of mathematics with a decidedly traditional bent. In her analysis of algebra in *Alice in Wonderland*, Bayley (2009) observes that the material in this children's book was a nineteenth century Victorian satire of the then "new math".

At this juncture, it is worth noting the mathematical elements encoded in these two works. There are several allusions to mathematics in Gardner's (Carroll 2000) annotated text. The first appears in the second chapter ("Pool of Tears") in which Alice does some multiplication tables. By the conventional base-10 model, her math is in error. Martin (Carroll 2000: 23) points out that Alice's calculations are, in fact, correct when other bases are used, for example, 18 and 21. Likewise, Pycior (1984: 165) notes that in this chapter, Alice engages in multiplication tables, which are in a different base system. Pycior (1984: 165) believes that Alice's problems with multiplication tables constitute Dodgson's (Carroll's) belief that mathematics is arbitrary. Pycior (1984: 165-166) strengthens her case for this claim by discussing the matter further when she states:

> Carroll's choice of a varying nondecimal base is designed, I believe, to indicate the arbitrariness of mathematics. Mathematicians are free to calculate in any base they choose; there is nothing sacred about the base ten. But in addition to the arbitrariness of mathematics, the passage alludes to its meaninglessness. Alice clearly states that "the Multiplication Table doesn't signify." Besides punning on the meaninglessness of mathematics, the verb "signify" for Carroll may also have been associated with mathematical writings of the symbolical algebraists, where it was frequently used. Their main point – and Alice's as expressed here – was that algebra (in Alice's case, arithmetic) is basically meaningless, that its symbols (here, numbers) stand for nothing in particular. Alice finds that mathematics is no mainstay of truth and certainty, and cannot save her from the madness of the underground world. Viewed from this perspective, the meaninglessness and arbitrariness of symbolical algebra provide a key to the meaninglessness and arbitrariness of Carroll's underground world.

With respect to the multiplication tables, Alice states that (Carroll 2000: 23, chapter 2, “Pool of Tears’; see also Taylor 1952: 47, Abeles 1976: 183):

> I’ll try to know all the things I used to know. Let me see: four times four is twelve, and four times six is thirteen, and four times seven is is–oh dear! I shall never get to twenty at that rate! However, the Multiplication-Table does not signify: let’s try Geography,

In his marginal annotation, Gardner (Carroll 2000: 23; Abeles 1976: 183) points out that the use of a different base system renders Alice’s multiplication correct.

In chapter 5 (“Advice from a Caterpillar”), Bayley (2009) notes that, in this episode, which involves Alice’s fluctuation in size because of her improper consumption of a mushroom to restore her to her normal size, is a subtle allusion to the Dodgson’s contempt for symbolic algebra. Gardner’s annotated *Alice* (Carroll 2000: 47-56) contains no reference to this aspect of Carroll’s mathematical allusion. As Bayley (2009) points out, Dodgson was a conservative mathematician who held traditional views about mathematics, especially Euclidean geometry. She notes that in several instances in the revised version of the original text, the author inserts subtle critiques of the new mathematics emerging in the nineteenth century. This includes Euclidean geometry, which was facing challenges from projective geometry.

In his online column, “Devlin’s Angle” (Devlin 2010), Keith Devlin discusses Bayley’s (2009) essay, and he concurs with her observations on Carroll’s (Dodgson’s) mathematical notions presented satirically and metaphorically in *Alice’s Adventures in Wonderland* (Carroll 2000) when he alludes to Bayley’s (2009) essay on the mathematical significance of Alice when Devlin (2010) states the following:

> In the caterpillar scene, Alice's height fluctuates between 9 feet and 3 inches. Alice, bound by conventional arithmetic where a quantity such as size should be constant, finds this troubling: "Being so many different sizes in a day is very confusing," she complains. "It isn't," replies the Caterpillar, who lives in this absurd world.
>
> The Caterpillar's warning, at the end of this scene, is perhaps one of the most telling clues to Dodgson's conservative mathematics, Bayley suggests. "Keep your temper," he announces. Alice presumes he's telling her not to get angry, but although he has been abrupt he has not been particularly irritable at this point, so it's a somewhat puzzling thing to say. But the word "temper" has another meaning of "the proportion in which qualities are mingled." So the Caterpillar could well be telling Alice to keep her body in proportion - no matter what her size. This may be another reflection of Dodgson's love of Euclidean geometry, where absolute magnitude doesn't matter: what's important is the ratio of one length to another. To survive in Wonderland, Alice must act like a Euclidean geometer, keeping her ratios constant, even if her size changes.

In chapter 6 (“Pig and Pepper”), Bayley (2009) argues that Carroll is satirizing projective geometry. As Bayley points out, in this chapter, Alice shrinks herself in order to enter a tiny house. In this chapter, the Duchess is nursing a baby, which transforms into a pig, when handed to Alice. Devlin (2010) goes on to explain in more detail the satire on projective geometry as follows:

> According to Bayley, the target of this scene is projective geometry, a subject that involved concepts that Dodgson would have found ridiculous, particularly the "principle of continuity." Jean-Victor Poncelet [1788-1867], the French mathematician who set out the principle, described it as follows: "Let a figure be conceived to undergo a certain continuous variation, and let some general property concerning it be granted as true, so long as the variation is confined within certain limits; then the same property will belong to all the successive states of the figure."
>
> When Poncelet talked of "figures", he meant geometric figures, of course, but Dodgson playfully subjects Poncelet's description to strict logical analysis and takes it to its most extreme conclusion. He turns a baby into a pig through the principle of continuity. Importantly, the baby retains most of its original features, as any object going through a continuous transformation must. His limbs are still held out like a starfish, and he has a queer shape, turned-up nose and small eyes. Alice only realizes he has changed when his sneezes turn to grunts.
>
> The baby's discomfort with the whole process, and the Duchess's unconcealed violence, signpost Dodgson's virulent mistrust of "modern" projective geometry, Bayley says. Everyone in the pig and pepper scene is bad at doing their job. The Duchess is a bad aristocrat and an appallingly bad mother; the Cook is a bad cook who lets the kitchen fill with smoke, over-seasons the soup and eventually throws out her fire irons, pots and plates.

Pycior (1984) provides a detailed discussion of Lewis Carroll's (2000) satire of symbolical algebra in *Alice's Adventures in Wonderland*. In particular, she points out that the mathematician Augustus De Morgan's (1806-1871) *Trigonometry and Double Algebra* (De Morgan 1849) with its presentation of symbolic, or abstract algebra, was at odds with Charles Lutwidge Dodgson's (Lewis Carroll) traditional view of algebra, so he parodied it in his popular children's book. Pycior (1984: 149) argues persuasively that "[t]he theme of meaninglessness, emphasized in earlier interpretations of the *Alices*, can be traced back to Dodgson's encounter with the symbolical approach; the roots of his nonsense verse may also be in symbolical algebra, which stressed in mathematics structure over meaning".

In that essay, Pycior (1984: 150) provides a historical overview of the then evolving "new math" of the mid nineteenth century. She (Pycior 1984: 154-159). further notes that there existed a rich tradition of mathematical humor during that time frame. Thus, Dodgson's (Carroll's) satire of the emerging math paradigm fit in with Victorian traditions. Pycior (1984: 151-154) goes on to illustrate that many of Alice's comments are clearly a parody of the new mathematics as illustrated in the passage below, which may be taken as a critique of the notion of negative numbers (Carroll 2000: 75; chapter 7, "A Mad Tea Party"):

> "Take some more tea," the March Hare said to Alice very earnestly.
>
> "I've had nothing yet, Alice replied in an offended tone: "so I ca'n't take more."
>
> "You mean you ca'n't take *less*," said the Hatter: "it's very easy take *more* than nothing."
>
> "Nobody asked *your* opinion," said Alice.

Later in the same work (Carroll 2000: 99; chapter 9, “The Mock Turtle’s Story”), the issue of negative numbers again appears:

> “And how many hours a day did you do lessons?” said Alice, in a hurry to change the subject.
>
> “Ten hours the first day,” said the Mock Turtle: “nine hours the next, and so on.”
>
> “What a curious plan!” exclaimed Alice.
>
> “That’s the reason they are called lessons,” the Gryphon remarked: “because they lessen from day to day.”
>
> This was quite a new idea to Alice, and she thought it over a little before she made her last remark.
>
> “Then the eleventh day must have been a holiday?”
>
> “Of course it was,” said the Mock Turtle.
>
> “And how did you manage on the twelfth?” Alice went on eagerly.

Pycior (1984: 153-154) further observes that the issue of negative numbers reappears in *Through the Looking-Glass* (Carroll 2000: 253-254) when the Red Queen says:

> “Can you do Addition?” the White Queen asked. “What's one and one and one and one and one and one and one and one and one and one?'
>
> “I don't know,” said Alice. “I lost count.”
>
> “She ca’n't do Addition,' the Red Queen interrupted. “Can you do Subtraction? Take nine from eight.”
>
> “Nine from eight I ca’n't, you know,” Alice replied very readily: “but –“
>
> “She can't do Subtraction,” said the White Queen. `Can you do Division? Divide a loaf by a knife -- what's the answer to *that*?”
>
> “I suppose –“ Alice was beginning, but the Red Queen answered for her. “Bread-and-butter, of course. Try another Subtraction sum. Take a bone from a dog: what remains?”
>
> Alice considered. “The bone wouldn't remain, of course, if I took it -- and the dog wouldn't remain; it would come to bite me -- and I'm sure *I* shouldn't remain!”
>
> “Then you think nothing would remain?” said the Red Queen.
>
> “I think that's the answer.”
>
> “Wrong, as usual,” said the Red Queen: “the dog's temper would remain.”
>
> “But I don't see how –“
>
> “Why, look here!” the Red Queen cried. “The dog would lose its temper, wouldn't it?”
>
> “Perhaps it would,” Alice replied cautiously.
>
> “Then if the dog went away, its temper would remain!” the Queen exclaimed triumphantly.
>
> Alice said, as gravely as she could, “They might go different ways.” But she couldn't help thinking to herself, “What dreadful nonsense we are talking!”
>
> “She can't do sums a bit!” the Queens said together, with great emphasis.

The satirical critique of negative numbers represents Dodgson's (Carroll's) dissatisfaction with the new abstract math, which permitted such notions. Lakoff and Núñez (2000: 424; see also Danesi 2008b: 20-22) provide their readers with a way to conceptualize this sometimes difficult notion even in today's world when they state that:

> Given the Numbers Are Points on a Line metaphor, we form the Number-Line blend, in which all real numbers–including the positives, the negatives, and zero–are conceptualized as spread out along a line with zero at a point called the origin. The positive numbers are conceptualized in this blend as being on one of the origin (zero) and the negative numbers on the other of the origin, with *-n* exactly as from zero as *+n*, for all nonzero real numbers *n*. That is, *-n* and *+n* are symmetrical points relative to the origin (zero).

The mathematics of this era was undergoing what Kuhn (1970) labeled a paradigm shift, that is, an essential and profound change in basic underlying assumptions of a scientific discipline. When a certain number of anomalies appear in an existing paradigm, there is a pivotal time in which a scientific genius offers a new approach to this incongruity by looking at the data in an entirely new way. In *The Structure of Scientific Revolutions,* Kuhn (1970) discusses three phases of a scientific revolution: (1) Pre-paradigmatic phase, or a lack of theoretical agreement; (2) normal science, or the resolution of longstanding problems within an agreed upon model; and (3) revolutionary science, or a period when long held assumptions are reviewed, challenged, and replaced during this critical phase.

In many ways, mathematics, in the middle of the nineteenth century was undergoing a sort of Kuhnian revolution, as Devlin (2002: 22) aptly describes. This mathematical revolution involved what Devlin (2002: 22) calls a shift from "… *doing* to *understanding*…". This revolution, as Devlin points out, was a quiet one led by the Mathematicians Johan Peter Gustav Lejeune Dirichlet (1805-1859), Julius Wilelm Richard Dedekind (1831-1916), and Bernhard Riemann (1825-1866) at the University of Göttingen in Germany. All three were involved in a transformation of mathematics from a concrete to an abstract theoretical approach, for example, abstract algebra and a shift away from Euclidean geometry. This revolutionary shift involved "the properties of abstract functions" (Devlin 2002: 26).

Carroll's (2000) *Alice's Adventures in Wonderland* and *Through the Looking Glass* thus offer insight into the profound transformations in mathematics in the nineteenth century. These two works present a conservative and reactionary view of mathematics at a time when this discipline was undergoing profound changes.

At about the same time of Carroll's now famous works on *Alice*, another nineteenth century Victorian writer and mathematician, Edwin A. Abbott (1838-1927) was also writing mathematical fiction about geometry and dimensions including the then popular theme of the fourth dimension.

Flatland

As noted previously, in his annotated edition of *Flatland: A Romance of Many Dimensions* (Abbott 2002: 33), Ian Stewart points out that an entire sub-genre of creative literature known as "mathematics fiction" exists. In one of his annotations (Abbott 2002: 33, note 1), Stewart states that:

> With his opening sentence [of his novel *Flatland*], Abbott transports his readers straight into a new universe. This is a science fiction type of opening, and ABBOTT, EDWIN (ABBOTT) is the second entry in *The Encyclopedia of Science Fiction*, edited by John Clute and Peter Nicholls [1993]. Though not part of the classic science fiction genre–in part because it is too early – *Flatland* is widely admired in science fiction circle and belongs firmly to the prehistory of science fiction….

> Indeed, *Flatland* is one of the earliest works of what might be termed mathematics fiction – speculative fiction with a mathematics theme. In this aspect, it was preceded by *Alice's Adventures in Wonderland* (1865 [Carroll 2000]) and *Through the Looking Glass and What Alice Found There* (1871 [Carroll 2000]) by Lewis Carroll (Charles Lutwidge Dodgson, 1832-1898). However, the mathematics in *Alice* is relatively well concealed: Dodgson was a mathematician in his "day job" and some of it rubs off in the narrative. In his poem *The Hunting of the Snark* (1876), the mathematical influence is more overt; see Martin Gardner's *The Annotated Snark* [Carroll 2006]. Dodgson and Abbott have some things in common: Both were clergymen, both enjoyed mathematics, both loved writing, and they lived at much the same time.

In his annotated *Flatland* volume, Stewart (Abbott 2002: 33-36) enumerates several examples of what he calls *Flatland* derivatives including *Sphereland: A Fantasy about Curved Spaces and an Expanded Universe* (Burger 1995), *The Planiverse: Computer Contact with a Two-Dimensional World* (Dewdney 2001, 1941-). In his first marginal note (Abbott 2002: 33-36), which extends for four pages, Stewart also alludes to short stories and other works, though he does not refer the reader to his own work *Flatterland: Like Flatland, Only More So* (Stewart 2001). Moreover, Stewart does not mention Rudy Rucker's (2002, 1946-) *Spaceland: A Novel of the Fourth Dimension* because it was probably not available to him at the time of publication of his annotated edition of *Flatland* (Abbott 2002). Segments of *Flatland The Movie* (2007) are available at two web sites: (Flatland the Movie 2011; Math in the Movies: Flatland the Movie, 2011; see References for the complete web site information).

In 2001, Ian Stewart (1945-), a professor of mathematics at the University of Warwick, UK, published *Flatterland: Like Flatland Only More So* (Stewart 2001) intended as a sequel to *Flatland*. Victoria Line, the great-great granddaughter of A[lbert]. Square comes upon a copy of the original *Flatland* in the basement of her home. This fortuitous encounter causes Vikki to call upon a sphere from "Spaceland" to visit her whereupon "Space Hopper", a creature capable of moving between Flatland and Spaceland arrives. It is in this new work that modern theories of geometry are presented including fractional dimensions (a geometrical shape capable of being split into parts), isolated points (a discrete countable set), as well as topology (spatial properties capable of preservation under continuous deformation, e.g., the Möbius strip with a single surface and a single edge), and hyperbolic geometry (non-Euclidean geometry).

As previously noted, various literary works in which mathematics plays an important role exist. In terms of geometry, there is Edwin Abbott. Abbott's (1838-1927) now classic *Flatland: A Romance of Many Dimensions* (1884, Abbott 2002) is a multifaceted novel. On the one hand, it was a social satire of Victorian England. On the other hand, some mathematicians and physicists, however, consider *Flatland* to be a clever examination of dimensionality.

The novel itself bears the the pseudonym "A. Square" instead of his own name. In his annotated version of *Flatland*, Ian Stewart (Abbott 2002: 1-2) argues that many believe this pseudonym is a pun on Abbott's own name (Edwin Abbott Abbott).

Various film versions now exist including the animated version of *Flatland* (1965) narrated by Dudley Moore (1935-2002), and Roddy Maude-Roxby (1930-). A short film of the same name appeared in 1982 directed by mathematician Michele Emmer (1945-), as well as *Flatland: The Movie* (2007), another animated film with the voices of Martin Sheen (1940-), Kristen Bell (1980-), Michael York (1942-), and Tony Hale (1970-). Selected television programs contain allusions to *Flatland* including *The Outer Limits* (1963-1965) episode entitled "Behold, Eck!" (October 3, 1964) with a two-dimensional creature named Eck who accidentally fell into our three-dimensional world. Finally, an episode ("The Psychic Vortex", January 11, 2010) of *The Big Bang Theory* (2007-) contains a reference to *Flatland* and this segment of the program is available on video on the Internet (Big Bang Theory Flatland, 2011: see the entire segment at the following web site: Big Bang Theory Flatland 2011, see the Reference section for complete web site information).

The original version of *Flatland* is based on concepts related to plane geometry defined as "[a] surface containing all the straight lines connecting any two points on it" (Morris 1979: 1002). In *The Annotated Flatland: A Romance of Many Dimensions*, Ian Stewart (Abbott 2002), author of *Flatterland: Like Flatland, Only More So* (Stewart 2001) provides a comprehensive commentary on Abbott's pseudonymous novella. In his introduction, Stewart observes that the slim volume consisted of two parts with the first a satire of Victorian England. The second, however, introduced the dimensional concepts (Abbott 2002: xvii) points out that:

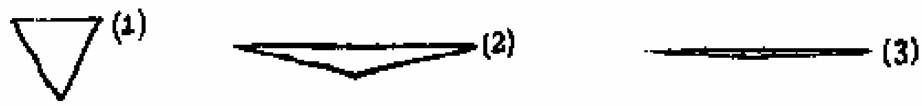
> The real aim of the second part of the book is to introduce the concept of the fourth dimension, which it does by analogy. The difficulties that a two-dimensional being experiences in comprehending a *third* dimension are used to help Victorians living in a three-dimensional space accustom themselves to the radical but enormously popular idea of a fourth dimension. The vehicle for the explanation of these ideas is to recount the adventures of a character named A. Square, who explores spaces of various dimensions: Flatland itself (two-dimensional, a perfect Euclidean plane), Lineland (one dimension), Pointland (no dimensions), and Spaceland (three dimensions).

In their discussion of narrative structure, Danesi and Perron (1999: 250-251) provide this insightful description of *Flatland* (Abbott 2002) in terms of narrative perspective:

> The characters of the novel are personified geometrical figures, known as Flatlanders, living in a two-dimensional universe called Flatland. Flatlanders can only see each other as dots or lines, even if they are, from our vantage point as readers, circles, squares, triangles, etc. The novel provides us with this perspective by projecting us into the mind of a Flatlander. To grasp what kind of perspectival view this entails, one should imagine Flatland as the flat surface of a table. A Flatlander can see figures in only one dimension: i.e., as dots or lines depending on their orientation (*looking from within*). For example, if one looks at a paper circle lying on the table, with the eyes level with the table's surface, s/he will see the edge of the circle as a line. The same applies to any other shape. The only way then to distinguish a circle from a straight line, an ellipse, or any other figure is to view Flatland from a vantage point above it, i.e. to look down at the shapes from above the table. This three-dimensional viewing of the figures constitutes a *looking from without* perspective. It literally provides the reader with a different view of Flatland and its inhabitants. Similarly, although the perspective in most novels is not 'virtual' as it is *Flatland*, the reader's understanding of any narrative is invariably conditioned by one of these two mental vantage points–*looking from within* and *looking from without*.

In his first chapter of Part I ("This World"), Abbott (2002: 34-35) provides a description and an accompanying graphic to describe Flatland as follows:

> I call our world Flatland, not because we call it so, but to make its nature clearer to you, my happy readers, who are privileged to live in Space.
>
> Imagine a vast sheet of paper on which straight Lines, Triangles, Squares, Pentagons, Hexagons, and other figures, instead of remaining fixed in their places, move freely about, on or in the surface, but without the power of rising above or sinking below it, very much like shadows - only hard and with luminous edges - and you will then have a pretty correct notion of my country and countrymen. Alas, a few years ago, I should have said "my universe": but now my mind has been opened to higher views of things.
>
> In such a country, you will perceive at once that it is impossible that there should be anything of what you call a "solid" kind; but I dare say you will suppose that we could at least distinguish by sight the Triangles, Squares, and other figures, moving about as I have described them. On the contrary, we could see nothing of the kind, not at least so as to distinguish one figure from another. Nothing was visible, nor could be visible, to us, except Straight Lines; and the necessity of this I will speedily demonstrate.
>
> Place a penny on the middle of one of your tables in Space; and leaning over it, look down upon it. It will appear a circle.
>
> But now, drawing back to the edge of the table, gradually lower your eye (thus bringing yourself more and more into the condition of the inhabitants of Flatland), and you will find the penny becoming more and more oval to your view; and at last when you have placed your eye exactly on the edge of the table (so that you are, as it were, actually a Flatlander) the penny will then have ceased to appear oval at all, and will have become, so far as you can see, a straight line.
>
> The same thing would happen if you were to treat in the same way a Triangle, or Square, or any other figure cut out of pasteboard. As soon as you look at it with your eye on the edge on the table, you will find that it ceases to appear to you a figure, and that it becomes in appearance a straight line. Take for example an equilateral Triangle - who represents with us a Tradesman of the respectable class. Fig. 1 represents the Tradesman as you would see him while you were bending over him from above; figs. 2 and 3 represent the Tradesman, as you would see him if your eye were close to the level, or all but on the level of the table; and if your eye were quite on the level of the table (and that is how we see him in Flatland) you would see nothing but a straight line.



Stewart (Abbott 2002: xxv) notes that in a two-dimensional world there are "…no knots, telephone lines cannot cross without intersecting; organisms cannot have (conventional) digestive tracts or they would fall apart; and if you drive a nail through a plank of wood, the woods splits into two separate pieces."

Flatland, the Movie. http://www.veoh.com/browse/videos/category/technology/watch/v809831x7RpfzdR Accessed 18 February 2011.

BigBang Theory_Flatland. http://www.youtube.com/watch?v=GmiXemW_oBQ Accessed 4 January 2011.

As the first novel to incorporate plane geometrical notions into an interesting narrative about a two-dimensional world, Edwin A. Abbott provides the reader with entertainment and education about the concept of dimensions. This work has been in print continuously since it first appeared in 1884. It thus provides the reader with an edifying introduction to nineteenth century Victorian England views about geometry and society, and the great interest in the fourth dimension, which would permit a view of the interior of solids (Abbott 2002:164-173).

The examples of creative literature in this section provide two instances of “mathematics fiction”. On the one hand Lewis Carroll offers a subtle critique of the “new mathematics” which troubled him a great deal, while Edward A. Abbott provides a critique of Victorian society and an innovative literary presentation of the three dimensions (length, width, depth). At the time of the publication of *Flatland* (1884), there was a great interest in the fourth dimension (time). The literature on mathematical fiction is now a rich one as Mann (2010) has illustrated.

Recreational Mathematics

Recreational mathematics refers to a wide assortment of mathematical puzzles and games. Historically, as Danesi (2002: 146) points out, recreational mathematics has existed for a very long period. Danesi cites Kasner and Newman (1940: 156) who state that the

"theory of equations, of probability, the infinitesimal calculus, the theory of point sets, of topology, all have grown out of problems first expressed in puzzle form."

Perhaps the earliest collection of recreational mathematical puzzles is a book (Bachet de Mézirac 1612) by Claude Gaspard Bachet de Mézirac (1581-1638). Since the nineteenth century, many more mathematicians have contributed to the field of recreational mathematics, for example, Lewis Carroll (Charles Lutwidge Dodgson, 1832-1998), Sam Loyd (1841-1911), François Edouard Anatole-Lucas (1842-1891), Henry E. Dudeney (1857-1930), W. W. Rouse Ball (1850-1925), Hubert Phillips (1891-1964), Raymond Smullyan (1919-), and Martin Gardner (1914-2010) are some of the best know practitioners of this domain.

Recreational mathematical magazines have also appeared with frequency during the twentieth century, for example, *Sphinx* (1931-1939), *Recreational Mathematics Magazine* (1961-1964), *The Journal of Recreational Mathematics* (1968-), *Eureka* (1978-), and *Games* (1977-1986, 1987-1990, 1991-1995, 1996-). The venerable *Scientific American* (1845-) featured a column by Martin Gardner entitled "Mathematical Games and Diversions" from 1956 to 1981. Various columns by Gardner have been collected into published books including one entitled *The Colossal Book of Mathematics: Classic Puzzles, Paradoxes, and Problems* (Gardner 2001), which contains fifty of his most popular columns from that journal. Douglas Hofstadter (1945-), author of the Pulitzer Prize winning volume *Gödel, Escher, Bach: An Eternal Golden Braid* (Hofstadter 1999 [1979]) likewise published a column in the same journal entitled "Metamagical Themas", an anagram of the earlier column "Mathematical Games" from 1981 to 1983.

Richard Restak (2010), a physician and author of many books on neuroscience, discusses his rationale for collaborating on a book entitled *The Playful Brain: The Surprising Science of how Puzzles Improve Your Mind* with Scott Kim (1955-), a well-known puzzleologist, when he (Restak 2010: 1) states that:

> My own interest in puzzles stems from my ongoing effort to develop new and innovative approaches to brain-performance enhancement. I've aimed at answering this question: What activities can my readers engage in that will enhance not just the whole brain but *distinct brain areas and processes*?

More than a decade before Restak's (2010) book on puzzle solving, Danesi (1997) produced a book entitled *Increase Your Puzzle IQ: Tips and Tricks for Building Your Logic Power*, which dealt with how to solve such puzzles in a systematic fashion. It contains a total of 105 puzzles with 10 chapters (deductive logic, truth logic, trick logic, arithmetic logic, algebraic logic, combinatory logic, geometrical logic, code logic, time logic, and paradox logic). In each chapter, Danesi (1997) provides a "how to" section, a set of puzzles, and answers with step-by-step solutions. In his book entitled *The Puzzle Instinct: The Meaning of Puzzles in Human Life* (Danesi 2002) provides a typology of mathematical and logical puzzles together with explanations on how to solve them.

Danesi (1985: 2) points out that psychologists define games

"… as problem-solving activities, that is, they are examples of behavior involving thought directed towards the attainment of a solution to a problem posed in some recreational fashion."

Palmer and Rodgers (1983:3) specify the five basic characteristics of what they label “gaming”:

1. Games are *competitive*, i.e., a person competes against another individual, time, personal performance, or a goal.
2. Games are *rule-governed*, i.e., principles determine the acceptable or unacceptable actions or behavior.
3. Games are *goal-defined*, i.e., these activities have their recognized and agreed upon objectives.
4. Games have *closure*, i.e., the participants know when the activity is completed according to pre-determined criteria.
5. Games are *engaging*, i.e., these pastimes are fun and the interactants derive amusement and stimulation from engaging in them.

In the following sections, selective examples of recreational mathematics will be discussed including *Sudoku*, *KenKen®*, and other print manifestations of mathematics (calendars, cartoons, sports mathematics, and various tangible manifestations of mathematical narrative puzzles.

Sudoku

The relatively new puzzle known as *Sudoku* (Japanese 'single number') now appears in many newspapers and magazines worldwide. Moreover, collections of these logic puzzles are in book form frequently arranged in terms of difficulty ("Easy", "Challenging", and so forth). It began to achieve widespread popularity in 1986 when a Japanese publishing organization named Nikoli began publishing puzzles in this format. By 2005, *Sudoku* had become an internationally popular numerical logic game. Its promotion in the west is due to the efforts of Wayne Gould (1945-) who convinced *The Times* of London to start publishing these puzzles. That newspaper published its first puzzles labeled *Su Doku* on November 13, 2004. It must be noted that number puzzles date back to the nineteenth century when some French newspapers published numerical versions of "magic squares", though these did not require logic to solve, but rather arithmetic. The first US paper to feature a *Sudoku* puzzle was *The Conway Daily Sun* (Conway, NH) in 2004. Its rapid growth in its popularity has led to competitive contests, the first of which took place in Lucca, Italy March 10-12, 2006. There was even a television program dedicated to this game in Britain, and first broadcast July 1, 2005.

The typical format of this type of puzzle is 9x9 grid. This larger grid contains sub-grids or boxes with 3x3 squares. Moreover, these 9 rows and the 9 columns contain the numbers 1 through 9. Each puzzle contains selected numbers between 1 and 9. The reader, or puzzle solver, must then fill in the empty slots with the appropriate missing digits. Additional restrictions include the requirement that no number from 1 to 9 may be duplicated in the sub-grids, columns, or rows.

Numerous books and articles have been published about the mathematical properties of *Sudoku*. Some are "how-to" books with examples (Wilson 2005) and others are actual collections of puzzles, which are graded from "easy" to "challenging" (Shortz 2005). After its initial appearance in the US in 2004, most of the books and articles dealt with how to solve these puzzles. Now that *Sudoku* is firmly entrenched in daily newspapers, magazines, and puzzle books, the instructions are minimal since most people are now familiar with its conventions and the strategies for solving them.

Variants of this popular numerical game now appear. They include larger grids, for example, 16x16, and even 25x25. They may also be used with words (*Wordoku*) rather than numbers. There is also a *Sudoku* cube, a version of the Rubik's Cube invented by Jay Horowitz (1947-). Moreover, there is a computer simulation of the *Sudoku* Cube that utilizes VPython, a program language with three-dimensional graphics.

Sudoku is a logical puzzle in the classical sense of the word, that is, it addresses the logical relationship of the numbers 1 to 9 so that there will be a predetermined and fixed outcome in the resolution of this type of puzzle. In this regard, the following set of instructions (*Soduko* 2010) is representative of those found in many daily newspapers.

> The goal is write the numbers in the boxes to satisfy only one condition: each row, column and 3 by 3 box must contain the digits 1 through 9 exactly once. What could be simpler? There is only one solution.

Figures 3 and 4 illustrate a typical *Sudoku* puzzle. Figure 3 is the stimulus and Figure 4 is the correct response.

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Figure 3. *Sudoko* Puzzle Stimulus

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

Figure 4. Solution to *Sudoku* Puzzle in Figure 3.

KenKen®

Robert Fuhrer , the founder of Nextoy (1981), purchased the rights to the trademark names *KenKen®* and *KenDoku*®. The first such puzzles appeared in *The Times* of London and then in *The New York Times*. In Stephey's (2009) *Time Magazine* interview with Will Shortz (1952-), the puzzle editor of *The New York Times*, Shortz speaks about the new puzzle sensation *KenKen*®. Will Shortz is the world famous puzzlelologist who received his B.A. degree from Indiana University at Bloomington in Enigmatology (the only one ever awarded), who is known for his excellent work in crossword puzzles. Moreover, Shortz was the subject of the 2006 documentary *Wordplay* (Creadon 2006) by Patrick Creadon (1967-) about him and his role in the 2005 American Crossword Tournament.

Shortz authored an essay about the new *KenKen*® puzzle in an article that appeared in *The New York Times* (Shortz 2009). Likewise, Shortz (2010) provides a quick step-by-step introduction to the simplicity of the *KenKen*® puzzle in a video entitled "Will Shortz on KenKen®" (2010).

The instructions for doing a *KenKen*® puzzle are simple as illustrated by the following directions from *The New York Times* (*KenKen*® 2010):

> Fill the grid with digits so as not to repeat a digit in any row or column, and so that the digits in each heavily outlined box will produce the target number shown, by using addition, subtraction, multiplication or division, as indicated in the box. A 4x4 grid will use the digits 1-4, a 6x6 grid will use the digits 1-6.

As was the case with the introduction of the Sudoku puzzle in 2004 in the US, the introduction of the *KenKen®* puzzle in 2008 in *The Times* of London and in *The New York Times*, numerous books and articles have addressed this new arithmetical and logic puzzle including "how-to" books (Miyamoto 2010), a "how-to" video (Shortz 2010), and *KenKen®* puzzle books (Shortz 2008)

KenKen® (Japanese, 'cleverness') is a logic and arithmetical puzzle invented by the Japanese mathematics instructor Tetsuya Miyamoto (1959-) to teach his students arithmetic, logic, and patience. It is a mathematical and logical puzzle in which the reader is required to fill a grid with numbers. The size of the grid depends upon the number of integers involved, for example, 3x3, 4x4, and 5x5. *KenKen®* puzzles appear in grids, or groups of cells labeled "cages" that are outlined with boldface type into cell groups. The numbers in each group must then produce a target number. The mathematical operations may involve addition, subtraction, multiplication, or division. In other words, a three-cell cage which has a target of 6 may allow repetition of a number so long as they do not appear in the same row or column. Figures 5 and 6 below illustrate a typical *KenKen®* puzzle.

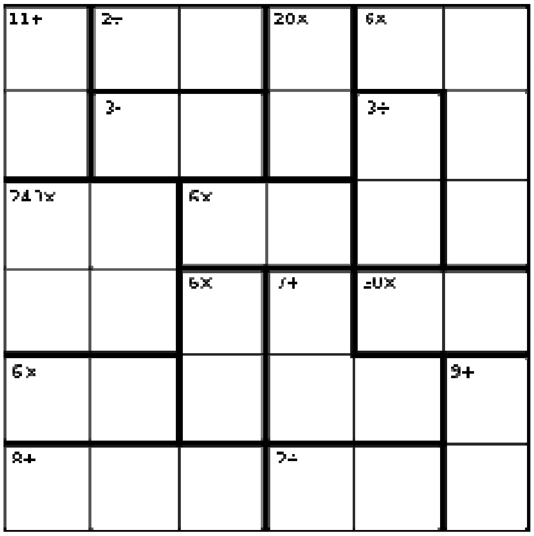


Figure 5: Sample *KenKen*® Puzzle Problem

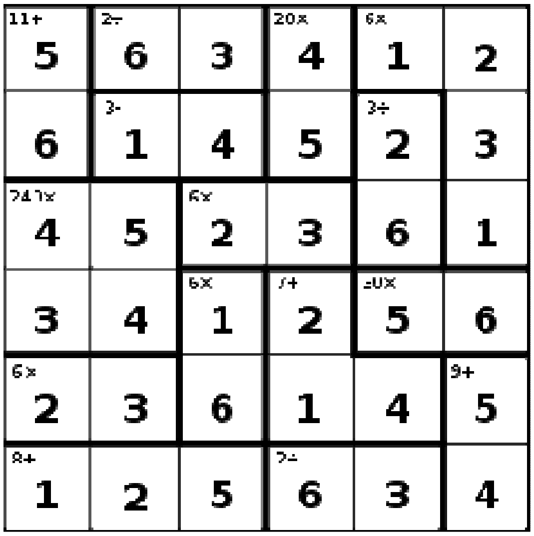


Figure 6: Solution to *KenKen*® Puzzle Problem in Figure 5

Shortz, Will. Will Shortz on KenKen®. http://www.KenKen®.com/latestnews_video.html. Accessed 25 December 2010. Video.

Mathematics in Calendars

As previously noted, print forms of recreational mathematics are widely available in book and magazine format. In recent times, another print manifestation of recreational mathematics has appeared in the form of calendars. Wall calendars and daily calendars have long been popular with the public. Over the years, many of these almanacs focus on special topics (sports, art, movie, television, and so forth), and, in this sense they mirror specialized magazines that address highly specialized individual tastes and interests.

Mathematician Theoni Pappas, for example, seeks to make mathematics accessible to the general public. One way that she does this is with her periodic publication *The Mathematics Calendar 2011 (*Pappas 2010b). As the author perceptively states on the back of the 2011 calendar:

> *The Mathematics Calendar* is designed to so that the problem appearing on each day has as its solution that date. The answer is only one small part in the process of solving a problem. The challenge is discovering how to arrive at the solution, and possibly discovering more than one method of solving it. The text and graphics accompanying each have a wealth of information and even a bit of humor.

The same author has produced a similar calendar entitled *Children's Mathematics 2011 Wall Calendar* (Pappas 2010a).

Another mathematically-based calendar is *Fractal Cosmos* by artist Alice Kelley (2010; see Kelley 2000 for a discussion of the theory behind fractal design and art), described as designs by Alice Kelley, with an intuitive glimpse into the infinite order that composes the natural world as well as proof that math is beautiful. The word “fractal” comes from the past participle of the Latin verb *frangere* (‘to break’).

While working at IBM’s Thomas J. Watson Research Center (Mandelbrot 1977; cf. Crilly 2008: 100-103), Benoît Mandelbrot (1924-2010) coined the expression “fractal” in 1975 to refer to a mathematical equation that undergoes iterative applications to create a recursive feedback. The mathematical notion of fractal had antecedents in the work of Gottfried Leibniz (1646-1916), Karl Weierstrass (1815-1897), Georg Cantor (1845-1918), and Felix Hausdorff (1868-1942) to name but a few.

The artistic creations of Alice Kelley illustrate the creative dimension of fractals. Approximate fractals also appear in nature (crystals, snowflakes, lighting, and so forth). Fractal patterns have been demonstrated to appear in the artwork of Jackson Pollock (1912-1956) and Max Ernst (1891-1976).

Mathematics in Comic Strips

FoxTrot, the highly successful comic strip by Bill Amend (1962-), started in 1988 as a daily and a Sunday strip, and, since December 31, 2006 as a Sunday only. The strip garnered its creator the Reuben Award for Outstanding Cartoonist of the Year (May 26, 2007).

The comic strip features a suburban family: Roger Fox (father), Andy (wife), Peter, Paige, and Jason (children). Jason represents a prototypical “geek” or “nerd”, a pubilectal slang expression (Danesi 1994: 95-123) for a person with a preoccupation for intellectual pursuits, who often sports identifying garb such as black horn-rimmed glasses, T-shirts with mathematical or scientific allusions, in-jokes, and so forth. In this sense, Jason Fox typifies this type of behavior and several of his strips contain mathematical allusions as illustrated in the following selected examples (Amend 2009).

In the *FoxTrot* comic that appeared on January 11, 2006 (Figure 7), Jason and his brother appear in the second panel of a two-panel strip, with Peter’s comment “Alpha-bits?” This is an allusion to the popular cereal Alpha-Bits® produced by Post Cereals, which contains alphabet-shaped corn cereal. Jason retorts with the single word “Alphabytes”, that is, he means that the bowl of cereal seen from a top view in the first panel contains a binary computer code. The computer code spells out an alphabetic message using the binary code. The *FoxTrot* cartoon encodes the message “YOU NERD” (Figure 8 contains the English alphabet in binary code).

The binary code is a way of representing a text on a computer by using a binary numbering system consisting of the digits 0 and 1 (Figure 8). The binary code was first introduced by the German philosopher and mathematician Gottfried Wilhelm Leibniz (1703; 1646-1716). His goal was to create a system that automatically convert logical statements into mathematical ones. Subsequent to his own system, Leibniz came upon the famed *I Ching* (‘Book of Changes’), one of the oldest of the ancient Chinese texts, and in his *Explication de l’Arithmétique Binaire* (1703), he used the Chinese text as a basis for claiming the universality of the binary system.

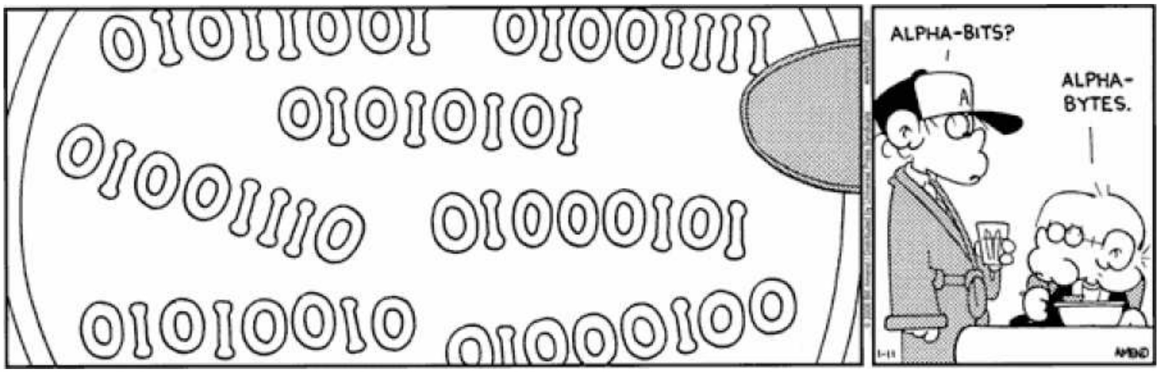


Figure 7. *FoxTrot*. January 11, 2006. Message in Binary Code = “YOU NERD”

The Alphabet in Binary Code

Letter	Binary Code
A	01000001
B	01000010
C	01000011
D	01000100
E	01000101
F	01000110
G	01000111
H	01001000
I	01001001
J	01001010
K	01001011
L	01001100
M	01001101
N	01001110
O	01001111
P	01010000
Q	01010001
R	01010010
S	01010011
T	01010100
U	01010101
V	01010110
W	01010111
X	01011000
Y	01011001
Z	01011010

Letter	Binary Code
a	01100001
b	01100010
c	01100011
d	01100100
e	01100101
f	01100110
g	01100111
h	01101000
i	01101001
j	01101010
k	01101011
l	01101100
m	01101101
n	01101110
o	01101111
p	01110000
q	01110001
r	01110010
s	01110011
t	01110100
u	01110101
v	01110110
w	01110111
x	01111000
y	01111001
z	01111010

Figure 8. The English Alphabet in Binary Code.

A second example of a set of mathematical problems appears in a numerical puzzle based on the classic word search format, which Jason labels a numerical word search appears in Figure 9. This *FoxTrot* strip appeared on November 6, 2005.

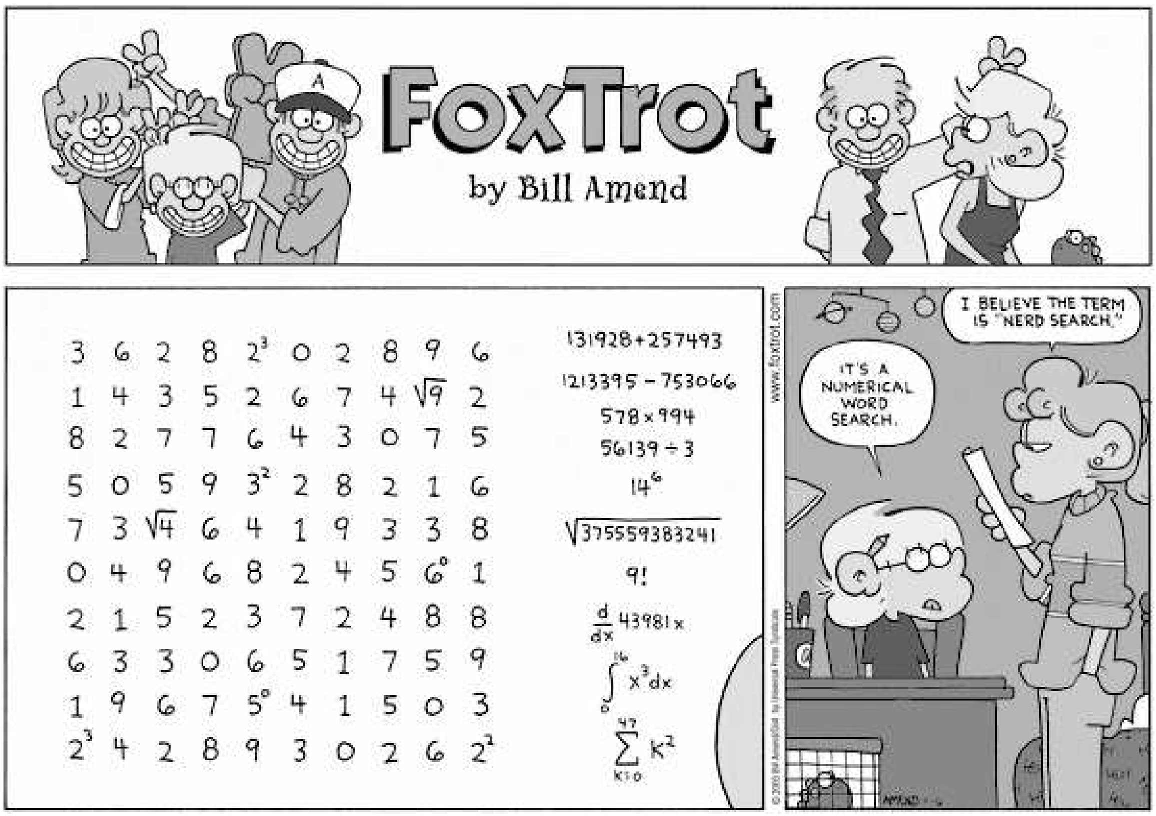


Figure 9. *FoxTrot*. November 6, 2005. Numerical Word Search.

A solution to this set of mathematical problems appears on Andy's *Foxtrot* Puzzle Solution Page by Robert Andrew Dunn (2011), who received his Ph.D. in physics from the University of Michigan, Ann Arbor in 1995. The specific answers are as follows (Figure 10).

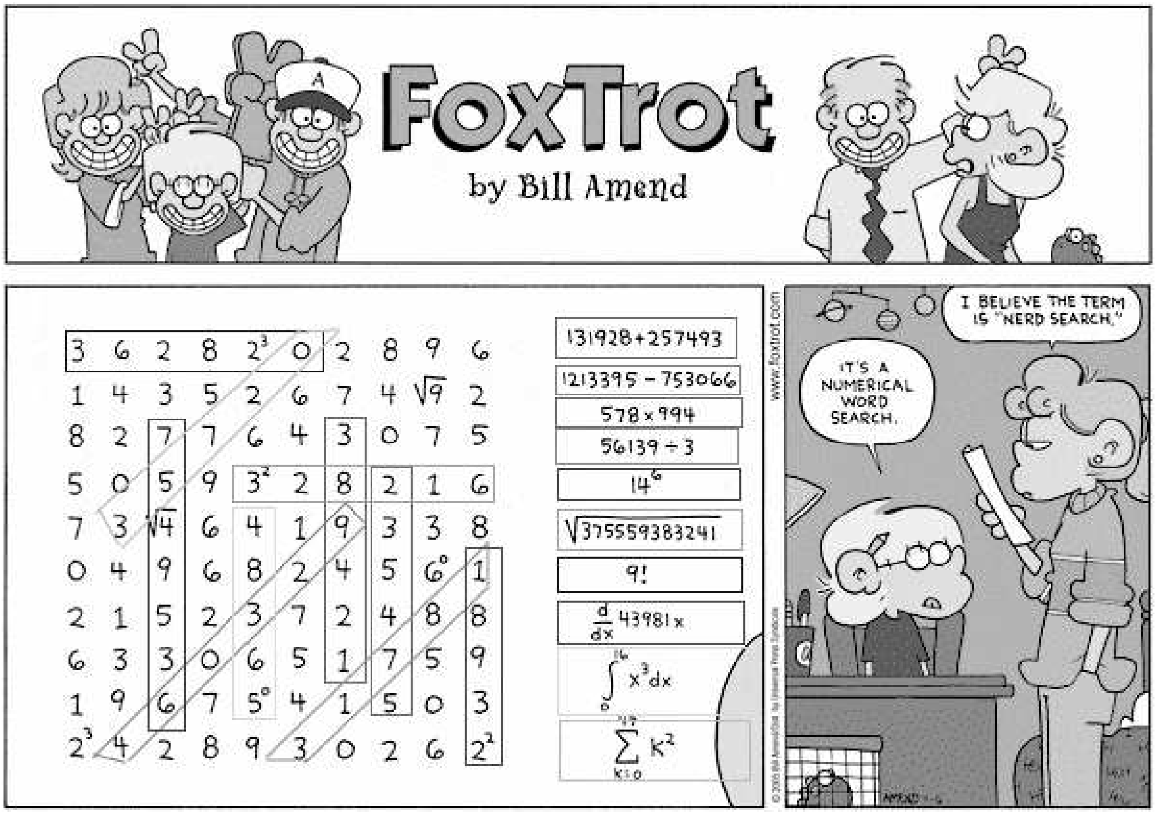


Figure 10. November 6, 2005. Answers to Numerical Word Search.

Paige Fox, Jason Fox's sister in the strip *FoxTrot* is not mathematically inclined and in one case (June 2, 1996), she has to take a final math exam pictured in Figure 11.

Figure 11. *FoxTrot*, June 2, 1996.Paige Fox Takes a Final Exam.

Eric W. Weisstein (2011) discusses this math problem in MathWorld–A Wolfram Web Resource examines the math problem and states that it involves the digamma function, or the logarithmic derivative of the gamma function.

It should be noted that the first two *FoxTrot* comics involve materials that appear in pop culture puzzles, namely, coded puzzles and word searches that are popular with readers as a form of recreation. The fact that these *FoxTrot* strips employ mathematical notions as a further step in these traditional lexical puzzles makes them all the more interesting (Danesi 2002: 52-59).

Mathematics in the Sports Media

Many children who grow up in the US and acquire a passion for sports. As a result, they seek to develop meaningful mathematical knowledge of the various popular sports (baseball, basketball, football, hockey, soccer, and so forth) by reading the daily newspapers and various sports-related weekly newspapers and magazines. The information contained in these stories frequently alludes to statistics. Children who have an interest in sports want to know as much as possible about their favorite players. Often a parent, a relative, or a sibling will explain some of the basic mathematical principles employed to describe a player's "stats", which reflect the relative quality of that individual's performance on the field, court, or rink. Certain statistical information thus provides information about a player's prowess in competition, and it may predict that player's future performance.

All sports have their own specific type of statistical information. In his book, *A Mathematician at the Ball Park: Odds and Probability for Baseball Fans*, Ken Ross (2004), a Professor emeritus of Mathematics at the University of Oregon addresses the mathematics of baseball. In that book, he discusses such topics as averages, odds and probabilities, and statistics in America's favorite pastime. Many children and adults are able to use basic arithmetic to determine important statistics of their favorite players. In baseball, for example, a player's batting average is significant because it tells how often that player is able to hit the ball over a particular period of time, for example a single season, or an entire career. A formula for determining this figure is the following: The average of a player is derived by dividing the number of hits by the number of times the player is at bat (Figure 12; Ross 2004: 1):

$$\text{Average} = \frac{\text{Hits}}{\text{At Bat}}$$

Figure 12. Baseball Batting Average Formula.

What a young child sees in a player's batting average is the result of this calculation. Unless it is decoded, however, its significance is lacking. Another individual will usually have to provide this formula and be able to explain its significance so that the child can both understand its meaning and determine how to calculate the mathematics him/herself. As Ross (2004: 2) correctly points out, the expression "batting average" should be replaced by the term "hitting proportion" because the latter tells a person the proportion of official at bats that resulted in hits. Ross then proceeds to cite Rob Neyer and Eddie Epstein (2000: 146) who pose the appropriate question:

> Why do people have such a hard time letting go of batting average? On-base percentage and slugging percentage are **more** important, **more** significant, **more** meaningful, **more everything** than batting average [boldface in original, FN].

Throughout his book, Ross provides the reader with ways to better understand the statistics that reveal a baseball player's athletic prowess by discussing significant statistics versus unimportant numerical data. Other sports have their own meaningful statistics, for example, basketball (points, blocks, steals), football (points, yardage, sacks, turnovers), hockey (goals, saves), and so forth. Although these statistics frequently appear in sport stories about specific players, the way in which they are calculated, and how they are interpreted is often not explained.

Tangible Manifestations of Recreational Mathematics

Recreational mathematics often starts out as a narrative about a specific mathematical problem. The Towers of Hanoi problem originated as a brief story description that situates the specific problem in a monastery in Hanoi, Viet Nam and is attributed to the Italian Renaissance mathematician Girolamo Cardano (1501-1576). Subsequently, many of these verbal descriptions were converted into tangible objects, or toys, that allowed an individual to interact physically with them. In this sense, recreational mathematics has assumed other concrete forms such as games that reflect underlying mathematical principles, these objects replicate basic, underlying mathematical notions in a recreational format. In playing with them, children and adults may learn some basic mathematical principles, albeit intuitively. In this regard, Danesi (2004a: 105-113, 125-133) has discussed how two mathematical puzzles have become popular and widely available toys, namely, The Towers of Hanoi and the Get Off the Earth puzzle. Several examples of such objectified mathematical problems will be discussed in this section, namely, the Towers of Hanoi, the Get Off the Earth Puzzle, Rubik's cube, and the 15 puzzle.

The Towers of Hanoi

The first example of a tangible puzzle is “The Towers of Hanoi” (Danesi 2004a: 105-113; see Figure 13). This puzzle contains three pegs on a block of wood. One of the pegs contains a set of disks with the largest one at the bottom of the peg. The challenge is to move all of the disks from the first one to the third one without allowing a larger disk to be placed on a smaller one. The idea for this toy derives from a puzzle developed by the Italian mathematician Girolamo Cardano (1501-1576), who provided a verbal description of his puzzle, which took place in a Hanoi monastery. That verbal description follows (Danesi 2004a: 109-110):

> A monastery in Hanoi has a golden board with three wooden pegs on it. The first of the pegs holds sixty-four golden disks in descending order of size–the largest at the bottom, the smallest at the top. The monks have orders from God to move all the disks to the third peg while keeping them in descending order, one at a time. A larger disk must never sit on a smaller one. All three pegs can be used. When the monks move the last disk, the world will end. Why?

As Danesi (2004a: 110) explains, the answer to the vexing conundrum of the Towers of Hanoi, involves the number of moves required to achieve the goal described in the story, that is, it would involve 2^{64}. In practical terms, it would take 582,000,000,000 years to complete this task described in the brief description.

It was the French mathematician François Anatole Lucas (1842-1891) who objectified this narrative puzzle by marketing it as tangible toy under the pseudonym N. Claus de Siam (an anagram of Lucas d’Amiens). There are various solutions to this puzzle. One solution is an iterative one, which involves the alternative moves of smallest pieces. A set of rules can be created to accomplish the moves in the shortest possible number of moves. Another solution utilizes a recursive algorithm. A non-recursive one may also be used. Other possibilities include a visual solution and a graphical one. Several mathematicians have addressed the Towers of Hanoi puzzle. Hinz (1989: 291) points out that the Towers of Hanoi problem has had a resurgence of interest because it is used to illustrate issues related to algorithms, discrete mathematics, artificial intelligence, and combinatorics. Gedeon (1996) offers an iterative solution by means of automatic program solution from a recursive solution. Chan (1988) also offers a graph solution (see also Romik 2006).

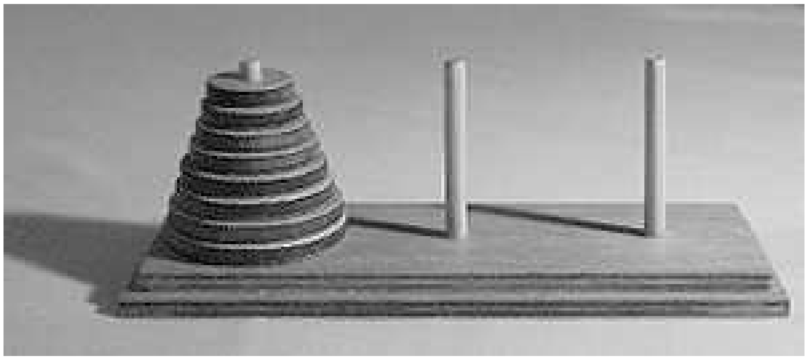


Figure 13. The Towers of Hanoi

Get Off the Earth Puzzle

A second toy puzzle is Sam Loyd's (1841-1911) "Get Off the Earth Puzzle" (Danesi 2004a: 125-140; see Figure 14) involves a visual illusion based on a "cut-and-slide" technique. This vanishing puzzle involves simple geometrical principles. Moreover, it is a puzzle type that includes "optical illusions". Danesi (2004a: 127) explains that this is a classic "cut-and-slide" trick:

> The Chinese warriors are made as assemblages of smaller pieces representing arms, legs, bodies, heads, and swords. When the earth is rotated, the pieces are rearranged in such a way that each Chinese warrior gains a sliver from his neighbor. For example, at the lower left, two warriors are next to each other. The top one is missing a foot. When the earth is rotated, he gains a foot from his neighbor on the right. That neighbor gains two feet (since he lost one) and a small piece of a leg. As a result of the rotation, one of the warriors will "lose" all his parts, making it seem that he has "disappeared."

Danesi (2004a: 134-137) points out that Sam Loyd's "Get Off the Earth Puzzle" belongs to a category of "*trompe-l'oeil*" (French, 'trick the eye') formats, variously known as optical illusions, ambiguous figures, or impossible figures.

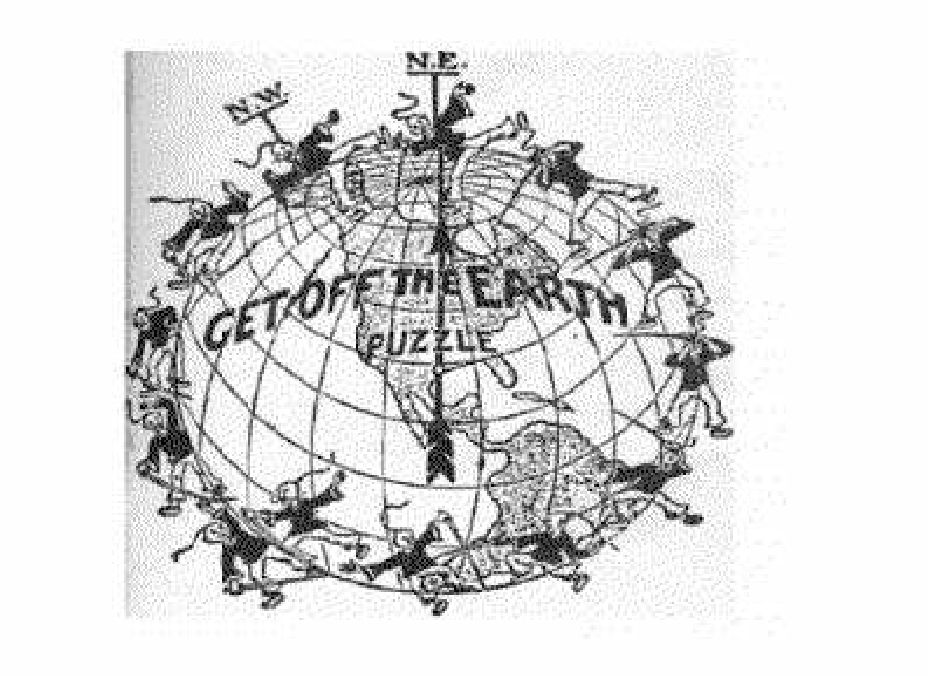


Figure 14. Sam Loyd's "Get Off the Earth Puzzle"

Rubik's Cube

The now famous Rubik's cube is a three-dimensional mechanical puzzle created in 1974 by the Hungarian Professor of Architecture Ernö Rubik (1944-) whose original name was "Magic Cube" (see Figure 15). Subsequently, the puzzle was licensed to Ideal Toys in 1980. More than 350 million copies of the puzzle have been sold already. It consists of six separate faces with nine stickers with six colors (blue, green, red, orange, yellow, white). A rotating mechanism allows the cubes to be turned independently, thus scrambling the colors. The standard cube measures approximately 2 and ¼ inches on each side. There are twenty-six miniature cubes, or cubelets. The original Rubik's Cube was 3x3x3 with eight corners and twelve edges. Fanatics of the Rubik's Cube have developed algorithms to solve the puzzle. One sequence of movements designed to return the cube to its original format with each color on one face of the device is called the "Singmaster notation" (Joiner 2002) after David Singmaster (1939-), a retired Professor of Mathematics (London South Bank University, UK). A number of solutions have been devised to allow the puzzle to be solved in 100 or fewer moves. In fact, there are competitions called "speedcubing". The fastest time for solving the mechanical puzzles is 6.77 seconds set in Melbourne, Australia. Variations of the original puzzle now exist (4x4x4, 5x5x5, 6x6x6, and 7x7x7). Several books have appeared that provide a simplified to the resolution of the Rubik cube® (Shah 2008, Slocum 2009).

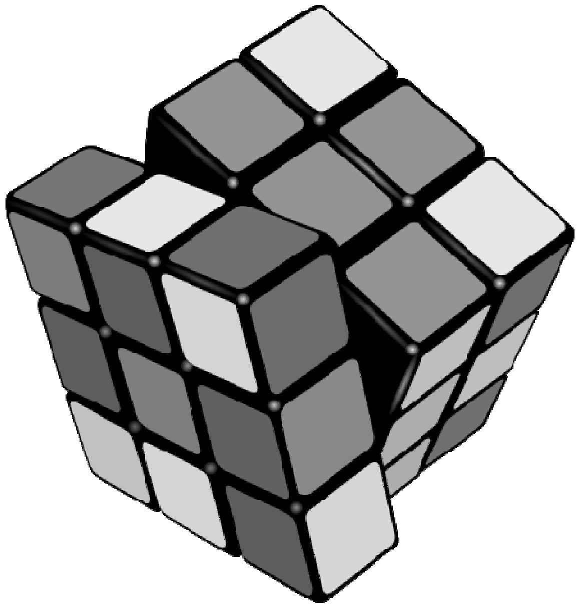


Figure 15. Rubik's Cube

The "15 Puzzle"

The "15 Puzzle", also called the "14-15 Puzzle", "Boss Puzzle", "Game of Fifteen", and "Mystic Square" (Figure 16), was a nineteenth century phenomenon, although this puzzle continues to be produced by contemporary toy makers. It consists of a square box with fifteen numbered compartments and one blank one. The object of the puzzle is to arrange the numbered square tiles into the order 1 to 15 with the final box left empty. The solutions to this puzzle involve algorithms and heuristics.

Its invention is attributed to Noyes Palmer Chapman (1811-1889) of Canastota, New York who first created it in1874. Production of this product began in 1879 by students at the American School for the Deaf in 1879. It was originally marketed as the "Gem Puzzle". By 1880, the game had become enormously popular in the US, Canada, and Europe. In 1891, the famous puzzle maker, Sam Loyd (1841-1911) claimed that he had invented the puzzle – an assertion that he made until the day he died (Slocum and Sonneveld 2006).

The puzzle received considerable notoriety when Ambrose Bierce (1842-1913) wrote a short note about the puzzle entitled "The Gem Puzzle" in the May 22, 1880 issue of the *Argonaut*, a San Francisco literary journal (Bierce 1880). In their article on Bierce's original essay, Berkove and Berkove (2010: 274-277) reproduce the entire text of the original Bierce note. In 1879, Johnson and Story published an article on how to solve this puzzle (Johnson and Story 1879). More than a century later, Archer (1999: 795-797) offers a step-by-step solution to this problem.

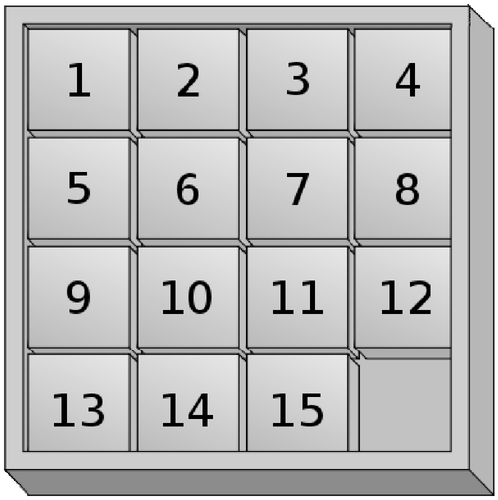


Figure 16. The “15 Puzzle”

Television

The police procedural is a type of crime drama that provides a sequential account of how police proceed in the investigation of a crime. This approach to fictional crime is often traced to the novel *V as in Victim* (1945) by Lawrence Treat (1903-1998) although earlier police procedurals exist, for example, Hillary Waugh's (1920-2008)) *Last Seen Wearing…* (1952), and Sir Basil Thomson's (1861-1939) novel *P. C. Richardson's First Case* (1933). In the domain of the comic strip, *Dick Tracy* (1931-) created by Chester Gould (1900-1985) is another well-known example of this genre.

In the history of television, several prominent examples exist including, but not limited to *Dragnet* and its various reincarnations 1951-1959, 1967-1970, 1989-1991, 2003-2004), which introduced the police procedural and established the prototype for this kind of drama. A few additional examples include *The Untouchables* (1959-1963), *Police Story* 1973-1978). More recently *CSI: Crime Scene Investigation* (2000-) and its spinoffs *CSI: Miami* (2002-), and *CSI: New York* (2004-). These shows are sometimes called "howdunits".

One television police procedural program, in particular, *NUMB3RS* (2005-2010) is a police procedural in which mathematics plays a primary role (Buckmire 2005). Its title, in fact, includes the number "3" as a replacement for the letter "e", which it resembles, albeit backwards. Several characters work at the fictional CalSci (California Institute of Science = the real California Institute of Technology, Pasadena, CA). Among the characters are several members of the Eppes family including Professor Charlie Eppes (David Krumholtz, 1978-), a mathematician, Don Eppes (Rob Morrow, 1962-), the main FBI agent, and Alan Eppes (Judd Hirsch, 1935-), father of Charlie and Alan. Other characters include Professor Larry Fleinhardt (Peter MacNicol, 1954-), a theoretical physicist and cosmologist, and Professor Amita Ramanujan (Navi Rawat, 1977-), another mathematician, and former student of Professor Charlie Eppes. The other principal characters are members of the FBI. The faculty members of CalSci are regularly called in to help solve cases by using their knowledge of mathematics and applying it to the resolution of various cases.

The titles of the episodes often suggest the mathematical principles involved (see Table 1). A few selected examples serve to exemplify this aspect of the program. It should be noted that most hour-long television programs contain approximately 42 minutes of actual program content and narrative structure. The remaining minutes are used for commercial announcements. This means that the actual plot and the insertion of mathematical information must conform to this time frame. The chief mathematical consultant firm is Wolfram Research (The Math Behind *NUMB3RS*, 2010 for season 1). Mark Bridger, a Professor at Northeastern University in Boston has a blog dedicated to the mathematics of each episode (Bridger 2010). Likewise, Devlin and Lorden (2007) have written a noteworthy book about the mathematics contained in the first three years of *NUMB3RS*. In the web site for *NUMB3RS* (2010), it is noted that the mathematical formulae contained on the blackboards as background for each episode are, in fact, correct. Nevertheless, the plot and not the mathematics is the most important part of the show for most of the audience.

Episode Title	**Mathematical Content**
"Uncertainty Principle" February 4, 2005	Heisenberg's (Werner Heisenberg, 1901-1976) uncertainty principle.
"Chinese Box" December 14, 2007	Chinese room, thought experiment devised by John Searle (1932-; Searle 1980). Chomp a two-player game in which a "chocolate bar" consisting of smaller square blocks. Cluster analysis, a method of unsupervised learning.
"When Worlds Collide" May 16, 2008	Byzantine fault tolerance, based on the Byzantine Generals' Problem (Pease, Shostak, and Lampor, 1980). Figure-Ground organization (faces-vase drawing). Wallpaper group (mathematical classification of a two-dimensional pattern). M. C. Escher (1898-1972; mathematically based woodcuts). Hyperbolic geometry (non-Euclidean geometry). Six degrees of separation or the Human Web.
"Conspiracy Theory" December 5, 2008	Simpson's (Edward H. Simpson, 1922-) Paradox (Decision making process).
"Frienemies" December 19, 2008	Group Dynamics and Three Way Duel.
"Angels and Devils" May 15, 2009	Angel problem, Burr puzzle (an interlocking puzzle made of wood or brass).

Table 1. *NUMB3RS* Episodes Whose Program Titles Suggest Their Content

The premise of each show centers on a specific crime (murder, rape, terrorism, robbery, sabotage, and so forth). Members of the FBI are often stymied by these crimes, and they call upon faculty members from the fictitious California Institute of Science (CalSci) modeled after the real California Institute of Technology often called Cal Tech.

In this drama, a few characters in the show are named for famous mathematicians, e.g., in the episode “In Plain Sight” (November 9, 2005), one of the characters is named Rolle (cf. Michel Rolle 1652-1719), a French mathematician known for his Rolle’s theorem (1691), and a co-inventor of the Gaussian elimination, an algorithm for systems of linear equations (1690). Moreover, Charlie Eppes’ father alludes to an encounter with a certain Robert Peterson (1839-1910), a Danish mathematician who worked in the area of geometry. In fact, several mathematicians work as consultants to this show, which makes the mathematics introduced into each episode credible.

In their outstanding book *The Numbers behind NUMB3RS: Solving Crime with Mathematics*, mathematicians Devlin and Lorden (2007), Professors at Stanford University and California Institute of Technology, respectively, discuss the ways in which the various disciplines of mathematics have served as a way to solve crimes in real life.

In their introductory statement, the co-authors (Devlin and Lorden 2997: ix) state that:

> Our book sets out to describe, in a nontechnical fashion, some of the major mathematical techniques currently available to the police, CIA, and FBI. Most of these methods have been mentioned during episodes of *NUMB3RS*, and while we frequently link our explanations to what was depicted on the air, our focus is on the mathematical techniques and how they can be used in law enforcement.

Their book covers the first three seasons (2005-2007).

In the following discussion, Devlin and Lorden's presentation of the intriguing pilot episode will be noted because it sets forth the basic premise that mathematics can be employed to solve crimes. In fact, that first episode is based on an actual person.

The initial episode of *NUMB3RS* called "Pilot" aired on January 23, 2005, and it featured a mathematical procedure known as "Criminal Geographic Targeting" (CGT) – one developed by Kim Rossmo (1955-), a former police officer with the Vancouver, Canada police force, who went on to earn a Ph.D. at Simon Fraser University in Burnaby, Vancouver, Canada. In this initial episode of the series (Devlin and Lorden 2007: 1-12), mathematician Charlie Eppes provides a mathematical solution to a vexing serial killer case in order to predict where the criminal will strike again. Eppes writes a complex formula on a blackboard. It is, in fact, the one that Rossmo developed to pinpoint the likely location of the killer based on a form of mathematical statistics known as multivariate analysis, that is, the analysis of more than one variable at a time. This episode is based on the actual case of a serial rapist in Lafayette, Louisiana in which Kim Rossmo employed his mathematical formula to solve the real case. This then became the premise of the pilot program of *NUMB3RS*. The formula that Eppes reproduces appears in Figure 17 (Rossmo's Formula 2010). It is a formula that concerns two elements. The first refers to a decreasing probability related to increasing distance while the second one refers to the radius of the buffer zone where the criminal is likely to be located. This specific episode is illustrative of the program's incorporation of mathematics into its plots.

$$p_{i,j} = k \sum_{n=1}^{\text{(total crimes)}} \left[\underbrace{\frac{\phi}{(|X_i - x_n| + |Y_j - y_n|)^f}}_{1^{\text{st}}\text{ term}} + \underbrace{\frac{(1-\phi)(B^{g-f})}{(2B - \mid X_i - x_n \mid - \mid Y_j - y_n \mid)^g}}_{2^{\text{nd}}\text{ term}} \right], \quad \text{Where } (X_i \neq x_n) \wedge (Y_i \neq y_n)$$

Figure 17. Kim Rossmo’s Formula for Geographic Profiling.

As Devlin and Lorden (2007) explain, there are many other mathematical components in other episodes of *NUMB3RS*, which had a run of six seasons (January 23, 2005 to March 12, 2010). Two more examples from Devlin and Lorden (2007) suffice to illustrate this point. The first involves codes, which represent a prototypical semiotic activity. Episode 5 of the first season of *NUMB3RS* entitled "Prime Suspect" (February 18, 2005). In this episode, Charlie Eppes provides mathematical assistance in the case of a kidnapped five-year old girl whose father is also a mathematician who is working on the Riemann hypothesis proposed by Bernhard Riemann (1826-1866; see Devlin 2002: 19-62) in 1859, and it concerns the distribution of prime numbers; see Devlin and Lorden 2007: 105-106). A definitive resolution to this hypothesis will shed light on our knowledge of prime numbers (Crilly 2008:200-203). The Clay Mathematics Institute located in Cambridge, MA established the Millennium Prize Problems May 24, 2000 with an award of one million dollars for the solution to seven classical mathematical problems. The Riemann hypothesis is one of these problems (Devlin 2002). The specific mathematics used in this episode involves cryptography, or the encoding of information to make it inaccessible to criminals and others who seek to decode such systems for malicious or destructive purposes. If the fictitious mathematician in the episode is able to solve Riemann's hypothesis, it would allow the villains to gain access to codes used by banks and allow them to have easy access to the money located there. As Danesi (2002: 53-59) explains in his book on recreational problem solving, cryptography dates back to Herodotus (484-425 BCE) who discussed such a code devised by a Greek spy named Demeratus as early as 480 BCE. A useful and accessible text on this topic is *Understanding Cryptography: A Textbook for Students and Practitioners* (Paar and Pelzl 2009).

Another example of mathematics employed in *NUMB3RS* involves Network theory, or the study of mathematical structures to determine the relationships of pairs in a larger group, in an episode entitled "Protest" (March 3, 2006). Network theory is an applied branch of graph theory. In this regard, Leonhard Euler (1707-1783) is considered to be the first mathematician to write on this subject. In 1736, Euler solved the problem known as the Seven Bridges of Königsberg (Danesi 2004a: 67-84), which asked the question of whether or not it is possible to follow a path, which crosses the seven bridges of this city only once. Euler proved that it was not. His resolution is now considered to be the first analysis of graph theory.

In this particular episode of *NUMB3RS*, graph theory is used to determine the network relationships involved in a protest. The approach to the solution of the crime in this story involves Network theory – a theory employed to determine the relationships of the terrorists involved in the 9/11 attacks against the US and described by Devlin and Lorden (2007: 137-151).

In their chapter on this episode and Network theory, the co-authors (2007: 147-149) allude to the now famous "six degrees of separation and the "small world phenomenon", which refers to Stanley

Milgram's (1933-1984) experiment (Milgram 1967) designed to measure the average length of paths in people's social networks.

The Milgram (1967) experiment was made famous by the film *Six Degrees of Separation* (1993), itself an adaptation of John Guare's (1938-) play (1990) of the same name. Its basic premise is that everyone is connected to everyone else through a chain with a maximum of six acquaintances. Allusions to this theory appear in J. D. Salinger's (1919-2010) *Catcher in the Rye* (1951). The "six degrees of separation" is sometimes labeled "the human web". There is even a popular game called the "Six Degrees of Kevin Bacon", which is a trivia entertainment pastime that seeks to apply the "small world phenomenon" whereby any actor can be connected to the actor Kevin Bacon (1958-) in six steps. Manifestations of this popular game have appeared in an episode of the television situation comedy *Mad About You* (November 19, 1996) entitled "Outbreak", which had Kevin Bacon as a guest star. There is even a book titled *Six Degrees of Kevin Bacon* written by Craig Fass, Brian Turtle, and Mike Ginelli (1996). Actors have been assigned a "Bacon number", which is a reference to the relative proximity or distance of one actor to Kevin Bacon, who has appeared in many films with numerous actors. Barbási's (2003) book *Linked: How Everything is Connected to Everything Else and What It Means for Business, Science, and Everyday Life* addresses this notion in a much broader context.

With respect to the "Bacon Number", mathematicians have developed their own special "number" as Devlin and Lorden (2007: 149) point out. One of the greatest mathematicians of the twentieth century, Paul Erdös (1913-1996), was a Hungarian mathematician who worked in graph theory and other mathematical domains. In their book on the television show *NUMB3RS*, Devlin and Lorden note that mathematicians who came of age in the twentieth century have Paul Erdös as their hero because he wrote more than 1500 papers and collaborated with at least 500 mathematicians. They speak of the "Erdös number", a reference to the shortest number of paths between a mathematician and Paul Erdös, which is an academic type of "Bacon number". Because of Erdös' prolific collaborative activity, some 200,000 mathematicians have been assigned an "Erdös number". Devlin and Lorden (2007: 149) point out that they both have an Erdös number of "2" by virtue of having co-authored an article with a person who co-authored a paper with Erdös.

As a final note, it should be pointed out that the following additional mathematical areas appeared in the first season of *NUMB3RS* during its six season run: 11-dimensional supergravity theory, Bayesian inference, Beale ciphers, Chapman-Kolmogorov equation, Combinatorics, Cryptography, Fibonacci sequence, Foucault pendulum, Geographic profiling, Golden ratio, Guilloché pattern, Heisenberg's uncertainty principle, Markov chain, Monty Hall problem. Prisoners' dilemma, Probability theory, Projectile motion, Sabermetrics, Schrödinger's cat, and Wavelet analsysis. Since this procedural crime drama is meant to be entertainment, the details of these mathematical issues are not elaborated since the average show has approximately 42 minutes of actual presentational time (the rest of the time is for commercial announcements). Nevertheless, the impact of this program in promoting interest in applied mathematics and mathematics to promote interest in this discipline cannot be understated. In fact, it is the only program that has incorporated this field in a systematic and meaningful way as dramatic entertainment.

Film

The classification of mathematics in film is similar to the previously system proposed by Mann (2010: 60) for mathematics in fiction. That schema is reproduced here for convenience.

1. Fiction by mathematicians
2. Fiction using mathematical structures
3. Fiction expounding mathematics
4. Other
5. Fiction about real mathematicians
6. Fiction about doing mathematics
7. Fictions about mathematical ideas
8. Fictions with mathematicians as characters

Danesi (2008b: vii) alludes to several movies that deal with mathematics in their narrative structure, namely, *A Hill on the Dark Side of the Moon* (1983), *Good Will Hunting* (1997), *Pi* (1998), *Conceiving Ada* (1997), and *Proof* (2005). It should be noted that placing any one of these films into any single category proposed by Mann (2010) is problematic at best because there tends to be category overlap in several of them. The following list contains a brief note about each of the films mentioned in Danesi's book (2008b). Nevertheless, it is worth reviewing these five films to consider their mathematical content, or lack thereof.

A Hill on the Dark Side of the Moon (1983) is a Swedish biographical film directed by Lennart Hjulstrom (1938-) about the Sonia Kovalevsky (1850-1891), a nineteenth century mathematician. Because the film focuses on her personal life (depression and her tragic love story), mathematics does not appear in the movie *per se*. It can be included in Mann's (2010) category 5, a film about a real mathematician.

In *Good Will Hunting* (1997), the main character, Will Hunting, solves a problem in algebraic graph theory that Professor Gerald Lambeau of Massachusetts Institute of Technology, and a Fields Medal winner in combinatorial theory, proposed to his graduate-level class. The characters are fictitious, and thus fits into Mann's (2010) category 7 of a fictional account of mathematical ideas. The film won two academy awards in 1997 (Best Supporting Actor, Robin Williams (1951-) and Best Writing (Original Screen Play, Ben Affleck (1972-) and Matt Damon (1970-)). Part of the narrative line of this film relates to the notion of an unsolvable math problem (The Unsolvable Math Problem 2011). George Bernard Dantzig (1914-2005), a doctoral candidate at the University of California, Berkeley, was able to solve two problems placed on the board in one of his mathematics classes, though at the time, he did not that they were "unsolvable" (see Albers and Reid 1986: 301 for an account). Clips from the movie are available on the Internet (Good Will Hunting 2011a,b; the complete web site information is available in the References section).

The main character in *Pi* (1998) is a number theorist who argues that all of nature can be comprehended in terms of numbers. The film contains the following allusions to mathematics: Gold spiral, nonlinear dynamics, and chaos theory. It also features the game *Go*. This movie, however, addresses the madness of the principle character and not mathematics. Using Mann's categorization, this film belongs to category 7 (Fictions about mathematical ideas). A brief clip from the movie is available (*Pi* 2011).

The film *Conceiving Ada* (1997) features the character Emmy Coer , who is creator of the first computer algorithm. The main protagonist devises a computer algorithm for Charles Babbage's (1791-1871) "difference engine", which is a mechanical calculator for computing polynomial functions. In fact, this film is a science fiction tale with mathematics as background to the primary story, and it falls into Mann's category 7 (Fictions about mathematical ideas).

Proof (2005). Anthony Hopkins (1937-) plays the brilliant mathematician, Robert. In fact, this film contains little mathematics, though the plot centers on efforts of various characters to discover some important theorem in Robert's notes. This film fits into Mann's (2010) category 8 (Fictions with mathematicians as characters). The trailer is available (Proof 2011).

A Beautiful Mind (2001) concerns John Forbes Nash, Jr. (1928-) who won the Nobel Prize in Economics in 1994 together with his colleagues John Harsanyi (1920-2000) and Reinhard Selten (1930-) in the area of game theory. This film focuses on Nash's medical problems (paranoid schizophrenia) and his personal life, and much less on his mathematical prowess. This film received four academy awards (best picture, best director, best adapted screenplay, and best supporting actress (Jennifer Connelly, 1970-) in 2002.

Finally, *Fermat's Room* (2007, *La habitación de Fermat*) is a Spanish horror and suspense film with English subtitles. In this thriller, three mathematicians and an inventor receive an invitation to a house in order to solve a major mathematical enigma. Once the four men arrive at their destination, they are trapped in a room and they must solve various puzzles to escape the room, which is slowly closing in on them. The puzzle master name is Fermat (Pierre de Fermat, 1601-1665). The characters receive pseudonyms of famous mathematicians Galois (Évariste Galois, 1811-1832), Hilbert (David Hilbert, 1862-1943), Pascal (Blaise Pascal 1623-1662). This film could be categorized as Mann's number 8 (Fiction with mathematicians as characters). There are, however, specific references to real mathematicians whose names are used as pseudonyms for the fictional characters. A short clip of the movie is available (Fermat's Room 2011).

An excellent website entitled “Mathematics in Movies (2011), which contains Flash and quicktime versions of segments from movies with mathematical content. Moreover, it contains brief annotations concerning the mathematical content of the film in question. Another helpful website is “Mathematics Goes to the Movies” (2011).

Mathematics Goes to the Movies. http://www.qedcat.com/moviemath/index.html Accessed 3 January 2011.

Mathematics in Movies. http://www.math.harvard.edu/~knill/mathmovies/ Accessed 3 January 2011.

Concluding Remarks

This paper deals selectively with the representation of mathematics in the print and non-print media. In this sense, it discusses popular cultural manifestations of mathematics. The way popular culture portrays, or fails to portray mathematics, provides an interpretive insight into the popular perception of this academic discipline. It has examined creative literature, recreational mathematics, television, and film with the intention of presenting materials that deal with mathematics in these popular formats in a credible fashion.

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Glossary

Abduction Term used by Charles Peirce to designate the form of reasoning whereby a new concept is inferred on the basis of an existing concrete, or already known concept; abduction is essentially a 'hunch' as to what something means or presupposes (Danesi and Perron 1999: 345)

Addition The process of computing with sets of numbers so as to find their sum (Morris 1979: 15)

Algebra A generalization of arithmetic in which symbols, usually letters of the alphabet, represent numbers or members of a specified set of numbers and are related by operations that hold for all members in the set (Morris 1979: 32)

Algorithm Any mechanical or recursive mathematical procedure (Morris 1979: 32)

Arabic numeral The numerical symbols 1, 2. 3. 4. 5. 6. 7. 8. 9, 0 (Morris 1979: 66)

Arithmetic The mathematics of integers under addition, subtraction, multiplication, involution, and evolution (Morris 1979: 71)

Calculation The act, process, or result of calculating (Morris 1979: 189)

Cardinal number A number such as 3 or 11 or 412 used to indicate quantity but not order (Morris 1979: 203)

Closed problem Type of problem that presents all the information needed for its solution in an unambiguous fashion (Danesi 2008: 219)

Code Any system of signs having the same properties and governed by the same rules of structure (for example the alphabetic code, the integer code, etc.) (Danesi 2008B: 219)

Computation The act, result or method of computing. (Morris 1979: 274)

Compute To determine by mathematics, especially by numerical methods. (Morris 1979: 274)

Cryptography The art or process of writing in or deciphering secret code (Morris 1979: 319)

Deduction The process of reasoning in which a conclusion follows necessarily from the stated premises; inference by reasoning from the general to the specific (Morris 1979: 344)

Digit Any one of the ten Arabic number symbols 0 through 9 (Morris 1979: 369)

Dimension A measure of spatial extent, especially width, length, or height (Morris 1979: 370)

Division The operation of determining how many times one quantity is contained in another (Morris 1979: 385)

Euclidean geometry Geometry developed by Euclid (*ca*. 300 BCE), author of the *Elements*, which involved the point, line, and plane. A system of axioms and theorems employed to create a comprehensive deductive logical system

Evolution The extraction of a root of a quantity (Morris 1979: 455)

Fourth dimension Time regarded as a coordinate dimension and required by relativisitic geometry, along with three spatial dimensions to completely specify the location of any event (Morris 1979: 520)

Fracatal A fractal is a geometric shape capable of being segmented into two parts, each one of which is a reduced replica of the original.

Game A set of rules completely specifying a competition, including the permissible actions of and information available to each participant, the probabilities with which chance events may occur, the criteria for termination of the competition, and the distribution of payoffs (Morris 1979: 541)

Geometry The mathematics of the properties of measurement, and relationships of points, lines, angles, surfaces, and solids (Morris 1879: 551)

Induction A principle of reasoning to a conclusion about all the members of a class from examination of only a a few members of the class; broadly reasoning from the particular to the general. *Mathematics* A deductive method of proof in which verification of a proposition consists of proving the first case and the case immediately following an arbitrary case for which the proposition is assumed to be correct (Morris 1979: 671)

Inference The act or process of inferring. A conclusion based on a premise (Morris 1979: 673)

Integer Any member of the set of positive whole numbers (1, 2, 3, …), negative whole numbers (-1, -2, -3, …) and 0 ((Morris 1979: 681-682)

Interpretant Process of adapting a sign's meaning to personal and social experiences (Danesi and Perron 1999: 355)

Involution The multiplying of a quantity by itself a specified number of times; raising to a power (Morris 1979: 690)

Irrational number A member of the set of real numbers which is not a member of the set of irrational numbers (Morris 1979: 692)

Logic The relationship of element to element to whole in a set of objects, individuals, principles, or events (Morris 1979: 787)

Magic square A square that contains numbers arranged in equal rows and columns in such a way that the sum of the numbers in each row, column, and diagonal is the same (Danesi 2004a: 239)

Mathematics The study of number, form, arrangement, and associated relationships, using rigorously defined linear, numerical and operational symbols (Morris 1979: 806)

Medium (singular of Media) A means of mass communication, such as newspapers, magazines, or television (Morris 1979: 815)

Multiplication The conjunction of two real numbers in which the number of times either is taken in summation is determined by the value of the other (Morris 1979: 862)

Non-Euclidean geometry This system of geometry involves hyperbolic (lines will not intersect with another line) and elliptic geometry (no lines that do not intersect). This type of geometry developed in the nineteenth century, though it was discussed much earlier.

Number A member of the set of positive integers; one of a series of symbols of unique meaning in a fixed order ((Morris 1979: 900)

Numeral A symbol such as a letter, figure, or word used alone or in a group to denote a number (Morris 1979: 900)

Object What a sign refers to (Danesi and Perron 1999: 360)

Open problem Type of problem that does not present all the information needed for its solution, requiring a more creative strategy (also known as a *puzzle*). (Danesi 2008: 220)

Ordinal number A number indicating position in a series or order. The ordinal numbers are first, second, third, etc. (Morris 1979: 925

Plane geometry The geometry of planar figures (Morris 1979: 1002)

Popular culture (Pop culture) The culture that has obliterated the distinction between high and low forms, allowing people themselves to evaluate artistic products and expressions (Danesi 2008a: 294)

Problem A question or situation that presents uncertainty, perplexity or difficulty (Morris 1979: 1043). A type of question that, consisting of information or a set of conditions designed to elicit a certain answer (Danesi 2008: 221)

Puzzle A toy, game or testing device that that tests ingenuity (Morris 1979: 1063). An *open problem* (Danesi 2008: 221)

Rational number Any number capable of being expressed as an integer or quotient of integers (Morris 1979: 983)

Recreation Refreshment of one's mind or body after after labor through diverting activity, play (Morris 1979:1090)

Representamen The physical part of a sign (Danesi and Perron 1999: 364)

Riemann's hypothesis A proposal that deals with prime numbers. It has yet to be resolved and it is one of the seven major mathematical problems to be addressed as established by the Clay Mathematics Institute (Cambridge, MA) (Crilly 2008: 200-203)

Roman numeral One of the letters employed in the ancient Roman system of numeration. The system is now used in certain formal texts as for dates and monuments; for distinctively separate numeration, as for the pagination of the front matter of a book; and as a subsidiary system in composite numeration (Morris 1979: 1126)

Semiotics Science that doctrine that studies signs and their uses in representation (Danesi and Perron 1999: 365)

Sign Something that stands for something or someone else in some capacity (Danesi and Perron 1999:366)

Signified That part of the sign referred to (Danesi and Perron 1999: 366)

Signifier The part of the sign that does the referring; the physical part of a sign (Danesi and Perron 1999: 366)

Spherical geometry The geometry of circles, angles, and figures on the surface of a sphere (Morris 1979: 1243)

Statistics The mathematics of the collection, organization, and interpretation of numerical data. (Morris 1979:1259)

Subtraction The arithmetic process or operation of finding a quantity that when added to one of two quantities produces the other (Morris 1979: 1284)

Sum The amount obtained as a result of adding (Morris 1979: 1288)

Symbol Something that represents something else by association, resemblance, or convention; especially a material-object used to represent something invisible (Morris 1979: 1302). Any sign that stands for something in a general or abstract way (for example 2n stands for any even number, no matter what value n takes on). (Danesi 2008: 221)