# THE SEMIOSIS OF MATHEMATICAL THOUGHT

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**Abstract.** This paper discusses mathematical thinking only insofar as that thinking provides an unmistakable prime example of anthroposemiosis in its species-specific difference from all the varieties of zoösemiosis. Thus, recurring to Euclid's triangle as a central example, my aim is to outline how relation as a mode of being exhibits a singularity that proves to be the basis for the prior possibility of semiosis in general, a singularity that mathematical objectivity makes particularly recognizable even though the feature in question extends to the full range of semiosis as an action transcending the contrast between mind-dependent and mind-independent being.

**Outline:**


"Rational animal" is perhaps the longest-standing definition of "human being" that we have. It is a definition not without problems, for we have learned, in the process of discovering that our world as including the animals is an evolutionary product, that "reasoning" is not a process restricted to humans, although *some types* of reasoning do seem to be species-specifically human, mathematics certainly one of the principal among them.

I am not, professionally speaking, a mathematician, though I was pretty good in mathematics through high school, up to and including calculus and the use of a slide-rule, which I am told is considered antiquated for students of mathematics today. So it is both a privilege and an embarrassment for me to be speaking here at the Fields Institute. Indeed, had the invitation (with accompanying friendly pressure) not come from Professor Danesi coded in Fibonacci numbers, I would have declined.

However, here I am; and I will do my best to address what seems to me, if not the heart of mathematics, certainly something very central thereto, namely, the concept of *relation as a mode of being* at work *singularly* in human reasoning.

## *Focusing the problem mathematically*

Where is the triangle demonstrated by Euclid to have angles equaling, or totaling, $180^\circ$? It is equally accessible to all of us, and yet it is nowhere in the usual sense of "where", namely, a location within our

physical surroundings that can be identified in terms, say, of longitude and latitude, a location we can circumscribe and travel to, a location where, once we have completed our travel thereto, we can look at and point to and say "Aha! There is our triangle!"

Well, hardly. And yet Euclid's triangle is not really his possession, nor could it be if he wanted to own it, for it is nowhere in space and time as measured by longitude and latitude or any version thereof.

And yet the triangle in question, the one whose angles can only equal $180^{o}$, is a *public* figure, one that all of us in this room, indeed, all of us on this planet or any planet, at this time or at any time past or future, can "relate to". Pretty strange, come to think of it! No wonder Plato, who believed that our souls are intelligences only temporarily occupying bodies, is said to have inscribed on the portal to his Academy "Let No One Who Knows Not Geometry Enter Here"; for he would rely on geometry to teach his disciples that "reality" is not something that can be seen or touched nor in any way grasped properly by the senses, but is something that can only be understood and, once understood, seen to belong to a realm in which sensation and material objects have no part.

Aristotle may have succeeded overall in reducing Plato's Ideas to footnotes on his own rather different "first philosophy" or "metaphysics", as it came to be known. But he did not succeed, so far as his surviving writings tell us, fully to explain how the objects of geometry, or, for that matter, any other objects, achieve by their nature the status of "public in principle". For that is what every object is, at least within the context of linguistic communication: something public in principle, in contrast with, say, a toothache which, fortunately for all save the one who has it, is a "private matter of awareness".

The idea that I want to present for your consideration is that the concept of relation, if developed properly (I say "if ", for one of the less-well-realized meanings of "rational animal" is "slow learner"), gives us the only hope there is of understanding how and why objects, as objects of awareness, are always and necessarily public in principle, precisely because relation is the only mode of being that transcends subjectivity — and, by that very fact, enables semiosis.

*Brief history of the concept of relation*

Aristotle learned from Plato that there is something irreducibly peculiar about relation. Substances are the true natural units of the physical world, or genuine "individuals"; and, like all individuals real or apparent, substances have accidents, or identifying and distinguishing characteristics, among which are relations as, for example, the relation which distinguishes you as a child of that parent and not a child of this parent, etc.

But as a characteristic of individuals, relations are already peculiar; because whereas all other characteristics of individuals are subjective features that exclude one another, relations somehow elude the principle of contradiction in that one and the same thing at one and the same time can enter into contrary relations: for while you cannot be overweight and underweight at one and the same time, yet at one and the same time you can be fat relative to Mary and skinny relative to Sue.

Plato never resolved the difficulty, for he never thematized the problem thus posed. Plato, it may be said, left the way open for the resolution proposed by William of Ockham: relations have no reality of their own, but amount to no more than *comparisons* made within and by some mind of subjects existing — substances — outside of the mind making the comparison. No, no, no! Aristotle would insist — followed later by Aquinas, both thinkers in advance of (but neither understood on the point by) Ockham. Relations have a being of their own that is — or, as we shall see, *can be* — independent of finite mind; but as a

"characteristic" of some individual, relation is not on the same footing, as it were, does not have the same standing, as do the *other* characteristics. For my personal being exists *in* me, as does my size, shape, weight, etc.; but my relative being exists most obviously *between* me and something else, some thing or things within my surroundings.

It was this contrast between existing or "being" *in* and existing or "being" *between* that Aristotle latched on to in defining relation as "that whose *whole being* consists in 'being toward' another".[1] He did not come by the formula easily. In fact, it took three tries, the first two of which failed to distinguish relation as a mode of being either from the other varieties of individual characteristics or, indeed, from substance itself — the actual individual characterized.

Unfortunately for the history of philosophy, Aristotle was so relieved to have finally hit upon a definition of relation as a 'category' or mode of mind-independent being that he then simply reverted to his original contrast between "substance" as a " being in itself " (or individual in the natural world) and "accident" as a characteristic of such an individual. I say "unfortunately", because the history of philosophy has taught us only all too well that anyone who approaches Aristotle's thought in terms of his distinction between "substance" as "being in itself" and "accident" as "being in another" has less than a 10% chance of understanding relation as an irreducibly distinct mode of being among the inherent accidents.[2]

For only *inherent* accidents fit the formula of "beings in another", that is to say, of characteristics of an individual which qualify or modify that individual in its here-and-now actual existence. Relations may or may not so qualify the individual in question. When they do, they are necessarily based or "founded" upon actual characteristics of the individual in question. But when they do not, there are two possibilities: either they *once did* because they were in fact founded upon characteristics of the individual, the case of "truth past"; or they *never did* but are *attributed* thereto upon some other founding basis, the case of "fiction", whether through mistake, intention (a writer of fiction), or deceit (a liar).

The first case, "when they do", is the only case that Aristotle pursued "to the end". He was interested in το ὀν, being as it exists independently of human thought, belief, opinion, or desire, and in relation as a mode of *that* being — *ens reale*, the Latins would say, being as (finite) "mind-independent ". Relieved as he was after two unsuccessful tries in arriving at a formula that would define relation without undermining his understanding of substance as "being in itself " (i.e., the individual unit of nature), and its contrast with accidents as ("characteristics of individuals"), he went on to other things; and to this day interpreters of

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[1] Cf. Aristotle c.360BC: *Categories* 8a28–34; Poinsot 1632: *Tractatus de Signis*, Second Preamble, Article 1, esp. 81/20–34.

[2] "So perhaps today, in order to get the point of postmodernity as a new epoch in philosophy's history as a whole, the single most important first step Aristotelians or anyone else talking about 'reality' has to take is to abandon the approach to the question of hardcore reality which begins by distinguishing between individuals and their characteristics, substance and accidents. ... The first step should not be [as it remained with Aristotle and the medieval mainstream] one that hides or buries relation in the order of subjectivity upon which relation depends, along with substance and its direct modifications (which is what all other accidents amount to and reduce to). The history of discussion on these questions amply demonstrates that the first step ought to be one that puts what is *different* about relation in contrast with *all* other accidents, and not merely with substance, rather than one which emphasizes rather what all accidents including relation have in common, namely, a dependence upon substance in order to be; for even in the mater of this shared characteristic of dependency it is important to note what is not noted in the traditional discussions — to wit, while all other accidents depend directly upon the being of substance, relation as such depends only indirectly *through* the other accidents. This is the reason why Aquinas terms relation *ens minimum* in the order of *ens reale*, but, as we will see, it is also the reason why relation turns out to be *ens maximum et solum* in the order of *ens rationis* in particular and *ens obiectivum* in general though not *solum* ..." (Deely 2010a: 66–67). Thus we must distinguish "esse in" from "esse ad", and only then *subdistinguish* substance ("esse in se") from accidents (as "esse in alio").

Aristotle begin their discussion of το ὀν with the substance/accident distinction, proceeding from there with very few exceptions to arrive at the conclusion of Ockham: relation has no being of its own in the order of what obtains independently of finite mind other than in its fundament, i.e., in the subjective characteristic of the individual which leads an observer to compare that individual with others, giving rise *in the comparing* to a relation over and above the individuals compared which relation *itself* in its "being toward" has no existence at all apart from the finite mind in which the comparison has been made.

On this common interpretation, then, there are actually not *two* cases, one where the relation as such is mind-independent and a second (subdividable[3]) wherein the relation is mind-dependent. There is "in reality" only the *one* case where the relation is a creation of finite mind comparing two or more things.

There remains the other interpretation, the only one that actually "gets the text of Aristotle right" in its treatment of relation: the view that relation has a positive being "toward" that is over and above the subjectivity of the individual (*esse in se*) and characteristic of that individual (*esse in alio*) upon which the relation depends in its own existence as *inter*subjective. And this term here, "intersubjective", marks a key point, for, in order to be in the order of *ens reale* (mind-independent being), relation required "reality" in the sense of an existing subject (a substance) both on the side of its foundation and on the side of its terminus. But this very fact, Aquinas observes,[4] should serve to notify us that *intersubjectivity*, while characteristic of relation in the order of *ens reale*, is not the characteristic that is essential for a relation to be a relation. For "being toward" is the one and only type of being that, from its positive nature *as* a type, does not require that it posit something realizable *only* mind-independently, whence occur instances of "being toward" which, as relations, are nothing in the mind-independent realm yet do obtain objectively and publicly within awareness, something that is impossible for any mode of "being in" or "subjectivity", whether "being in itself" (substance, the individual) or "being in another" (accidents, the inherent characteristics of individuals).

If we now ask, in the light of this consideration, what is the "positive and defining character" of relation, we find that it is *not* intersubjectivity directly, but simply *suprasubjectivity*, the fact that relation *always* transcends the order of subjectivity to which substance and inherent accidents necessarily belong and, *as a consequence of this transcendence*, is also *indifferent* to being realized intersubjectively! Any given relation will, as a relation, *always* exist suprasubjectively but only *sometimes* will it exist intersubjectively. To exist intersubjectively is necessarily to exist suprasubjectively, but to exist suprasubjectively is not necessarily to exist intersubjectively. Thus to say that relation as such is *intersubjective*, to characterize relation in this way (as is commonly done, for example, in contemporary phenomenological analyses), is, more than a mistake, a distortion. For this characterization does not hold true of relation as such, i.e., of relation in the *singularity* of its being, but applies only to relation in its uniqueness as one among the modes of *ens reale*.

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[3] I.e., in sensation the human animal experiences both subjectivities and intersubjectivities. In perception and intellection, through the formation of concepts, the human animal introduces via the objects of those concepts various mind-dependent beings formed in the course of interpreting and on the basis of what is presented in sense. When the mind-dependent being so formed is based on the sensory pattern of subjectivity it constitutes an object that *is not* what its pattern is, for the object in this case is purely objective and as such does not participate directly in any subjectivity. But when the mind-dependent being so formed is based on the sensory pattern of intersubjectivity, then both the objectivity formed and pattern after which it is formed are both relations.

[4] Cf. Thomas Aquinas 1257: *Quodlibet Nonum*, "Quaesitum est de primo de Christo capite, deinde de membris"; *Quaestio 2*, "De Christo quantum ad unionem humanae naturae cum divina"; *Articulus 3*, "Utrum in Christo sit una tantum filiatio", *corpus* (Busa ed. vol. 3. p. 489, co. 3): "in hoc differt ad aliquid ab aliis generibus: quod alia genera ex propria sui ratione habent quod aliquid sint; sicut quantitas ex hoc ipso quod est quantitas, aliquid ponit; et similiter est de aliis; sed ad aliquid ex propria sui generis ratione non habet quod ponat aliquid, sed ad aliquid: unde inveniuntur quaedam ad aliquid quae nihil sunt in rerum natura, sed in ratione tantum: quod in aliis generibus non contingit."

Intersubjectivity concerns relation only as occurring in the order of *ens reale*, whereas objectively relation is suprsasubjective regardless of whether it is or is not intersubjective at any given time. Intersubjectivity concerns directly the *circumstances* of relation itself and only indirectly the positive being of relation as indifferent to the circumstances which make it one time to be intersubjective as well as objective, and another time to be purely objective and without a positive intersubjective realization.

*Implications for the distinction between objects and things*

Let us return to Euclid's triangle, the one we found to be unlocatable in space and time, unlike any given person's thought or "idea" of that triangle, which has a very definite location in space and time. Does Euclid's triangle exist whether it is thought about or not? Plato thought so; Aristotle thought not. Aristotle thought the triangle of Euclidean geometry is brought into being by the intellectual process that he called "abstraction", wherein the human being abstracts from the world of experienced things a *pure object* that has no existence as such independently of being thought.

Fine as far as it goes, but an object "pure" in this sense, how is it public in principle and the same for all? Triangular shapes are found in the material world, no doubt; but they lack the perfection of Euclid's triangle, nor can they be said to be of two dimensions only. Well, here we encounter in a potentially clarifying way the difference in principle between an *object* and a *thing*. A thing exists whether or not anyone thinks of it. It just is what it is, and it remains what it is should someone happen to notice it, regardless of any interpretation — sane or zany — that the noticer may make of the thing. But an object which no one is aware of cannot properly be said to be an *object*, even though it may well be a *thing*.

With Euclid's triangle, however, we have an object that is *never* a thing. The "common usage" in modern English according to which "object" is just another way of saying "thing" is, quite simply, mistaken — indeed, delusionally so. For while an object, *pace* Kant, *may be* and *often enough is* also a thing, what makes a thing be a thing is existing in itself or in another, in other words, belonging to the order of mind-independent being; while an object, be it a thing or not, cannot exist as object except in relation to some knower, some "state of awareness".

"In relation to" is the key phrase here. Let us put it this way. A thing is what it is regardless of whether anyone happens to be aware of it or not. But an object is an object only when and to the extent that someone is aware of it. In other words, "relation to a knower", incidental to being a thing, is essential to being an object.

Now "relation" is a fairly simple mode of being, inasmuch as it involves three factors, namely, 1. a foundation, basis, or *fundament*; 2. the relation itself, the "being toward" or "respecting" which makes the fundament be a fundament; and 3. the terminus or "that toward which" the relation exists. So when we say that being an object necessarily involves a relation to a knower, we are saying that being an object essentially consists in one or another of these three factors, fundament, relation, terminus; and once the matter is stated in these terms it seems clear that it is the third factor, the terminus, that will be the *object* within any relation of direct cognition, while either the relation itself or the fundament as fundament can be made into "objects" in their own right only by means of a critically controlled and *reflexive* awareness on the part of some knower.

To appreciate this point — that the difference in principle between an object and thing is that an object necessarily while a thing only contingently terminates some awareness of a finite mind — one needs to realize that the being of a fundament *as such*, regardless of whether it is *also* a subjective characteristic of

some thing, and likewise the being of a terminus *as such*, regardless of whether it *too* has a subjective dimension in its existence, derives from the positive being proper to relation as suprasubjective.

Take the simplest example of two triangular figures, A and B, both existing in a given area. They are and can be seen to be "similar" on the basis of their shape. The shape is a subjective characteristic, existing *in* each of the triangles, but it is *also* fundament and terminus in the relation of similarity. Destroy either A or B, and the other is no longer *similar* thereto, for there is no longer something to be "similar to". *Yet the subjective characteristic of shape as a subjective characteristic is in no way changed by the fact of its ceasing to be either a fundament or a terminus.* The status of fundament as fundament and terminus as terminus, thus, results from the relation itself, not from the subjectivity of the characteristic viewed as "founding" (*or* "terminating") a relation.[5]

But what this means is that even when the fundament of a given relation happens to be an idea subjectively present in and modificative of some knower, the terminus of that relation will be, respecting that fundamental subjectivity, *over and above it*, precisely as *terminating* the relation *as relation*, i.e., as suprasubjective respecting the order of any and all subjective being. Where ideas overlap in their content subjectively, things will overlap objectively insofar as they are known. Hence, just as any two things can be related to a common third, so any two knowers can share in awareness of a common object. Their respective *ideas* are not common, but are subjective and private to each; but these same ideas as founding a relation to an object give to that object an existence suprasubjective respecting the subjective dimension of the ideas as private (as "my idea" or "your idea"), and hence public in principle in the way that the ideas themselves are not and cannot be. As terminating relations, objects participate in the suprasubjectivity of the relations they terminate, and it is for this reason that objects are public in principle and able to be shared in cognitively and cathectically communicative interactions.

*Objects that don't exist in the material world*

Well, we are perhaps making some progress in understanding Euclid's triangle, for we now can see that an object is always public in principle, even when it does not or cannot have a subjective existence in the world of things existing as material substances with their distinguishing characteristics. But how does all this — all that we have uncovered about relation so far — help us to understand where these "pure" objects, such as Euclid's triangle among many others, *are* in relation to those aware of them?

It seems to me that the key to resolving this question is to dissolve it, on the basis of another feature of relation that has no counterpart in the other modes of mind-independent material being.

When we say that a being is "material", or a "material substance", we are saying in ordinary terms that it is something that can be seen and touched, something that can be exemplified or instantiated before our eyes. And yet relations, even those obtaining mind-independently and in the material, physical order of our

[5] The late Latin Age Ockhamites, I am told by Professor Timothy Noone, used this fact to support their view that relation as a "being toward" exists positively, formally and actually, only within a cognitive comparison, while the basis for such comparison, the "fundament", exists independently of any such comparison, i.e., exists as a mode of mind-independent being. What they never stopped to consider, it seems, was that the denial of relation as anything more or other than a mind-dependent mode of being locked the subjectivity of the knower within itself exactly as Kant would formally proclaim, as this realization "slow by slow" came to the fore among modern thinkers. On the solipsistic implication of Ockham's denial of any mind-independent status for relations, see Deely 2008: *Descartes & Poinsot, the crossroad of signs and ideas*, esp. Chap. 5; on the formal recognition of the solipsistic consequence of reducing relations to the purely objective order, see Deely 2001: *Four Ages of Understanding*, Chap. 13, esp. pp. 553–572.

universe, precisely cannot be seen and touched. They are "immaterial" in this sense, that they involve directly no secondary matter.

I say "secondary matter" in contrast to the Aristotelian idea of "primary matter", which is the potentiality within every material substance to cease existing and to have its body assimilated to entirely other things, such as the lion who kills and eats its hunter: the hunter's individual being ceases with death and the visible, tangible parts of what used to be his or her body become assimilated to the being of the lion, of carrion birds, of worms, of the earth, and whatnot. To say that the hunter is no more is to say that the form of his body which made it his no longer controls the matter of that body but has been displaced and replaced by other forms in the ongoing interaction of physical beings that constitutes the physical universe. Yet these displacing substances now controlling in various ways the formerly bodily parts of the hunter are themselves subject to being displaced and so on, *ad infinitum* as long as there is a physical universe of interacting finite substances.

Thus Aristotle distinguished the matter as seeable and touchable from the matter as underlying and subject to generations and corruptions. The latter we can know to be there only by means of critical analysis; we cannot "isolate" or remove it from a body, but without it we cannot explain continuity in the physical universe across generations of new individuals. The former, by contrast, is matter as directly experienced, matter that can be seen and touched. Bodily substances and characteristics are precisely of that former sort. Relation is not.

Relation, then, in its formal and distinctive being as suprasubjective involves neither primary matter nor secondary matter.

By reason of the first, it is in the finite order a dependent sort or mode of being: for relation to exist suprasubjectively, there must exist subjectively an individual upon some characteristic of which the relation is founded. Suprasubjective in its own being, relation nonetheless depends upon a subjective foundation or ground.

By reason of the second, relation does not belong to the order of what can be seen or touched[6] and, once arisen, is unaffected by distance — a feature which is manifested most dramatically, I would guess, in the phenomena that current physics gathers under the label of "quantum weirdness". Individuals need physical proximity to interact. But relations are not interactions; they are rather the children of interactions, and as such continue to exist once the interaction has ceased. "For far or near, a son is in the same way the son of his father."[7]

And there is a third feature of relation to be noted, quite related to the second and key for mathematics but key, more generally, for semiosis of any sort. Relation is not like a clothesline between two poles. Relation is rather a field concept: one relation can have many terms. Thomas Aquinas uses the example of parenthood: no matter how many children the couple generates, they each and all constitute the terminus of one same relation.[8]

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[6] This ethereal nature of relation, no doubt, contributed substantially (no pun intended) to the Ockhamite error — shared in by Hobbes, Locke, Berkeley, Hume, Descartes, Spinoza, Leibniz, Kant: the whole genealogy of modern philosophy's mainstream (see Weinberg 1965) — of thinking that only in thought can relations, as distinct from existing things, occur.

[7] Poinsot 1632: 85/9–12: "... nihil conducit vel obstat distantia, quia relationes istae non dependent a locali situatione; eodem enim modo est filius sui patris filius distans et indistans."

[8] Ibid. 886/31–387/7: "potest una relatio attingere omnes terminos suae speciei tamquam inadaequatos suae extensioni, licet quilibet sufficiat ad suam existentiam; et sic licet haec relatio numero respiciat determinate hunc terminum, non tamen adaequate, sed omnes illos, in quibus est formalitas talis termini."

When we consider psychological states in the light of this third feature, we find an unexpected and unexpectedly simple resolution to the centuries old dispute over nominalism. For what distinguishes a psychological state, cognitive or cathectic, is that it cannot exist save by being "of " or "about" something other than itself. This is but to say that every psychological state exists by virtue of founding a relation the terminus of which, in contrast to the psychological state as a feature of the subjectivity of the knower, provides that knower with the *object* of its cognition and affection.

*Why and how objects are public in principle regardless of their nonexistence independent of mind*

Let us try to resolve the problem over which Descartes stumbled, the problem which sent modern philosophy down the "way of ideas" incompatible with the way of signs. And again Euclid's triangle provides us with the perfect example. In semiotics, it is necessary to distinguish objects as termini of relations from things, insofar as involvement with relations to a finite mind is not essential to being a thing, as we have noted above. In sensation it is aspects of things that we become aware of, that we "objectify". These aspects the animal must then *interpret* in order to survive, structuring the realm of sensations as a world (an Umwelt) of objects to be sought (+), avoided (–), or ignored (0), and it is only here that mental representations, or concepts, enter in. In sensation, things insofar as they are partially apprehended become objects, that is to say, *represent themselves* insofar as they have entered awareness. Thus an object is first of all a *self-representation*. But these self-representations of sensation as interpreted become *now further* objects of (i.e., based on) *other-representations*, that is to say, objects of perception and intellection as well as of sensation.

Moreover, once the interpreted objects are attained by perception and intellection they can be thought of even when they are no longer present physically, as even when they are no longer *existent* physically. Moreover again, when the concept presenting an object *as* this or that, i.e., as an object of interpretation, *misrepresents* that object, in the sense that the concept itself is a *misinterpretation* or even an act of "wishful thinking", there remains the distinction and formal difference between an object as a *self*-representation and a concept as an *other*-representation.

It is this distinction between self-reference and other-reference that is collapsed when the very psychological states that we call "ideas", "images", or "concepts" are (mis)taken for the objects represented.[9] That every psychological state of necessity is "of" or "about" something is just another way of saying that every psychological state of its proper nature is an *other-representation* rather than a self-representation. Collapsing other-representation into self-representation, conflating the two, is the opening to the Way of Ideas.

Thus the *idea* of triangle is something formed in and by each of us whenever we think of a triangle. As such it is part of our subjectivity, but a part which perforce *gives rise to* a *relation* over and above our subjectivity as separating us from our surroundings which, as a relation, gives rise also — that is, suprasubjectively — to an *object* as its terminus; and it is *this* object, not the idea thereof, that we analyze in demonstrating the necessity of its angles equaling $180^{o}$. The objective triangle, then, even though as a pure terminus of a relation — even though as a purely objective being — it has no material component locating it here or there in space and time, is the focus of Euclidean geometry; and so it is with all mathematical

[9] Poinsot 1632: 382/4–8: "... distinguendum est, quod terminus vel sumitur formalissime in ratione termini oppositi, vel fundamentaliter ex parte absoluti fundantis istam rationem terminandi."

objects prior to their "application" to quantitative aspects of our physical universe. And just as any two things can be related to a common third while yet remaining distinct as things, so two thinkers having the idea of triangle are related to that common third which is the objective triangle *in* neither of them. The triangle in its objectivity, not the respective ideas in their subjectivity, establish between the two thinkers a "common ground" which is in itself neither spatial nor temporal, for it has in itself no secondary matter.

Distance making no difference to relations in their positive being of "toward another" is the key to how an object which has no mind-independent existence can yet be common between two or more thinkers. For the difference in principle between *objects* as self-representations and *things* as existing independently of a relation to finite mind (which yet allows — requires — a coincidence in fact of the two within sensation prescissively considered), becomes a difference in fact *as well as in principle* when concepts as other-representations are introduced into perception and intellection to interpret the things sensed as objects of this or that kind. And it is just this difference in principle between objects and things that allows for interpreted objects to be known even when there is no immediate sensory ground for their consideration, as it is also this difference which allows us to construct objects which *have* no sensory instantiation.

For it is we who form concepts, but these concepts are not what we know directly, any more than the concept of a clock which alone enables us to see a given sensory pattern *as* a clock is what we first and directly see when we recognize a sensory pattern as instantiating a clock. The formation of a concept within our minds serves in every case immediately and directly to relate us to something other than itself, some object, regardless of whether that object here and now has or could have some manner and measure of sensory instantiation or not.

Mathematics thus forms a central case in anthroposemiosis, for it is not communicable zoösemiotically, and for a very good reason. Animals capable only of perception are restricted to awareness of material objects as related, but have no way to distinguish between the objects related and the *relations themselves*. Yet it is the relations themselves that formally constitute signs, as we saw above, and these relations are no less invisible to sense perception than they are to sensation itself directly and prescissively considered. Sense provides the first sign vehicles, and perception further elaborates these signs into an objective world of desirable, undesirable, and safely ignored features of the surroundings, weaving a semiotic web that enables the animal to orientate itself and move around in its surroundings as it goes about the business of seeking to achieve and ensure its well-being and flourishment.

But human understanding goes a further step. The animal through its experience in forming its Umwelt adds to its surroundings many relations that would not otherwise be there, and it is just these relations that transform the simple physical surroundings, on their side indifferent to the animal, into objects related to the animal as desirable, undesirable, or ignorable. The further step human animals take is to add to the world of objects the mind-dependent relation of self-identity transforming objects into *things* independent of their relation to me as +, –, 0 objects, independent of whether they are mind-dependent or mind-independent: a being is what it is, regardless of me as a specific instance of animal, over and above my interests in it. That recognition is what opens the way to the investigations we call philosophy, or cenoscopic science, just as it also opens the way to the further investigations we call science in the ideoscopic sense.

But notice that the relation of self-identity is a pure construct of understanding: a thing is not *identical* with itself, it just *is* itself. Yet to see an object for what it is in itself *apart* from its relation to my animal self requires to see it as *more than* an object. This opening *within* objectivity to thinghood as something

investigatable in its own right *beyond* objectivity distinguishes human understanding,[10] and results from the ability of understanding to grasp relations even though they are in their formal being invisible to sense. Other animals cannot, even preconsciously, introduce into their Umwelt the relation of self-identity making that world a world of things *as well as* of objects, because they cannot organize their objects on the basis of anything other than the directly sensible properties basing their interpretations of +, –, 0. That is why Umwelt is a generically animal notion, but the Umwelt as species-specifically human is also termed a Lebenswelt, an objective world open further to the recognition and investigation of things as things and the recognition and evaluation of purely objective elements transforming animal social interaction into cultural interaction as well in the human case.

Thus, by reason of its capacity to understand the difference between a relation and a related object or thing, the human animal not only depends (like all animals) on the *use* of signs but further comes to recognize *that there are* signs, that is to say, invisible triadic relation upon which the whole world of objects (both insofar as it does and insofar as it does not involve things) depends. It is in this sense at least, we should note here, that mathematics presupposes semiosis: for the objects of mathematics, pure or applied, are precisely *objects*, and every object as such presupposes the action of signs in order to exist as something apprehended. Significate, as I have explained at book-length,[11] resisted by English dictionary-makers since the distortion of the subject/object distinction introduced by philosophy's deviation from the way of signs unto the dead-end by-way of "epistemology" or the *way of ideas*,[12] is not only a synonym for the term "object" but has the further advantage of placing in the foreground of our consciousness what the term "object" conceals and obfuscates, namely that, apart from semiosis, apart from the action of signs, there actually are no objects, no objects at all; while within semiosis even virtual objects attain to actuality.[13]

Thus for animals objects are inextricably mixed with things. But human animals can transcend this extrication to deal not only with material objects, whether correctly or incorrectly interpreted, but also with *pure objects recognized as such*, objects which have never yet or even never could be things within the physical surroundings. Euclid's triangle is just such an object, but so is that whole realm of relations that we call "pure mathematics". Other animals indeed recognize and deal with objects in their plurality, but they cannot *number* them (because "1" as a number is not this or that material object, but *any object at all* regarded in a particular way, i.e., under the unity of a specifying relation), nor can they abstract from the material object its quantitative dimension upon which geometry is based. Thus arithmetic and geometry have their roots in the world of material objects as multiple and quantified, but their flower comes about only through the ability of human understanding to deal with relations in their difference from and irreducibility to related objects and things.

Is it coincidence that Pythagoras (c.582–495BC), the first thinker to recognize the importance of mathematics for the understanding of nature, was also, according to the ancient Greeks (who are in the best position to know, after all!), the coiner of the term "philosophy"? I think hardly.

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[10] See the extended analyses in Deely 1982, 1994, 2002, 2007.

[11] Deely 2009: *Purely Objective Reality*; and cf. on YouTube since 2010: "Semiotic Sign" <http://www.youtube.com/view_play_list?p=E9651802BCDC14BF>.

[12] See Cobley ed. 2009; Deely 2008.

[13] Poinsot 1632: 126/3–5: "sufficit virtualiter esse signum, ut actu significet", where "signum" has the sense of the sign-vehicle: a formula with a decisive import for the notion of physiosemiosis.

*In consideration of Kalevi's question, "What mathematical structure is semiosis, if any?"*

I have written this paper with three books to my left on the desktop: the 1984 translation by G. J. Toomer of *Mathematike Syntaxis* from 150AD, better known as *Ptolemy's Almagest*; the 1978 Rosen translation of Copernicus' *De Revolutionibus Orbium Caelestium* of 1543AD; and the 1687AD *Philosophiae Naturalis Principia Mathematica* of Isaac Newton in the 1999 translation of Cohen and Whitman. I lacked perhaps only key works of Einstein and Planck! These were my inspiration, but not my focus.

My focus has been narrower, to wit, the singularity of relation which alone among the varieties of mind-independent being is able to exist also purely objectively, the singularity which lies at the basis, as constituting the prior possibility, of semiosis, the action of signs, without which there would be no mathematics, and probably no transformation of the universe in the direction of intelligent life, as the anthropic principle of modern physics gropingly indicates.

When Professor Kull asked us earlier in this week "what mathematical structure is semiosis", then, just as when he unquestioningly embraces the assumption that semiosis is criterial of life, I suggest he may be having us look in the wrong direction. For our universe, though perfused with signs and unknowable without them, yet does not consist exclusively of signs; semiosis is essential to the whole story, but it is *far* from being itself the whole story. Indeed objects, even though they presuppose signs and normally themselves soon become signs, commonly participate in the physical reality of the universe; yet just as things are in principle distinct from objects, so objects do not simply reduce to things: Umwelt includes something of, but itself transcends, physical surroundings. And even the pure objects do not *reduce* to semiosis. Thus it is not semiosis that presupposes mathematics *or* life, but the reverse. It is in the singularity of relation as a mode of being knowable only reflexively and not by direct sensation or perception that we must look if we are to discern the heart of semiosis — the basis, as Heidegger might have put it,[14] of the prior possibility of the action of signs; and while the action of signs is everywhere at work, that action is not by any means every thing. Still, nothing better illustrates that action from within anthroposemiosis than the development of mathematics, applied, yes, but especially pure mathematics.

I leave it to our colleague Marcel Danesi to explain in Fibonacci numbers this conclusion of a recovering slide-rule addict.

[14] To judge from his essay of 1943 "On the Essence of Truth".

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