The Fields Institute

Tsunamis

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University of Colorado
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Questions about tsunamis

- What is a tsunami?
- How do tsunamis work?
- Does soliton theory describe tsunamis?
- How to protect people from the dangers of tsunamis?
- Why are only some tsunamis deadly?

show video

http://www.youtube.com/watch?v=tu056xg4hc8

Questions about tsunamis

- What is a tsunami?

An enormous *volume of water*, displaced by a sub-marine earthquake or landslide.

A Richter scale reading does not predict the *volume of water* generated in a tsunami.

- How do tsunamis work?
- Does soliton theory describe tsunamis?
- How to protect people from the dangers of tsunamis?
- Why are only some tsunamis deadly?

Equations of water waves

(i) On bottom, z = -h(x,y)

$$\vec{u} \cdot \nabla(z + h(x,y)) = 0$$

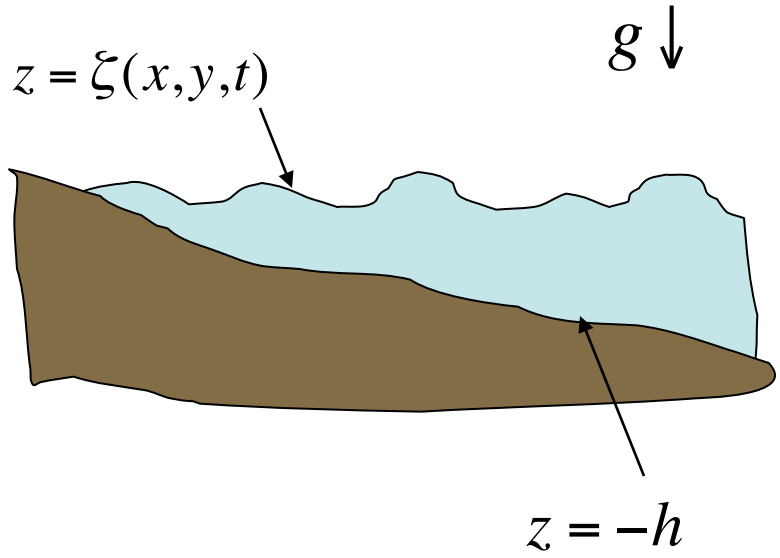
(ii) In fluid, -h < z < ζ(x,y,t)

$$\vec{u} = \nabla\phi, \qquad \nabla^2\phi = 0$$

(iii) At free surface, z = ζ(x,y,t)

$$\partial_t\zeta + \nabla\phi \cdot \nabla\zeta = \partial_z\phi,$$

$$\partial_t\phi + \tfrac{1}{2}|\nabla\phi|^2 + g\zeta = 0.$$



(iv) Ignore viscosity, surface tension, variable density, fish, …

Basic facts about wave propagation

(according to linear theory)

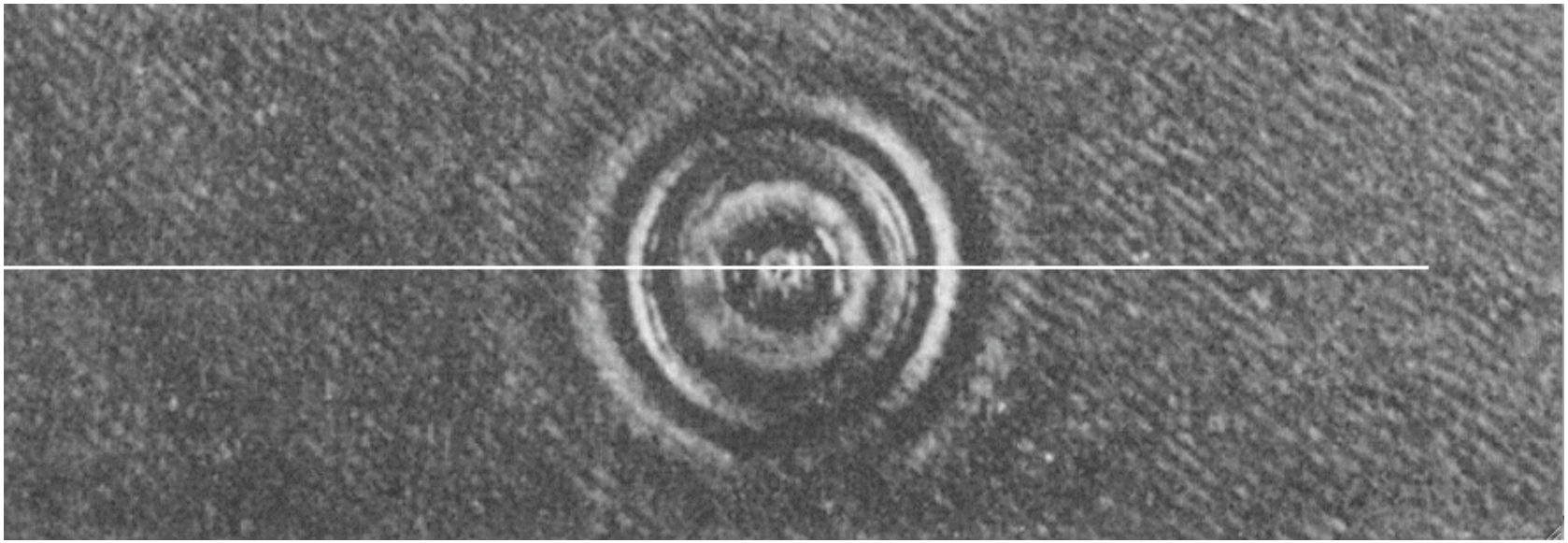
Sound waves

- All travel at the same speed (speed of sound)

Water waves

- Longer waves travel faster than shorter waves (for gravity-induced surface water waves)

Long waves travel faster than short waves



from Stoker's *Water Waves*, 1957

Basic facts about wave propagation

(according to linear theory)

Sound waves

- All travel at the same speed (speed of sound)

Water waves

- Longer waves travel faster than shorter waves (for gravity-induced surface water waves)

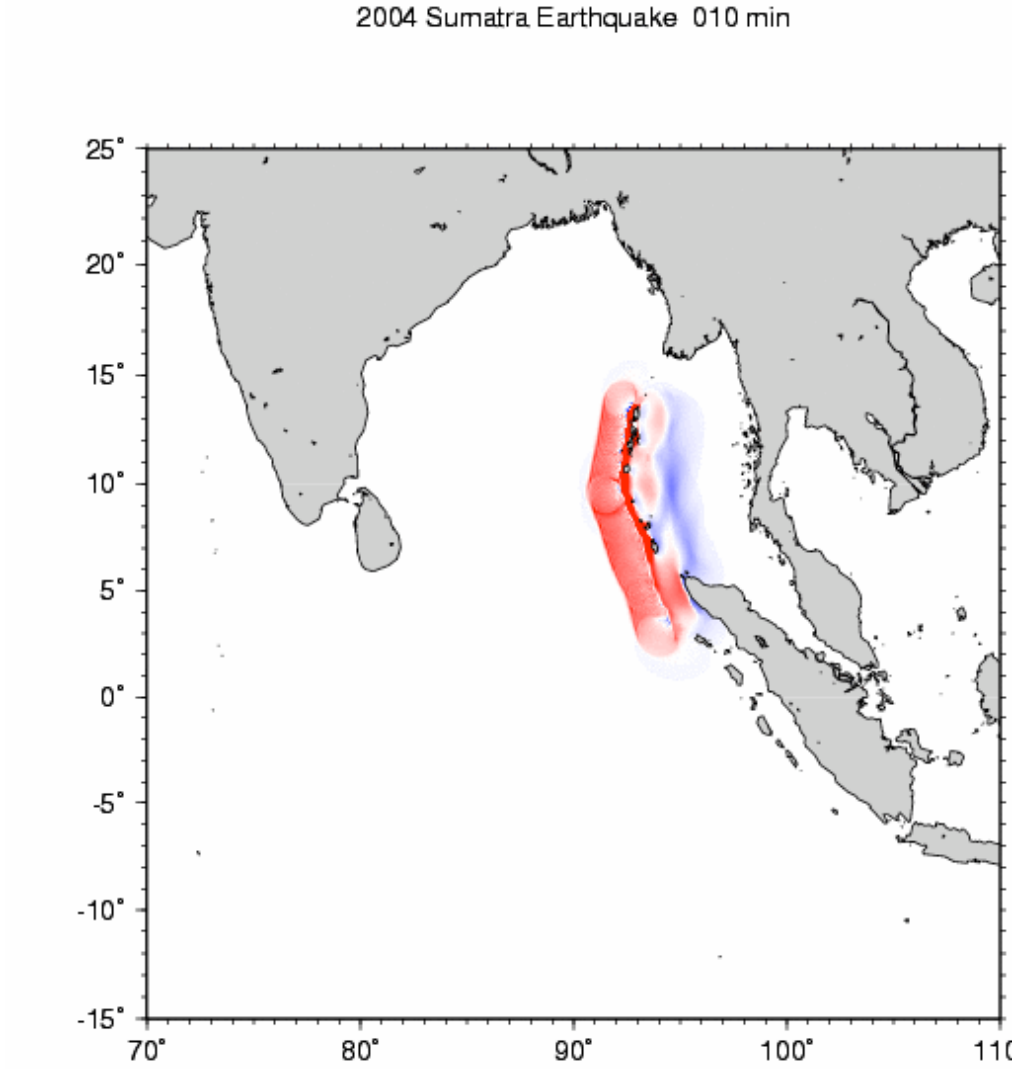
- But all very long waves all travel at about speed

$$\sqrt{gh}$$

gravity fluid depth

How do tsunamis work?

Map of Indian Ocean



Seismology?

Tsunami of 2004
- approximate scales

- Water depth,

  h = 3 km (Bay of Bengal, west)

  = 1 km (Andaman Sea, east)

- For the tsunami,

  λ = 100 km (wavelength)

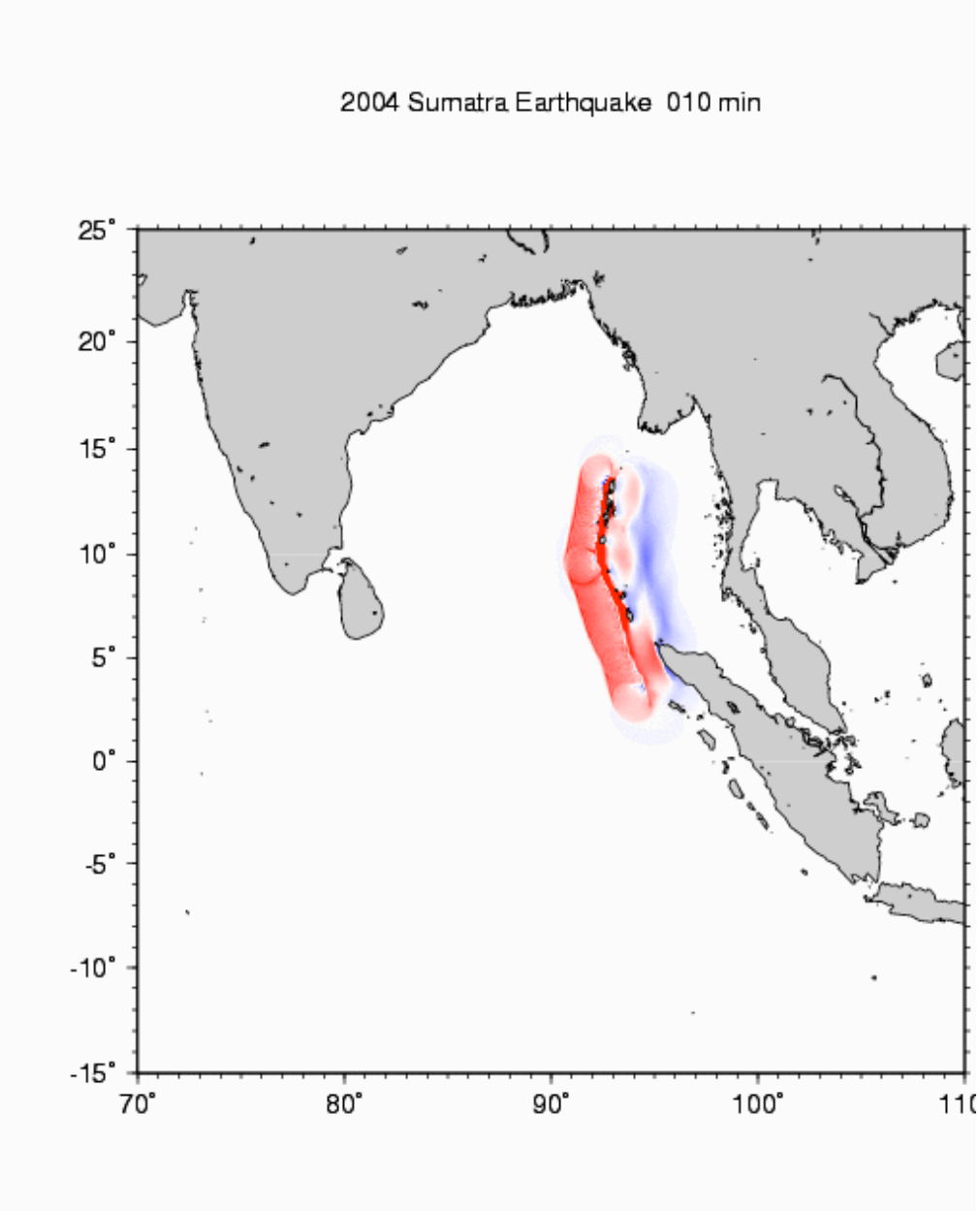
  a = 1 m (wave height)

  L = 1000 km (lateral extent)

  $c = \sqrt{gh}$ = 620 km/hr (wave speed)

  (360 km/hr in Andaman Sea)

Computer animation of 2004 tsunami



Kenji Satake, Japan
http://ioc.unesco.oro/itsu/

or, see
Steven Ward, US
http://www.es.ucsc.edu/~ward/

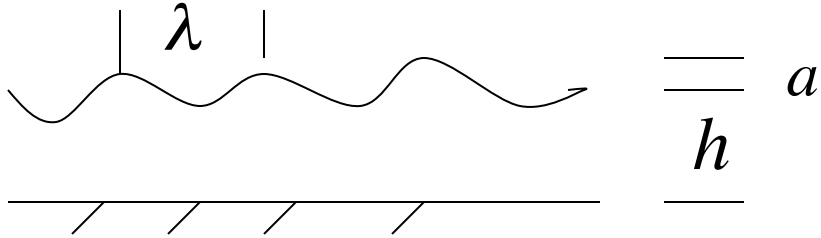
What happens to very long waves?

D.J. Korteweg & G. deVries (1895) derived their equation to describe the motion of long waves of moderate amplitude in shallow water.

Is the KdV equation relevant to the tsunami of 2004?

To derive KdV or KP

Assume:



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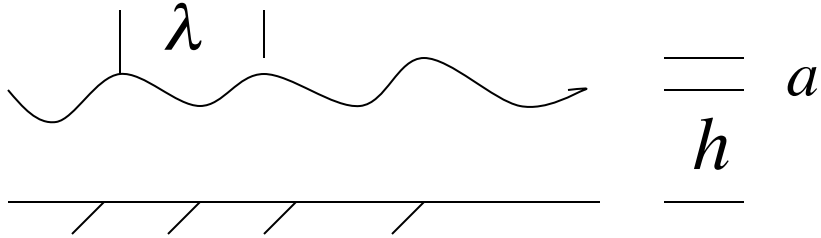

- Long waves ($\lambda >> h$)
  (shallow water)

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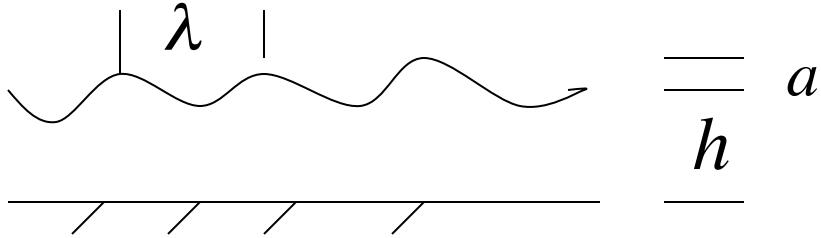

- Long waves ($\lambda >> h$)
  (shallow water)
- Small amplitude ($h >> a$)

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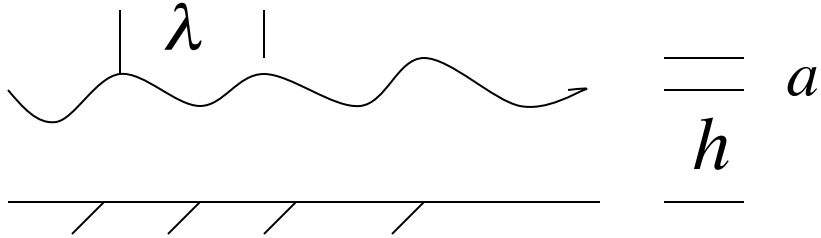

- Long waves ($\lambda >> h$)
  (shallow water)
- Small amplitude ($h >> a$)
- Motion primarily in one direction ($L >> \lambda$)
  if exactly so, ➔ KdV
  if approximately so, ➔ KP

To derive KdV or KP

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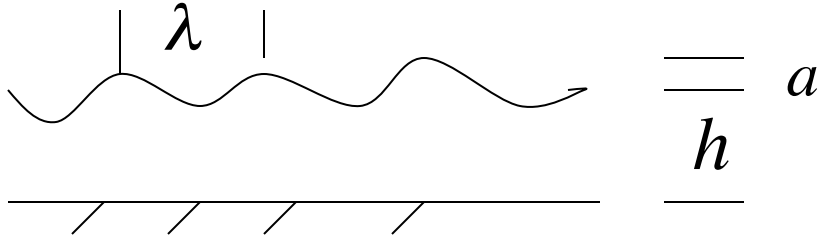

- Long waves ($\lambda >> h$)
  (shallow water)
- Small amplitude ($h >> a$)
- Motion primarily in one direction ($L >> \lambda$)
  - if exactly so, ➔ KdV
  - if approximately so, ➔ KP
- All small effects are comparable

At leading order (with constant *h*):

- Wave equation in 1-D:

$$\partial_t^2 \eta = c^2 \partial_x^2 \eta \quad \text{with} \quad c^2 = gh$$

➔ $$\eta = \varepsilon[F(x - ct; y, \varepsilon t) + G(x + ct; y, \varepsilon t)] + O(\varepsilon^2)$$

At leading order:

- Wave equation in 1-D:

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➔ $$\eta = \varepsilon[F(x - ct; y, \varepsilon t) + G(x + ct; y, \varepsilon t)] + O(\varepsilon^2)$$

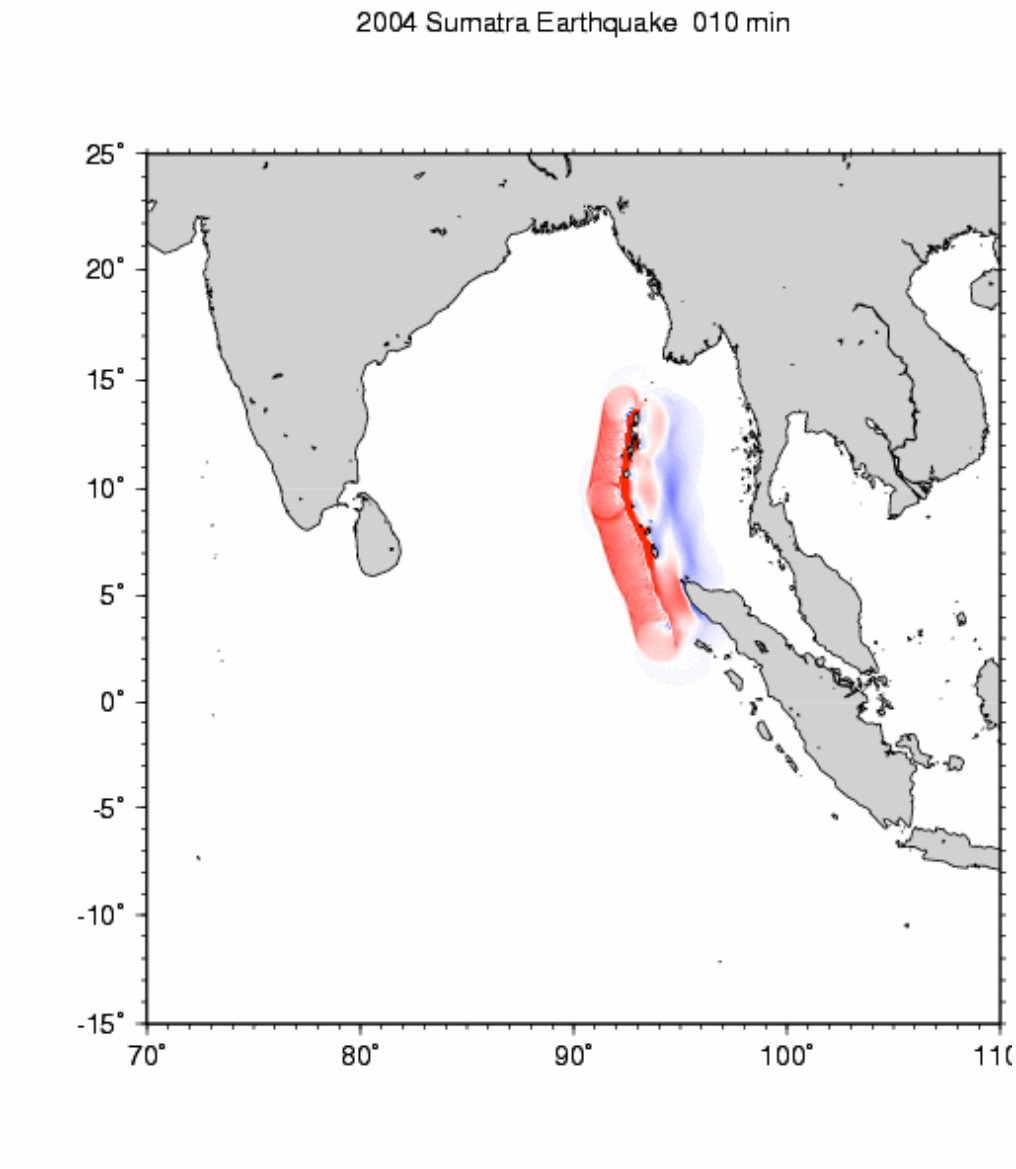
At next order, $F(\xi; y, \tau)$ satisfies either

$$\partial_\tau F + F \partial_\xi F + \partial_\xi^3 F = 0 \qquad \text{KdV}$$

or

$$\partial_\xi (\partial_\tau F + F \partial_\xi F + \partial_\xi^3 F) + \partial_y^2 F = 0 \qquad \text{KP}$$

Length scales for 2004 tsunami



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Tsunami of 2004 - approximate scales

- In the Bay of Bengal,

  h = 3 km (fluid depth)

- For the tsunami,

  λ = 100 km (wavelength)

  a = 1 m (wave height)

  L = 1000 km (lateral extent)

  $c = \sqrt{gh}$ = 620 km/hr (wave speed)

- Scales:

$$\frac{a}{h} \sim 10^{-3}, \quad (\frac{h}{\lambda})^2 \sim 10^{-3}, \quad \frac{\lambda}{L} \sim 10^{-1}$$

KdV model (or KP, or …)

- Includes nonlinearity, frequency dispersion and (perhaps) 2-D surface patterns
- Requires (nearly) uniform depth
- Requires long distances

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with $\varepsilon = \dfrac{amplitude}{depth}$, need propagation distance

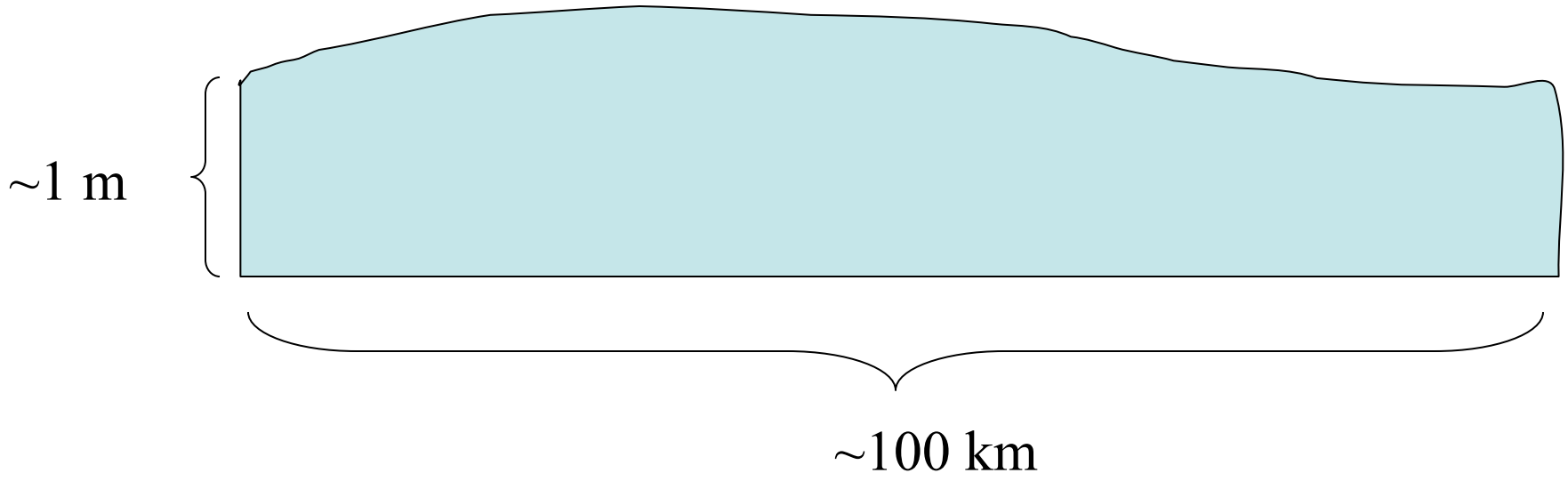
$$D \sim \frac{1}{\varepsilon} \cdot wavelength$$

For 2004 tsunami, need ~ 100,000 km

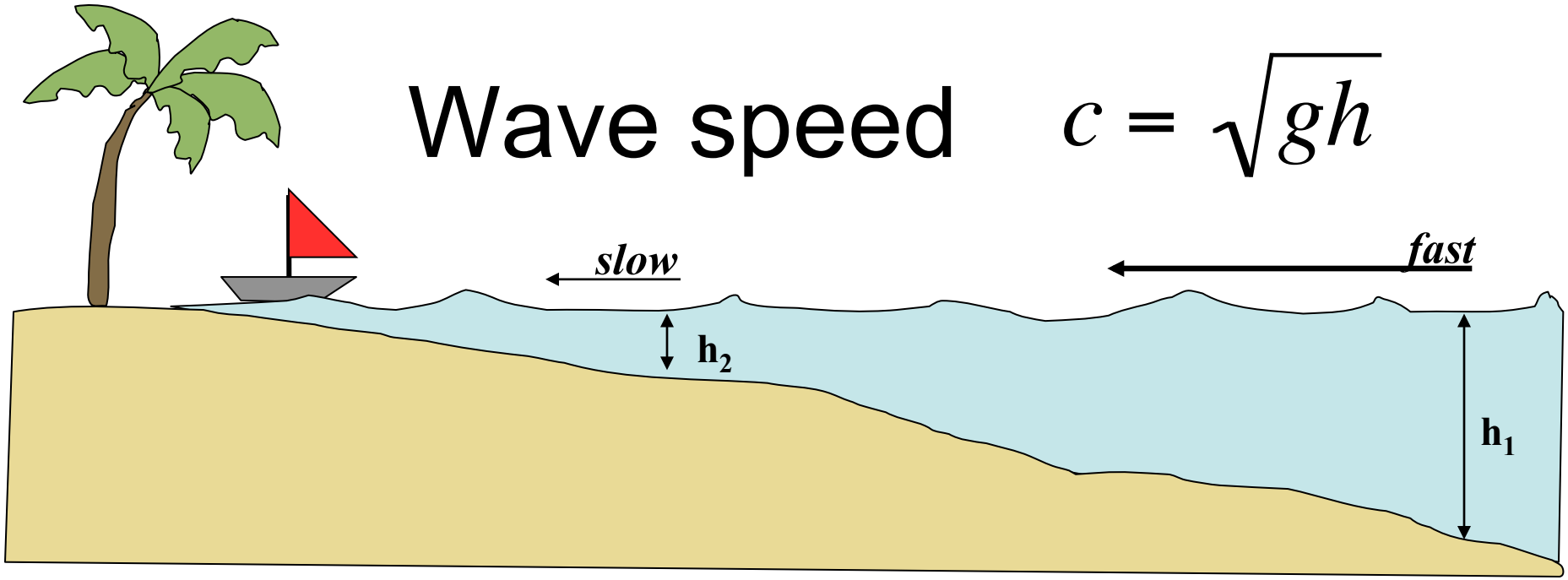
Distance across Bay of Bengal ~ 1300 km

What makes tsunamis dangerous?

Volume of water in 2004 tsunami (per width of shoreline)



Wave speed $c = \sqrt{gh}$



- Open ocean
  - c = 620 km/hr
  - *A wave 100 km long passes by in about 10 minutes*

- Near shore
  - Front of wave slows as it approaches the shore
  - Back of the wave is still in deep water
  - Consequence: *Wave compresses horizontally and grows vertically*

How do long, negative waves evolve?
(in water of uniform depth)

J.L. Hammack, 1944-2004

How do long, negative waves evolve?
(in water of uniform depth)

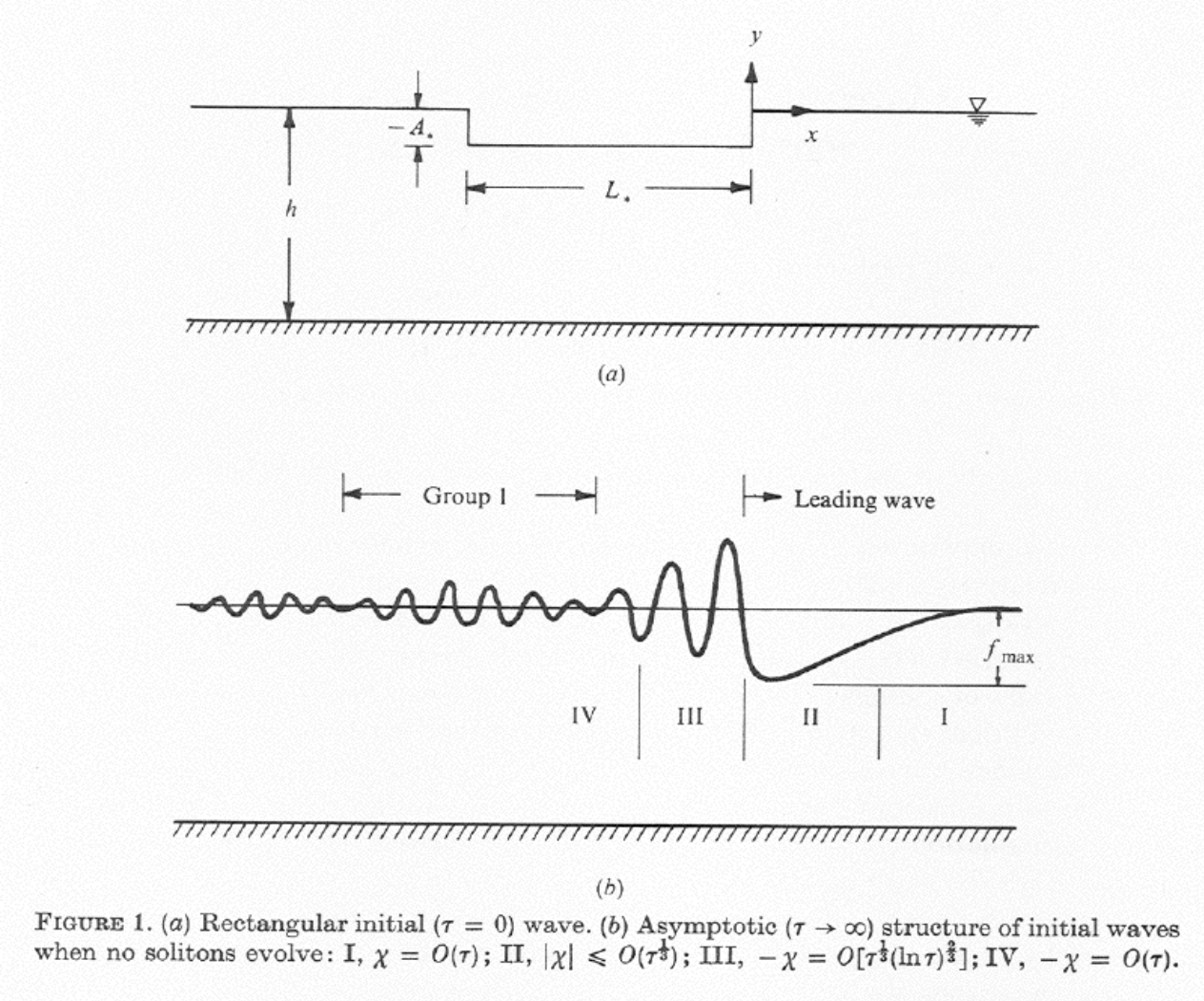
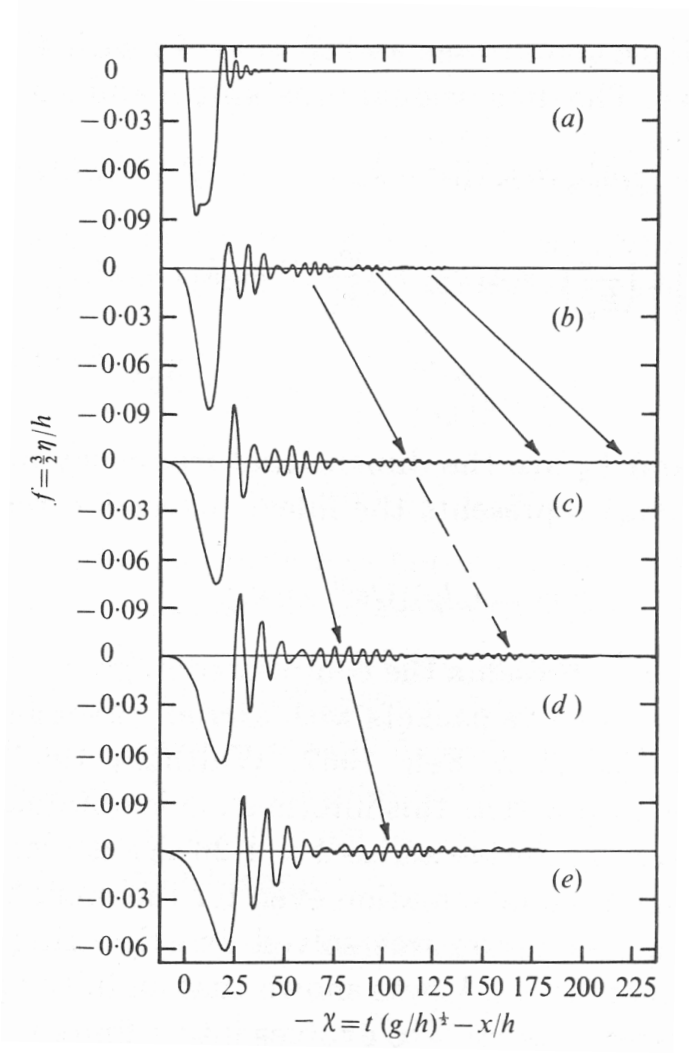


FIGURE 1. (a) Rectangular initial ($\tau = 0$) wave. (b) Asymptotic ($\tau \to \infty$) structure of initial waves when no solitons evolve: I, $\chi = O(\tau)$; II, $|\chi| \leqslant O(\tau^{\frac{1}{3}})$; III, $-\chi = O[\tau^{\frac{1}{3}}(\ln\tau)^{\frac{2}{3}}]$; IV, $-\chi = O(\tau)$.

(Hammack & Segur, 1978)

Experimental data on negative waves



(Hammack & Segur, 1978)

What happened in Thailand?

2004 (negative) wave front reaches Thailand

(photos by Anders Grawin (copyright 2006)

copied from Constantin & Johnson (2008))

What happened in Thailand?

2004 (negative) wave front reaches the shore of Thailand

. The tsunami of 26 December 2004 approaching Hat Ray Leah beach on the Krabi coast, Thailand. (Copyright Scanpix

Note two breaking wave crests (from Constantin & Johnson, 2008)

What happened in Japan on March 11, 2011?

Seismology

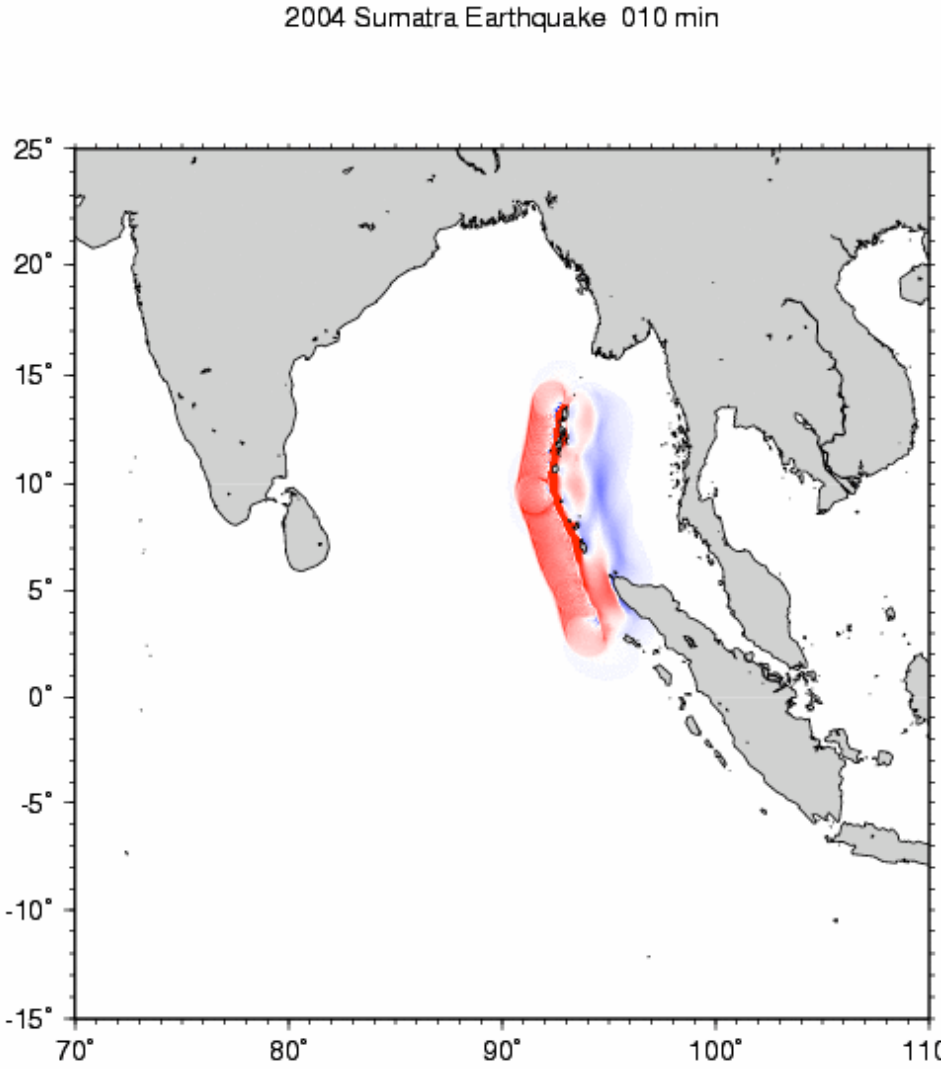
What happened in Japan on March 11, 2011?

Seismology

- The epicenter of the earthquake was 72 km offshore
- A 400 km stretch of coastline dropped 0.6 m
- Parts of Japan moved 2.4 m to the east

(http://en.wikipedia.org/wiki/2011_Tohoku_earthquake_and_tsunami)

How to protect people from the dangers of tsunamis?



a) near the epicenter

b) away from the epicenter

c) near shore

How to protect people from the dangers of tsunamis?

(my personal opinions)

Away from the epicenter and well away from shore

Solve $\partial_t^2 \phi = \nabla \cdot (gh(x,y) \nabla \phi)$

Need decent information about initial data

Volume of water displaced is important

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Q: What measurement estimates the volume of water displaced in a tsunami?

How to protect people from the dangers of tsunamis?

(my personal opinions)

Near shore

Modify the wave eq’n in 1-D to include:

Variable topography => wave refraction
Wave reflection (also from topography)
Nonlinear effects, including wave breaking
Return flow, with vorticity
Wave damping, from bottom friction & return flow
Other (please specify)

How to protect people from the dangers of tsunamis?

(my personal opinions)

Near shore: zoning

How to protect people from the dangers of tsunamis?

(my personal opinions)

Near shore: zoning

High dwellings are the peace and harmony of our descendents. Remember the calamity of the great tsunamis. Do not build any homes below this point.

Why are only some tsunamis deadly?

Dec 26, 2004 Richter scale: 9.0

Earthquake near Banda Aceh, Indonesia

Tsunami killed more than 200,000 people near Indian Ocean

Why are only some tsunamis deadly?

Dec 26, 2004 Richter scale: 9.0

Earthquake near Banda Aceh, Indonesia

Tsunami killed more than 200,000 people near Indian Ocean

March 28, 2005 Richter scale: 8.7

Earthquake 250 km SE of Banda Aceh

Tsunami killed no one

Why are only some tsunamis deadly?

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Earthquake near Banda Aceh, Indonesia

Tsunami killed more than 200,000 people near Indian Ocean

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Earthquake 250 km SE of Banda Aceh

Tsunami killed no one

May 22, 1960 Richter scale: 9.5 (**)

Earthquake 900 km S of Santiago, Chile

Tsunami killed 61 people in Hawaii, 197 more in Japan

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Tsunami killed no one

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Earthquake 900 km S of Santiago, Chile

Tsunami killed 61 people in Hawaii, 197 more in Japan

February 27, 2010 Richter scale: 8.8

Earthquake 335 km SW of Santiago, Chile

Tsunami killed no one outside of Chile

Why are only some tsunamis deadly?

Dec 26, 2004	Richter scale: 9.0	offshore from Sumatra
March 28, 2005	Richter scale: 8.7	offshore from Sumatra
May 22, 1960	Richter scale: 9.5	offshore from Chile
February 27, 2010	Richter scale: 8.8	offshore from Chile

Why are only some tsunamis deadly?

Hypothesis:

Dec 26, 2004 Richter scale: 9.0 offshore from Sumatra
Lateral extent of disturbance: 1000 km

March 28, 2005 Richter scale: 8.7 offshore from Sumatra

May 22, 1960 Richter scale: 9.5 offshore from Chile
Lateral extent of disturbance: 800 km

February 27, 2010 Richter scale: 8.8 offshore from Chile

Thank you for your attention