Topics related to tsunamis generated by rock slides

Hazards and challenges in computations and experiments

G. Pedersen

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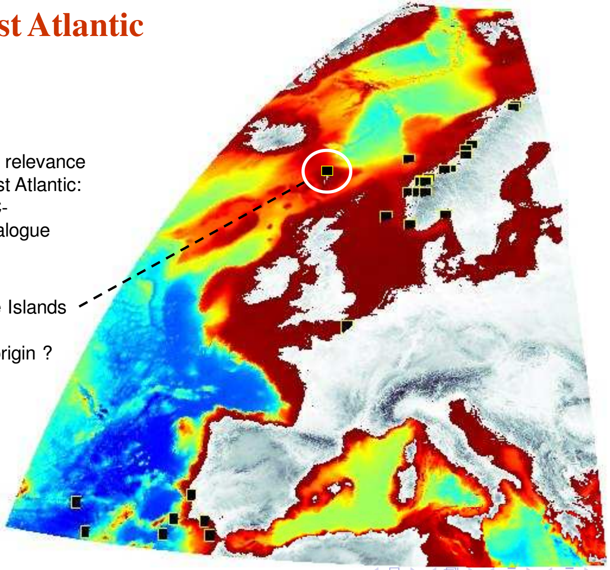
Fields Institute, Toronto, June 13-16, 2011

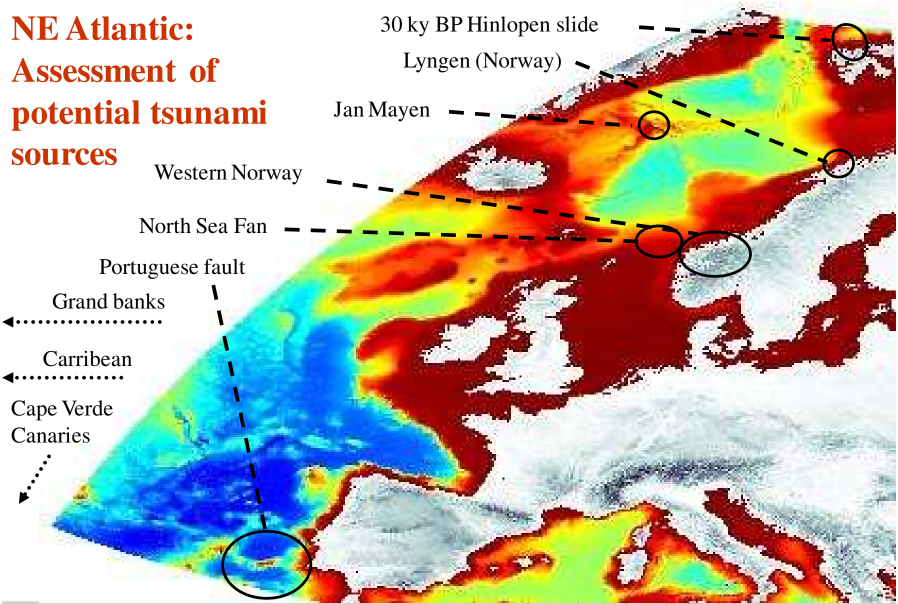
North East Atlantic region

Past events with relevance for the North East Atlantic: From the GITEC-TRANSFER catalogue

Event on Faeroe Islands
28. May 2008
Metereological origin ?



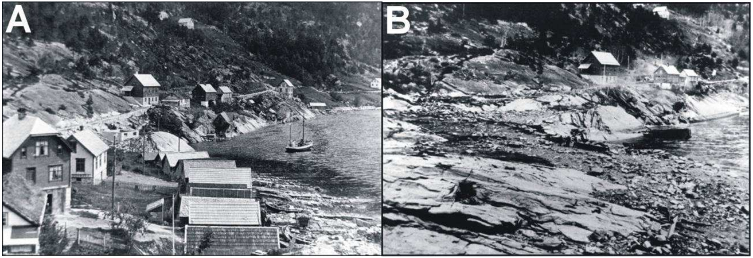
NE Atlantic: Assessment of potential tsunami sources



Waves from rock-slides in fjords and lakes

Local events, but huge waveheights.
Exposed: Alaska, Norway, Chile, Canada, Greenland, alpine regions...

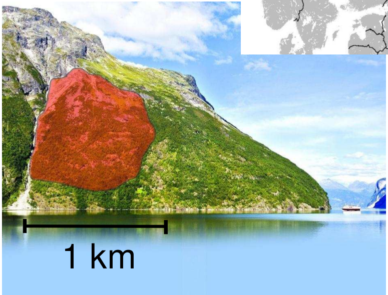
Example: Fjøra 1934, Tafjord, Norway. Wave generated by $1.5 - 3\mathrm{Mm}^3$ slide. 41 perished.



The Åkneset case

Åknes/Norway, <u>future</u> event

- Storfjorden, Western Norway
- Max. volume > 50 Mm^3
- 150 to about 900 m.a.s.l.
- Largest movements at the upper western part
- Numerical models compared to 1:500 scale model



Back fracture expands 4-12 cm per year

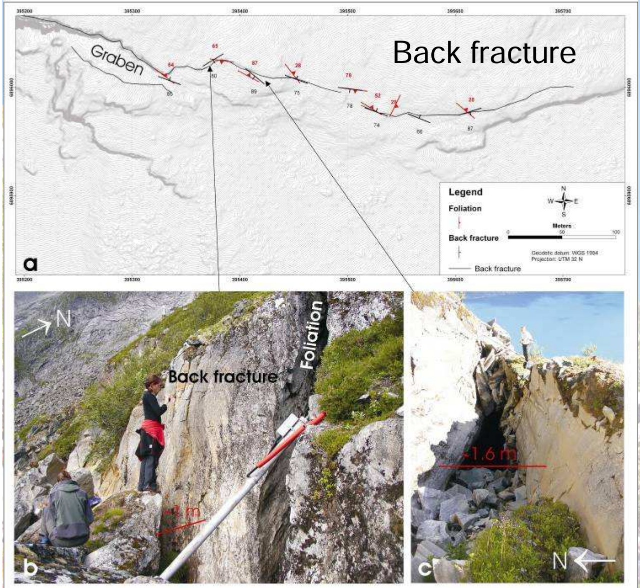
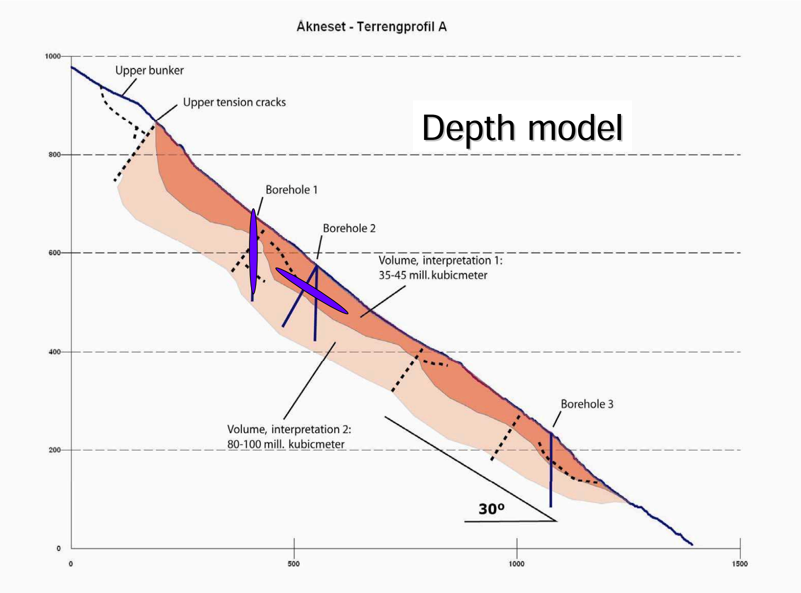


Figure:
Guri Venvik
Ganerød, NGU

Åknes/Tafjord
project

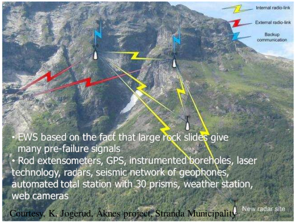
Core samples with breccia at $\sim 50\,\mathrm{m}$ (Blikra)

Constructed slide profile



Risk assessment and mitigation

- Rock slide tsunamis affect the entire fjord system or region
- The risk is here larger than accepted by the Norwegian Building Act
- Mitigation:
  - Inter municipalital preparedness centre
  - Monitoring of the rock slope
  - Tsunami warning
  - Evacuation plans
  - Emergency exercises
  - Drainage?
  - Openness
    - Public meetings, media, stakeholders



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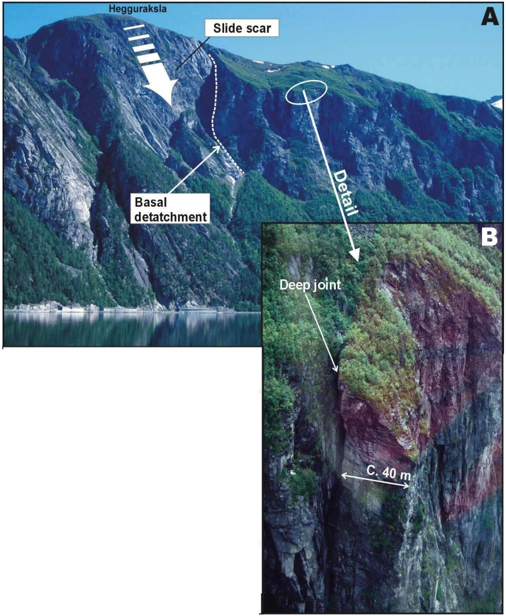
Digression:
A few other cases

Fig. 4

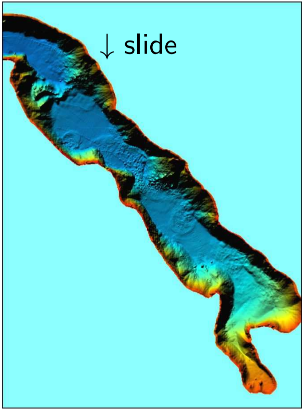


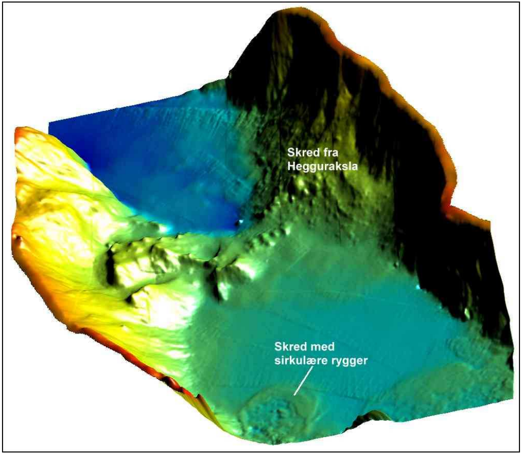
Instrumentation and monitoring in Tafjord

Plans:

- Ground-based radar
- Accellerator
- Possible GPS
- Continous monitoring

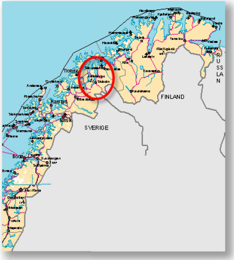
Deposits in the Tafjord, below Hegguraksla





NE Atlantic
Rockslide sources in Norwegian fjords

- Lyngen, northern Norway
- Potential slide from the mountain Nordnes, 7 km from the village Lyngseidet
- Two scenarios:
  1. 7 Mm^3, impact velocity of 45 m/s
  2. 11 Mm^3, impact velocity 55 m/s



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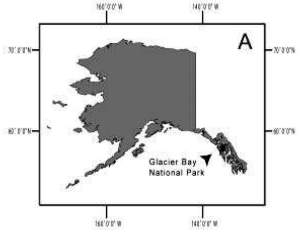
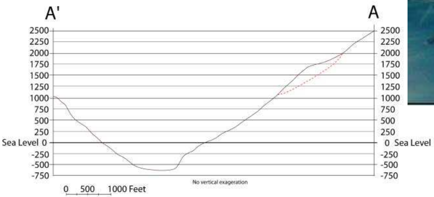
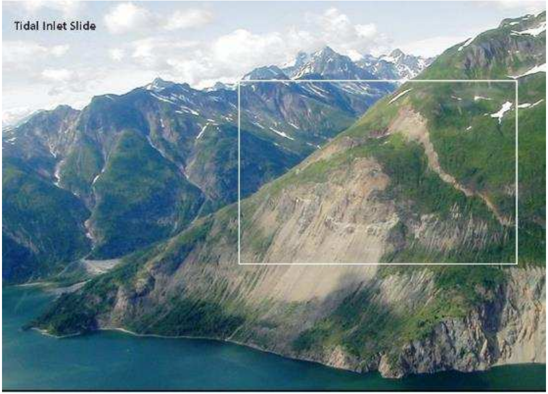
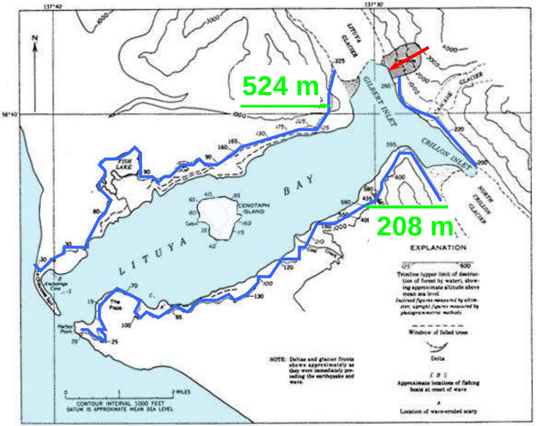




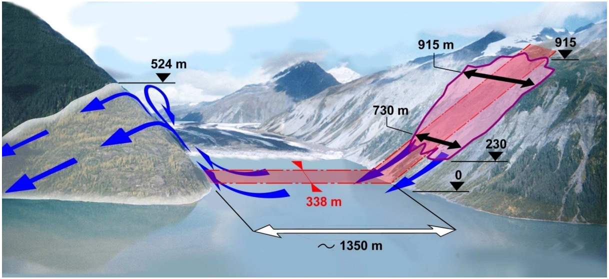


Volume $5 - 10\,\mathrm{Mm}^3$



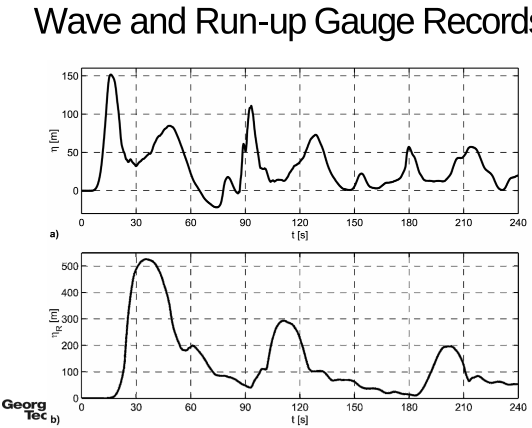
Slide released by $M = 7.9$ earthquake.

Lituya Bay
impact and run-up site



Slide volume and height above water level comparable to Åkneset
Water more shallow than for Åkneset

Overtopped ridge facing slide



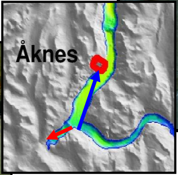
Very high amplitude, steep slopes and (some) breaking.
2D experiments modelled by Weiss et al. 2009 (G. Res. Lett.)

Analysis of the Åknes tsunami

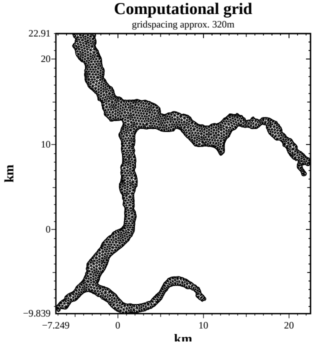
Åknes 3D experiments

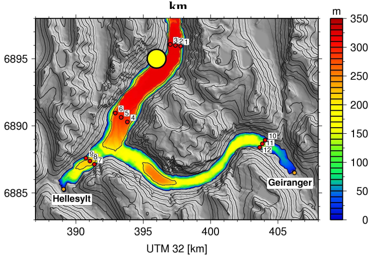
3D laboratory experiments

- Coast and Harbour Research Laboratory at SINTEF, Trondheim, Norway
- Scale 1:500, 30 m x 40 m
- Instrumentation and setup is based on numerical simulations and the 2D laboratory experiments



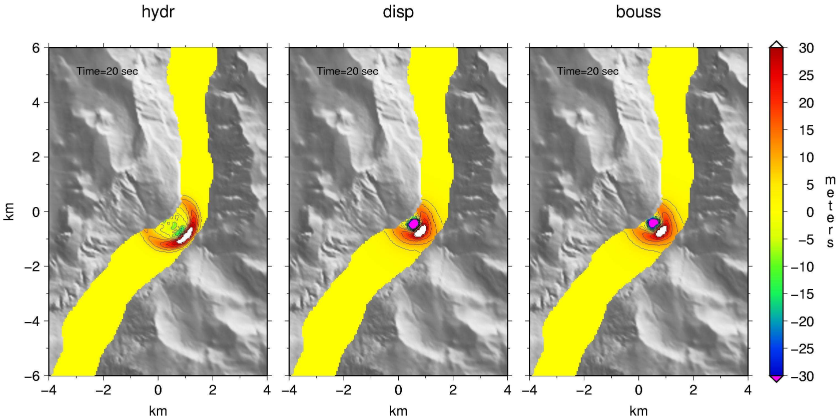
Computations





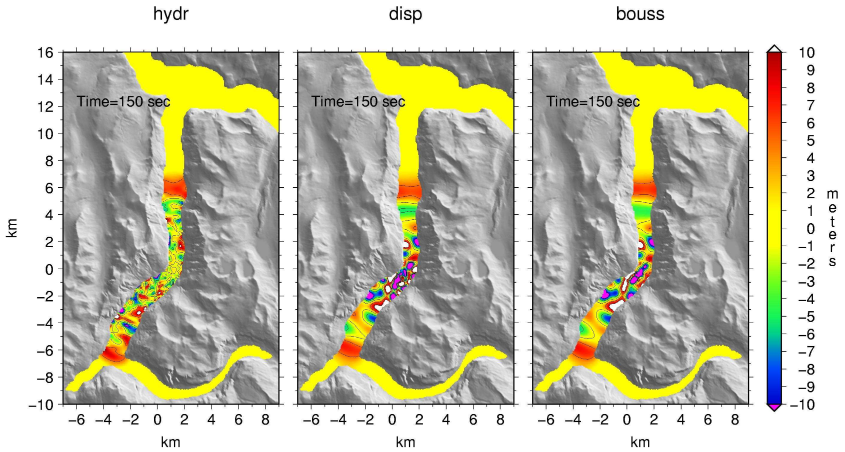
1. Generation by equivalent slide
   or
   input from experiments (gauges 4-6 left)
2. Propagation by Boussinesq models
   FEM (grid to the left)
   FDM, GloBouss
3. Runup: coupling with NLSW model (MOST/COMMIT)

Generation



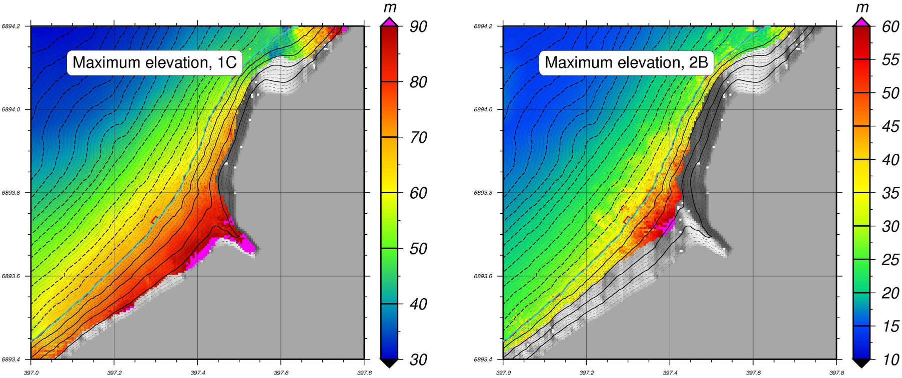
hydr: LSW, disp: linearized Boussinesq
Simple slide model applied.
Dispersion significant.

Propagation



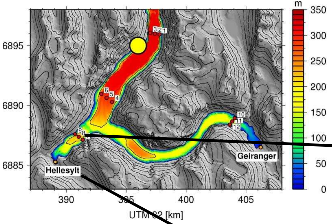
hydr: LSW, disp: linearized Boussinesq
Some effect of nonlinearity

Runup across the fjord; two scenarios

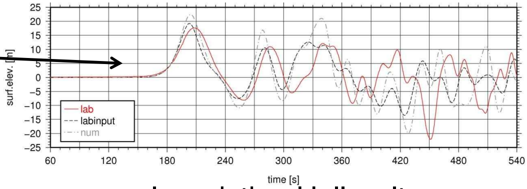


Up to 100m runup; smaller than for the Litua Bay case, but still appreciable...

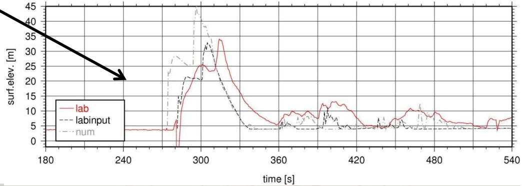
Comparison to laboratory experiments
Scenario: 54 Mm^3



Surface elevation outside Hellesylt

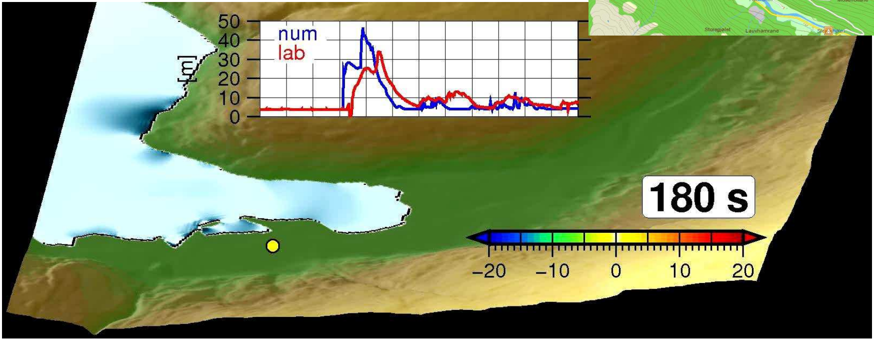


Inundation Hellesylt



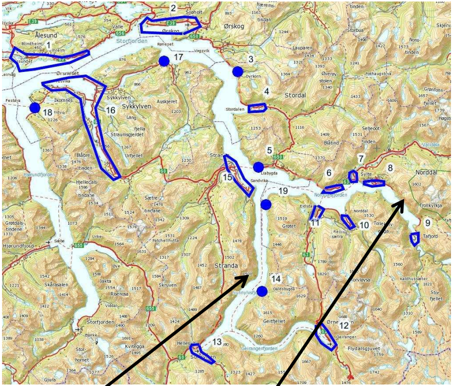
- Lab. experiments – **"lab"**
- Numerical model with input from lab – **"labinput"**
- Numerical model including generation phase – **"num"**
- Leading waves well reproduced
- Inundation: MOST

Run-up Hellesylt, 54Mm^3



- Combination of GloBouss and ComMIT/MOST
- Mariogram extracted at yellow dot
- Numerical model (blue line) and laboratory exp. (red line)

Calculated run-up heights (m)



Åknes:
1C=54Mm3
2B=18Mm3

Hegguraksla:
H2=2.0Mm3
H3=3.5Mm3

Location		Scenarios			
Name	***no***	***1C***	***2B***	***H2***	***H3***
Dyrkorn	3	5	2	-	-
Eidsdal	11	7	3	-	-
Fjøra	8	5	3	17	20
Geiranger	12	65	25	-	-
Gravaneset	5	6	2	-	-
Hellesylt	13	85	35	-	-
Hundeidvik	18	1	<1	-	-
Linge	6	6	2	-	-
Magerholm	1	2	<1	-	-
Norddal	10	15	6	-	-
Oaldsbygda	14	100	70	-	-
Ørskog	2	6	3	-	-
Ramstadvika	17	3	1	-	-
Raudbergvika	19	18	7	-	-
Stordal	4	8	3	-	-
Stranda	15	6	2	-	-
Sykkylvsfjorden	16	3	<1	-	-
Tafjord	9	13	5	8	14
Valldal	7	8	3	6	10
Vegsundet	1	3	2	-	-
Vika	8	9	4	8	13

Models employed.

Some issues with long wave modeling in fjords.

Key points: steep slopes and large amplitudes.

Standard long wave scaling, used in Boussinesq equations

$$x^\star = \ell\hat{x}, \quad y^\star = \ell\hat{y}, \quad t^\star = \ell(gd)^{-\frac{1}{2}}\hat{t},$$
$$\eta^\star = \epsilon d\hat{\eta}, \quad \mathbf{v}^\star = \epsilon(gd)^{\frac{1}{2}}\hat{\mathbf{v}}, \quad h^* = d\,h,$$

d, ℓ and ϵ are typical depth, wavelength, and amplitude factor, respectively.

Expansion parameters $\mu^2 \equiv d^2/\ell^2$ and ϵ.
NLSW: relative errors μ^2.
Standard Boussinesq : relative errors $\epsilon\mu^2, \mu^4$

Alternative scaling, used in figures++

Put $\ell = d$, don't extract ϵ; use of dimensional quantities $(*)$.
Some confusion may follow; sorry.

FEM model

Boussinesq set subjected to weighted residuals

ϕ is a velocity potential (nonlinear and dispersive terms):

$$\frac{\partial \eta}{\partial t} = -\nabla \cdot \left[(h + \epsilon\eta)\nabla\phi + \mu^2 \left(h \left(\frac{1}{6}\frac{\partial \eta}{\partial t} - \frac{1}{3}\nabla h \cdot \nabla\phi \right) \nabla h \right) \right],$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}\epsilon(\nabla\phi)^2 + \eta - \mu^2 \left(\frac{1}{2} h \nabla \cdot \nabla (h \frac{\partial \phi}{\partial t}) - \frac{1}{6} h^2 \nabla^2 \frac{\partial \phi}{\partial t} \right) = 0.$$

Well suited for FEM, but

1. Cannot easily include: Coriolis effects, bottom drag, bore treatment.
2. Terms with explicit ∇h makes equations prone to instabilities linked to depth gradients.

In short: not the best choice for tsunami modeling
(but efficient for potential flow over mild slopes)

FDM Boussinesq model.

GloBouss: Generalization of standard Boussinesq equations

- Model developed for large scale dispersive tsunami simulations
- Enhanced linear dispersion, like FUNWAVE/COULWAVE
- Less general, but much simpler and efficient than FUNWAVE/COULWAVE
- Geographic coordinates: x -longitude, y -latitude
  u and v are corresponding velocity components
  Scaled and dimensionless equations
- Rotational effects (Coriolis) included (useless in fjords)

Continuity equation (identical with NLSW model)

$$c_\phi \frac{\partial \eta}{\partial t} = -\frac{\partial}{\partial x}\{(h+\epsilon\eta)u\} - \frac{\partial}{\partial y}\{c_\phi(h+\epsilon\eta)v\},$$

where $c_\phi = \cos\phi$ is a map factor

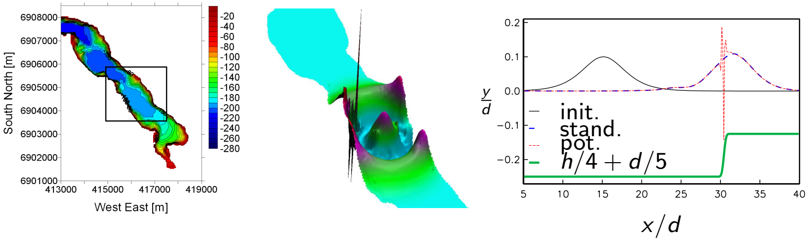
Momentum equations

$$
\begin{aligned}
\frac{\partial u}{\partial t} + \epsilon\left(\frac{u}{c_\phi}\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) &= -\frac{1}{c_\phi}\frac{\partial \eta}{\partial x} + fv - \gamma\mu^2 h^2 \frac{1}{c_\phi}\frac{\partial D_\eta}{\partial x} \\
&\quad + \frac{\mu^2}{2}\frac{h}{c_\phi^2}\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left(h\frac{\partial u}{\partial t}\right) + \frac{\partial}{\partial y}\left(c_\phi h\frac{\partial v}{\partial t}\right)\right] \\
&\quad - \mu^2\left(\frac{1}{6}+\gamma\right)\frac{h^2}{c_\phi^2}\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right) + \frac{\partial}{\partial y}\left(c_\phi\frac{\partial v}{\partial t}\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial v}{\partial t} + \epsilon\left(\frac{u}{c_\phi}\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) &= -\frac{\partial \eta}{\partial y} - fu - \gamma\mu^2 h^2 \frac{\partial D_\eta}{\partial y} \\
&\quad + \frac{\mu^2}{2}h\frac{\partial}{\partial y}\left[\frac{1}{c_\phi}\frac{\partial}{\partial x}\left(h\frac{\partial u}{\partial t}\right) + \frac{1}{c_\phi}\frac{\partial}{\partial y}\left(c_\phi h\frac{\partial v}{\partial t}\right)\right] \\
&\quad - \mu^2\left(\frac{1}{6}+\gamma\right)h^2\frac{\partial}{\partial y}\left[\frac{1}{c_\phi}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right) + \frac{1}{c_\phi}\frac{\partial}{\partial y}\left(c_\phi\frac{\partial v}{\partial t}\right)\right]
\end{aligned}
$$

LSW, with Coriolis terms (f), nonlinear terms, dispersion terms,
Dispersion correction terms: D_η is Laplacian of η and $\gamma = -0.057$
Moderately lengthy appearance of equations, but structure well suited for simple, implicit numerical methods.

Instability for potential model (fjord and idealized shelf)



Standard formulation stable.

All other investigated version may be unstable, but instability not easily triggered in FUNWAVE/COULWAVE and GloBouss (Løvholt & Pedersen 2008).

With Coriolis terms even LSW may be unstable (Espelid & Berntsen 2007).

Improved Boussinesq equations; an example

$H = h + \epsilon\eta$ (flow depth), $\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \epsilon u \frac{\partial}{\partial x}$, $h' = \frac{\mathrm{d}h}{\mathrm{d}x}$

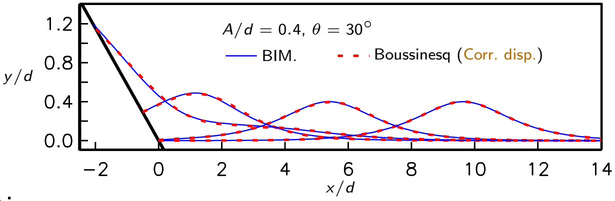
Fully nonlinear equations (used in Lagrangian models)

$$\frac{\mathrm{D}H}{\mathrm{D}t} = -H\frac{\partial u}{\partial x},$$

$$\left(1 - \tfrac{1}{2}\mu^2 Hh'' - \kappa\epsilon\mu^2 h'\frac{\partial\eta}{\partial x}\right)\frac{\mathrm{D}u}{\mathrm{D}t} = -\frac{\partial\eta}{\partial x} - \frac{\mu^2}{3}\left[H\frac{\partial}{\partial x}\left(\frac{\mathrm{D}^2 H}{\mathrm{D}t^2}\right) + 2\frac{\partial H}{\partial x}\frac{\mathrm{D}^2 H}{\mathrm{D}t^2}\right]$$
$$+\epsilon\mu^2\left[h'H\left(\frac{\partial u}{\partial x}\right)^2 - (1-\kappa)h'\left(\frac{\partial\eta}{\partial x}\right)^2 + h''Hu\frac{\partial u}{\partial x} + \left(\epsilon\frac{\partial\eta}{\partial x}h'' + \tfrac{1}{2}Hh'''\right)u^2\right]$$
$$-\gamma\mu^2\frac{\partial}{\partial x}\left[H^2\frac{\partial^2\eta}{\partial x^2} + 2\left(\frac{\mathrm{D}H}{\mathrm{D}t}\right) - H\frac{\mathrm{D}^2 H}{\mathrm{D}t^2}\right] + O(\mu^4).$$

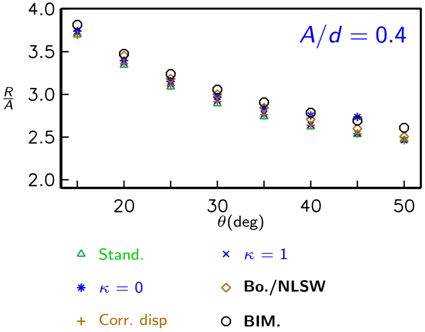
NLSW+linear dispersive $O(\mu^2)$+nonlinear dispersive $O(\epsilon^n\mu^2)$
+dispersion correction $O(\mu^4)$.
Comparable to formulation in FUNWAVE/COULWAVE. Ambiguity, for instance choice of κ.

Test for usefulness: solitary waves on steep slopes

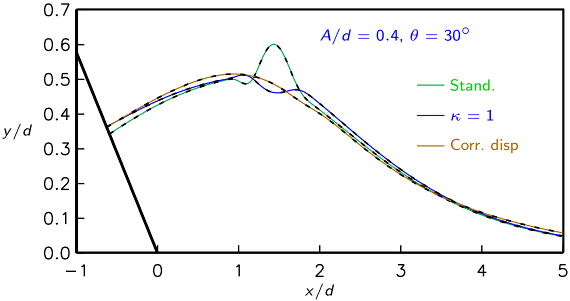


Simulations

- Primarily linear dispersion terms (Stand.)
- Models for ($\kappa = 0$) and ($\kappa = 1$).
- Model for $\kappa = 1$ and (Corr. disp.)
- Combined standard Boussinesq (finite h) and NLSW (**Bo./NLSW**)
- Boundary integral method for full potential theory (**BIM**). Reference solution.
- To be includeed: Coulwave, Funwave, NS solvers ...



Not much difference, really.
Some deviation from deep water properties of models (solitary wave shape).
Observation: Models struggle most with transition constant depth/beach, even if smoothed; important in its own right.

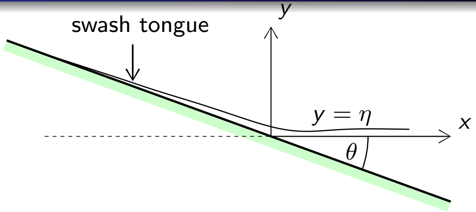


Dashes correspond to half the resolution

Artifacts linked to transition flat bottom ⇒ beach.
Full nonlinearity helps; somewhat.
Corrected dispersion almost remove artifact; strange since this correction is designed from *flat* bottom properties only.

Higher order performance of Boussinesq models for variable bottom is not much studied.

Thin swash zone approximation (with dimensions)



Vanishing flow depth; NLSW equation

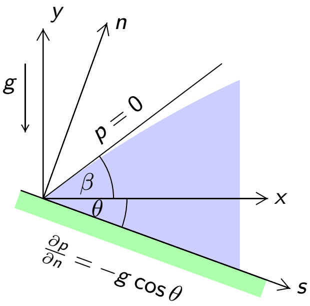
$$a_{NLSW} = \frac{\mathrm{D}u}{\mathrm{D}t} = -g\frac{\partial \eta}{\partial x} = g\frac{\partial h}{\partial x} = g\tan\theta,$$

Interpretation: gravity dominates over momentum transfer due to pressure; fluid moves as independent particles.
Correct acceleration (a_s : alongshore component)

$$a_x = \cos\theta a_s = -g\sin\theta\cos\theta = \cos^2\theta a_{NLSW}$$

θ	5°	10°	20°	30°	40°
$\frac{a_{NLSW}-a_x}{a_x}$	0.008	0.031	0.132	0.333	0.704

Shoreline acceleration (with dimensions)



Geometry yields shoreline gradient

At surface $p = 0 \Rightarrow \cos\beta\frac{\partial p}{\partial x} + \sin\beta\frac{\partial p}{\partial y} = 0$

$$\frac{\partial p}{\partial x} = \frac{\rho g \tan\beta}{1 - \tan\beta\tan\theta}$$

$$\frac{\mathrm{D}u}{\mathrm{D}t} = -\frac{g\frac{\partial \eta}{\partial x}}{1 - \frac{\mathrm{d}h}{\mathrm{d}x}\frac{\partial \eta}{\partial x}}$$

$\theta + \beta \to \frac{\pi}{2} \Rightarrow$ inifinite acceleration; dambreak analogy.

$\beta = -\theta \Rightarrow$ thin swash approximation reproduced

NLSW: $\frac{\mathrm{D}u}{\mathrm{D}t} = -g\frac{\partial \eta}{\partial x} \Rightarrow$ half of correct value for $\theta = 30^\circ$, $\beta = 45^\circ$.

Standard Boussinesq equations fare no better than NLSW!

If R is radii of curvature for beach:

$\frac{\partial p}{\partial n} = -g\cos\theta + v_s^2/R$.

Shoreline properties and Boussinesq equations

Full potential theory; non-dimensional.

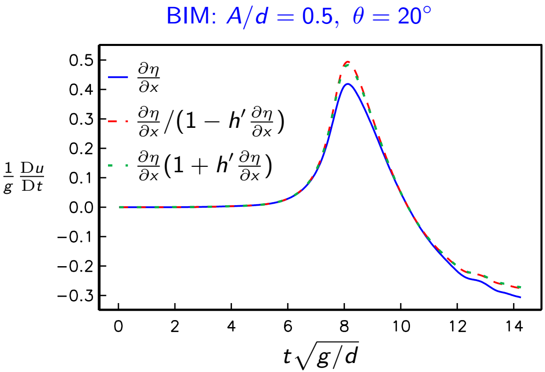
$$\frac{\mathrm{D}u}{\mathrm{D}t} = \frac{-\frac{\partial\eta}{\partial x}}{1-\mu^2\epsilon\frac{\partial\eta}{\partial x}h'} = -\frac{\partial\eta}{\partial x} - \mu^2\epsilon\left(\frac{\partial\eta}{\partial x}\right)^2 h' + \ldots$$

Boussinesq equation; $H = \frac{\mathrm{D}H}{\mathrm{D}t} = 0$ at shoreline

$$\frac{\mathrm{D}u}{\mathrm{D}t} = \frac{-\frac{\partial\eta}{\partial x} - (1-\kappa)\epsilon\mu^2\left(\frac{\partial\eta}{\partial x}\right)^2 h'}{1-\kappa\mu^2\epsilon\frac{\partial\eta}{\partial x}h'} = -\frac{\partial\eta}{\partial x} - \mu^2\epsilon\left(\frac{\partial\eta}{\partial x}\right)^2 h' + \ldots$$

Only $\kappa = 1$ reproduce "correct" shoreline behaviour (and reward you with model breakdown for $\theta + \beta = \frac{\pi}{2}$).
What is the significance for real applications of fully nonlinear Boussinesq equations?

Why the soliton test was that indiscriminate



- Steep slope and long wave $\Rightarrow$ moderate $\frac{\partial\eta}{\partial x}$. In addition deviation in $\frac{\mathrm{D}u}{\mathrm{D}t}$ presumably counterbalanced.
- Real waves may be more extreme (as in Litua Bay). Other incident waves must be employed.

Future work

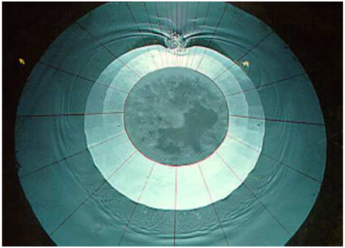
2011-2014 project granted by the Norwegian Research Council

- New experiments in 1:500 fjord model; focus on generation.
- 3D Navier-Stokes model for wave generation; measurements used for verification
- Assessment of long wave models for propagation. May Boussinesq type models be made to work properly ?
- Dynamic coupling NS generation model/long wave propagation model

Runup experiments; scale effects

Experiments

Circular island (1995)

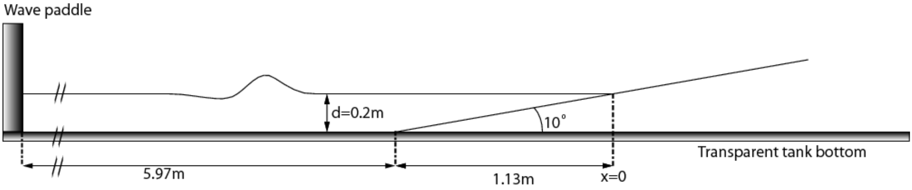


Impact

Åkneset

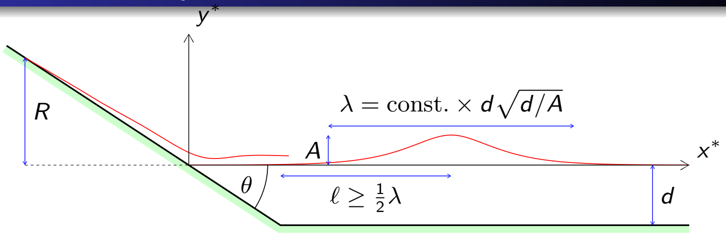
Small scale experiments often used for model validation; but do they reproduce the full-scale case ?

Recent experimental runup investigation UiO



- Experiments on inclined plane (10°) ; A basic runup experiment revisited 2010 and 2011 (less relevant exp. 2008).
- Inundation measured by video and edge detection.
- Flow depth measured by accoustic gauges
- Velocities measured by PIV.
- Solitary incident waves with A/d up to 0.5: no clear breaking during runup, although roughly vertical front at initial shoreline for $A/d \approx 0.5$.

Solitary wave runup



General relation (from dimension analysis)

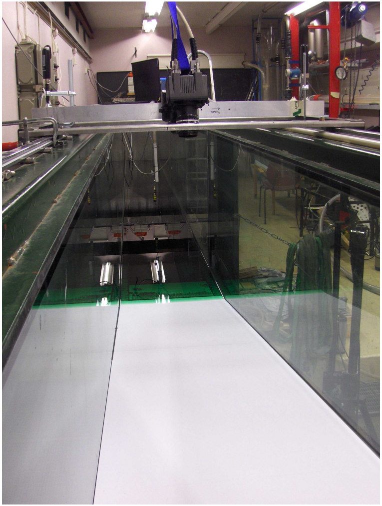
σ is surface tension, … : contact point effects etc.

$$\frac{R}{A} = F\left(\theta, \frac{A}{d}, \frac{\ell}{d}, \frac{d\sqrt{gd}}{\nu}, \frac{\sigma}{\rho g d^2}, \ldots\right),$$

Non-viscid and non-dispersive solution (Synolakis 1987)

$$\frac{R}{A} = 2.831(\cot\theta)^{\frac{1}{2}}\left(\frac{A}{d}\right)^{\frac{1}{4}} \quad \text{for} \quad \sqrt{\frac{A}{d}}\cot\theta \to \infty, \ \frac{A}{d} \to 0,$$

Runup measurements



Shoreline tracing

- camera synchronized with paddle and gauges
- dyed water, edge detection for shoreline
- complete history pieced together from several recordings from different camera locations
- some transverse variation observed

Maximum runup heights

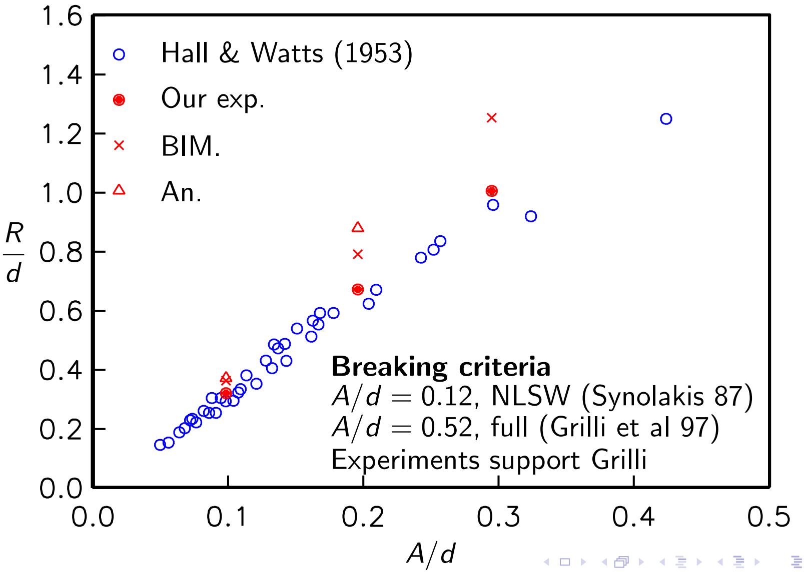
	R/A for $\theta = 10^\circ$				
A/d	Exp.	BIM	Bouss.	(N)LSW	An.
0.098	3.10±0.03	3.69	3.67	3.92	3.77
0.195	3.37±0.03	4.04	4.00	4.61[a]	4.48[a]
0.292	3.46±0.02	4.25	4.19	5.07[a]	4.96[a]
0.390	3.52±0.03	4.47[b]	4.44[b]	5.43[a]	5.33[a]

- Experimental R much less than theoretical ones. Why ?
- Shallow water solutions overpredict R even more; too much steepening of incident waves.

[a]: (N)LSW is breaking, contrary to exp. and dispersive models

[b]: Contact angle slightly surpasses 90°, formal validity questionable

Comparison to Hall & Watts 1953, $\theta = 10°$



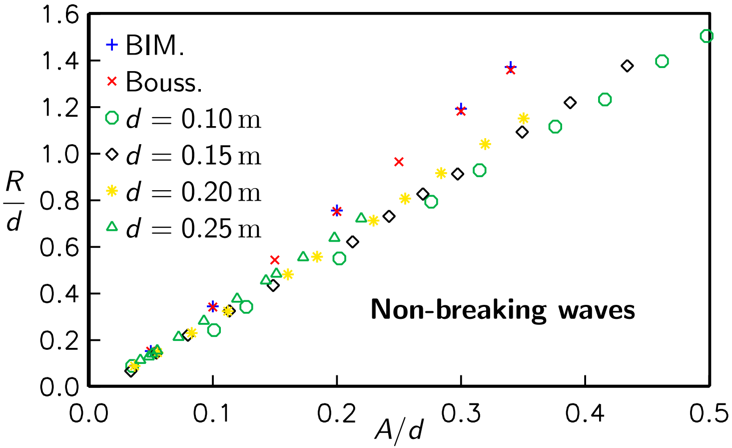
Breaking criteria

$A/d = 0.12$, NLSW (Synolakis 87)

$A/d = 0.52$, full (Grilli et al 97)

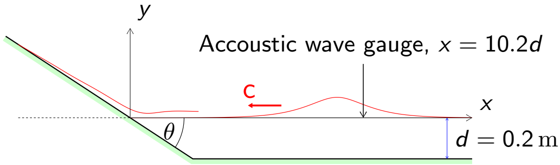
Experiments support Grilli

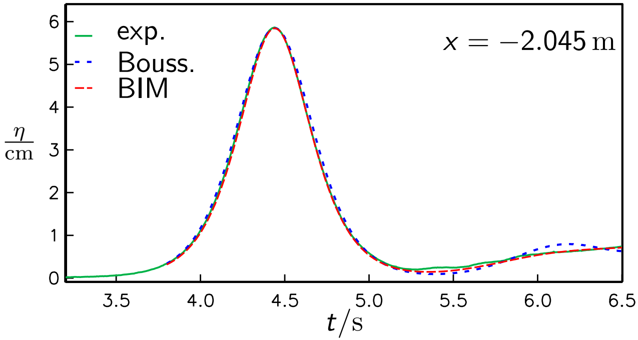
Langsholt 1981, $\theta = 12^\circ$



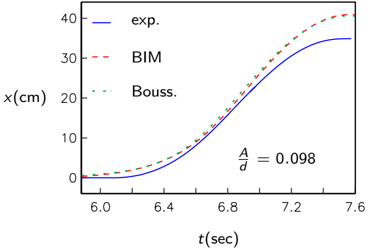
R and A from resistance gauges; experiments not fully published.
Trend : R/A increases with $d \Rightarrow$ viscous or cappilary effects ?

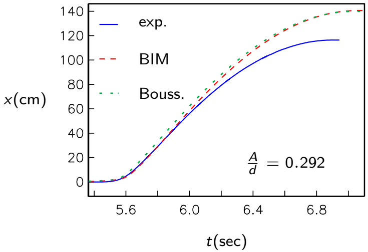
Synchronization of experiments and computations





Reduced experimental runup





Observations

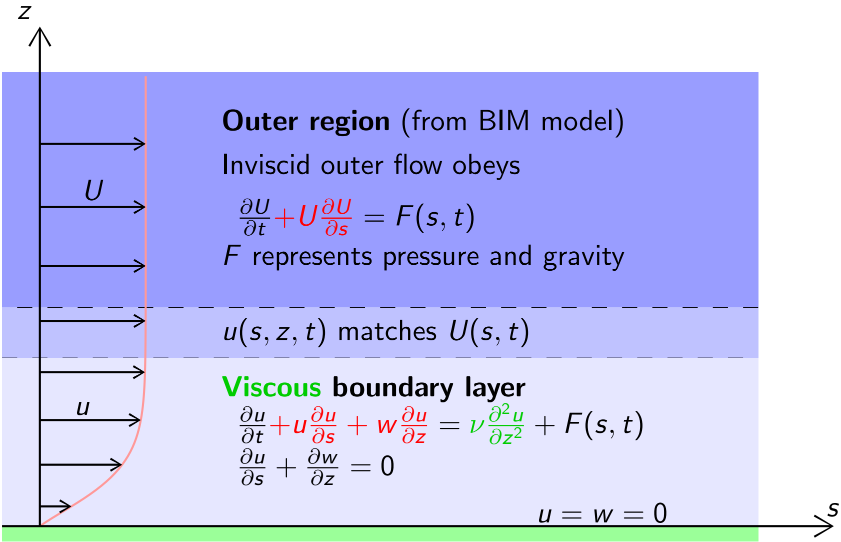
-Early delay for small amplitudes due to capillary effects.

-Main deviation develops later and is presumably due to viscous effects.

-Capillary effects less important for higher amplitudes.

-Both capillary and viscous effects are highly scale dependent.

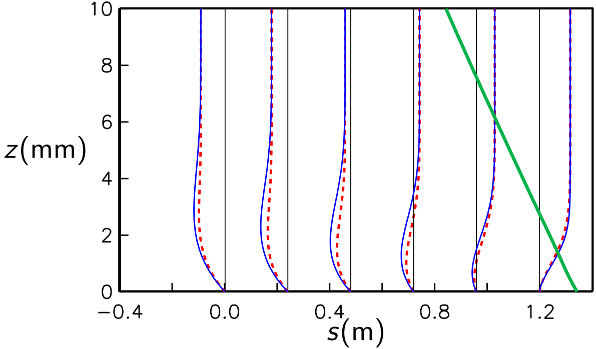
Boundary layer theory



Boundary layer equations solved numerically.

Computed velocity profiles

$A/d = 0.295$, before max. runup (solid/dashed: linear/nonlinear)



(s along beach, 0.1 m on axis is 0.58 m/s)

First reversal of flow in boundary layer.

Experience from stationary flow: acceleration stabilizes; retardation destabilizes; boundary layers separate for zero wall stress ($\frac{\partial u}{\partial z} = 0$) – not appropriate for strongly transient flows.

Boundary layers under solitary waves on constant depth

Recent investigations

- Liu (2006) computations
- Liu et al. (2007) experiments and computations
- Vittori and Blondeaux (2008) computations
- Sumer et al. (2010) experiments

Reynolds number

$Re = \frac{UL}{\nu}$, where
U is maximum velocity,
L is particle displacement in outer flow.

Regimes

$Re < 4 \times 10^5$	laminar,
$4 \times 10^5 < Re < 10^6$	vortex tubes in retardation phase,
$10^6 < Re$	transitional.

Comparison with runup

Constant depth: Acceleration of outer flow in wave front, retardation in rear part.

Runup: Short period of strong acceleration, then retardation.

Drawdown: Accelerated downward flow on beach.

$\alpha = A/d$	0.1	0.2	0.3	0.5
Re_c (constant depth)	$2.0 \cdot 10^4$	$5.8 \cdot 10^4$	$1.1 \cdot 10^5$	$2.3 \cdot 10^5$
Re_R (runup)[1]	$3.8 \cdot 10^5$	$1.4 \cdot 10^6$	$3.0 \cdot 10^6$	$7.7 \cdot 10^6$

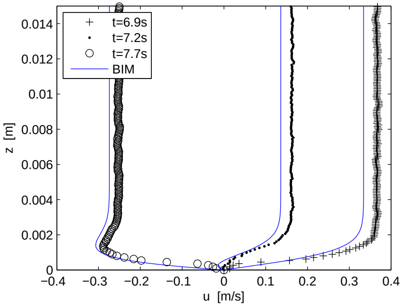
Runup Reynolds numbers in transitional range of Sumer et al. Preliminary observations: $\alpha = 0.1$, remains laminar; $\alpha = 0.3$ non-laminar in later stages, at least up-beach; $\alpha = 0.5$ mostly non-laminar.

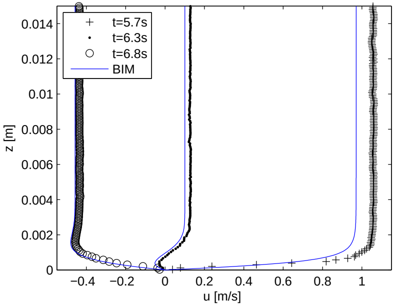
[1]: $U = \sqrt{gR}$, $L = R \cot\theta$, depth $d = 0.2\,\mathrm{m}$, R from Synolakis' formula.

Measured Boundary layers on beach

Measurements $\sim 7\,\mathrm{cm}$ inland; averaged over $\sim 0.5\,\mathrm{cm}$ along beach.

$A/d = 0.098$ $\qquad$ $A/d = 0.292$





Surprisingly large discrepancy in outer flow
Good agreement for profile shapes and boundary layer thicknesses
No apparent sign of transition to non-laminar flow.

Feedback from the boundary layer on the outer flow

$A/d \leq 0.3$: laminar boundary layers in region behind shoreline

Volume transport deficiency

$$\Delta q = \int_0^\infty (U - u)dz$$

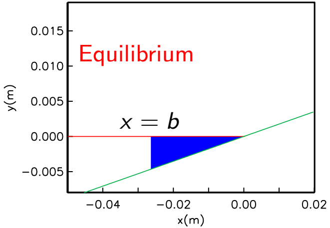
If flow had been stationary: $\Delta q / U =$ displacement thickness.

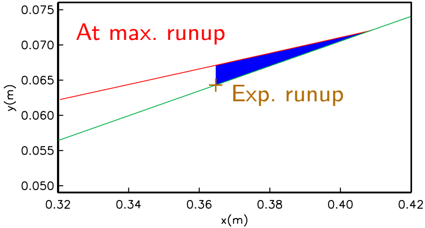
Divergence of Δq may be incorporated in depth-integrated continuity equation (Liu & Orfila 2004: linear boundary layer)

Inclusion is not straightforward in either the boundary integral method (BIM) or Lagrangian Boussinesq model with runup $\Rightarrow$ effect of boundary layer only assessed through estimation of volume loss, feedback not included in wave models

Accumulated volume loss

Wedge shape fluid body that surpasses observed inundation is traced in BIM model.



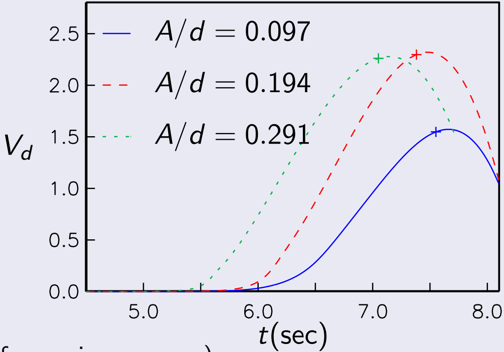


Relative accumulated loss of volume

$$V_d = \frac{\int_0^t \Delta q(x(b,t),t)dt}{\frac{1}{2}\tan\theta b^2},$$

where $x(b,t)$ is rear end of wedge shaped fluid volume.

Integrated volume transport defects



(Marks: time for maximum runup)

Normalized volume transport defects of order one

Values are sensitive to parameters

Consistent behaviour for momentum and energy (not shown)

Remarks

Scale dependent features in experiments ($d = 20\,\mathrm{cm}$)

- Surface tension, contact point properties.
  Visible in start of runup
  Most important for small amplitudes
- Viscous boundary layers in swash region.
  Observed in experiments, thickness $\sim 3 - 5$ mm May explain deviations between experiments and theory,

Such effects not appreciated in the literature on runup.

Comparison of models to small scale experiments

Computed R may be reduced by other effects, such as

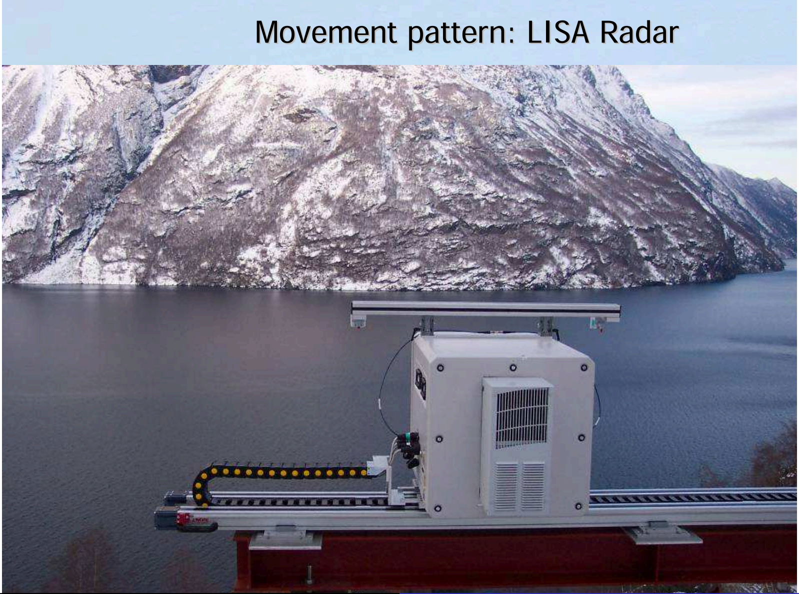
- artificial damping in general
- premature (non-physical) breaking in NLSW models
- inaccurate methods for shoreline tracing

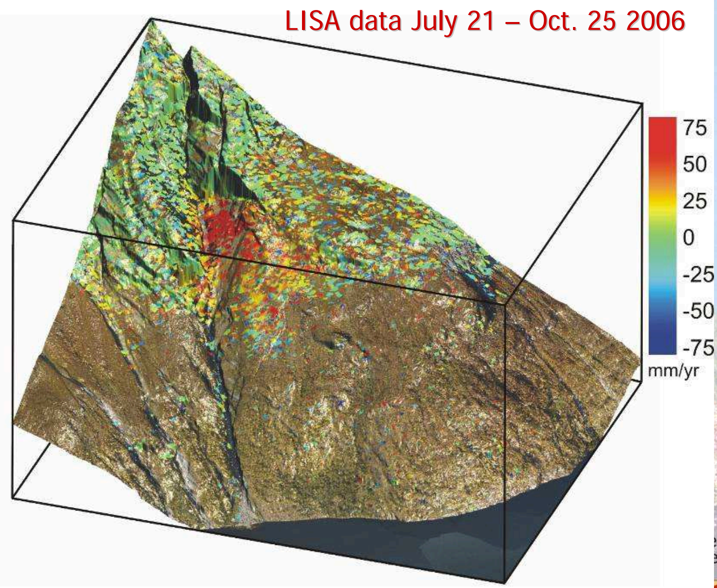
Good agreement may not be conveyed to a full scale tsunami (flow regime: fully turbulent swash zone, bottom roughness ...)

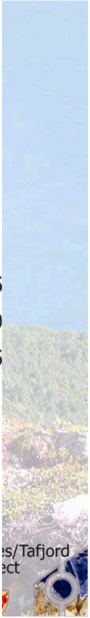
- Proper simulations with Navier-Stokes models addressing scale dependence, transition to turbulence (and breaking) ?
- Real tsunami runup is complex (obstacles, sediments, heavy debris..)
  The simple cases should be sorted, and scale effects pointed out, before complex experiments are attempted.

Extra slides

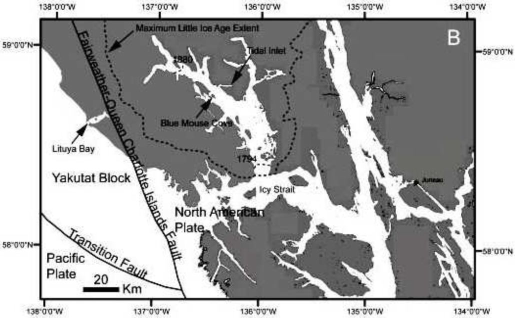
Surveillance by radar (from Blikra)

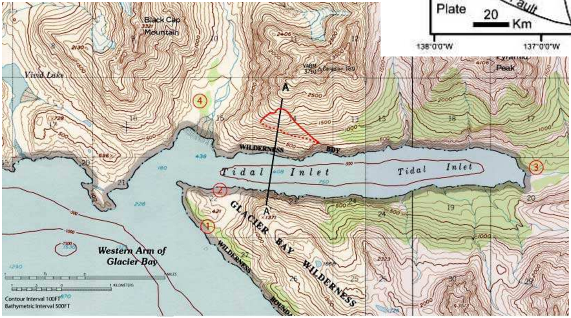



LISA data July 21 – Oct. 25 2006
75
50
25
0
-25
-50
-75
mm/yr

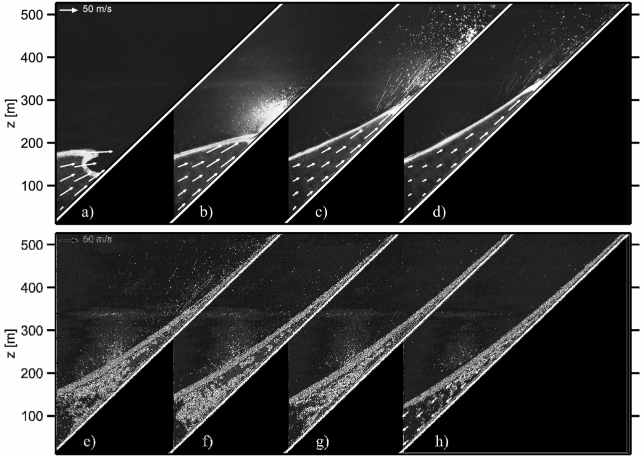


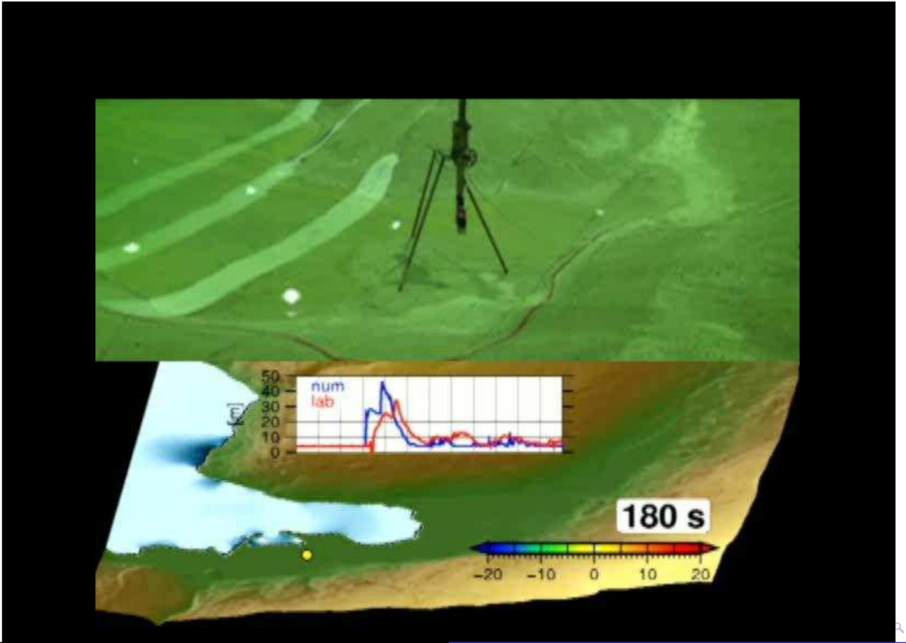
Glacier Bay, location (from USGS)





530m Tsunami Wave Run-up




50
40
30
20
10
0
num
lab
180 s
−20
−10
0
10
20

The z_α formulation

Popular formulation from Nwogu, later extended by others (Kennedy, Kirby, Wu, Liu, Lynett..)
Velocity profile

$$\vec{v} = \vec{v}_s + \mu^2 (z_\alpha \nabla_h \frac{\partial \eta}{\partial t} - \frac{1}{2} z_\alpha^2 \nabla_h \nabla_h \cdot \vec{v}_*) + O(\mu^4),$$

$\vec{v}_s$ = surface velocity, $\vec{v}_*$ velocity at any depth.
Velocity at $z_\alpha(x, y)$
$\mathbf{v}(x, y, t) \equiv \vec{v}(x, y, z_\alpha(x, y), t)$,
used as unknown. Optimization of disperson on flat bottom $\Rightarrow$
$z_\alpha = -0.531h$

Extra nonlinearities, $O(\mu^2 \epsilon)$, may be kept in derivation.

Generalized Boussinesq equations

Hsiao et al. (2002):

$$\eta_t = -\nabla_h \cdot \left[(h+\epsilon\eta)(\mathbf{v}+\mu^2\mathbf{M})\right] + O(\mu^4),$$

$$\mathbf{v}_t + \frac{\epsilon}{2}\nabla_h(\mathbf{v}^2) = -\nabla_h\eta - \mu^2\left[\tfrac{1}{2}z_\alpha^2\nabla_h\nabla_h\cdot\mathbf{v}_t + z_\alpha\nabla_h\nabla_h\cdot(h\mathbf{v}_t)\right]$$
$$+\epsilon\mu^2\nabla_h(D_1+\epsilon D_2+\epsilon^2 D_3) + O(\mu^4) + \mathbf{N} + \mathbf{E},$$

where index t denotes temporal differentiation and

$$\mathbf{M} = [\tfrac{1}{2}z_\alpha^2 - \tfrac{1}{6}(h^2 - \epsilon h\eta + \epsilon^2\eta^2)]\nabla_h\nabla_h\cdot\mathbf{v}$$
$$+[z_\alpha + \tfrac{1}{2}(h-\epsilon\eta)\nabla_h\nabla_h\cdot(h\mathbf{v})].$$

Extra nonlinearities marked with blue.

Furthermore...

$$
\begin{aligned}
D_1 &= \eta\nabla\cdot(h\mathbf{v}_t) - \tfrac{1}{2}z_\alpha^2\mathbf{v}\cdot\nabla\nabla\mathbf{v} - z_\alpha\mathbf{v}\cdot\nabla\nabla\cdot(h\mathbf{v}) - \tfrac{1}{2}(\nabla\cdot(h\mathbf{v}))^2,\\
D_2 &= \tfrac{1}{2}\eta^2\nabla\cdot\mathbf{v}_t + \eta\mathbf{v}\nabla\nabla\cdot(h\mathbf{v}) - \eta\nabla\cdot(h\mathbf{v})\nabla\cdot\mathbf{v},\\
D_3 &= \tfrac{1}{2}\eta^2\left[\mathbf{v}\cdot\nabla\nabla\cdot\mathbf{v} - (\nabla\cdot\mathbf{v})^2\right],\\
\mathbf{E} &= H^{-1}\nabla_h(\nu(x,y,t)\nabla_h(H\mathbf{v}),\\
\mathbf{N} &= -\tfrac{\epsilon}{\mu}\tfrac{K}{H}|\mathbf{v}|\mathbf{v}.
\end{aligned}
$$

Unsystematic terms:

E is dissipation term for capturing of breaking waves

N is bottom drag.

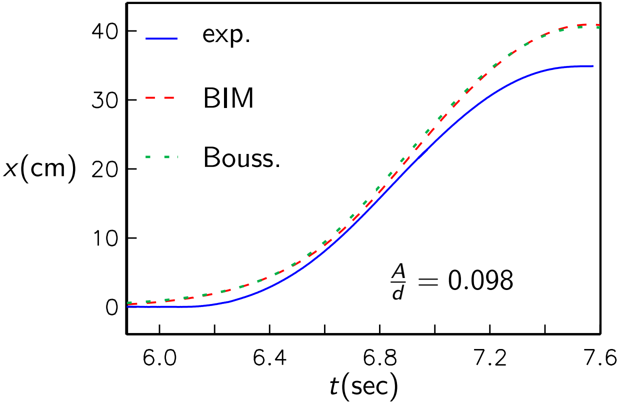
Programs freely available on WEB (Funwave and Coulwave).

Models employed

- Full inviscid theory. Boundary integral method with special design for runup. **BIM**
- Weakly dispersive and fully nonlinear inviscid theory; Boussinesq equations with Lagrangian grid. **Bouss.**
- LSW in constant depth region combined with NLSW on slope **(N)LSW**
- Analytical solution of Synolakis (1987) **An.**

Measured runup digitized from video.

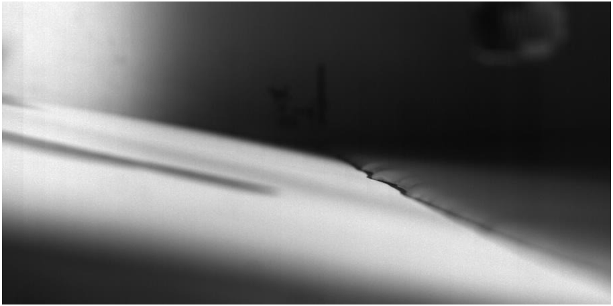
Inundation; small amplitude ($A/d \sim 0.1$)



- Early delay in exp. shoreline $\Rightarrow$ surface tension effect
- Then experiments nearly catches up
- Last stage: experiment steadily lags behind

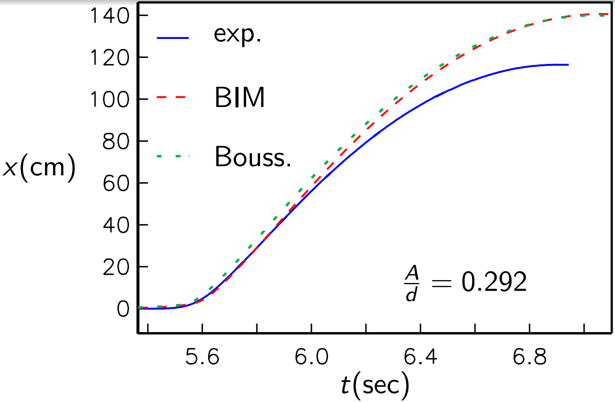
Experimental shoreline

$A/d = 0.0985$; before shoreline starts to move



Steep front and shadow effect visible in focused region
Small scale non-uniformity in lateral direction

Inundation; larger amplitude ($A/d \sim 0.3$)



- Early discrepancy relatively reduced
- Deviations develop at increasing rate during runup
- Max. runup eached earlier in experiment
- Boussinesq earlier than BIM; similar maximum

Reynolds numbers (Re), comparison with Blasius flow

Blasius profiles; uniform flow along flat plate

L travel distance(from leading edge), U free-stream velocity

$$Re_B = UL/\nu.$$

$Re \gtrsim 5 \cdot 10^4$: instability, $Re \gtrsim 3 \cdot 10^6$: fully turbulent
Accelerating flow more stable
Retarding flow more unstable

Solitary waves on constant depth and runup

Naive "wavetank" Reynolds number: $Re_d = \frac{\sqrt{gd}d}{\nu}$.

Using max. velocity and particle travel distance and $\alpha = A/d$

Constant depth	$U = \alpha\sqrt{gd}$,	$L = \frac{4}{\sqrt{3}}A\alpha$,	$Re_c = \frac{4}{\sqrt{3}}\alpha^{\frac{3}{2}}Re_d$
Runup	$U = \sqrt{gR}$,	$L = \frac{R}{\sin\theta}$,	$Re_R = (\frac{R}{A})^{\frac{3}{2}}\frac{\alpha^{\frac{3}{2}}}{\sin\theta}Re_d$

Synolakis' formula then used for R/A.

Reynolds numbers for wavetank

Depth $d = 0.2\,\mathrm{m}$, slope $\theta = 10^\circ$, $Re_d = 2.8 \cdot 10^5$

$\alpha = A/d$	0.1	0.2	0.3	0.5
Re_c (constant depth)	$2.0 \cdot 10^4$	$5.8 \cdot 10^4$	$1.1 \cdot 10^5$	$2.3 \cdot 10^5$
Re_R (runup)	$3.8 \cdot 10^5$	$1.4 \cdot 10^6$	$3.0 \cdot 10^6$	$7.7 \cdot 10^6$

Comparison with Blasius; observations

($Re \gtrsim 5 \cdot 10^4$: instability, $Re \gtrsim 3 \cdot 10^6$: fully turbulent)

- Flat bottom numbers below or in lower part of transition range; laminar layers are measured by Liu & Orfila (2004)
- Runup: lowest amplitude in low transition range; higher amplitudes near turbulent range
  qualitative observations: $A/d \sim 0.1$ laminar boundary layers, $A/d \sim 0.3$ and higher switch to turbulens at some stage
- Comments: Re_c and Re_R defined somewhat high, retarded flow in most of the runup phase (destabilizes), rapid transient evolution may prevent instability

Accumulated dissipation and momentum loss

Dissipation and drag per length

$$\mathcal{D} = \int_0^\infty \mu \left(\frac{\partial u}{\partial z}\right)^2 dz, \quad \text{and} \quad \sigma = \mu \frac{\partial u}{\partial z}|_{z=0},$$

Integrated in wedge and "normalized" (t_a onshore)

$$R_d = \frac{\int_0^t \left[\int_{x(b,t)}^{x(0,t)} \mathcal{D}(\cos\theta)^{-1} dx\right] dt}{\frac{1}{2}\rho \tan\theta b^2 g \Delta R}, \quad \sigma_d = \frac{\int_{t_a}^t \left[\int_{x(b,t)}^{x(0,t)} \sigma(\cos\theta)^{-1} dx\right] dt}{\frac{1}{2}(t-t_a)\rho \tan\theta b^2 g \sin\theta}.$$

These are more uncertain (large contribution from vicinity of moving shoreline, less clear interpretations), but yield values consistent with the volume loss.

Dispersion relation for single harmonic mode

Mode

$$\eta = A\cos(kx - \omega t)$$

Full potential theory ($k = 2\pi/\lambda$, $\omega = ck$)

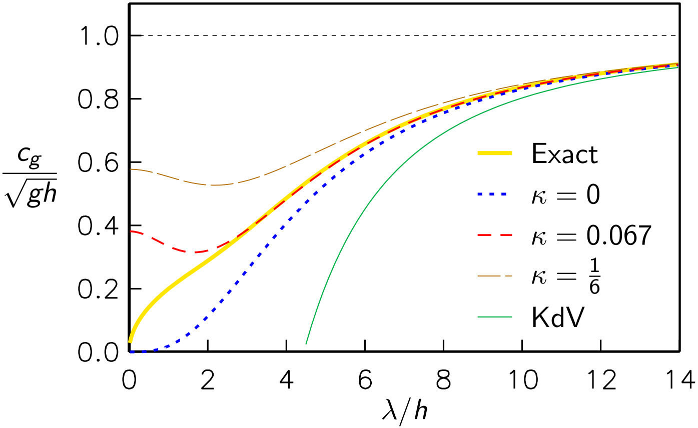
$$c^2 = \frac{g}{k}\tanh(kh) = gh\left(1 - \frac{1}{3}(kh)^2 + \frac{2}{15}(kh)^4 + \ldots\right)$$

Many Boussinesq models fulfill

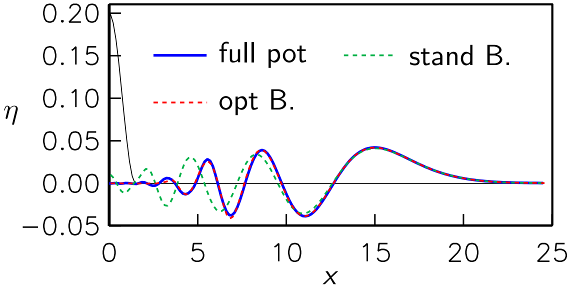
$$c^2 = \frac{gh(1+\kappa h^2k^2)}{1 + (\frac{1}{3}+\kappa)h^2k^2} = gh\left(1 - \frac{1}{3}(kh)^2 + \ldots\right),$$

Standard Boussinesq with averaged velocity	$\kappa = 0$
Optimized GloBouss ($\gamma = -0.057$)	$\kappa = 0.067$
Optimized FUNWAVE/COULWAVE ($z_\alpha = -0.531h$)	$\kappa = 0.067$
Generalized model ($z_\alpha = -h$)	$\kappa = \frac{1}{6}$

Dispersion properties



$\kappa = \frac{1}{6} \rightarrow u$ at bottom, $\kappa = 0 \rightarrow$ averaged u,
$\kappa = 0.067 \rightarrow$ optimal choice
$c_g = \mathrm{d}\omega/\mathrm{d}k$ – group velocity



Evolution from short initial elevation

Front: Good agreement for all Boussinesq formulations

Rear: Improved model superior, standard B. too dispersive

Observe: No corresponding improvement for steep bottom gradients

Artificial dispersion FDM/FEM solutions

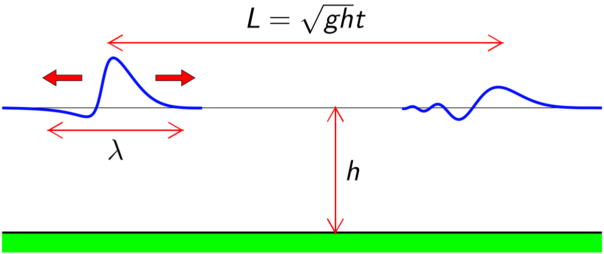
One horizontal dimension for simplicity
Finite Δx and Δt $\Rightarrow$ artificial dispersion.
Numerical solutions converge as $\Delta x, \Delta t \to 0$
Plane waves, second order method: $(h = 1)$

$$c = 1 + \left(\frac{\mu^2}{6} + \kappa \Delta x^2 + \gamma \Delta t^2 \right) k^2 + k^2 O(k^2, \Delta x^2, \Delta t^2),$$

Method in GloBouss: $\kappa = -\gamma = \frac{1}{24}$
Leap frog $\kappa = -\gamma = \frac{1}{6}$
$O(k^2 \Delta x^2), k^2 \Delta t^2$ errors often removed by h.o. methods or correction terms

A FDM/FEM method defines a discrete medium with optical properties of its own

Dispersion effects in tsunami propagation



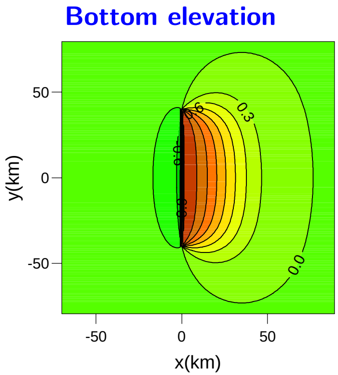
Dispersion parameter ($T = \lambda/c_0$, $c_0 = \sqrt{gh}$)

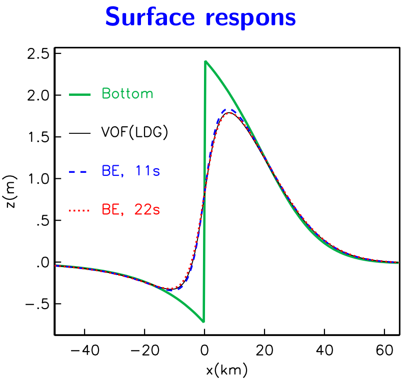
$$\tau = \frac{c_0 h^2}{\lambda^2} \cdot t \cdot \frac{1}{\lambda} = \frac{h^2 L}{\lambda^3} = \frac{h^2 L}{\lambda^3} = \frac{ht}{gT^3}$$

Even very weak dispersion important for "trans-ocean" tsunamis

Strong sensitivity with respect to wavelength

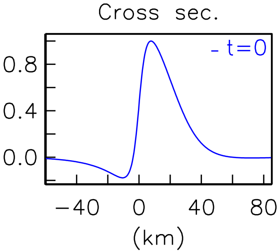
Choice of λ ambiguous $\Rightarrow$ comparisons difficult





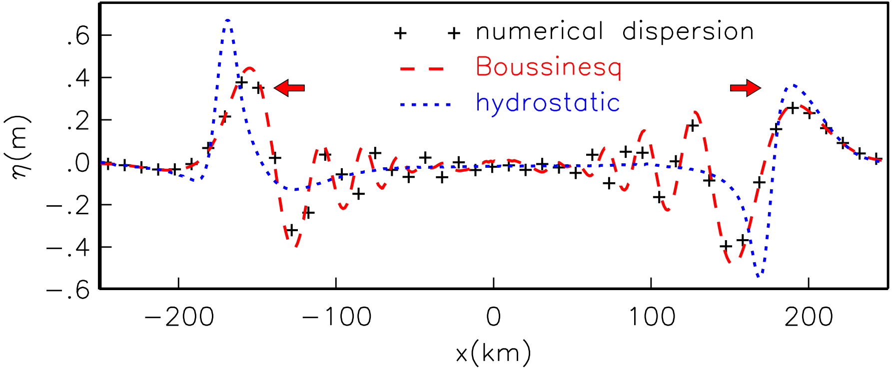
Magnitude: $M_s = 7.9$, $h = 5000\,\mathrm{m}$, inverse thrust fault, large dip angle $\approx 50^\circ$, fault length $\approx 70\,\mathrm{km}$ $\Rightarrow$ rather confined bottom uplift
Co-seismic uplift from Okada's formula
VOF, BE in right panel different models

Dispersion (Portugal 1969)



Dip. $\approx 50^\circ$, $H = 5\,\mathrm{km}$
$t = 13\,\mathrm{min}$, $\tau = 3.5 \cdot 10^{-2}$
Marked dispersion already
Portugal (1969)

Numerical dispersion; Portugal (1969)



Cross section of 2HD simulation (1969 tsunami) in 5 km depth.
Dashes: Converged Boussinesq and Shallow water eq.
Marks: coarse grid ($\Delta x = 10.6\,\mathrm{km}$) shallow water simulation with numerical dispersion that mimic the true one.