UNITO

ROGUE WAVES AND MODULATIONAL INSTABILITY

1) TRIGGERING BREATHERS IN AN OPPOSING CURRENT
2) CROSSING SEAS: THE FORECASTING OF THE LOUIS MAJESTY ACCIDENT

M. Onorato

Università di Torino – Dip. di Fisica Generale – Italia

Collaborators: D. Proment, A. Toffoli, L. Cavaleri

MECHANISM OF FORMATION OF ROGUE WAVES:

- **LINEAR SUPERPOSITION**
- **MODULATIONAL INSTABILITY**
- **INTERACTION WITH CURRENTS/TOPOGRAPHY**
- **CROSSING SEAS**

Taken aboard the SS Spray (ex-Gulf Spray) in about February of 1986 (best recollection), in the Gulf Stream, off of Charleston. Circumstances: A substantial gale was moving across Long Island, sending a very long swell down our way, meeting the Gulf Stream. We saw several rogue waves during the late morning on the horizon, but thought they were whales jumping. It was actually a nice day with light breezes and no significant sea. Only the very long swell, of about **15 feet** high and probably 600 to 1000 feet long. This one hit us at the change of the watch at about noon. The photographer was an engineer (name forgotten), and this was the last photo on his roll of film. We were on the wing of the bridge, with a height of eye **of 56 feet**, and this wave broke over our heads. This shot was taken as we were diving down off the face of the second of a set of three waves, so the ship just kept falling into the trough, which just kept opening up under us. It bent the foremast (shown) back about 20 degrees, tore the foreword firefighting station (also shown) off the deck (rails, monitor, platform and all) and threw it against the face of the house.

WAVE-CURRENT INTERACTION AND FORMATION OF FREAK WAVES

Lavrenov, *Nat. Haz.*, 1998

White and Fornberg, *JFM,* 1998

Heller, Kaplan and Dahlen, *JGR*, 2008

Hjelmervik and Trulsen, *JFM,* 2009

BREATHERS: EXACT SOLUTION OF THE NLS

N. Akhmediev,et al. (1987) E. Kuznetsov, (1977) - Y. Ma, (1979)

$$A(x,t) = A_0 \exp[-i\epsilon^2 k_0 x] \times$$
$$\times \left[\frac{\rho^2 \cosh(\chi x) - i\gamma \sinh(\chi x)}{\cosh(\chi x) - \sqrt{1-\rho^2/2}\cos(\omega_0 t/N)} - 1 \right]$$

with

$$\rho = \frac{1}{N\epsilon}, \qquad \gamma = \frac{\sqrt{2}}{\epsilon N}\sqrt{1 - \left(\frac{1}{\sqrt{2}\epsilon N}\right)^2}, \qquad \chi = \gamma k_0 \epsilon^2.$$

Two remarks:

1) $A(x \to -\infty, t) = A_0 \exp(i\phi)(1 + \delta \cos(\omega_0 t/N)),$

2) The solution depends on steepness and N

MAXIMUM AMPLITUDE

$$\frac{A_{max}}{A_0} = 1 + 2\sqrt{1 - \left(\frac{1}{\sqrt{2}\varepsilon N}\right)^2}.$$

1) It depends on the product εN

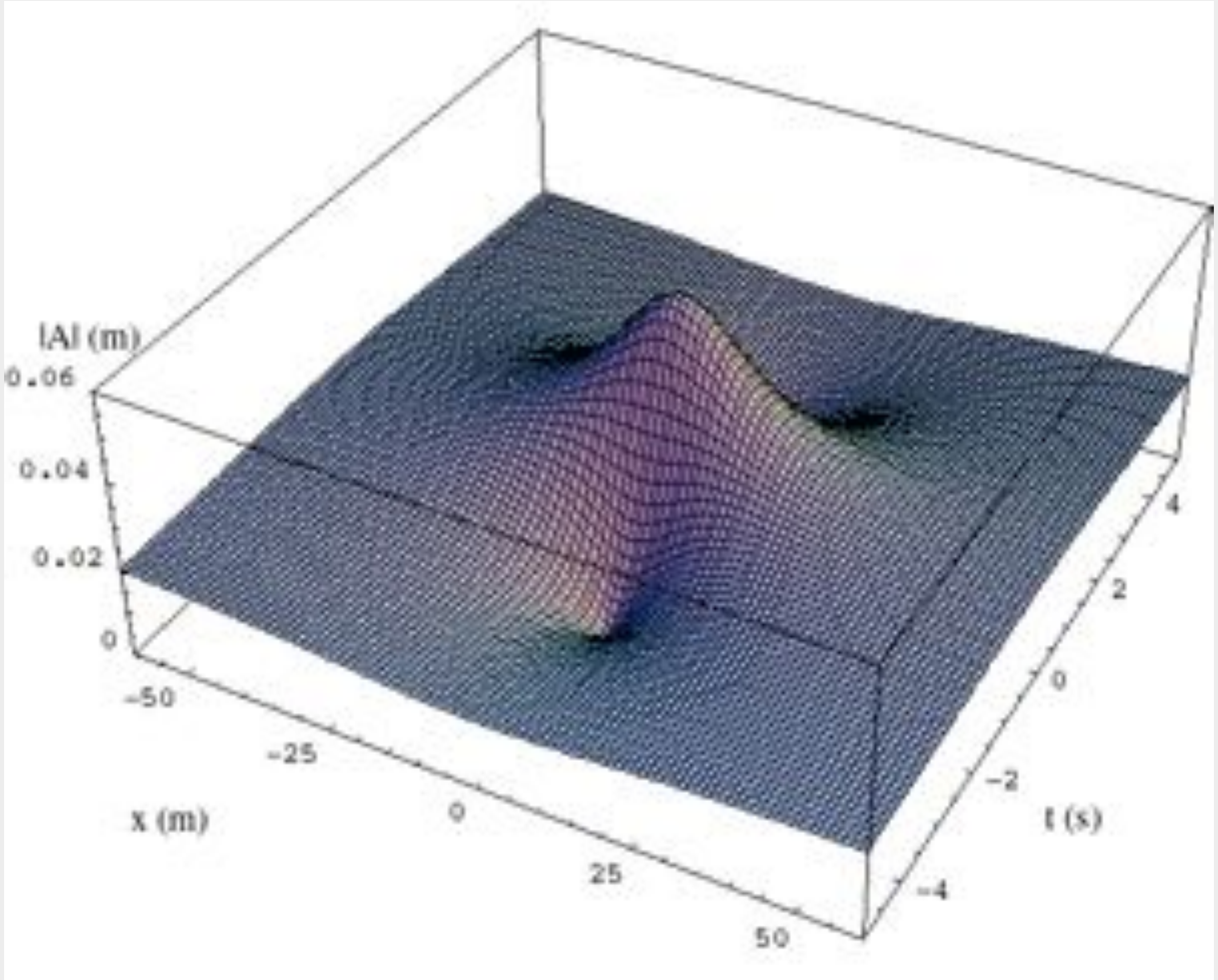
2) Maximum amplitude is 3 -> The Peregrine solution

Such solutions have been tested experimentally in a number of wave tank and fully nonlinear computations

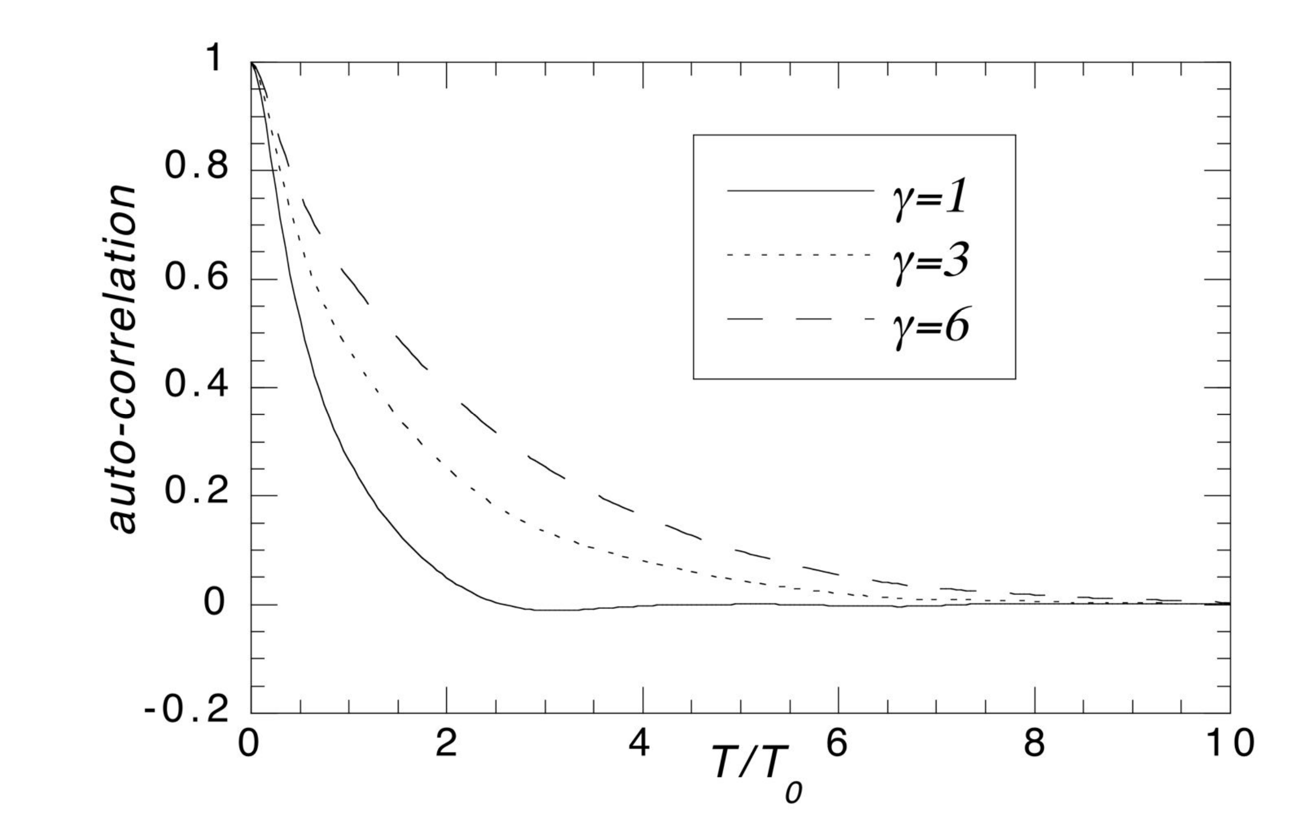
THE PEREGRINE SOLUTION

A_0=0.02 m

ε=0.1



OCEAN WAVES ARE CHARACTERIZED BY JONSWAP SPECTRUM



Example: N=7, ε= 0.1 – Wave group is stable

➔BREATHERS ARE RARE OBJECTS

THE MODEL

Hjelmervik and Trulsen, *JFM,* 2009

Assumptions:

$$\varepsilon = k_0 A_0 << 1$$

$$U / c = O(\varepsilon)$$

$$W / c = O(\varepsilon^2)$$

$$1/(k_0 \Lambda) = O(\varepsilon)$$

$$\left[\frac{\partial A}{\partial x} + \frac{1}{c_g}\left(1 - \frac{3}{2}\frac{U}{c_g}\right)\frac{\partial A}{\partial t}\right] + i\frac{k_0}{\sigma_0^2}\frac{\partial^2 A}{\partial t^2} + ik_0^3|A|^2 A = \\ = -\frac{1}{2c_g}\frac{dU}{dx}A - ik_0\frac{U}{c_g}\left(1 - \frac{5}{4}\frac{U}{c_g}\right)A \tag{2}$$

$$\eta = \frac{1}{2}\left(A \exp[i(k_0 x - \sigma_0 t)] + c.c.\right)$$

USE THE FOLLOWING TRANSFORMATION

$$A = B \exp\left[-\int_0^x ik_0 \frac{U}{c_g}\left(1 - \frac{5}{4}\frac{U}{c_g}\right) - \frac{1}{2c_g}\frac{dU}{dX} dX\right]$$

$$x' = x, \qquad t' = t - \int_0^x \frac{1}{V(X)} dX$$

To obtain

$$\boxed{\frac{\partial B}{\partial x} + i\alpha \frac{\partial^2 B}{\partial t^2} + i\beta(x)|B|^2 B = 0}$$

$$\frac{1}{V(x)} = \frac{1}{c_g}\left(1 - \frac{3}{2}\frac{U}{c_g}\right), \quad \boxed{\beta(x) = k_0^3 exp\left[-\frac{\Delta U}{c_g}\right]}$$

$$E(x) = \int |A|^2 dt = \exp\left[-\frac{\Delta U}{c_g}\right] \int |B|^2 dt.$$

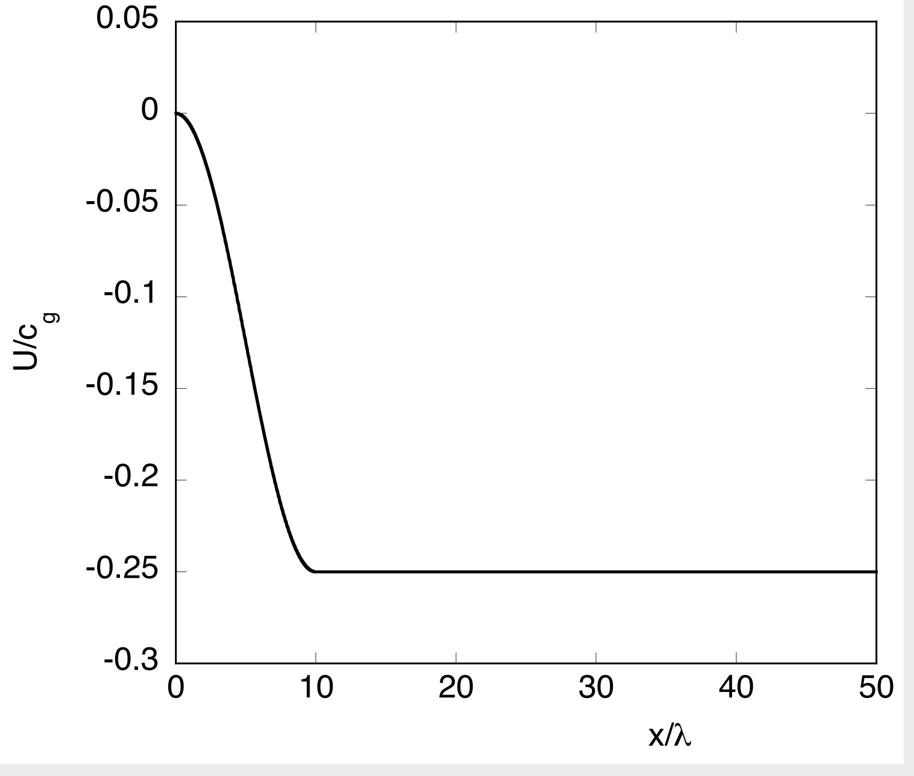
Remark:

The non linear coefficient depends on space

SPECIFICATION OF THE OPPOSING CURRENT

$$U(x) = \begin{cases} 0 & \text{if } x < x_0 \\ U_0 \sin^2\left[\frac{\pi}{2\Lambda}(x - x_0)\right] & \text{if } x_0 \leq x < x_0 + \Lambda \\ U_0 & \text{if } x \geq x_0 + \Lambda \end{cases}$$

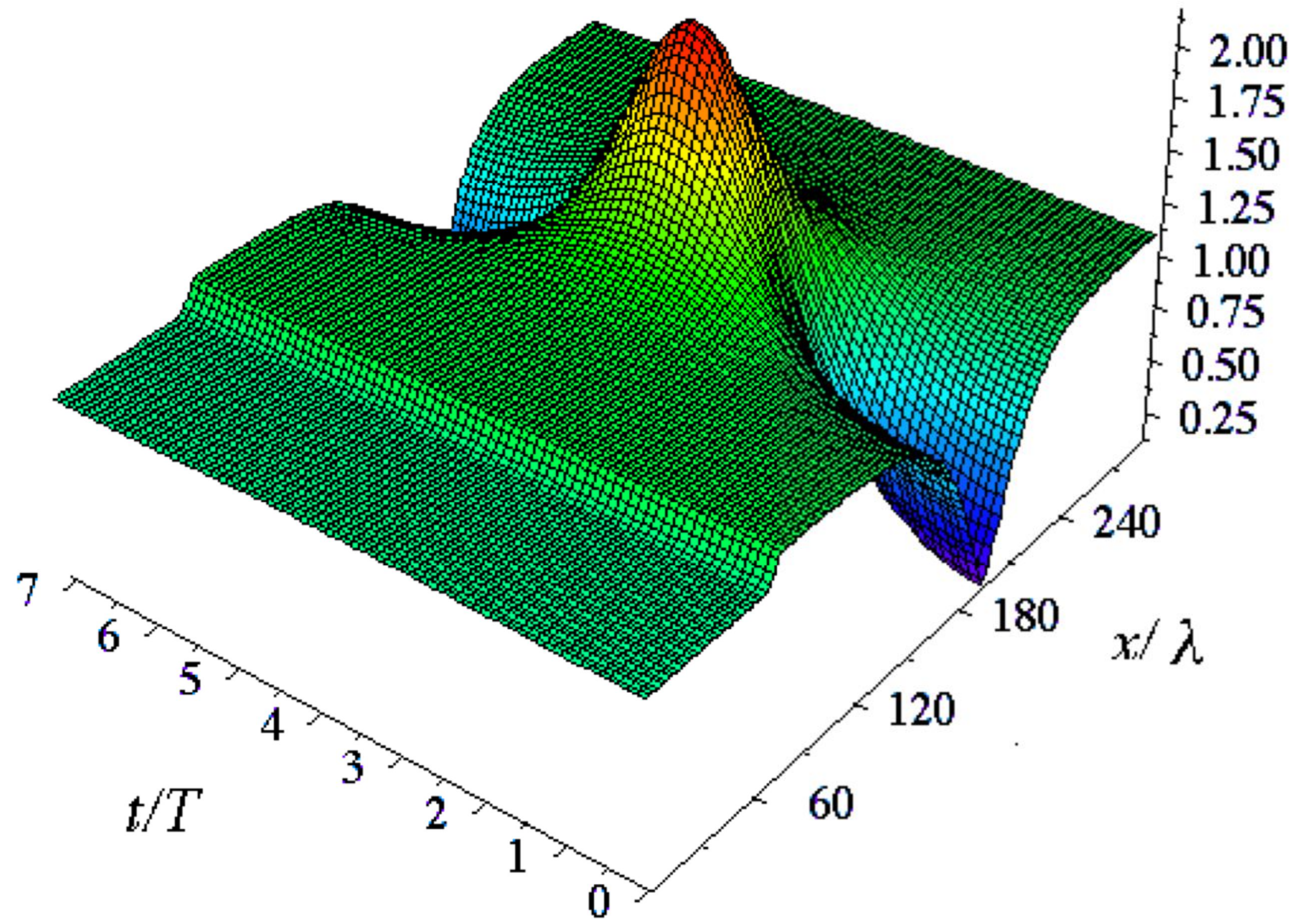
Example



TRIGGERING A BREATHER

Initial conditions:

plane wave perturbed: N=7, $\varepsilon=0.1$, U_0/c_g=-0.2

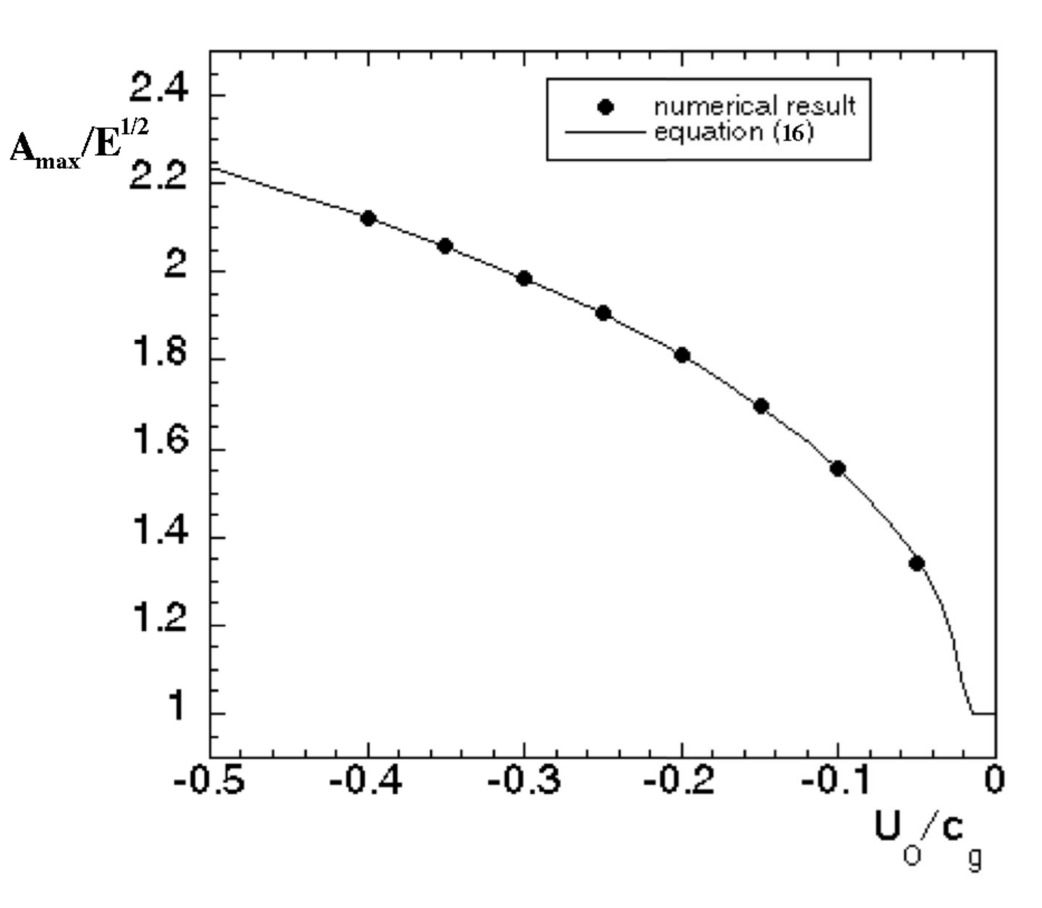


MAXIMUM AMPLITUDE

$$\frac{A_{max}}{A_0} = 1 + 2\sqrt{1 - \left(\frac{1}{\sqrt{2}\varepsilon N}\right)^2}.$$

Heuristic prediction

$$\frac{A_{max}}{\sqrt{E(x_0+\Lambda)}} = 1 + 2\sqrt{1 - \left(\frac{\exp[U_0/(2c_g)]}{\sqrt{2}\varepsilon N}\right)^2}.$$



RANDOM SEA STATE

- Spectrum for the envelope

$$P(\omega) = \frac{E}{\Delta\Omega\sqrt{2\pi}} \exp\left[-\frac{\omega^2}{2\Delta\Omega^2}\right]$$

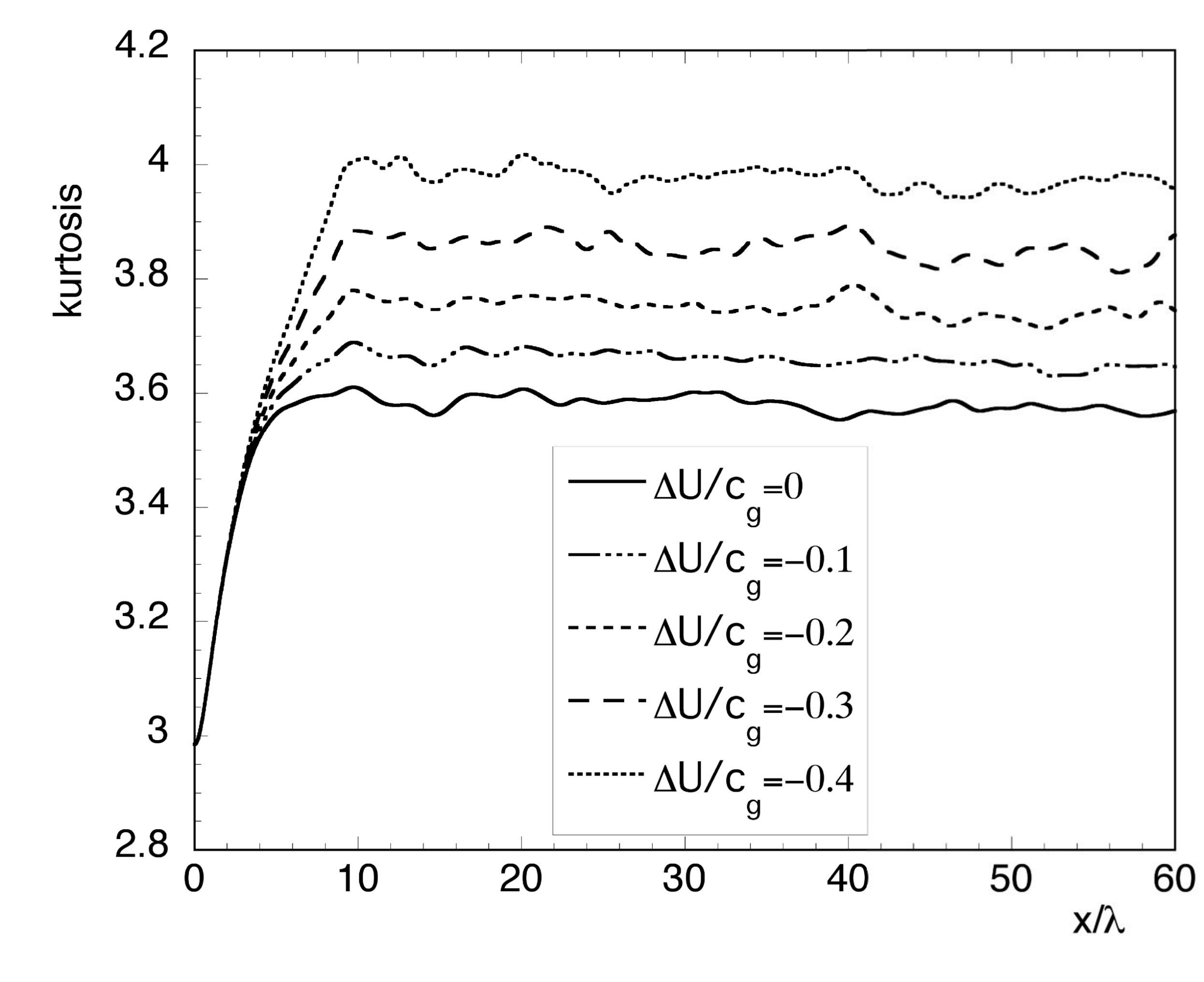
- Random phases

- ε=0.15, $\Delta\Omega/\sigma_0$=0.2

- -0.4≤U_0/c_g≤0

- 600 realizations for each configuration ➔ stable statistics

EVOLUTION OF THE KURTOSIS



PREDICTION OF THE KURTOSIS

$$\kappa = \frac{<\eta^4>}{<\eta^2>^2} = \frac{3}{2}\frac{<|B|^4>}{<|B|^2>^2}$$

$$<|B|^4> = \int <\hat{B}_1^*\hat{B}_2^*\hat{B}_3\hat{B}_4 Exp[(\omega_1+\omega_2-\omega_3-\omega_4)t]> d\omega_{1,2,3,4}$$

$$<\hat{B}_1^*\hat{B}_2^*\hat{B}_3\hat{B}_4> = <\hat{B}_1^*\hat{B}_3><\hat{B}_2^*\hat{B}_4> + <\hat{B}_2^*\hat{B}_3><\hat{B}_1^*\hat{B}_4> + C_{1,2,3,4}$$

$C_{1,2,3,4}$ fourth order cummulant

Use NLS to find the evolution of $C_{1,2,3,4}$

Split the sixth order correlator as sum of second order correlators

Integrate in space, assuming the spectrum changes on a larger scale with respect to the resonance function

$$C_{1,2,3,4} = 2Exp\left[-\frac{\Delta U}{c_g}\right]\delta(\omega_1+\omega_2-\omega_3-\omega_4)G(\Delta k,x)(P_1P_2(P_3+P_4)-P_3P_4(P_1+P_2))$$

$$G(\Delta k,x) = i\int_0^x Exp(i\Delta k(X-x))dX = \frac{1-\cos(\Delta kx)}{\Delta k} + i\frac{\sin(\Delta kx)}{\Delta k}$$

$$\kappa = 3 + Exp\left[-\frac{\Delta U}{cg}\right]\int P_1P_2P_3\frac{1-\cos(\Delta kx)}{\Delta k}\delta(\omega_1+\omega_2-\omega_3-\omega_4)d\omega_{1,2,3,4}$$

For Gaussian shape spectrum and for large space the integral can be computed analytically

PREDICTION OF THE KURTOSIS

$$\kappa = \frac{<\eta^4>}{<\eta^2>^2} = \frac{3}{2}\frac{<|B|^4>}{<|B|^2>^2} \approx 3 + \frac{\pi}{\sqrt{3}} Exp\left[-\frac{U_0}{c_g}\right] BFI^2$$

$$BFI = \frac{\sqrt{E}\,k_0}{\Delta\Omega/\sigma_0}$$

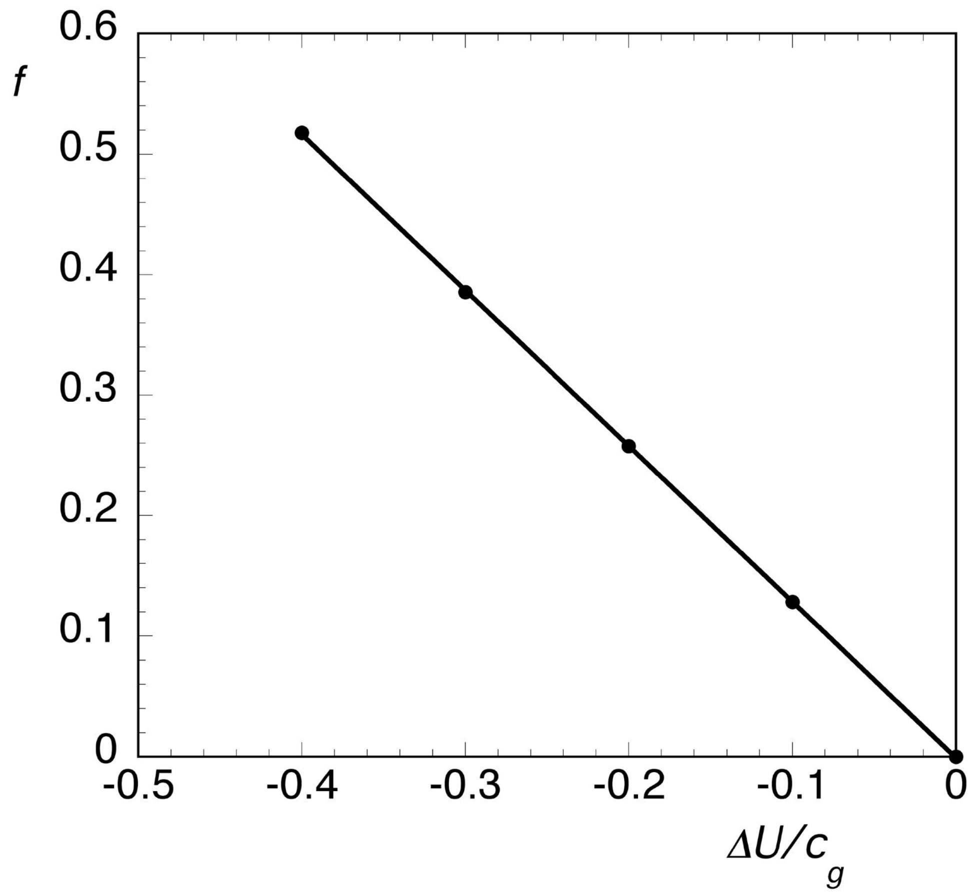
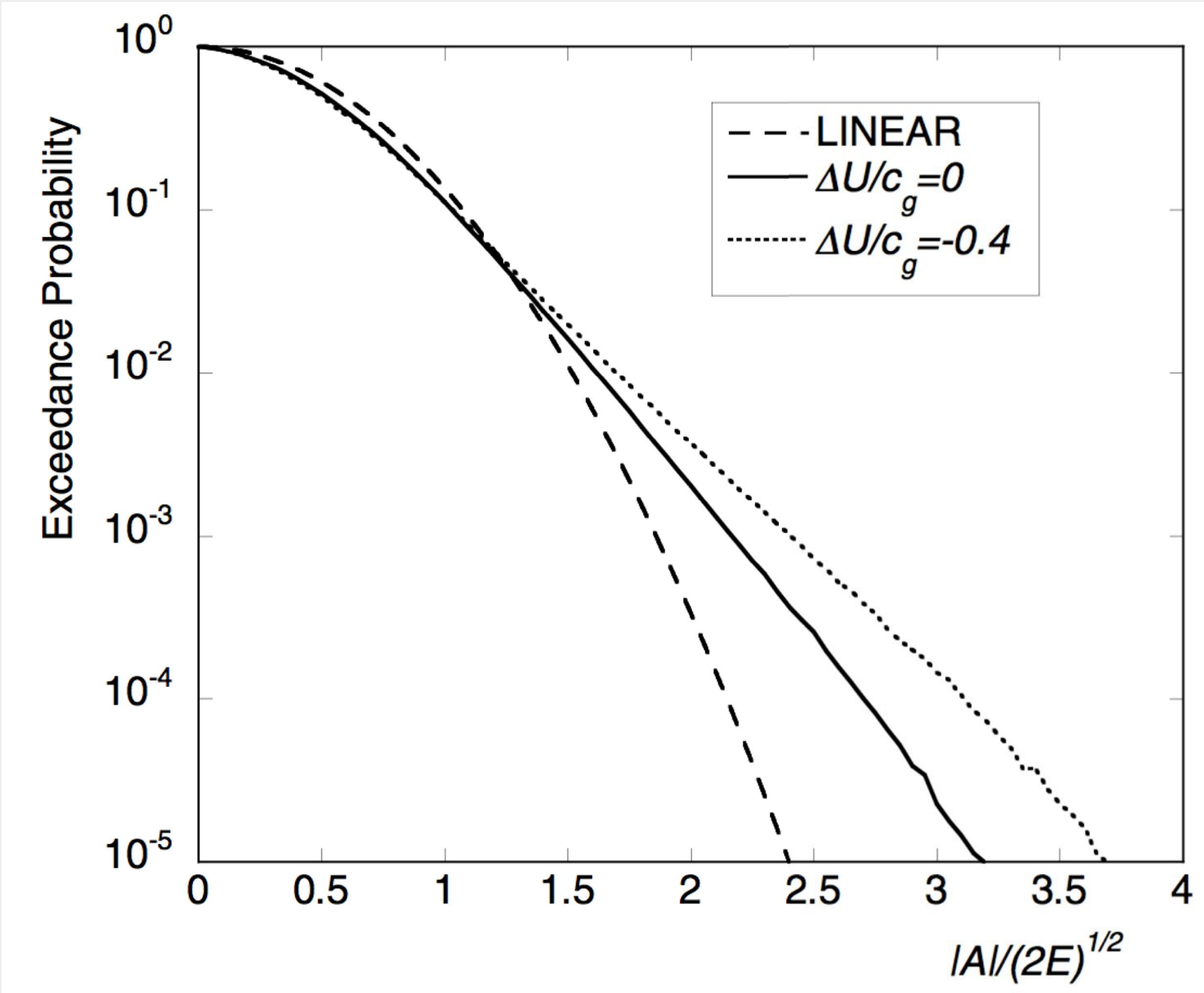


FIG. 4: $f(\Delta U/c_g) = \ln[(\kappa - 3)/(a\pi BFI^2/\sqrt{3})]$ as a function of $\Delta U/c_g$. Dots are the results from the numerical computation and the solid line is obtained by a linear fit.

PROBABILITY OF EXCEEDENCE



RESULTS

1) Opposing current can trigger breather solutions of the NLS Equation

2) A stable wave group can be destabilized once enters into a current: breathers can be triggered

3) The maximum amplitude of the breather is related to the ratio between the current and the group velocity

4) Breathers are also triggered in random waves

5) The statistics of random waves depends also on the ratio between the current and the group velocity

UNITO
Dipartimento di Fisica Generale

CROSSING SEAS: THE FORECASTING OF THE LOUIS MAJESTY ACCIDENT

Collaborators: Gigi Cavaleri and Luciana Bertotti
- CNR VENZIA -

THE ACCIDENT

On March 03, 2010 at 15:20 the Louis Majesty has been hit by a wave whose height was estimated to be 8 – 10 meters

The wave broke the glass windshields in the forward section on deck five

THE FORECASTING MODEL

The wave fields are the result of the forecasting of Nettuno model from the Italian National Meteorological Service

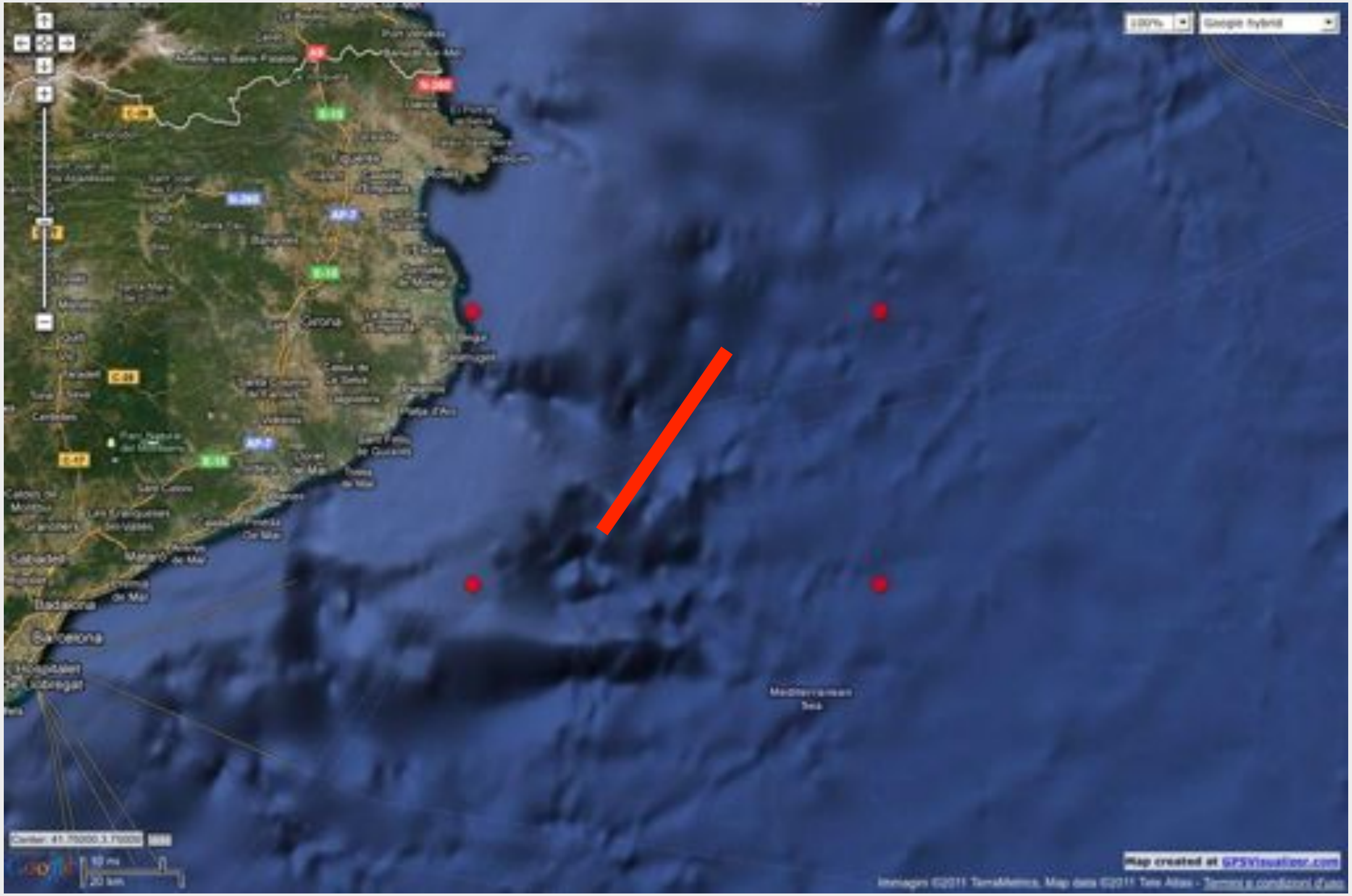
Resolution of the meteorological model: 7 km

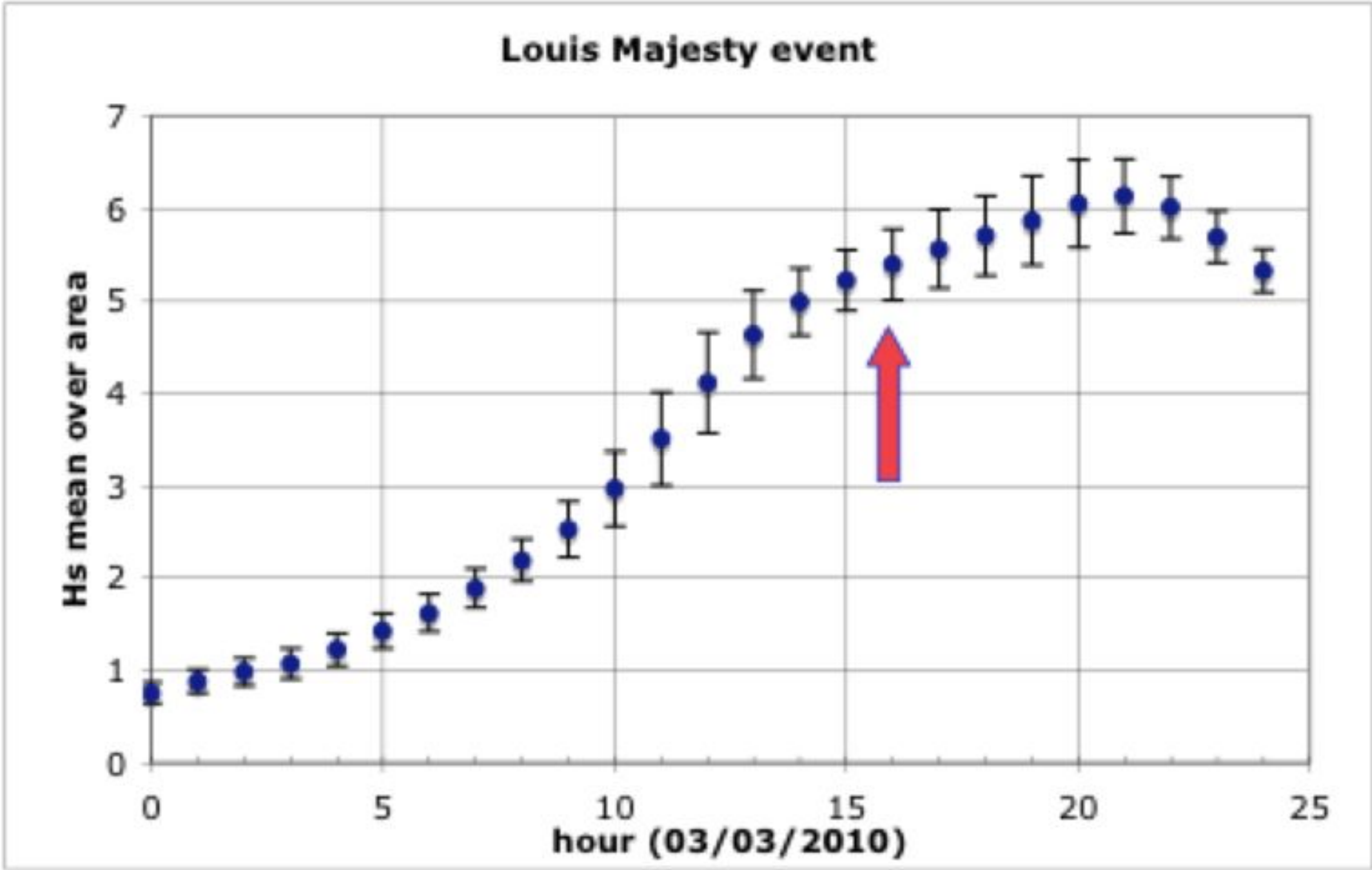
Resolution of the wave model in space (WAM): $1/20^0$

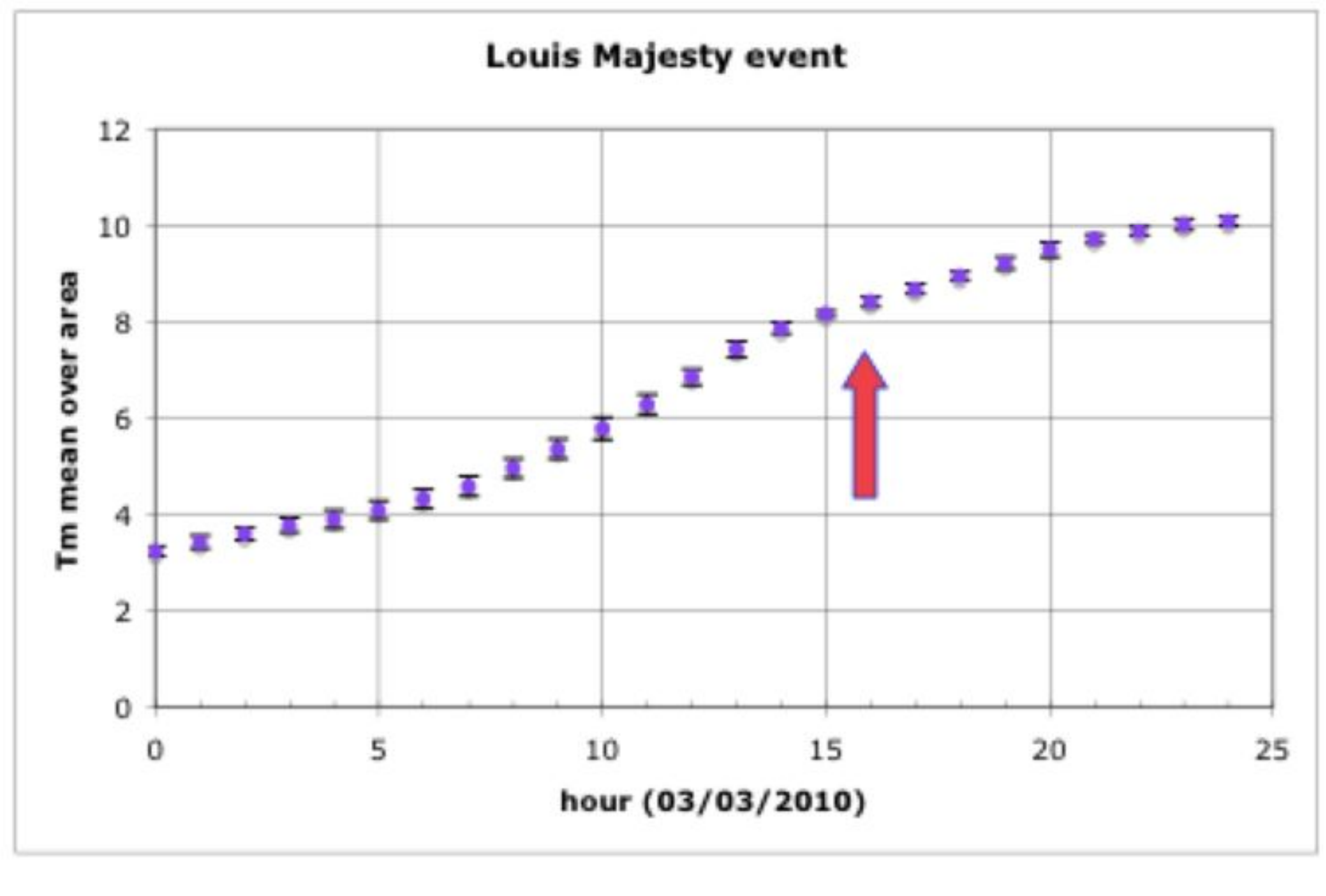
Spectral Resolution of the wave model:

number of frequencies: 30
number of directions: 36

REGION OF THE FORECAST
long 3 deg 15' - 4 deg 15'
lat 41 deg 30' - 42 deg 00'

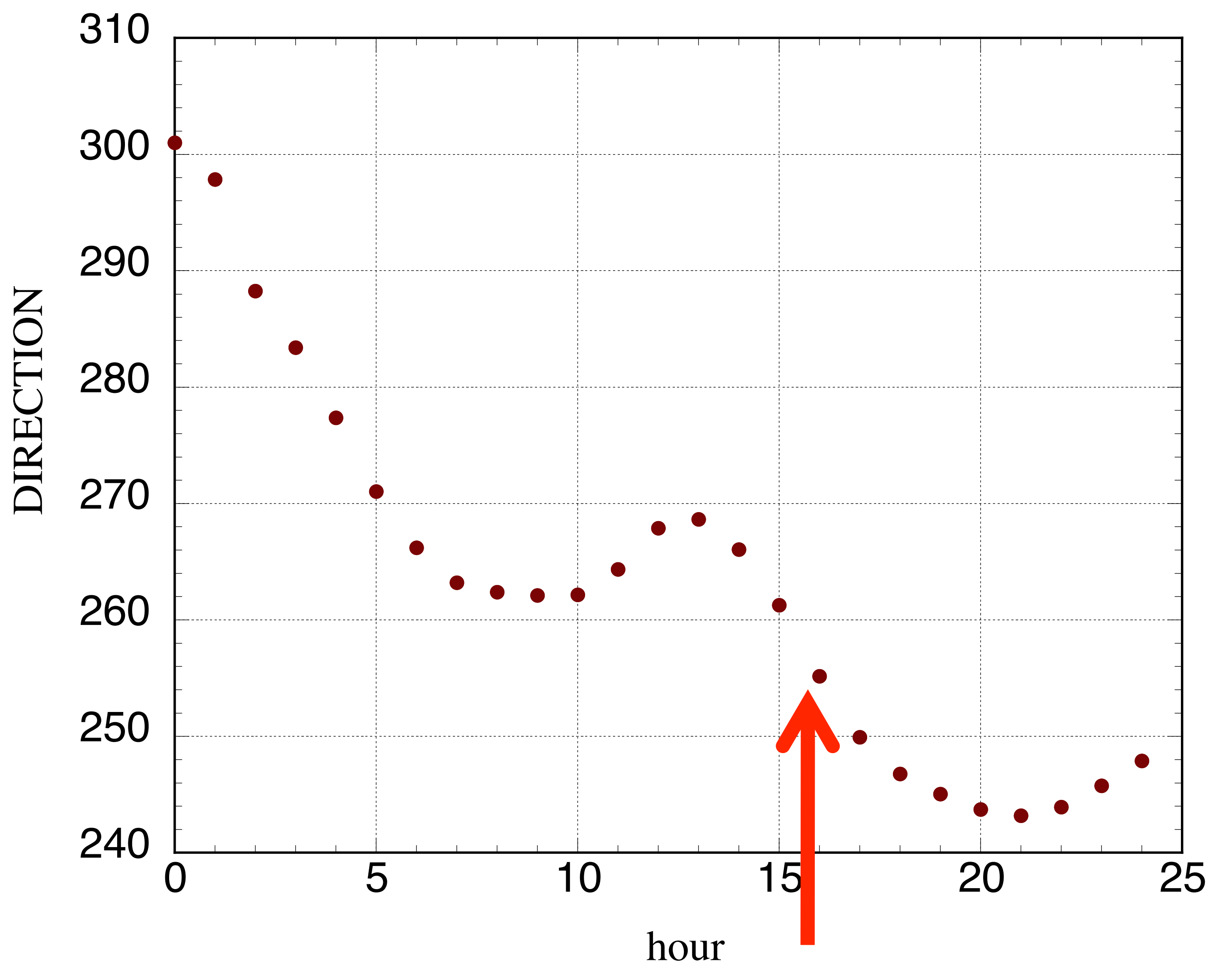



Louis Majesty event
Hs mean over area
hour (03/03/2010)
0
1
2
3
4
5
6
7
0
5
10
15
20
25

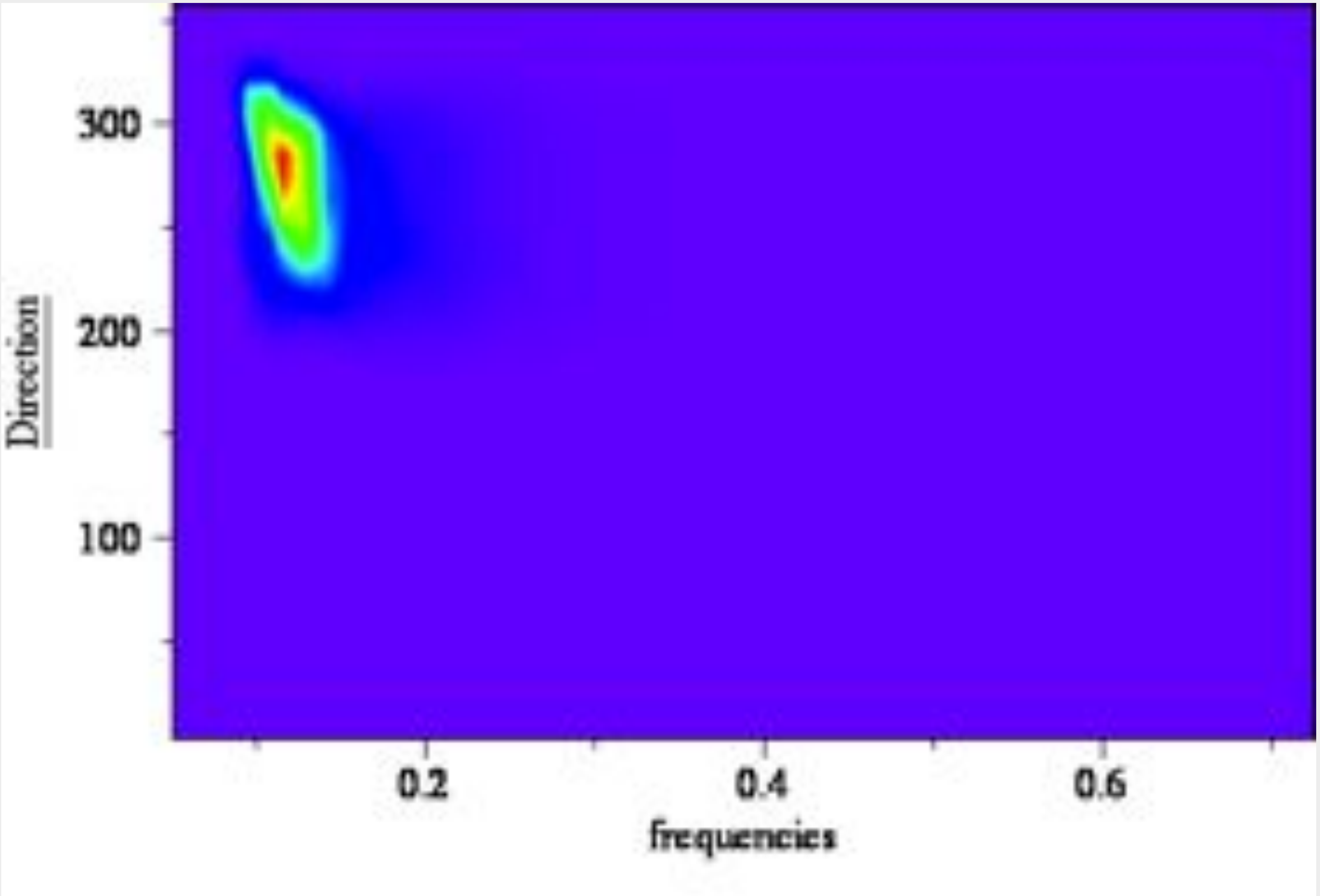


Wavelength at the moment of the accident: 100 m

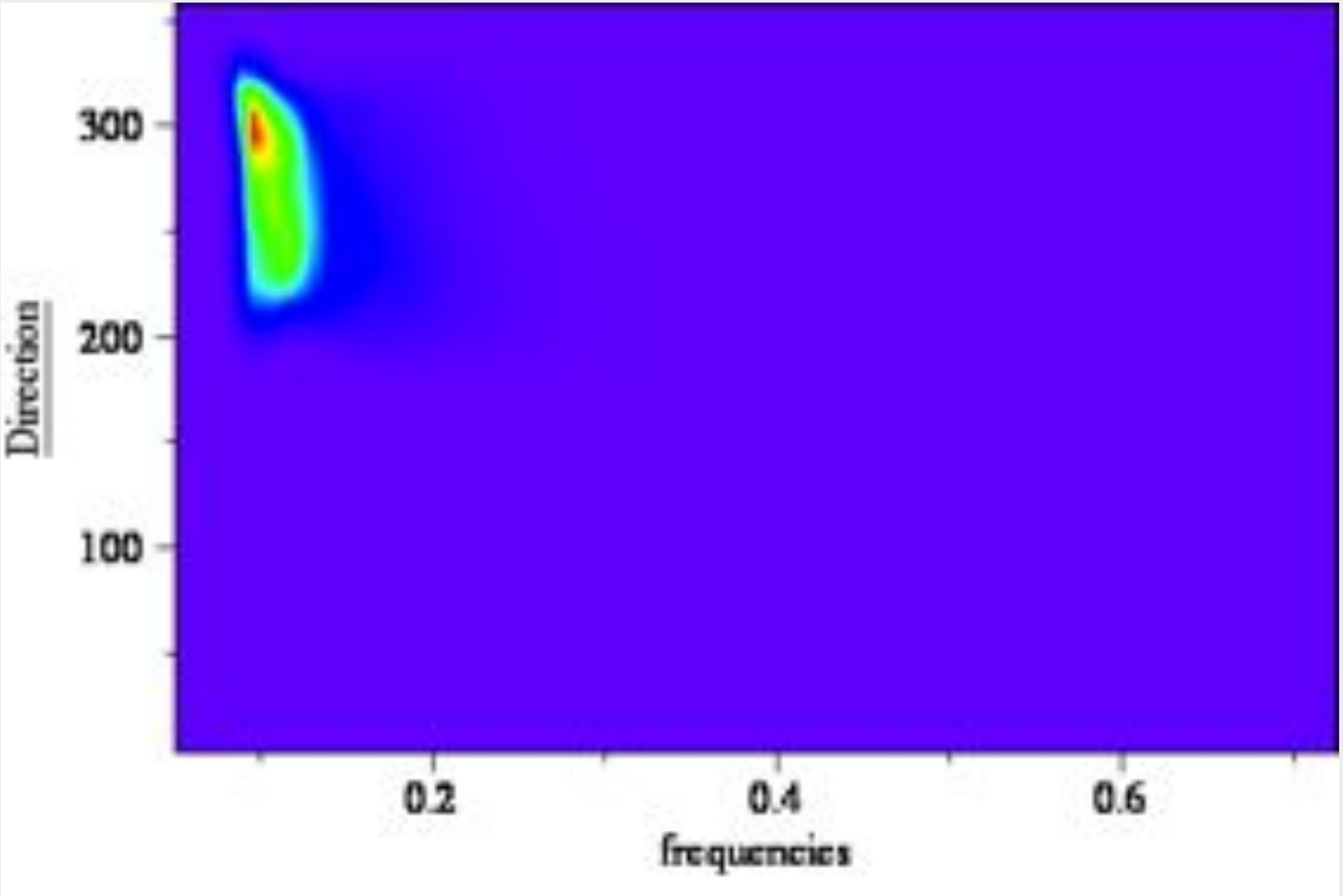
Length of the ship: 207 m


310
300
290
280
270
260
250
240
DIRECTION
0
5
10
15
20
25
hour

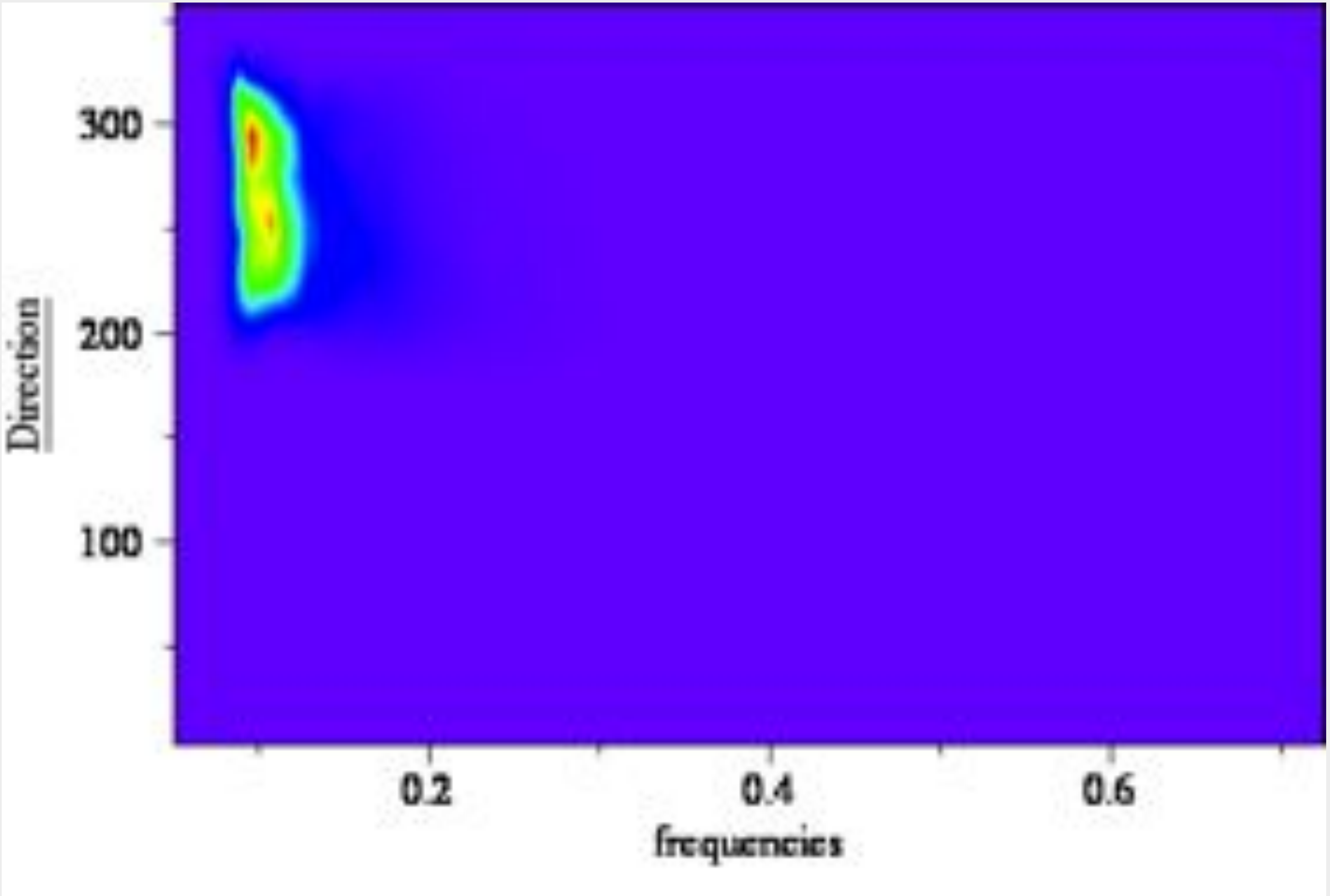
DIRECTIONAL SPECTRUM
Time 12:00



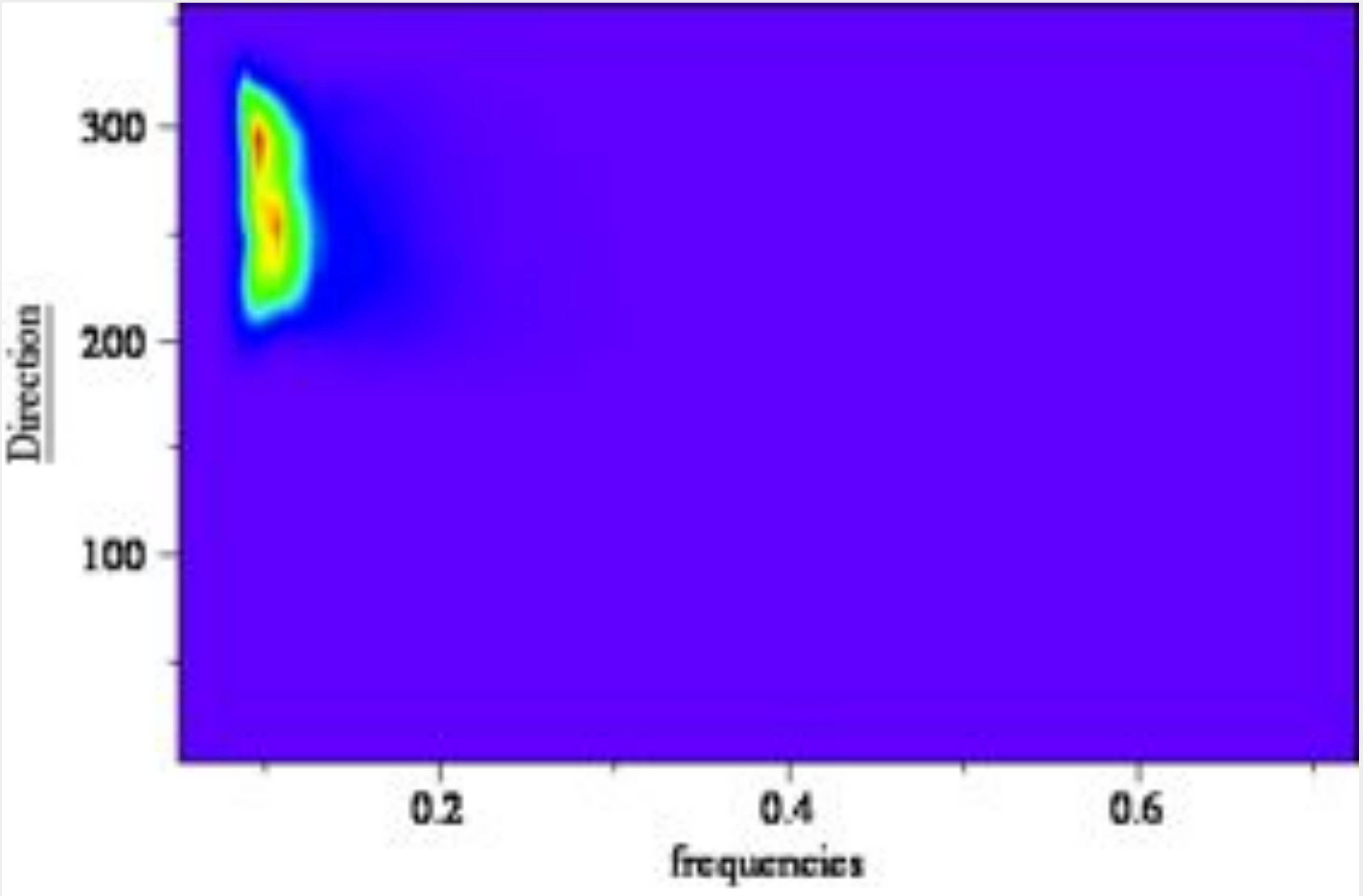
DIRECTIONAL SPECTRUM
Time 13:00



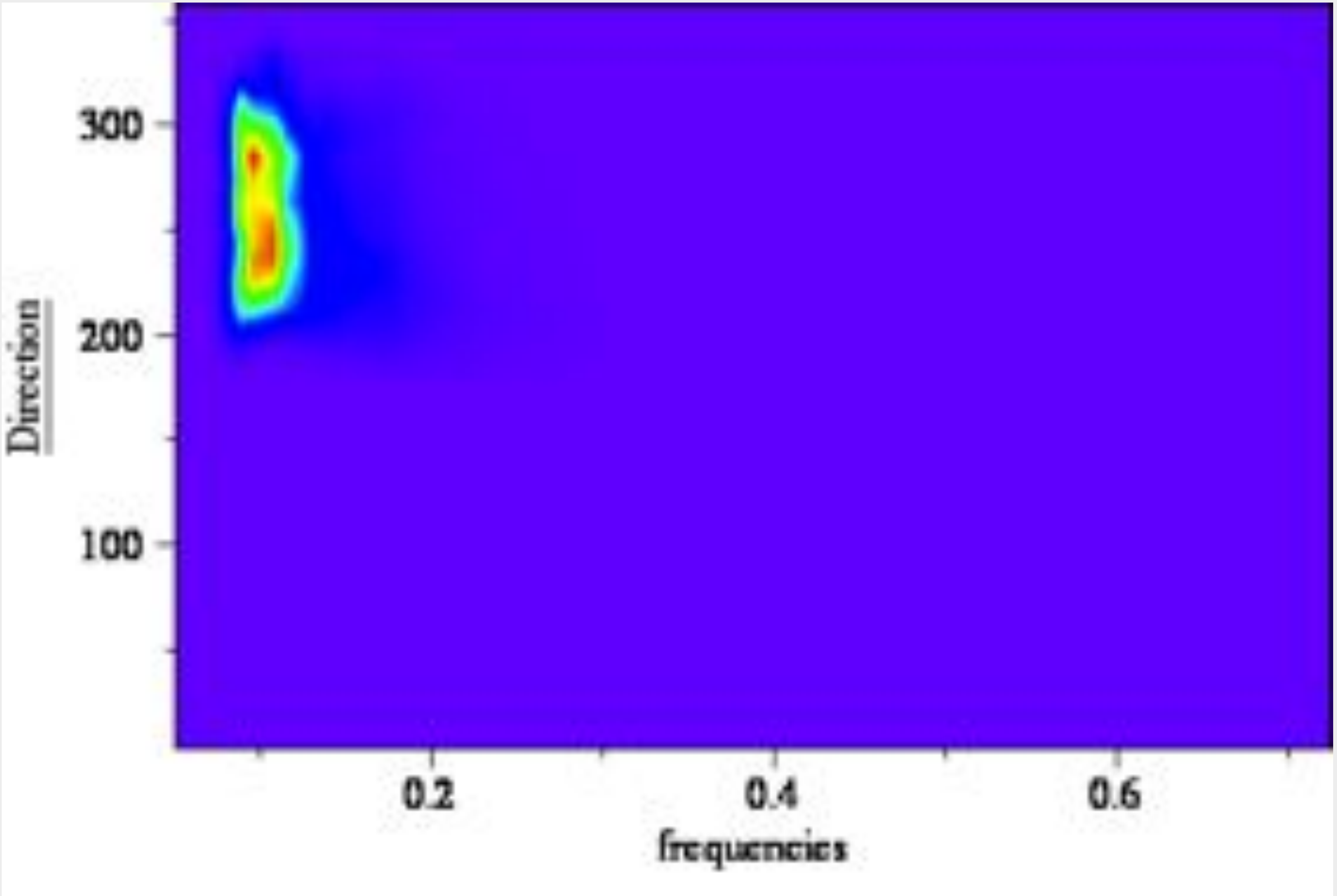
THE DIRECTIONAL SPECTRA
Time 14:00



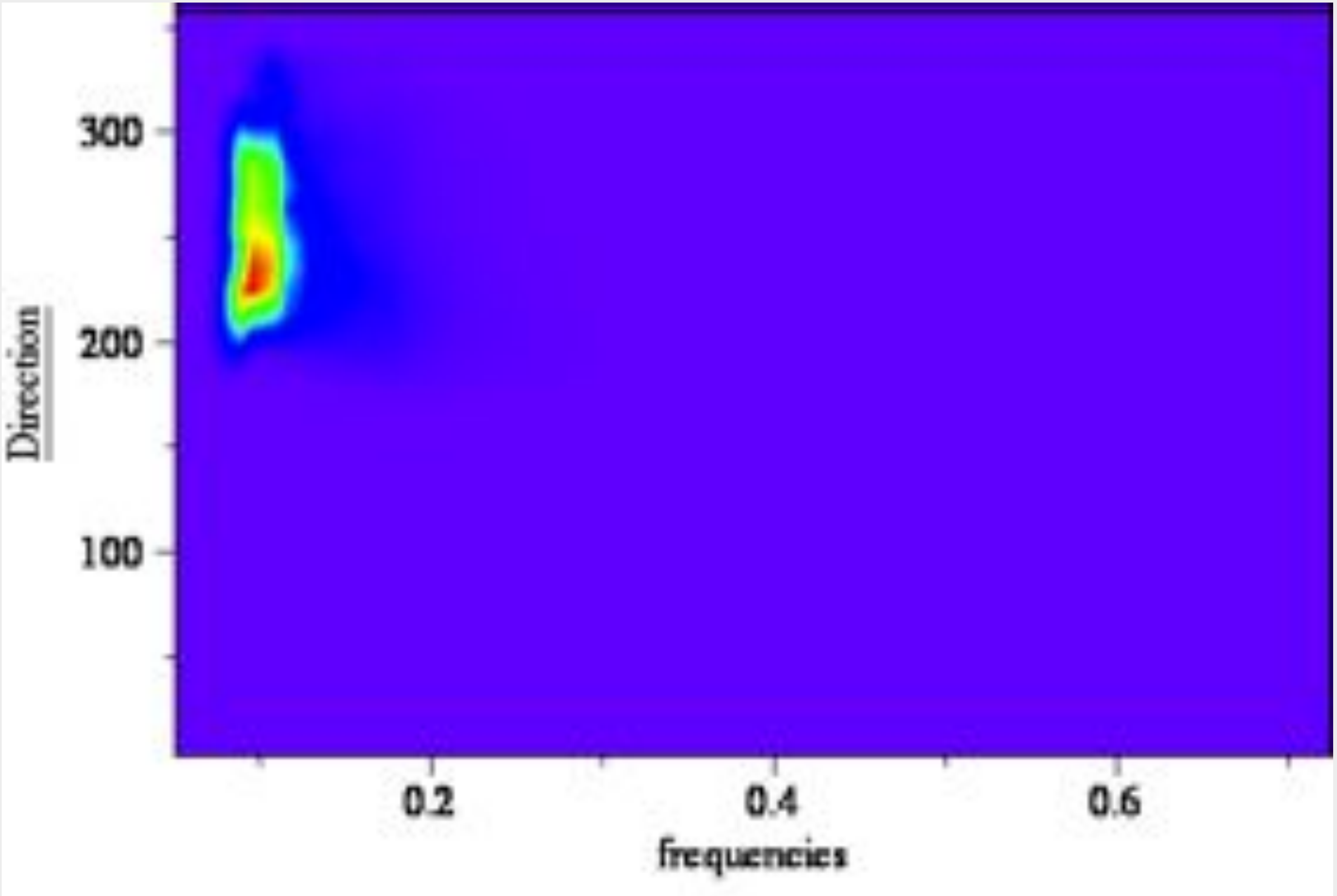
DIRECTIONAL SPECTRUM
Time 15:00



THE DIRECTIONAL SPECTRA
Time 16:00



THE DIRECTIONAL SPECTRUM
Time 17:00

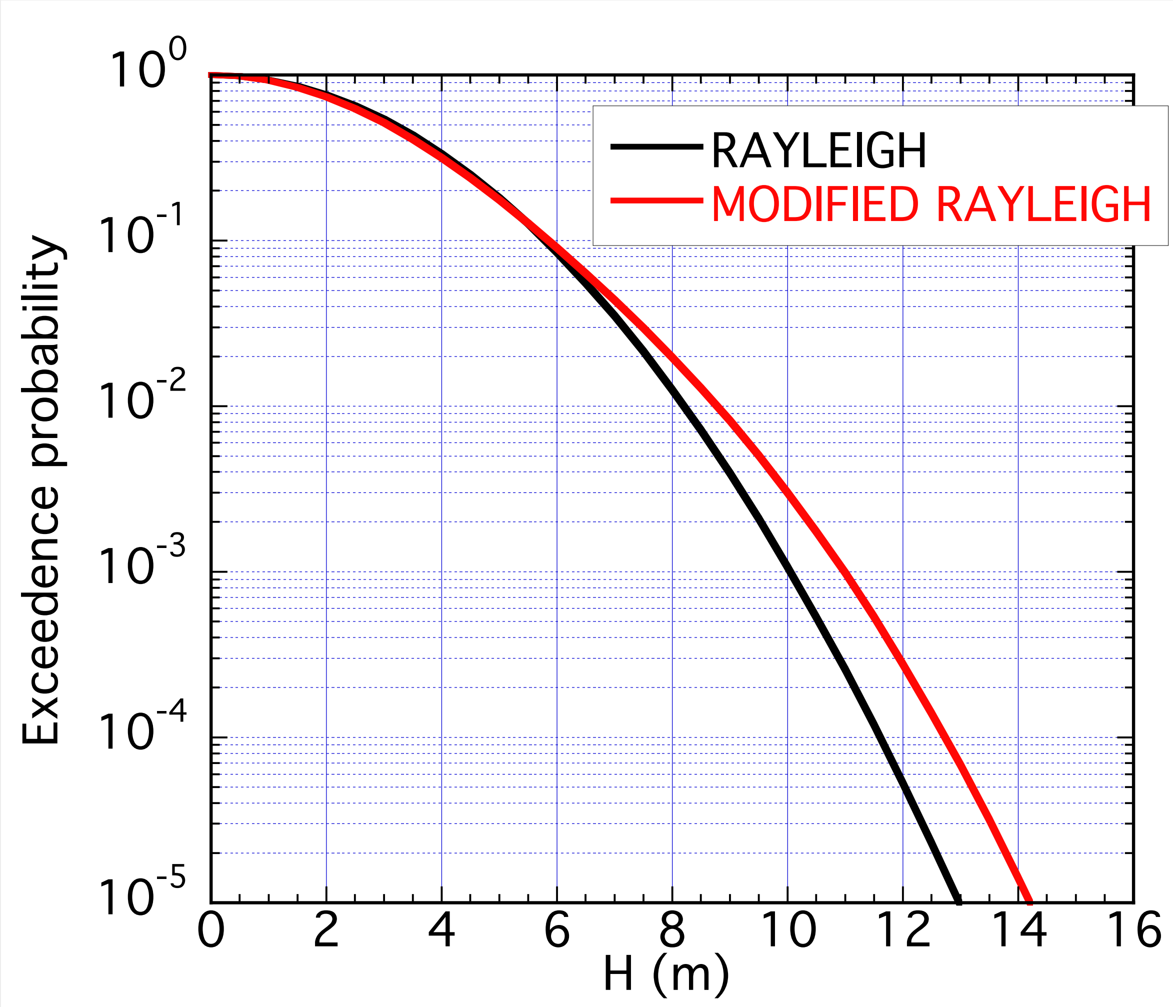


PROBABILITY OF EXCEEDENCE AT THE TIME OF THE ACCIDENT

Linear case:

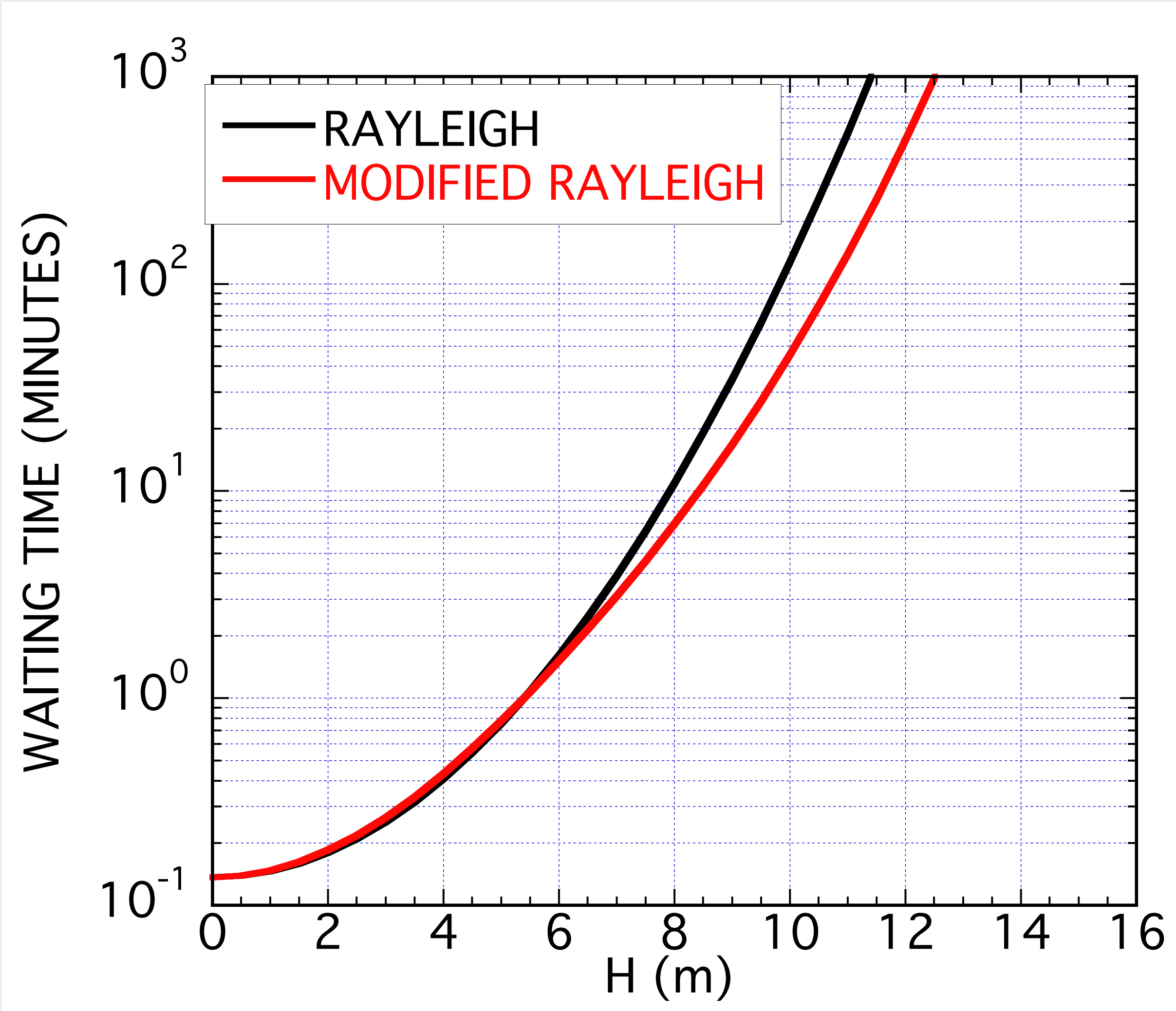
$$P_H(H) = e^{-\frac{1}{8}H^2}$$

PROBABILITY OF EXCEEDENCE AT THE TIME AND PLACE OF THE ACCIDENT



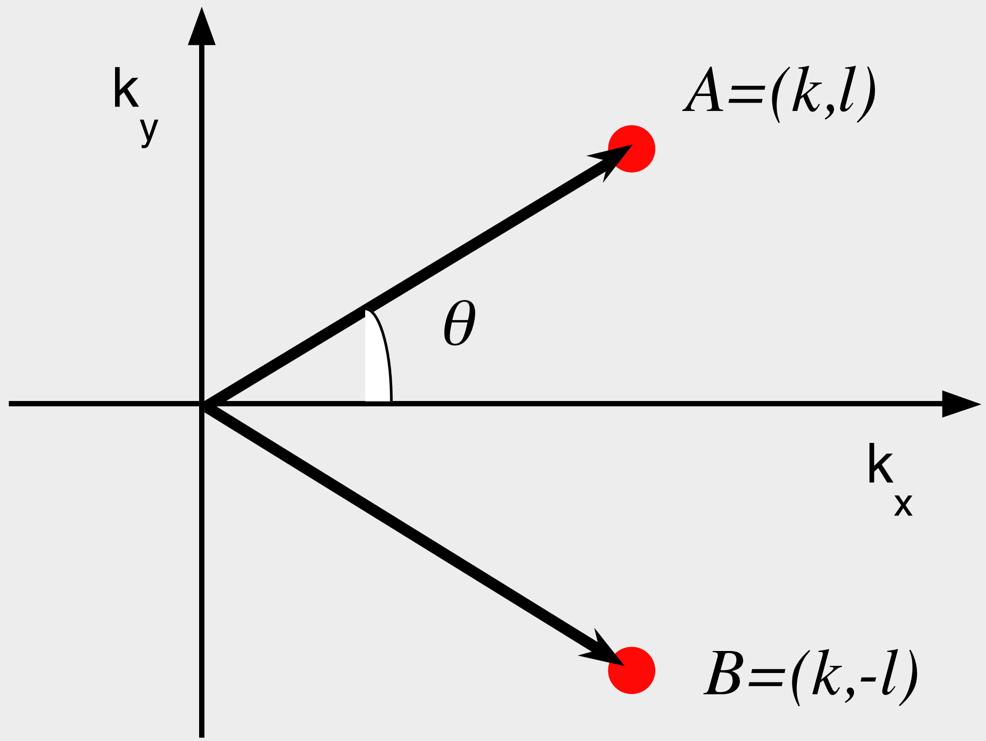
Hs=5.11 m

WAITING TIME AT THE TIME AND PLACE OF THE ACCIDENT



Hs=5.11 m

CROSSING SEAS: THE SIMPLEST CASE



COUPLED NONLINEAR SHRODINGER EQUATION

Zakharov equation

$$i\frac{\partial b_o}{\partial t} = \omega_o b_o + \int T_{0,1,2,3} b_1^* b_2 b_3 \delta\left(\mathrm{k}_o + \mathrm{k}_1 - \mathrm{k}_2 - \mathrm{k}_3\right) d\mathrm{k}_1 d\mathrm{k}_2 d\mathrm{k}_3$$

- consider the following decomposition

$$b(\boldsymbol{k}) = A(\boldsymbol{k} - \boldsymbol{k}_A)e^{-i\omega(k_A)t} + B(\boldsymbol{k} - \boldsymbol{k}_B)e^{-i\omega(k_B)t}$$

with $\boldsymbol{k}_A = (k, l)$ $\quad \boldsymbol{k}_B = (k, -l)$

- suppose that both spectral distribution are narrow banded

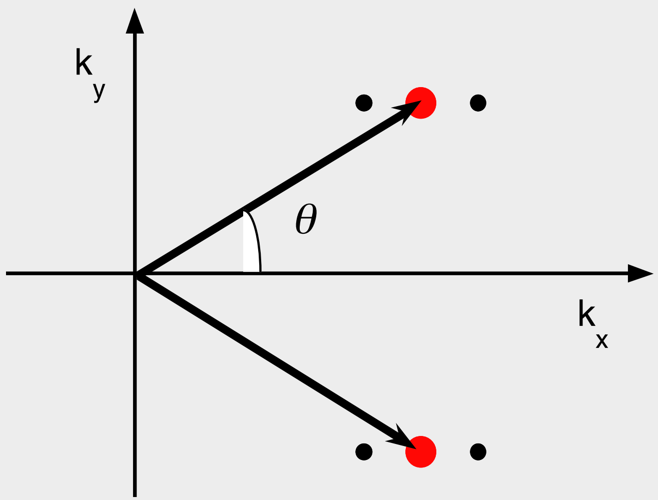
COUPLED NLS EQUATIONS

$$\frac{\partial A}{\partial t} + C_x \frac{\partial A}{\partial x} + C_y \frac{\partial A}{\partial y} - i\left[\alpha \frac{\partial^2 A}{\partial x^2} + \beta \frac{\partial^2 A}{\partial y^2} - \gamma \frac{\partial^2 A}{\partial x \partial y}\right] + i\left[\xi |A|^2 + 2\zeta |B|^2\right] A = 0$$

$$\frac{\partial B}{\partial t} + C_x \frac{\partial B}{\partial x} - C_y \frac{\partial B}{\partial y} - i\left[\alpha \frac{\partial^2 B}{\partial x^2} + \beta \frac{\partial^2 B}{\partial y^2} + \gamma \frac{\partial^2 B}{\partial x \partial y}\right] + i\left[\xi |B|^2 + 2\zeta |A|^2\right] B = 0$$

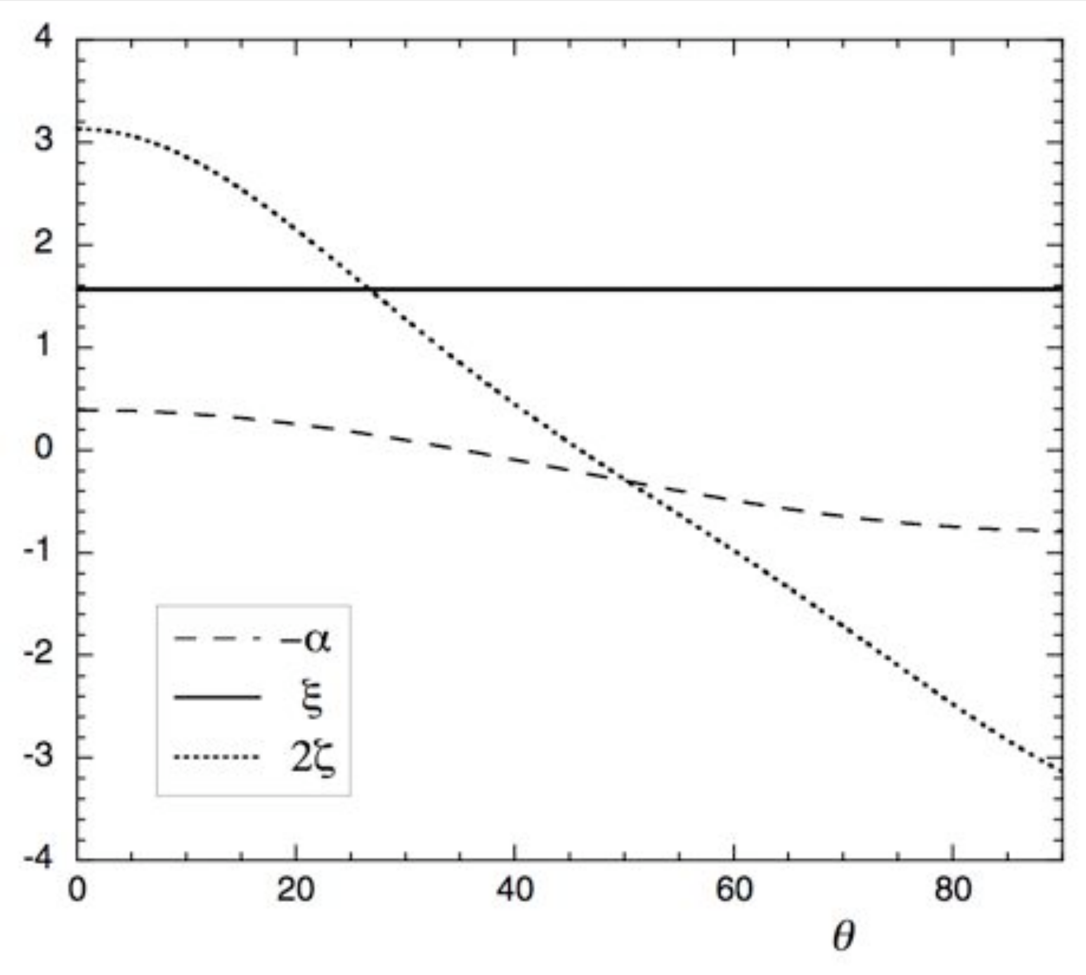
Coefficients are a function of *k* and *l*

- Consider perturbations only a function of k_x



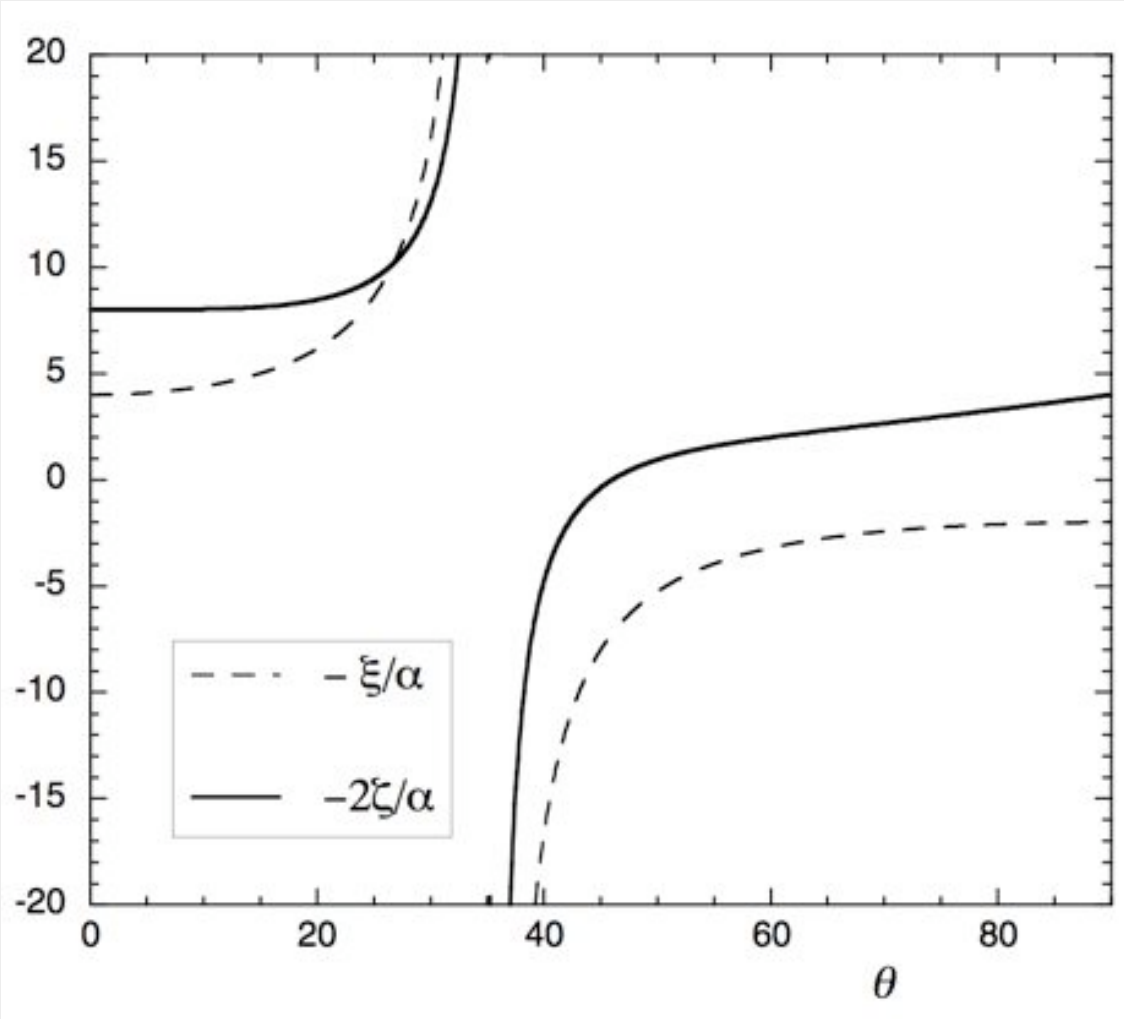
$$\boxed{\begin{aligned} \frac{\partial A}{\partial t} - i\alpha \frac{\partial^2 A}{\partial x^2} + i\left[\xi |A|^2 + 2\zeta |B|^2\right]A = 0 \\ \frac{\partial B}{\partial t} - i\alpha \frac{\partial^2 B}{\partial x^2} + i\left[\xi |B|^2 + 2\zeta |A|^2\right]B = 0 \end{aligned}}$$

ANALYSIS OF THE COEFFICIENTS



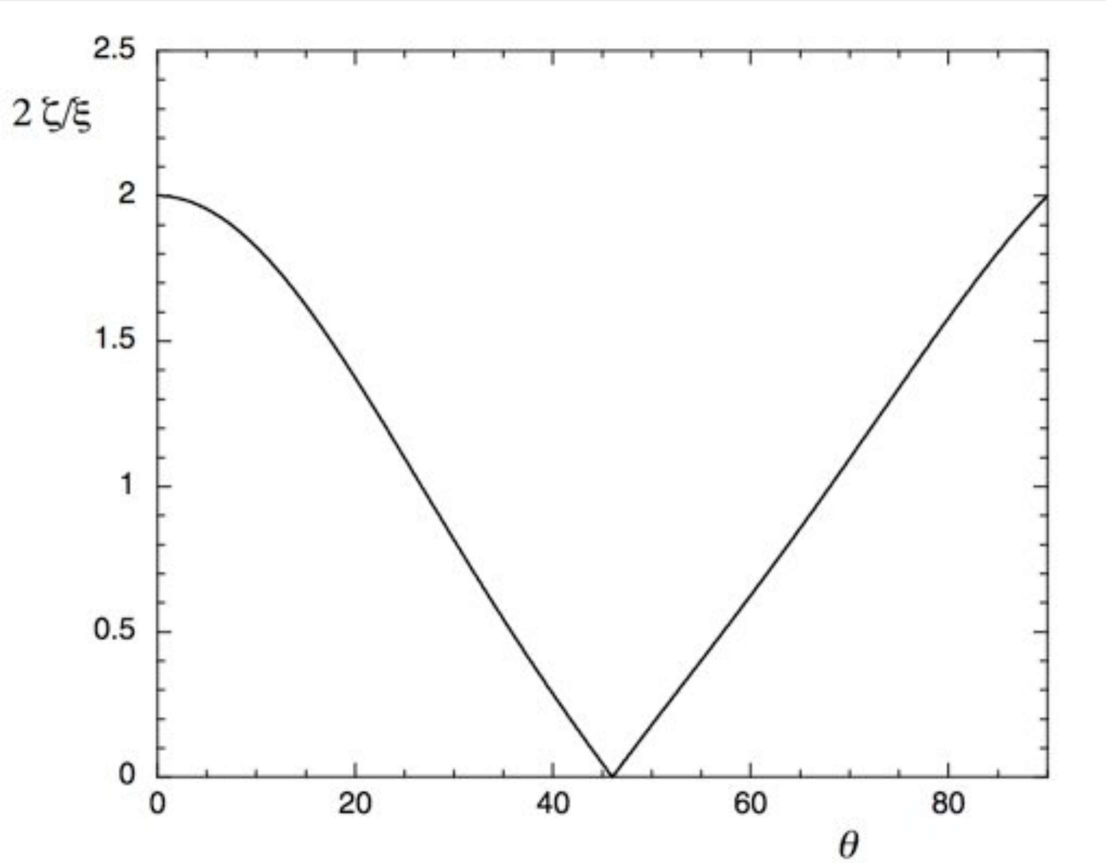
α= dispersive term
ξ= self-interaction term
ζ=cross-interaction term

ANALYSIS OF THE COEFFICIENTS



α= dispersive term
ξ= self-interaction term
ζ=cross-interaction term

ANALYSIS OF THE COEFFICIENTS



α= dispersive term
ξ= self-interaction term
ζ=cross-interaction term

SUMMARY OF THE RESULTS

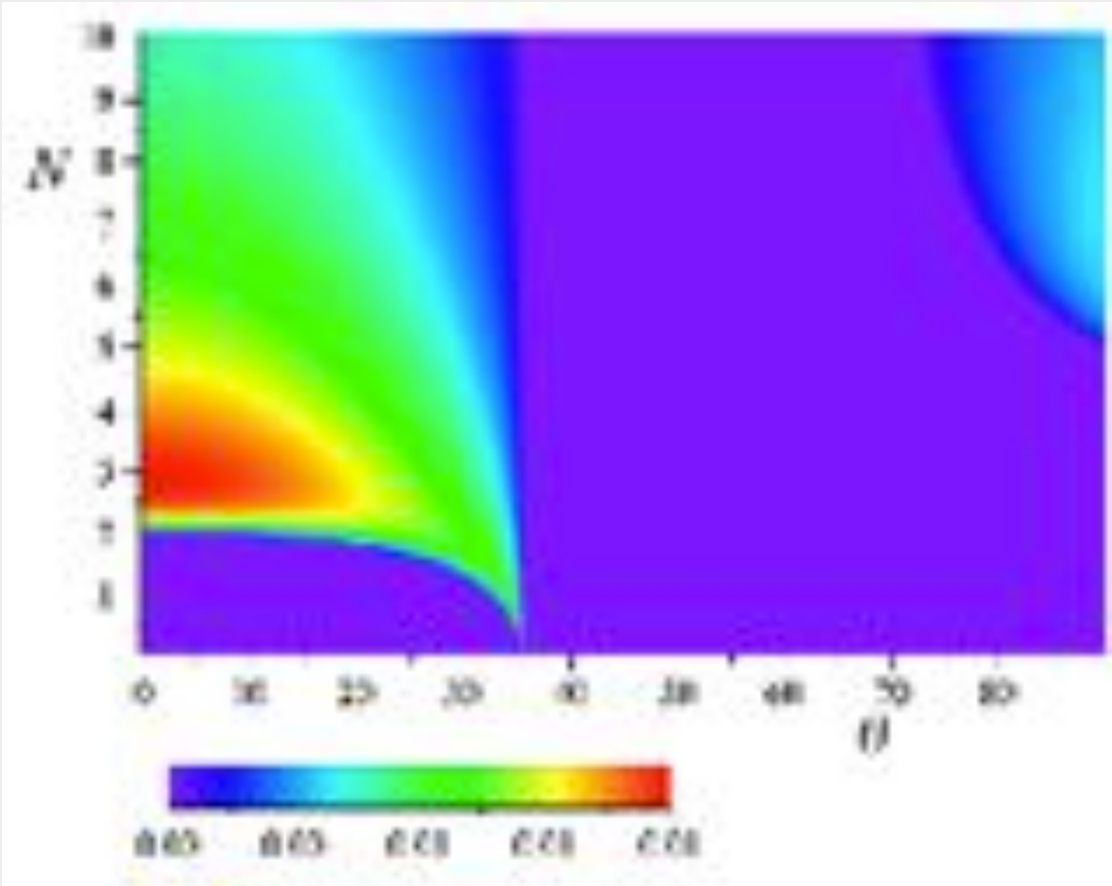
- For θ <35.3^0, dispersive and both nonlinear terms have the same sign

- The ratio between nonlinearity and dispersion becomes larger as θ approaches 35.3^0 (this is valid for both self-interaction and cross-interaction nonlinearity)

- The cross-interaction nonlinearity is stronger than the self Interaction one for angles between 0^0 and 26.7^0

DISPERSION RELATION FOR PERTURBATION

$$\Omega = \sqrt{\alpha K^2\left[2(\xi + 2\zeta)A_0^2 + \alpha K^2\right]}$$



AMPLIFICATION FACTOR FOR BREATHER SOLUTIONS

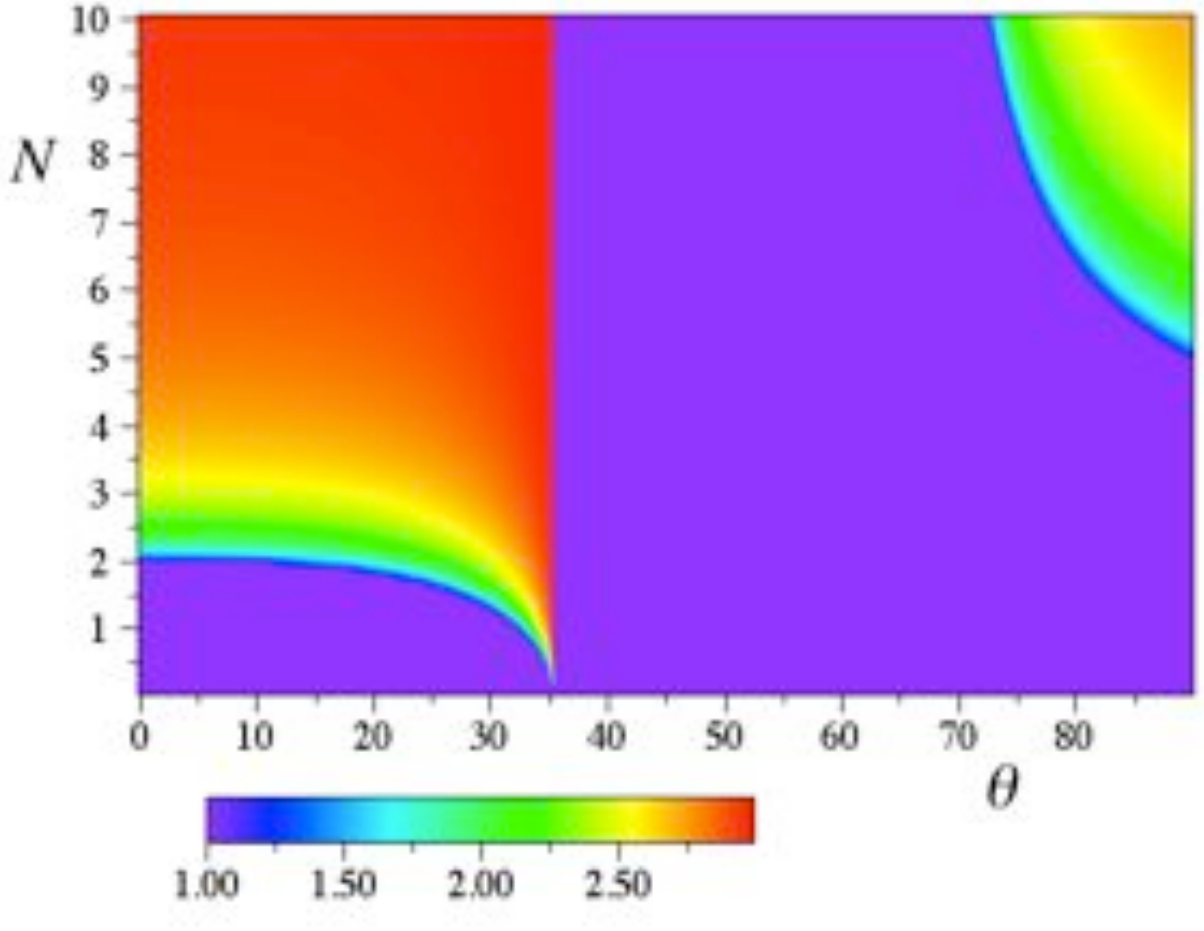
$$\frac{A_{\max}}{A_0} = 1 + 2\left[1 - \left(\sqrt{\frac{\alpha}{\xi + 2\zeta}}\,\frac{\kappa^2}{\varepsilon N}\right)^2\right]^{1/2}$$

N = number of waves under the envelope

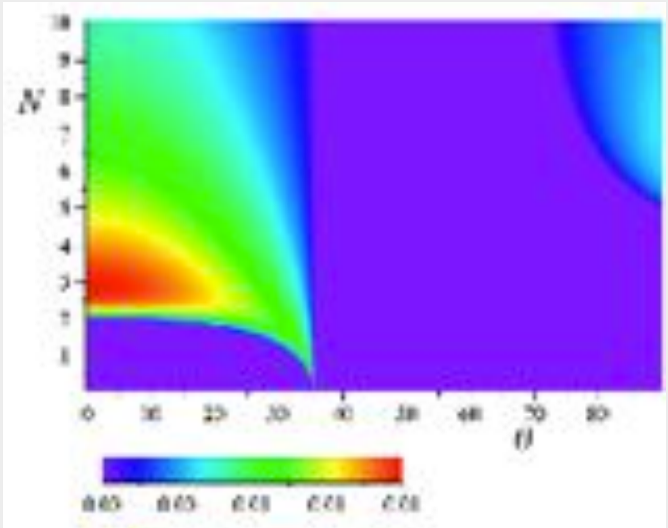
ε = initial steepness

κ = modulus of the wave number

AMPLIFICATION FACTOR



GROWTH RATE



SUMMARY OF THE RESULTS:

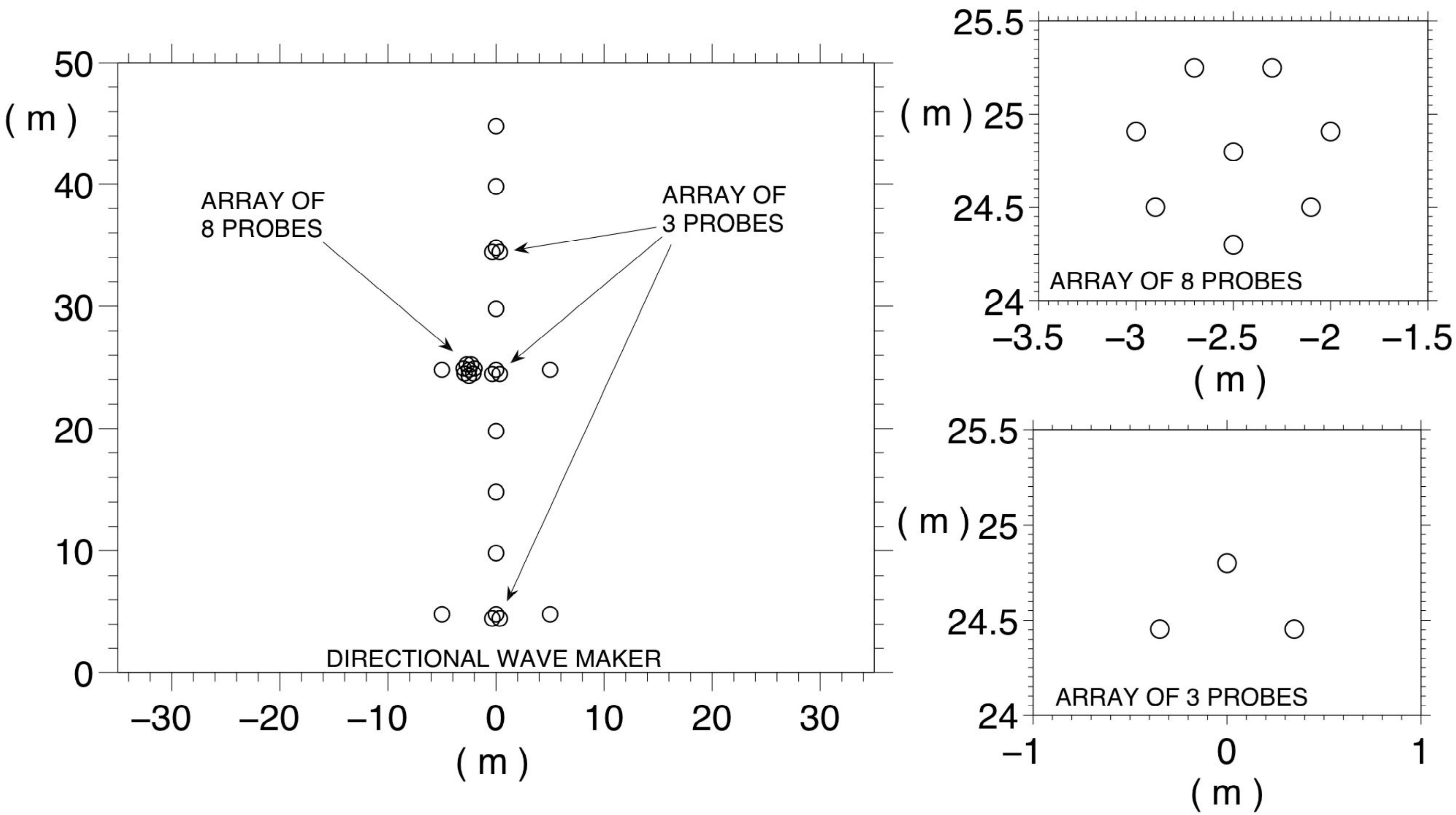
- The maximum amplification is for $\theta \to 35.3^0$ and large N
- The maximum growth rate is for $\theta=0^0$ and $N\approx3$

CONSIDERATIONS:

Extreme waves are the result of a maximum amplification factor in a reasonable time scale

WE EXPECT LARGE EXTREME WAVE ACTIVITY AT ANGLES OF $\theta\approx20^0$- 30^0

EXPERIMENTS: MARINTEK FACILITY



DESCRIPTION OF THE EXPERIMENT

SUM OF TWO JONSWAP SPECTRA:

$$E(\omega,\theta) = E_1(\omega,\theta) + E_2(\omega,\theta)$$

with

$$E_1(\omega,\theta) = \frac{\alpha g^2}{\omega^5} Exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right] \gamma^{Exp\left[-(\omega-\omega_p)^2/(2\sigma^2\omega_p^2)\right]} \delta(\theta-\theta_0)$$

$$E_2(\omega,\theta) = \frac{\alpha g^2}{\omega^5} Exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right] \gamma^{Exp\left[-(\omega-\omega_p)^2/(2\sigma^2\omega_p^2)\right]} \delta(\theta+\theta_0)$$

NUMERICAL SIMULATIONS

- HIGHER ORDER SPECTRAL METHOD (THIRD ORDER IN NONLINEARITY)

- BOX PERIODIC IN x AND y COORDINATES

- INITIAL CONDITIONS PROVIDED BY TWO JONSWAP SPECTRA TRAVELLING AT AN ANGLE

RESULTS ON MAXIMUM KURTOSIS