TSUNAMIS AS NORMAL MODES OF THE EARTH, and *A Venture into Extracurricular Geophysics*

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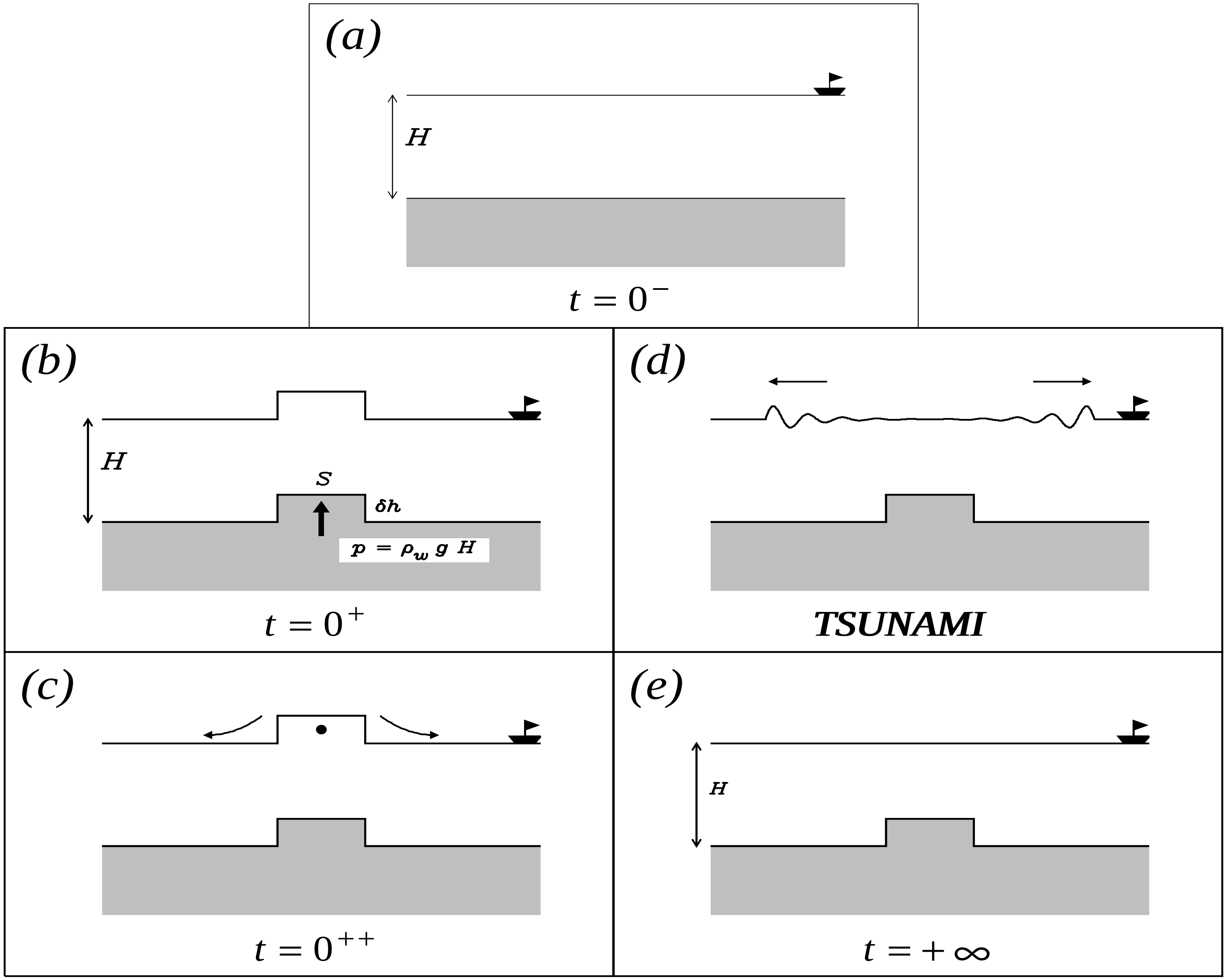
Ocean Waves Workshop
Fields Institute, Toronto
Monday, 13 June 2011

TSUNAMI GENERATION by EARTHQUAKE SOURCES

CLASSICAL APPROACH



HYDRODYNAMIC SIMULATIONS

The Classical Approach

1. Obtain model of Earthquake Rupture

2. Compute Static Deformation of Ocean Bottom

3. Use as Initial Conditions of

Vertical Surface Displacement with Zero Initial Velocity

4. Run Hydrodynamic Model (*e.g.*, **MOST**)

5. Propagate, up to and including

INUNDATION of Receiving Shore

STATIC DEFORMATION OF OCEAN BOTTOM

Straightforward, if somewhat arcane analytical formulæ

[*Mansinha and Smylie*, 1971; *Okada*, 1985]

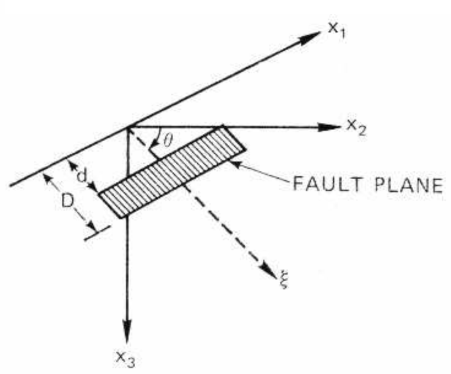
(1) Displacements

For strike-slip

$$\begin{cases} u_x = -\dfrac{U_1}{2\pi}\left[\dfrac{\xi q}{R(R+\eta)} + \tan^{-1}\dfrac{\xi\eta}{qR} + I_1 \sin\delta\right]\Bigg\| \\ u_y = -\dfrac{U_1}{2\pi}\left[\dfrac{\tilde{y}q}{R(R+\eta)} + \dfrac{q\cos\delta}{R+\eta} + I_2 \sin\delta\right]\Bigg\| \\ u_z = -\dfrac{U_1}{2\pi}\left[\dfrac{\tilde{d}q}{R(R+\eta)} + \dfrac{q\sin\delta}{R+\eta} + I_4 \sin\delta\right]\Bigg\| . \end{cases}$$

For dip-slip

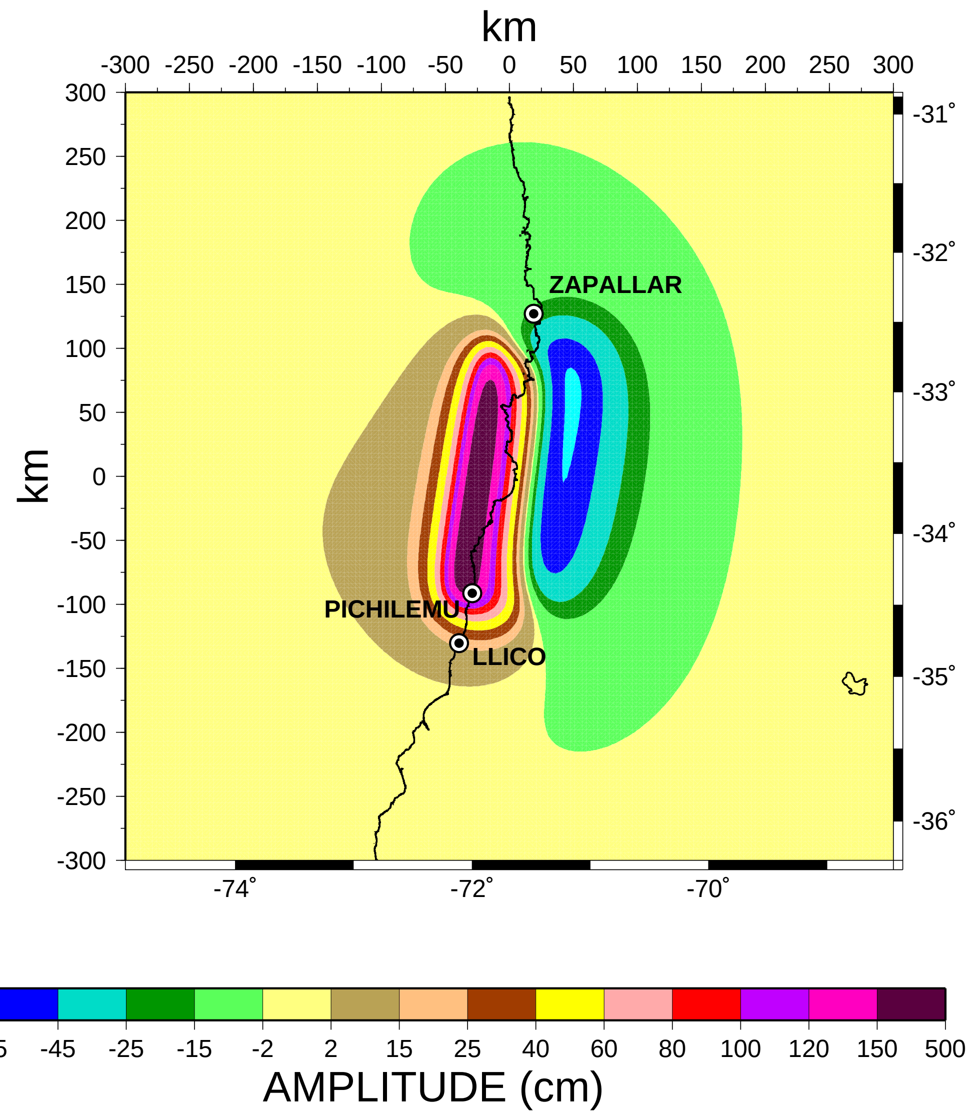
$$\begin{cases} u_x = -\dfrac{U_2}{2\pi}\left[\dfrac{q}{R} - I_3 \sin\delta\cos\delta\right]\Bigg\| \\ u_y = -\dfrac{U_2}{2\pi}\left[\dfrac{\tilde{y}q}{R(R+\xi)} + \cos\delta\tan^{-1}\dfrac{\xi\eta}{qR} - I_1 \sin\delta\cos\delta\right]\Bigg\| \\ u_z = -\dfrac{U_2}{2\pi}\left[\dfrac{\tilde{d}q}{R(R+\xi)} + \sin\delta\tan^{-1}\dfrac{\xi\eta}{qR} - I_5 \sin\delta\cos\delta\right]\Bigg\| . \end{cases}$$



where

$$\begin{cases} I_1 = \dfrac{\mu}{\lambda+\mu}\left[\dfrac{-1}{\cos\delta}\dfrac{\xi}{R+\tilde{d}}\right] - \dfrac{\sin\delta}{\cos\delta} I_5 \\ I_2 = \dfrac{\mu}{\lambda+\mu}[-\ln(R+\eta)] - I_3 \\ I_3 = \dfrac{\mu}{\lambda+\mu}\left[\dfrac{1}{\cos\delta}\dfrac{\tilde{y}}{R+\tilde{d}} - \ln(R+\eta)\right] + \dfrac{\sin\delta}{\cos\delta} I_4 \\ I_4 = \dfrac{\mu}{\lambda+\mu}\dfrac{1}{\cos\delta}[\ln(R+\tilde{d}) - \sin\delta\ln(R+\eta)] \\ I_5 = \dfrac{\mu}{\lambda+\mu}\dfrac{2}{\cos\delta}\tan^{-1}\dfrac{\eta(X+q\cos\delta)+X(R+X)\sin\delta}{\xi(R+X)\cos\delta} \end{cases}$$

1906 CHILEAN EVENT



MOST Hydrodynamic Code

(Method **O**f **S**plitting **T**sunamis**)**

[*Titov and Synolakis*, 1998]

Solves the Non-Linear Shallow Water Equations

$$\frac{\partial}{\partial t}(\eta + h) + \frac{\partial}{\partial x}[(\eta + h)\bar{u}] + \frac{\partial}{\partial y}[(\eta + h)\bar{v}] = 0$$

$$\frac{\partial}{\partial t}[(\eta + h)\bar{u}] + \frac{\partial}{\partial x}[(\eta + h)(\bar{u})^2] + \frac{\partial}{\partial y}[(\eta + h)\bar{u}\,\bar{v}] = -g\frac{\partial \eta}{\partial x}\cdot(\eta + h)$$

$$\frac{\partial}{\partial t}[(\eta + h)\bar{v}] + \frac{\partial}{\partial x}[(\eta + h)\bar{u}\,\bar{v}] + \frac{\partial}{\partial y}[(\eta + h)(\bar{v})^2] = -g\frac{\partial \eta}{\partial y}\cdot(\eta + h)$$

by Finite Differences, using the Method of Splitting Integration Steps

- MOST calculates the onland run-up by introducing additional grid points during inundation and dropping them during down-draw.

CREATING A GRID

A critical step in running a hydrodynamic code is to obtain an adequate bathymetric grid (and a topographic one for run-up extensions).

In practice, several grids of increasingly finer resolution are used. The coarsest grid is typically on the order of 1 km.

Due to the general unavailability of homogeneous bathymetric databases, the grid often must be compiled by combining various sources, which can be a grueling process.

For basin-wide calculations, the sphericity of the Earth must be taken into account by re-casting the SWA equations into spherical coordinates.

The grid must be smoothed to eliminate sharp gradients leading to instabilities in the computation.

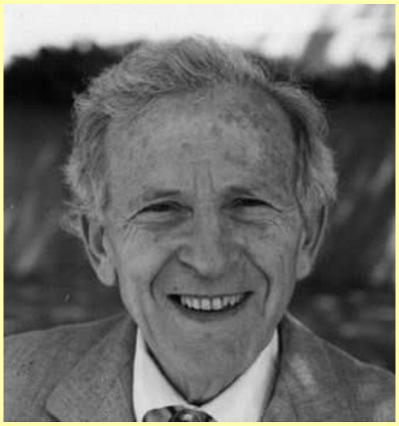
Finally, the time step of the computation, Δt, must satisfy the CFL condition of stability of finite difference approximations

$$\Delta t \ < \ \frac{1}{C} \Delta x$$

This condition involves the velocity phase velocity C of the wave and must be carefully assessed throughout the grid, notably at high latitudes. In very general terms, Δt for the coarse grid can be ~ 10 s, much less on the finer ones.

R. Courant

K. Friedrichs



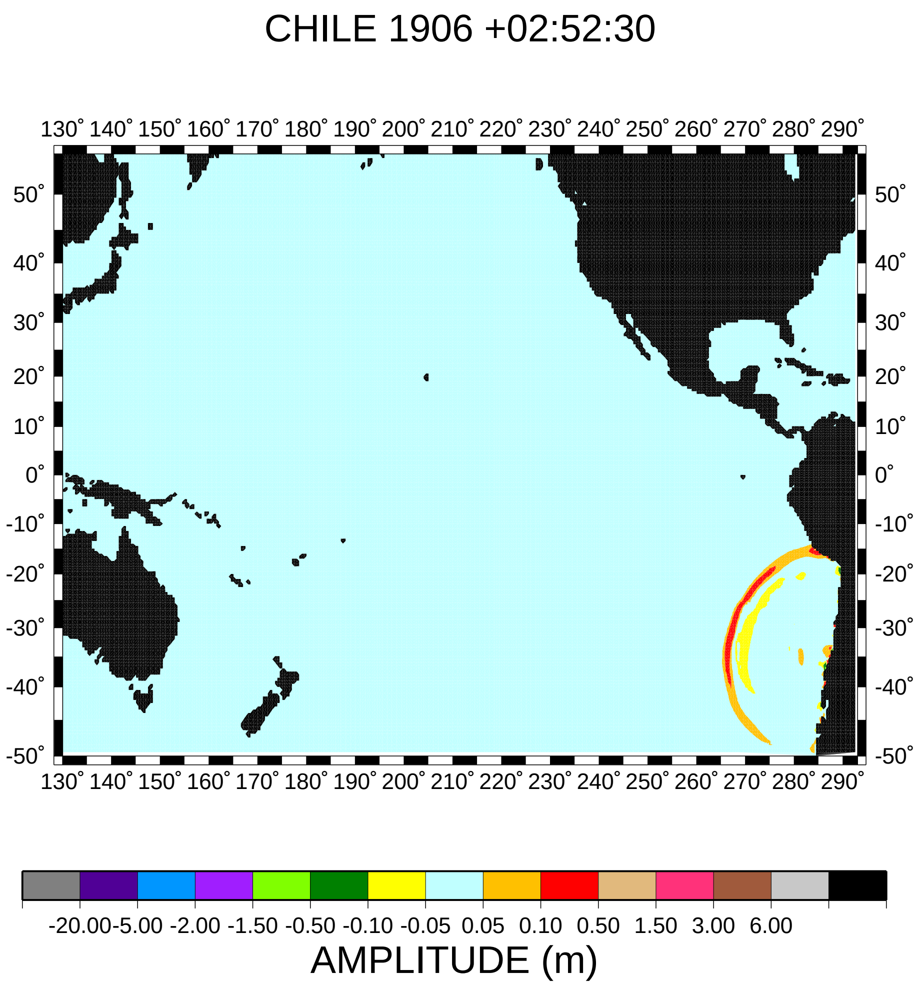
H. Lewy

MOST PRODUCTS

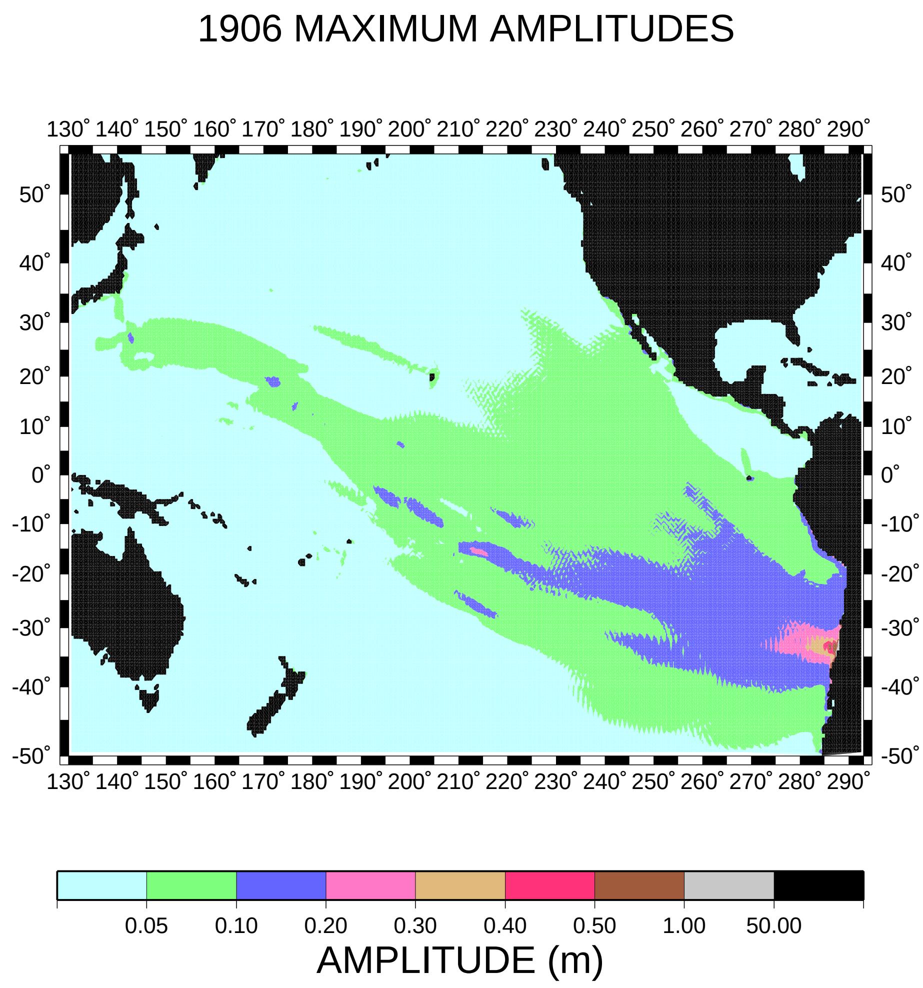
Example: *1906 Valparaiso, Chile Tsunami*

[*Okal*, 2005]

INSTANTANEOUS SURFACE SNAPSHOT



MAXIMUM SEA SURFACE AMPLITUDES



HYDRODYNAMIC SIMULATIONS

The Classical Approach

INVOLVES SEVERE, INCOMPATIBLE, APPROXIMATIONS

1. Obtain model of Earthquake Rupture

2. Compute Static Deformation of Ocean Bottom

IGNORES WATER COLUMN !

3. Use as Initial Conditions of

Vertical Surface Displacement with Zero Initial Velocity

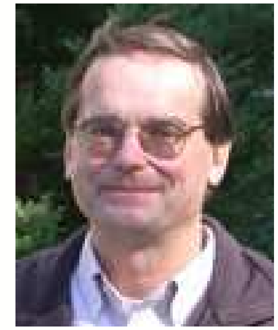
4. Run Hydrodynamic Model (*e.g.*, **MOST**)

5. Propagate, up to and including

INUNDATION of Receiving Shore

IGNORES FINITE BOTTOM RIGIDITY !

TSUNAMIS: The NORMAL MODE FORMALISM

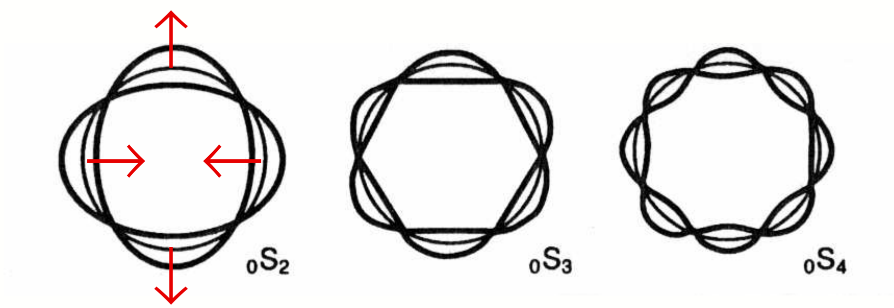


[*Ward*, 1980]

- At very long periods (typically 15 to 54 minutes), the Earth, because of its finite size, can ring like a bell.

- Such *FREE OSCILLATIONS* are equivalent to the superposition of two progressive waves travelling in opposite directions along the surface of the Earth.

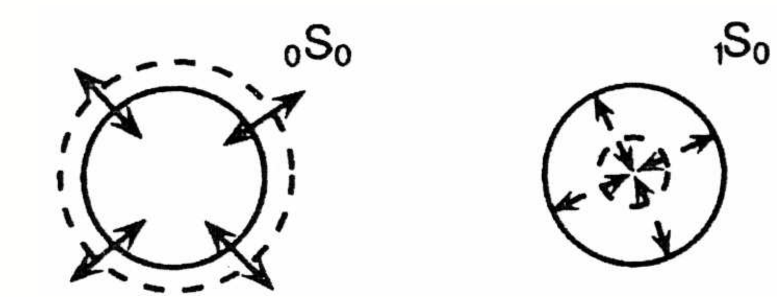
T = 54 minutes



"FOOTBALL Mode"

[After *Lay and Wallace*, 1995]

T = 21.5 minutes



"BREATHING Mode"

Ward [1980] has shown that **Tsunamis come naturally as a special branch of the normal modes of the Earth,** provided it is bounded by an ocean, and gravity is included in the formulation of its vibrations.

In the normal mode formalism, the solution of the vertical displacement (both in the water and solid Earth) is sought as

$$u_z(\mathbf{x};t) = u_z(r,\theta,\phi;t) = y_1(r)\cdot Y_l^m(\theta,\phi)\exp(i\omega t) = y_1(r)\cdot P_l^m(\cos\theta)\cdot e^{i\,m\,\phi}\cdot\exp(i\omega t)$$

where Y_l^m is a *spherical harmonic* of order l and degree m; P_l^m is the Legendre polynomial of order l and degree m; and $\{r,\theta,\phi\}$ is a system of spherical polar coordinates.

This allows for the ***separation*** of the variables $\{r,\theta,\phi\}$.

The problem is complemented by similar expressions for the overpressure $p = -y_2$ in the tsunami wave, the horizontal displacement $u_x = l\cdot y_3$, and the change in the gravity potential y_5.

Under the linear approximation, the equations of hydrodynamics transform into a system of linear differential equations of the first order.

For any given l, *i.e.*, wavenumber $k = (l + 1/2)$ (a radius of the Earth), the system has non trivial solutions for only one value of ω. The relationship between l and ω is the

Dispersion Relation of the Tsunami.

SPHEROIDAL MODE HAS 6−COMPONENT EIGENFUNCTION SATISFYING:

$$\left|\begin{matrix} \frac{dy_1}{dr} \\ \frac{dy_2}{dr} \\ \frac{dy_3}{dr} \\ \frac{dy_4}{dr} \\ \frac{dy_5}{dr} \\ \frac{dy_6}{dr} \end{matrix}\right| = \left|\begin{matrix} \frac{-2\lambda}{(\lambda+2\mu)r} & \frac{1}{(\lambda+2\mu)} & \frac{L^2\lambda}{(\lambda+2\mu)r} & 0 & 0 & 0 \\ -\omega^2\rho+\frac{4\mu(3\lambda+2\mu)}{(\lambda+2\mu)r^2}-\frac{4\rho g}{r} & \frac{-4\mu}{(\lambda+2\mu)r} & L^2\left[\frac{\rho g}{r}-\frac{2\mu(3\lambda+2\mu)}{(\lambda+2\mu)r^2}\right] & \frac{L^2}{r} & 0 & -\rho \\ \frac{-1}{r} & 0 & \frac{1}{r} & \frac{1}{\mu} & 0 & 0 \\ \frac{\rho g}{r}-\frac{2\mu(3\lambda+2\mu)}{(\lambda+2\mu)r^2} & \frac{-\lambda}{(\lambda+2\mu)r} & -\omega^2\rho+\frac{4\mu L^2(\lambda+\mu)}{(\lambda+2\mu)r^2}-\frac{2\mu}{r^2} & \frac{-3}{r} & \frac{-\rho}{r} & 0 \\ 4\pi G\rho & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-4\pi L^2 G\rho}{r} & 0 & \frac{L^2}{r^2} & \frac{-2}{r} \end{matrix}\right| \cdot \left|\begin{matrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{matrix}\right|$$

y_1 : Vertical displacement $\quad$ y_2 : Normal stress $\quad$ y_5 : Gravity potential

y_3 : Horizontal displacement $\quad$ y_4 : Tangential stress $\quad$ y_6 : Auxiliary gravity

e.g., Field of Vertical Displacement of the Tsunami :

$$\eta(r,\theta,\phi;t) = y_1(r)\cdot P_l^m(\cos\theta)\cdot e^{i\,m\,\phi}\cdot[1-\cos\omega_l\,t]$$

EASILY SOLVED WITH APPROPRIATE BOUNDARY CONDITIONS

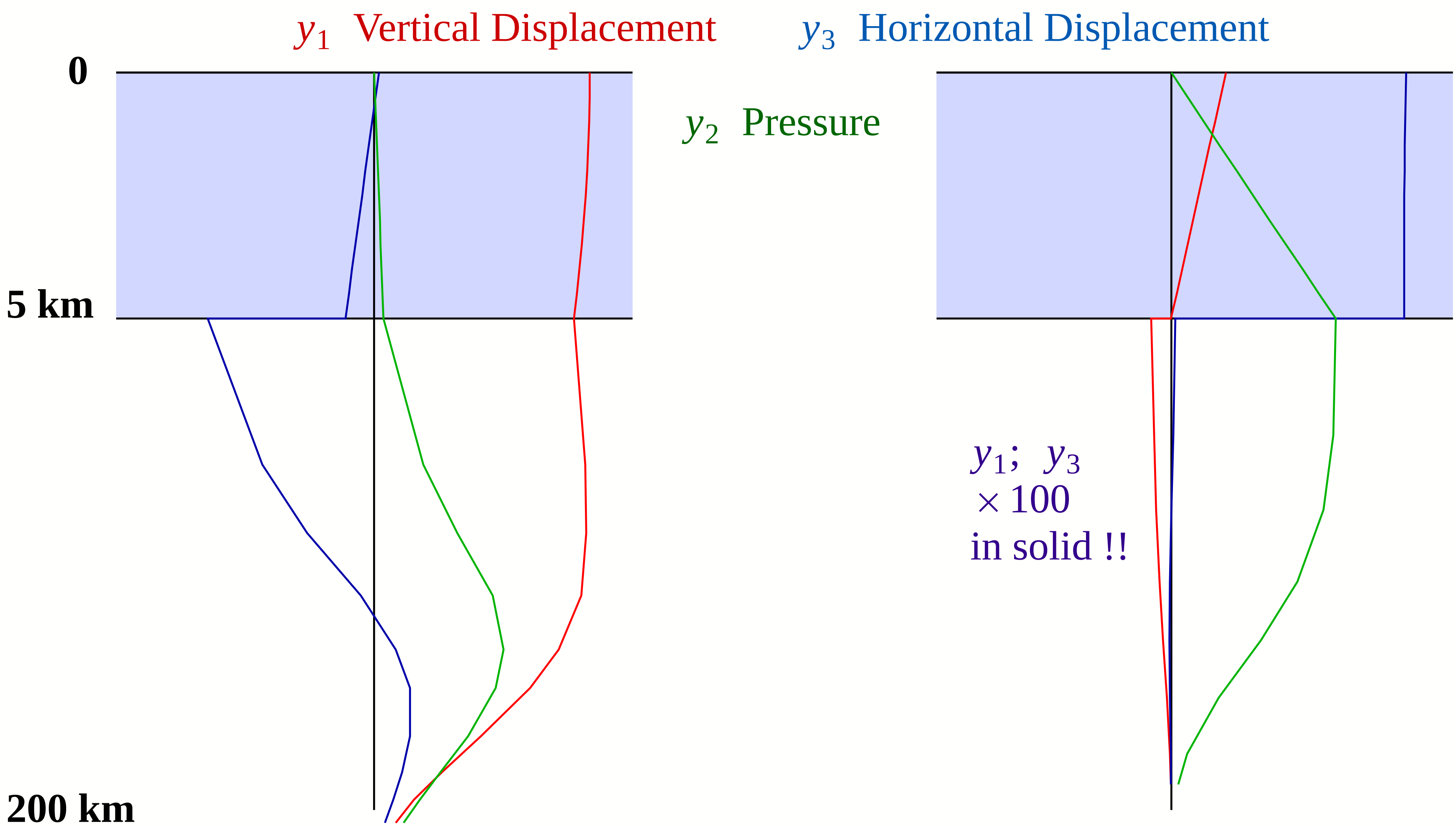
TSUNAMI as SPHEROIDAL MODE : STRUCTURE of the EIGENFUNCTION

Rayleigh Mode

$l = 200$; $T = 52$ s

Tsunami Mode

$l = 200$; $T = 908$ s



TSUNAMI EIGENFUNCTION is CONTINUED (SMALL) into SOLID EARTH

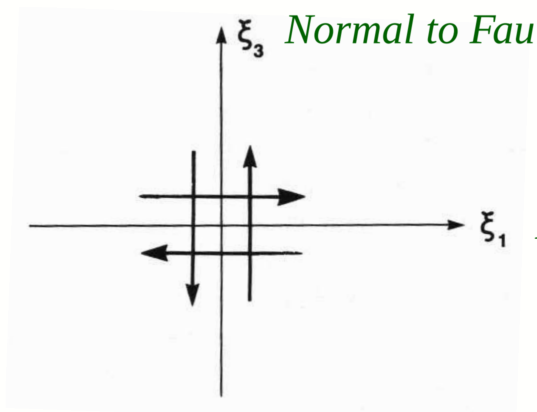
EXCITATION OF TSUNAMI in NORMAL MODE FORMALISM

- *Gilbert* [1970] has shown that the response of the Earth to a point source consisting of a single force **f** can be expressed as a summation over all of its normal modes

$$\mathbf{u}(r,t) = \sum_N \mathbf{s}_n(\mathbf{r}) \left(\mathbf{s}_n^*(\mathbf{r_s}) \cdot \mathbf{f}(\mathbf{r_s}) \right) \cdot \frac{1 - \cos\omega_n t \exp(-\omega_n t/2Q_n)}{\omega_n^2} \quad ,$$

the *EXCITATION* of each mode being proportional to the *scalar product of the force* **f** *by the eigen-displacement* **s** *at location* $\mathbf{r_s}$ *.*

- Now, an *EARTHQUAKE* is represented by a system of forces called a ***double – couple***:



The response of the Earth to an earthquake is thus

$$\mathbf{u}(r,t) = \sum_N \mathbf{s}_n(\mathbf{r}) \boxed{\left(\varepsilon_n^*(\mathbf{r_s}) : \boldsymbol{M}(\mathbf{r_s}) \right)} \cdot \frac{1 - \cos\omega_n t \exp(-\omega_n t/2Q_n)}{\omega_n^2}$$

where the *EXCITATION* is the *scalar product* of the earthquake's **MOMENT *M*** with the local *eigenstrain* ε at the source $\mathbf{r_s}$.

This formula is directly applicable to the case of a tsunami represented by normal modes of the Earth.

ADVANTAGES of NORMAL MODE FORMALISM

- Handles any Ocean-Solid Earth Coupling
  Including Sedimentary Layers
- Works well at Higher Frequencies
  No need to assume Shallow-Water Approximation

IMMEDIATE RESULTS

- Eigenfunction very small in Solid
  *Requires **HUGE** Earthquake*
- Eigenfunction decays slowly in Solid
  *Depth has **minimal influence** on tsunami excitation* ($h \leq 70$ *km*)
- y_3 present in solid. *All geometries, **including strike – slip** excite tsunamis.*

DRAWBACKS of NORMAL MODE FORMALISM

- Must assume Laterally Homogeneous Structure
- Linear Theory -- Does not allow for Large Amplitudes

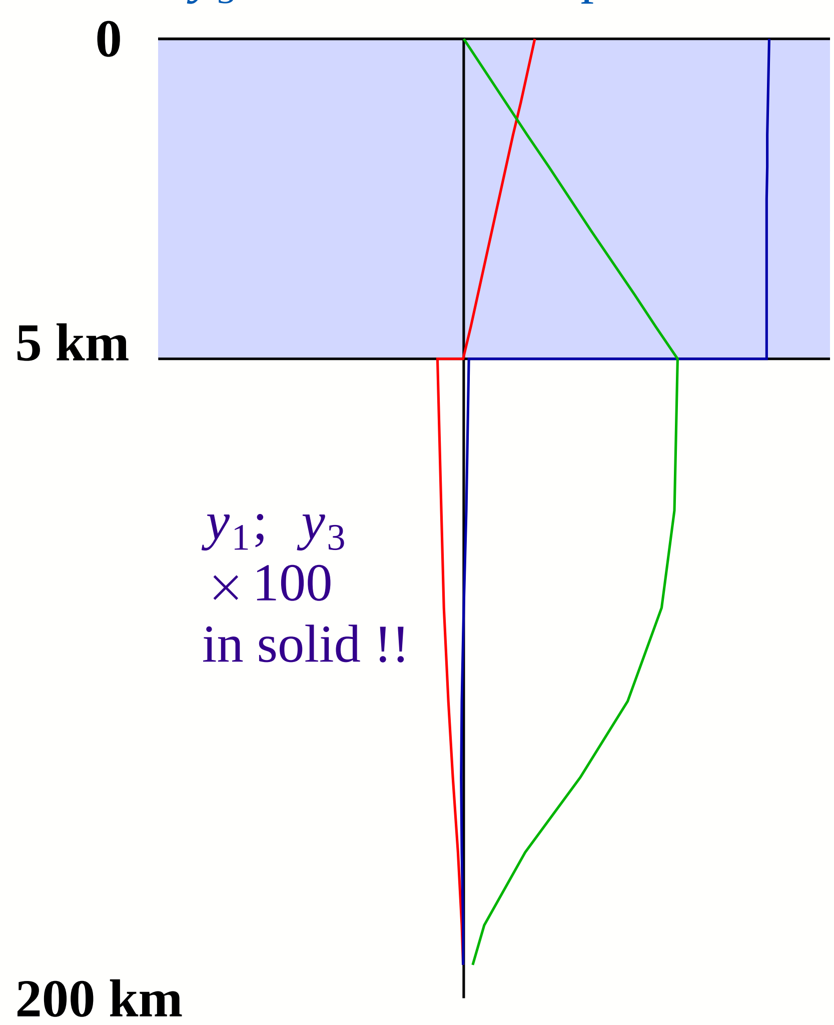
Tsunami Mode

$l = 200; \ T = 908$ s

y_1 Vertical Displacement

y_2 Pressure

y_3 Horizontal Displacement

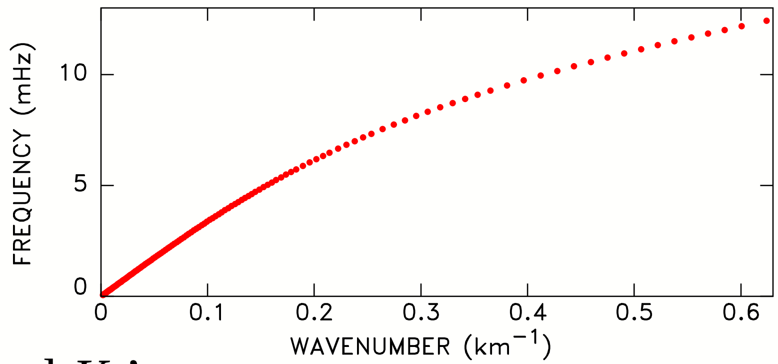


NORMAL MODE TSUNAMI SYNTHETICS

In practice:

- Compute tsunami modes at representative frequencies. Store eigenfucntion and excitation coefficients K_0, K_1, K_2 in the formalism of *Kanamori and Cipar* [1974].

- Build synthetic in Fourier Domain:



At each [angular] frequency ω,

Interpolate ω into catalogued eigenfrequencies.

Derive appropriate values of equivalent l, U, C, and K_i's.

Obtain spectral amplitude of vertical displacement of sea surface as:

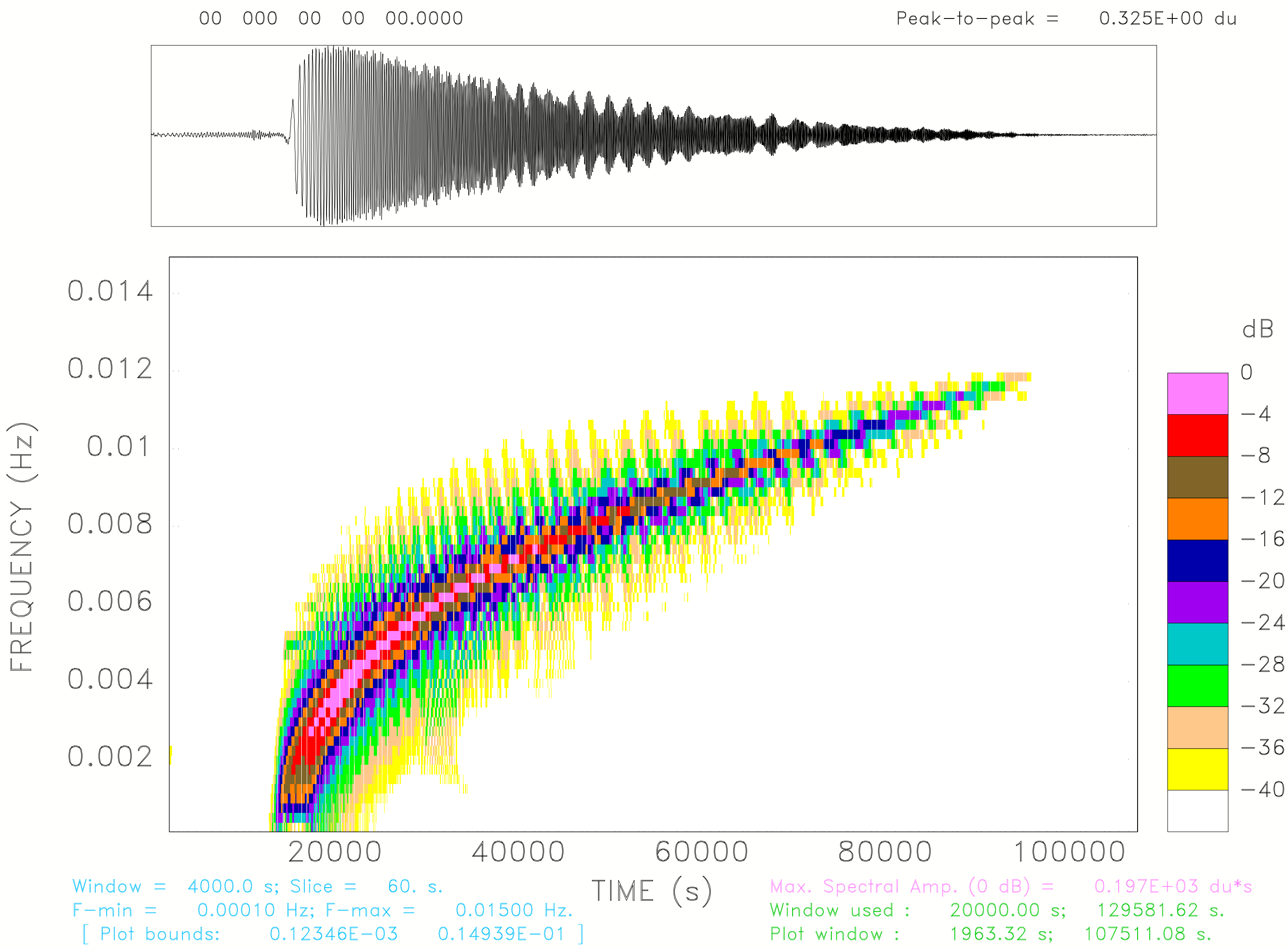
$$X(\omega) \;=\; \frac{M_0}{\sqrt{\sin\Delta}}\,\frac{a}{U}\sqrt{\frac{\pi}{2l}}\left(s_R\,K_0 - i\,q_R\,l\,K_1 - p_R\,l^2\,K_2\right)\cdot e^{i\pi/4}\cdot e^{-i\,\omega\,a\,\Delta/C}$$

(This formula is just an asymptotic interpolation of Gilbert's excitation, recast into a notation emphasizing the physical source angles [*Kanamori and Stewart*, 1976].)

- Fourier-transform back into time domain.

EXAMPLE of NORMAL MODE TSUNAMI SYNTHETIC

$$\Delta = 70^\circ$$



The spectrogram illustrates the dispersion of the tsunami outside the Shallow-Water Approxiamtion.

Note that high-frequency components ($f = 10$ mHz or $T = 100$ seconds) take *close to one day* to reach the receiver.

This computation is equivalent to a

LINEAR, DISPERSIVE technique.

THE [possible] INFLUENCE of SEDIMENTARY LAYERING

The conventional algorithm uses a *Homogeneous Half-Space* structure.

- Would layering (in particular with *mechanically deficient* **("Sedimentary")** *layers* affect tsunami excitation ?

→ Of course, putting the source *inside* the sedimentary layer will *increase* tsunami excitation, but

What about in the case of a source in the hard rock,

***overlain by a sedimentary layer* ?**

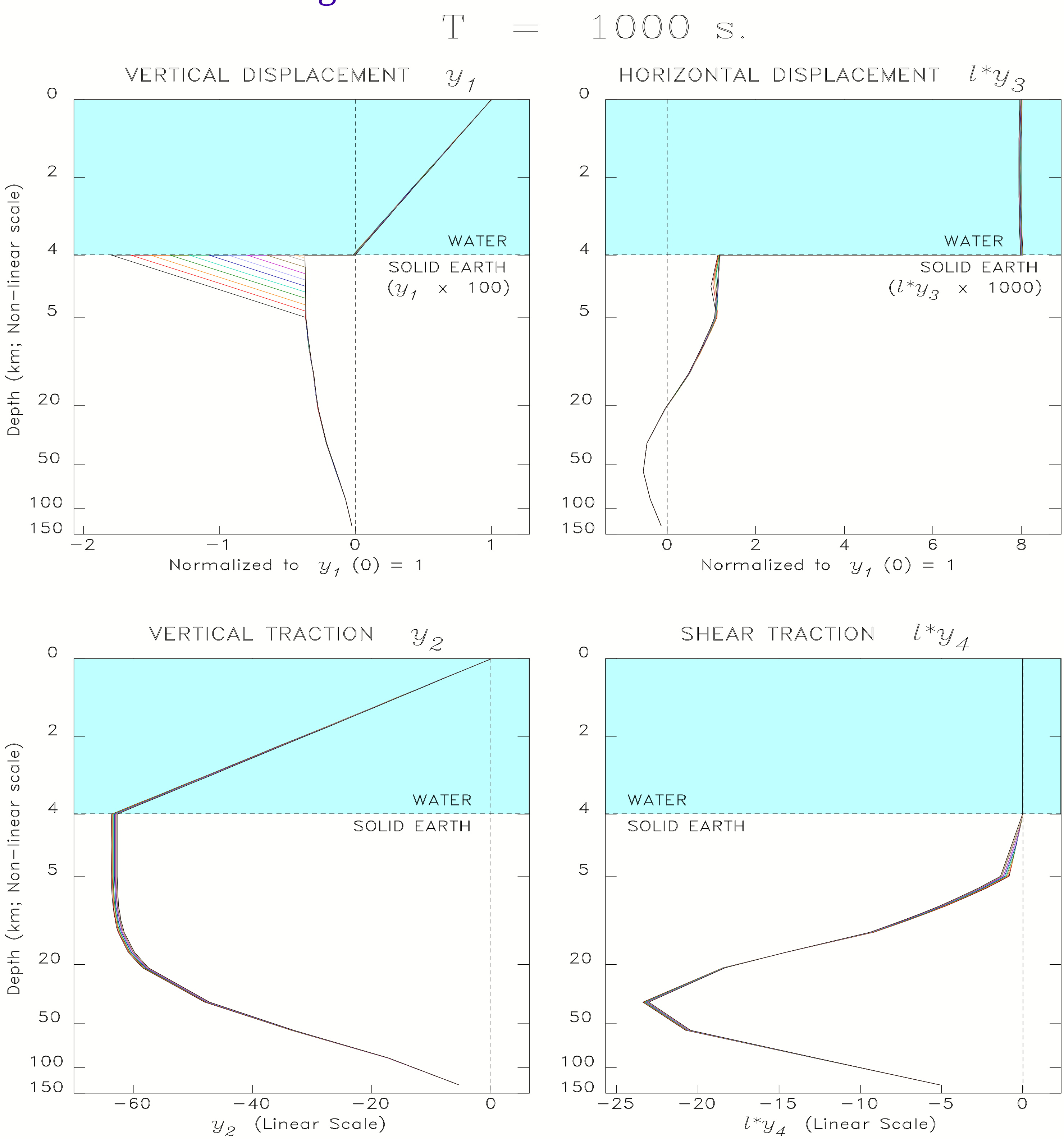
- We use normal mode theory and consider 12 crustal models featuring a sedimentary layer varying from 0 to 1 km in thickness; we then

- examine the behavior of the relevant components of the eigenvector $\{y_i\}$;
- examine the behavior of the point-source excitation coefficients;
- place a source *below the sediments* and build several million synthetic tsunami waveforms to study the effect of sedimentation through the *change in amplitude* relative to the un-sedimented model, while maintaining all other parameters constant.

EFFECT of SEDIMENTS on EIGENFUNCTION

1. T = 1000 s (Shallow Water)

- Sedimentary layering (color lines on figures) enhances vertical displacement in the soft layer, but leaves all components essentially unchanged in the hard rock.



EFFECT of SEDIMENTS on EXCITATION COEFFICIENTS

1. $T = 1000$ s (Shallow Water)

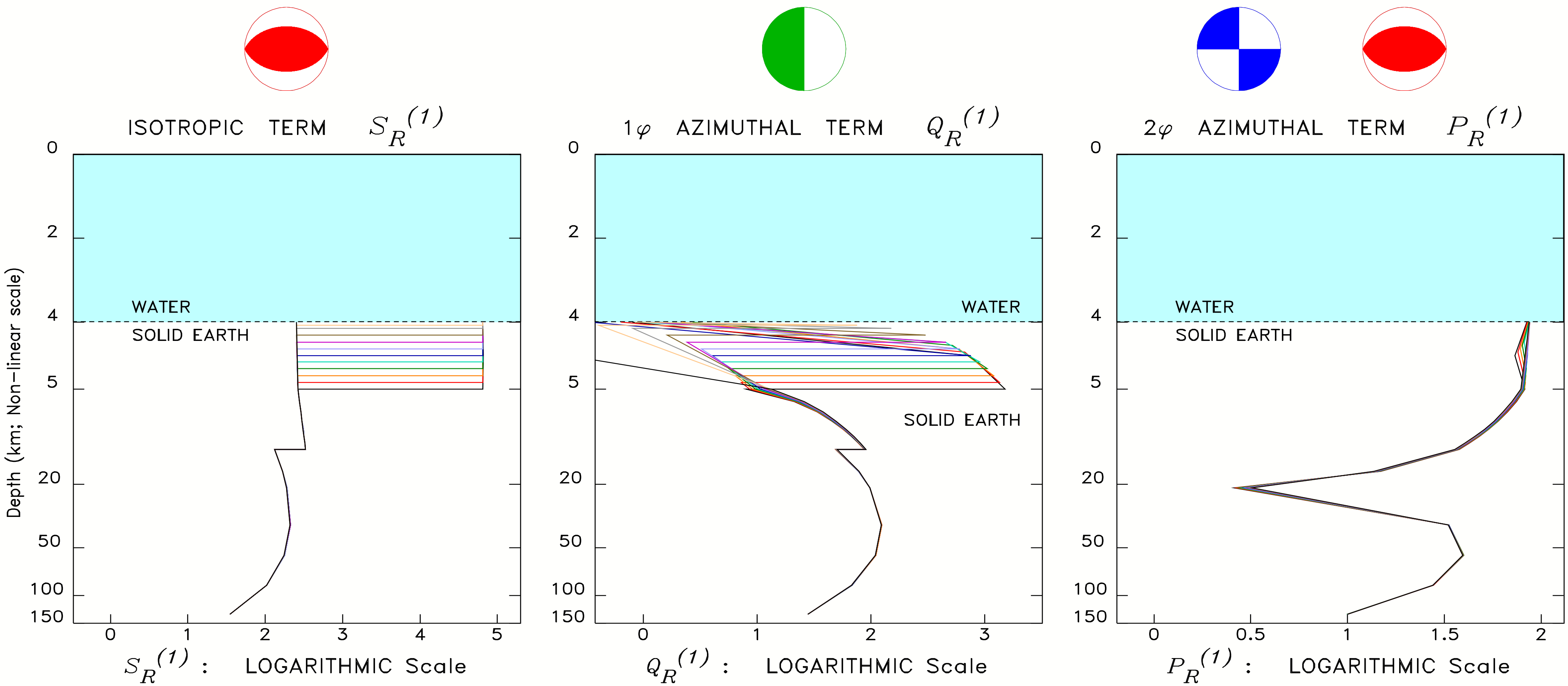
- We study excitation coefficients in the formalism of *Kanamori and Stewart* [1976].

→ Sediments affect only very marginally the coefficients $Q_R^{(1)}$ and $P_R^{(1)}$ in the hard rock.

Excited by Thrust faults

Excited by Vertical Dip-Slips

Excited by Strike-Slip [and Thrust] faults



EFFECT of SEDIMENTS on TSUNAMI WAVEFORMS

1. The Far Field ($10° \leq \Theta \leq 145°$)

We examine this question by comparing the *maximum amplitude* of tsunami waveforms built in the far field for each of the 12 structural models.

In each plot, the polar angle is the azimuth of the receiver relative to the fault line (taken as E−W) and the radius shows the distance to the receiver, increasing from 10° (inner pixel) to 145° (outer pixel).

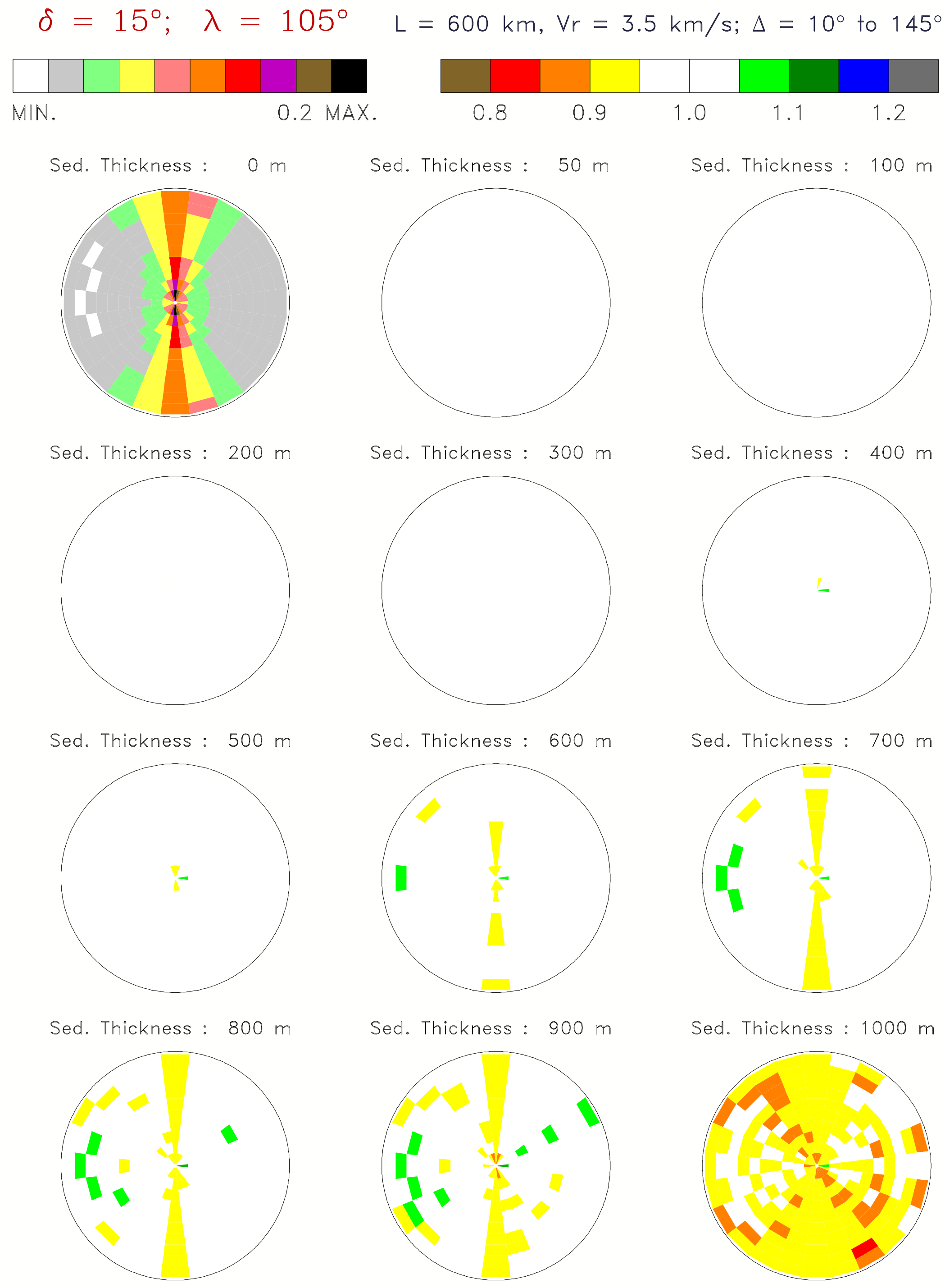
The first plot (upper left) shows the abolute distribution of amplitudes in the absence of sediments (left color palette), and expresses mostly the well-known effect of *directivity*.

The other plots show, for each pixel, the relative amplitude of the tsunami in the presence of sediments, *scaled to the reference un-sedimented case*, and uses the right color palette.

White means less than 5% change; cold colors mean an *increase* in amplitude; hot colors a *decrease* in amplitude.

Among the more than 200 available such plots, we present here one typical of a mega-event on a shallow dipping thrust fault:

Tsunami amplitudes are generally *decreased* by the presence of sediments but the magnitude of the effect is only marginal, reaching 15% only for the thickest layers.



EXTRACURRICULAR GEOPHYSICS

The occurrence of exceptional events, such as the 2004 Sumatra earthquake, occasionally gives rise to the recording of physical phenomena by instruments not designed for that purpose.

For example, a seismometer may record an air wave, a hydrophone may record a tsunami...

Such recording by *"unprepared" or "incompetent"* **instruments often times illustrates a physical coupling between the medium of the phenomenon and that where the instrument is supposed to operate.**

Such coupling being generally weak, requires a very large event (Sumatra, Maule...) to be detectable.

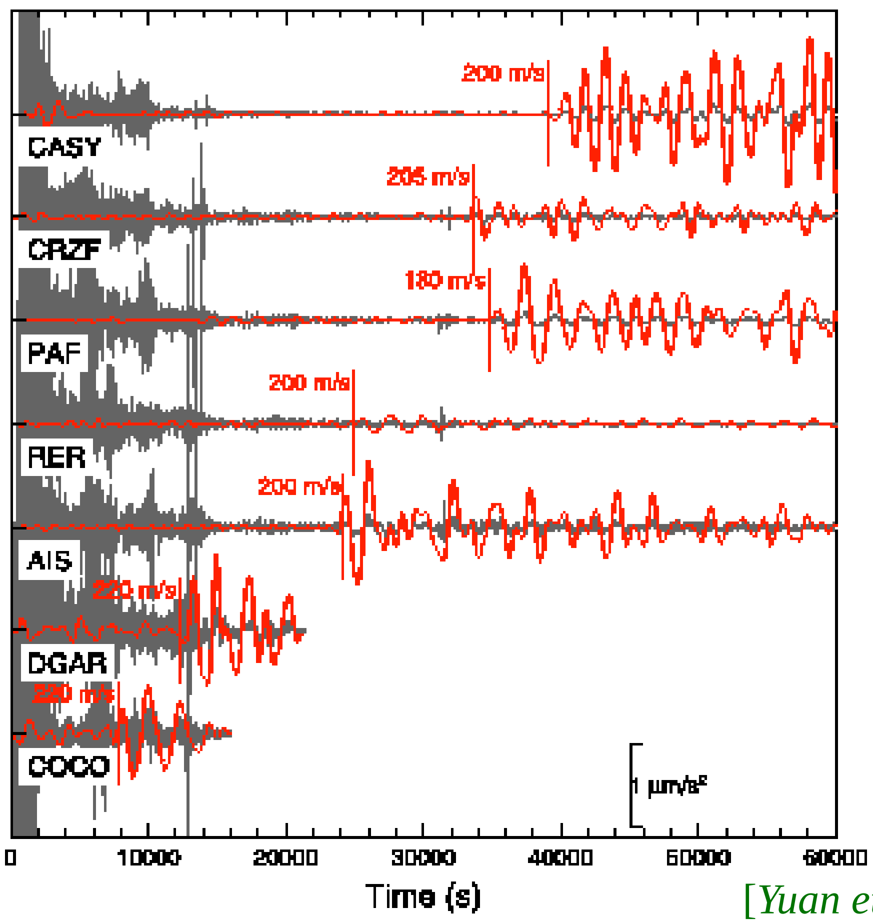
However, such instances of coupling are precious, since they shed light on some unsuspected properties of the physical waves and media involved.

SEISMOMETERS DETECT TSUNAMIS

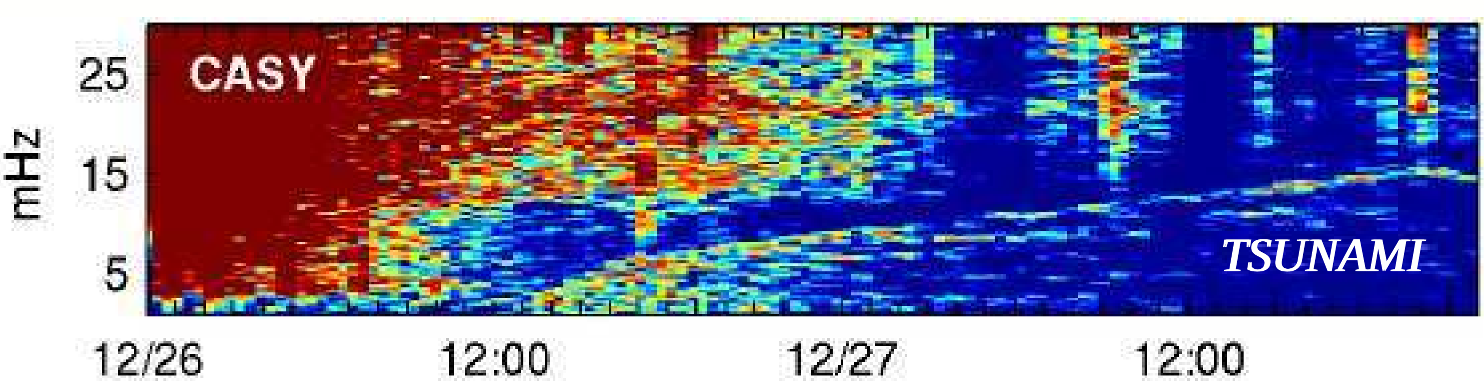
(The Seismic "DART" ?)

TSUNAMI RECORDED ON SEISMOMETERS

- Horizontal long-period seismometers (GEOSCOPE, IRIS...) record ultra-long period oscillations following arrival of 2004 tsunami at nearby shores [*R. Kind*, 2005].
- Energy is mostly between 800 and 3000 seconds
- Amplitude of equivalent displacement is **centimetric**



[*Yuan et al.*, 2005]



[*Hanson and Bowman*, 2005]

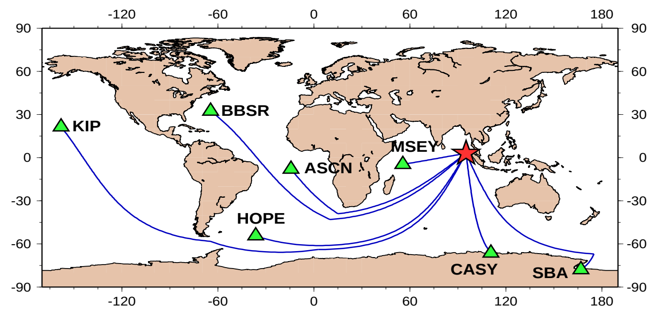
TSUNAMI RECORDED ON SEISMOMETERS (ctd.)

Enhanced Study [*E.A. Okal*, 2005−06].

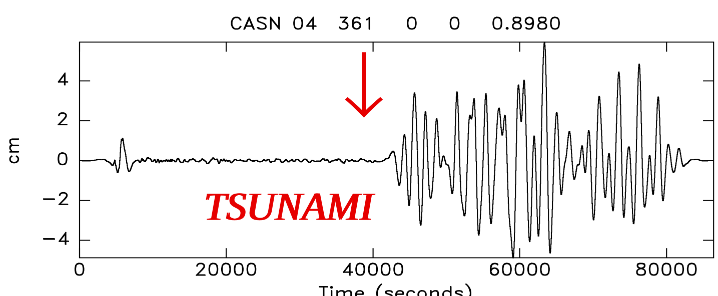
- *RECORDED* **WORLDWIDE** (On Oceanic shores)
- ***HIGHER FREQUENCIES*** (up to 0.01 Hz) *PRESENT* (in regional field)
- Tsunami detectable during **SMALLER EVENTS**
- *CAN BE* **QUANTIFIED**

SUMATRA 2004: TSUNAMI RECORDED ON SEISMOMETERS

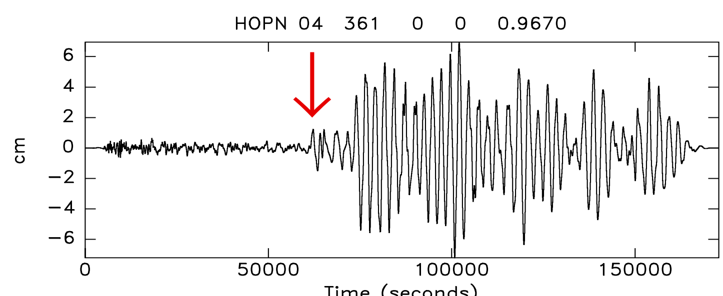
- Recording by shoreline stations is **WORLDWIDE** including in regions requiring strong refraction around continents (Bermuda, Scott Base).



Casey, Antarctica, 8300 km

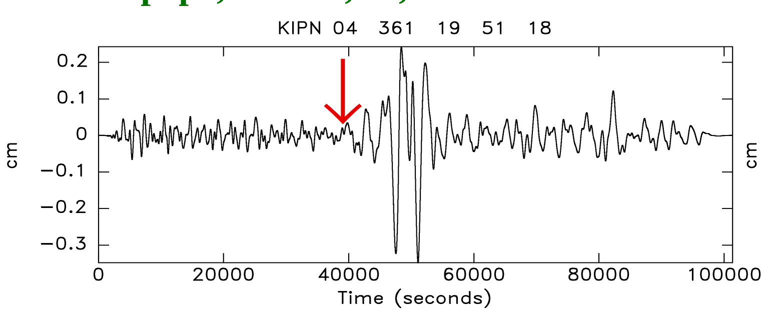


Hope, South Georgia, 13100 km

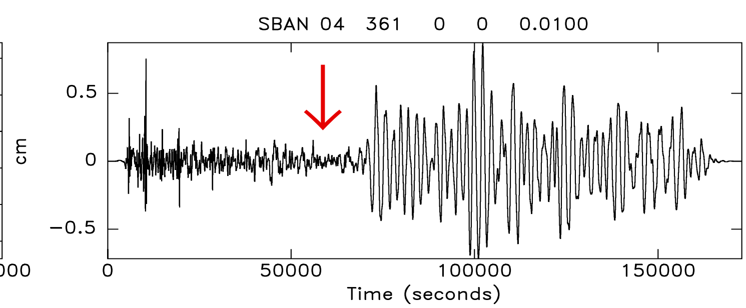


Filtered $100 < T < 10000$ s.

Kipapa, Hawaii, 27,000 km

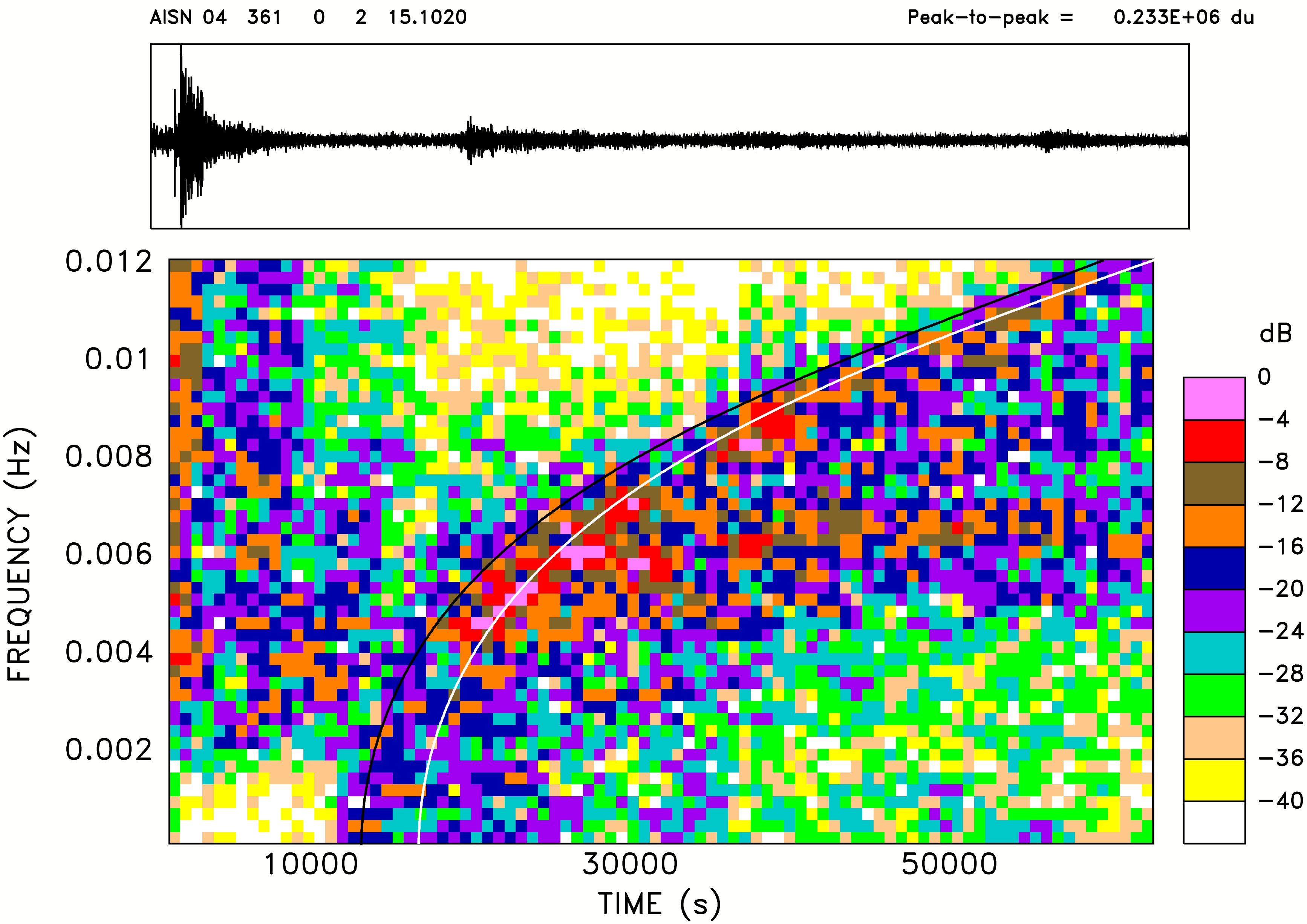


Scott Base, Antarctica, 10400+ km



Dispersed energy resolved down to T = 80 s.

Ile Amsterdam, 26 Dec. 2004



NOTE STRONG HIGH-FREQUENCY TSUNAMI COMPONENTS

QUANTIFYING the SEISMIC RECORD at CASY

- Assume that seismic record (*e.g.*, at CASY) reflects response of seismometer to the *deformation of the ocean bottom.*

FORGET THE ISLAND (or continent) !

- Use *Gilbert*'s [1980] combination of displacement, tilt and gravity;

Apparent Horizontal Acceleration (*Gilbert*'s [1980] Notation):

$$AV = \omega^2 V - r^{-1} L\,(g\,U + \Phi)$$

or (*Saito*'s [1967] notation):

$$y_3^{APP} = y_3 - \frac{1}{r\,\omega^2} \cdot (g\,y_1 - y_5)$$

- Use *Ward*'s [1980] normal mode formalism;

Evaluate Gilbert response on solid side of ocean floor, and derive equivalent spectral amplitude of surface displacement $y_1(\omega) = \eta(\omega)$.

- Use *Okal and Titov*'s [2005] Tsunami Magnitude, inspired from *Okal and Talandier*'s [1989] M_m ;

- Apply to CASY record at maximum spectral energy ($S(\omega)$ = 4000 cm*s at T = 800 s).

$\rightarrow$ Find $\mathbf{M_0 = 1.7 \times 10^{30}}$ **dyn – cm.**

Published: 1.15×10^{30} dyn*cm [*Stein and Okal*, 2005; *Tsai et al.*, 2005]

Acceptable, given the extreme nature of the approximations.

$\rightarrow$ Suggests that the signal is just the expression of the horizontal deformation of the ocean floor, and that

CASY functions in a sense like an OBS !!

QUANTIFICATION of SEISMIC TSUNAMI RECORDS

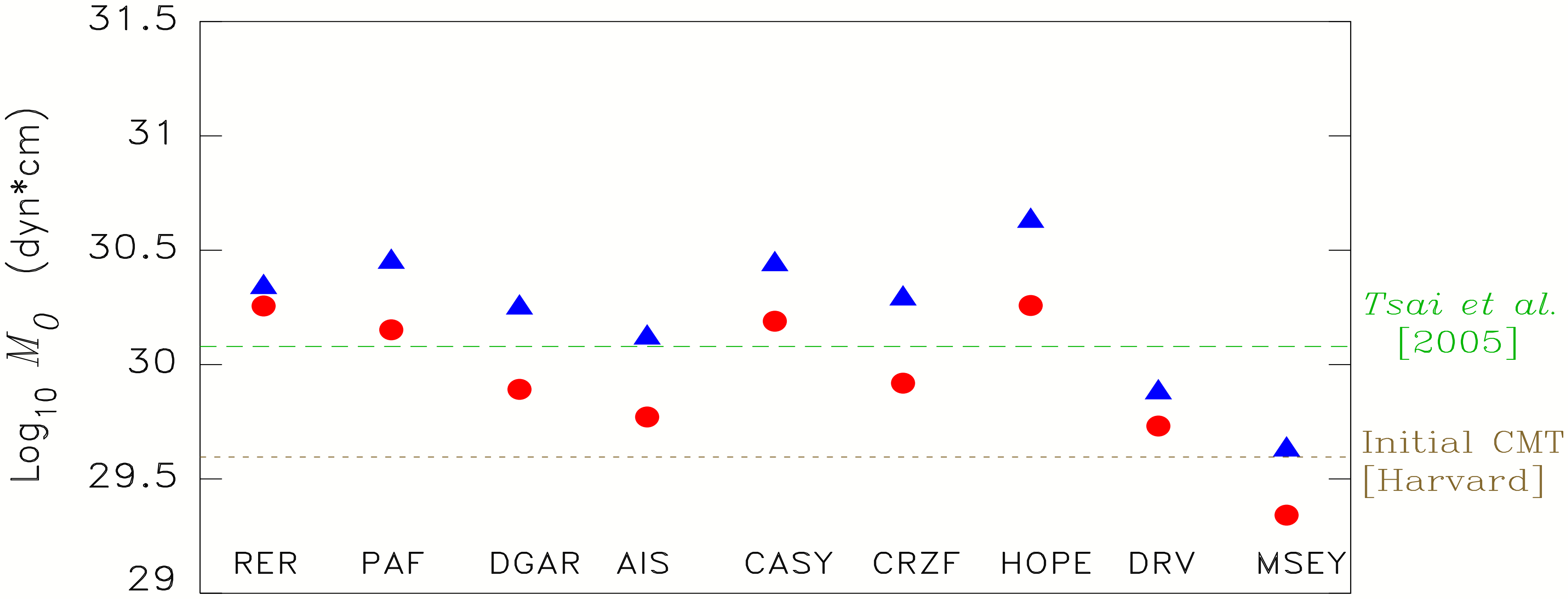
- Apply technique to dataset of 10 stations with direct great circle paths
- Use either Full Source computation (**Red Symbols**)

$$\overline{M_0} = 1.6 \times 10^{30} \text{ dyn} - \text{cm}$$

or M_{TSU} magnitude approach (**Blue Symbols**)

$$\overline{M_0} = 2.1 \times 10^{30} \text{ dyn} - \text{cm}$$

In good agreement with *Nettles et al.* [2005] and *Stein and Okal* [2005] (green dashed line)



STATIONS

NOTE: DRV and MSEY affected by *substantial continental shelves.*

USING AN ISLAND SEISMOMETER AS A "DART" SENSOR?

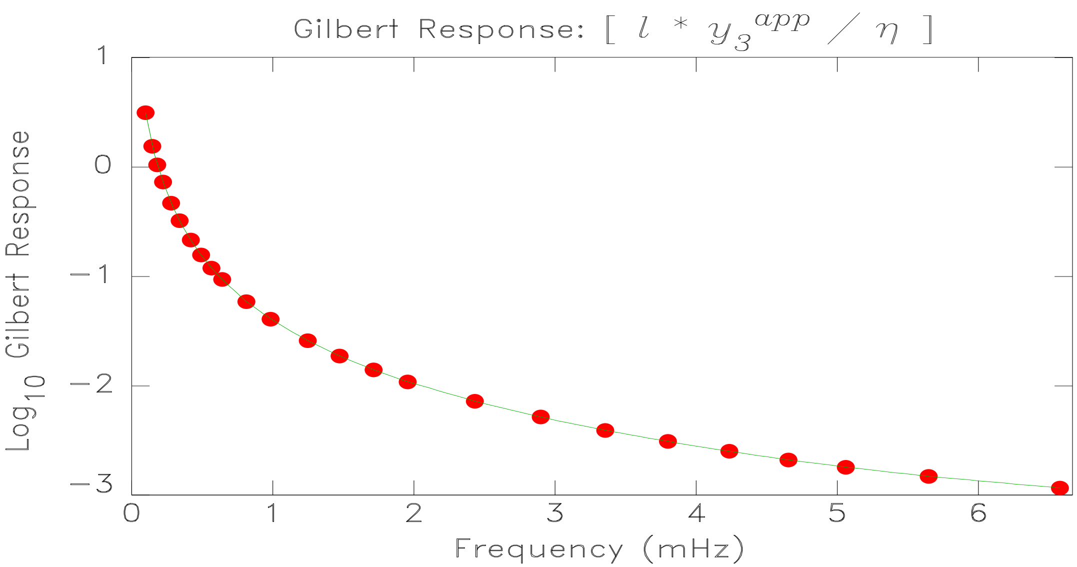
Example: Ile Amsterdam, 26 DEC 2004 (d= 5800 km)

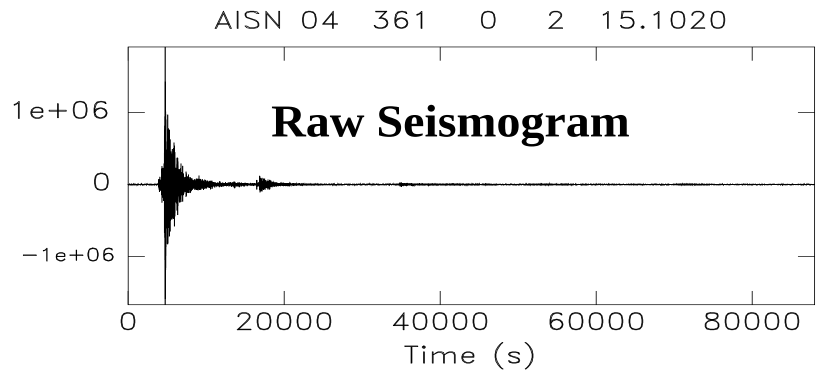
- A horizontal seismometer at a shoreline location can record a tsunami wave.

- Once the instrument is deconvolved, we obtain an apparent horizontal ground motion of the ocean floor

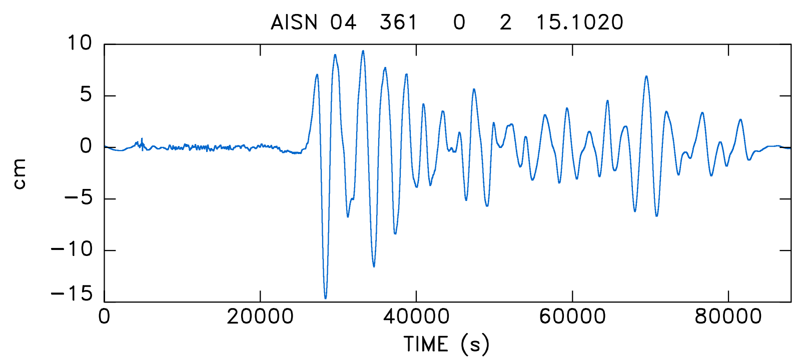
- Further deconvolve the *"**G**ilbert **R**esponse **F**actor"* $[l\, y_3^{app} / \eta]$ and obtain the time series of the surface amplitude of the tsunami.

- The ***G R F*** can be computed from normal modes

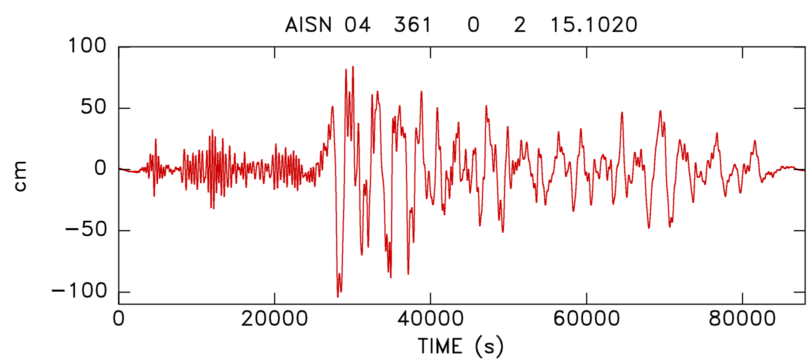




Deconvolve Instrument: **Apparent Ground Motion**

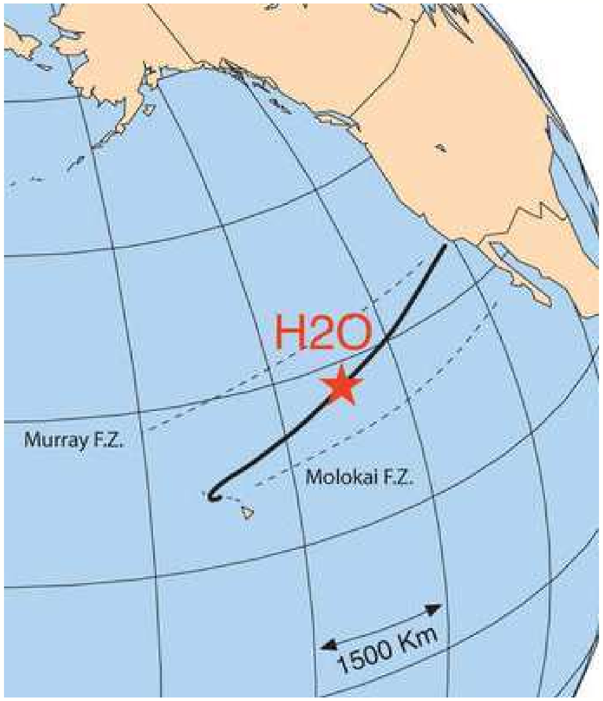


Deconvolve *GRF:* **"Tsunami Record"**



We find an amplitude $\eta \approx 80$ cm; Satellite Altimetry measured 70 cm in Bay of Bengal

TSUNAMIS RECORDED ON O.B.S.

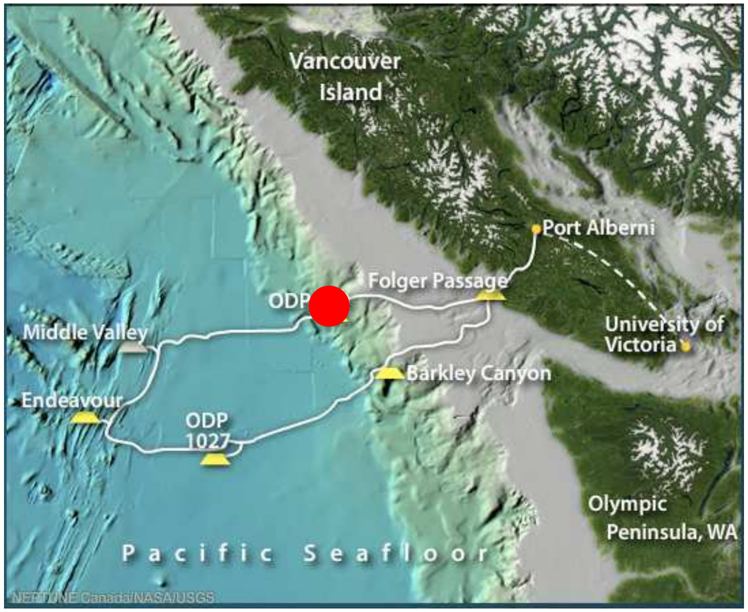


The mourning of H 2 O

5 October 1999 — 26 May 2003

The Rise of Neptune

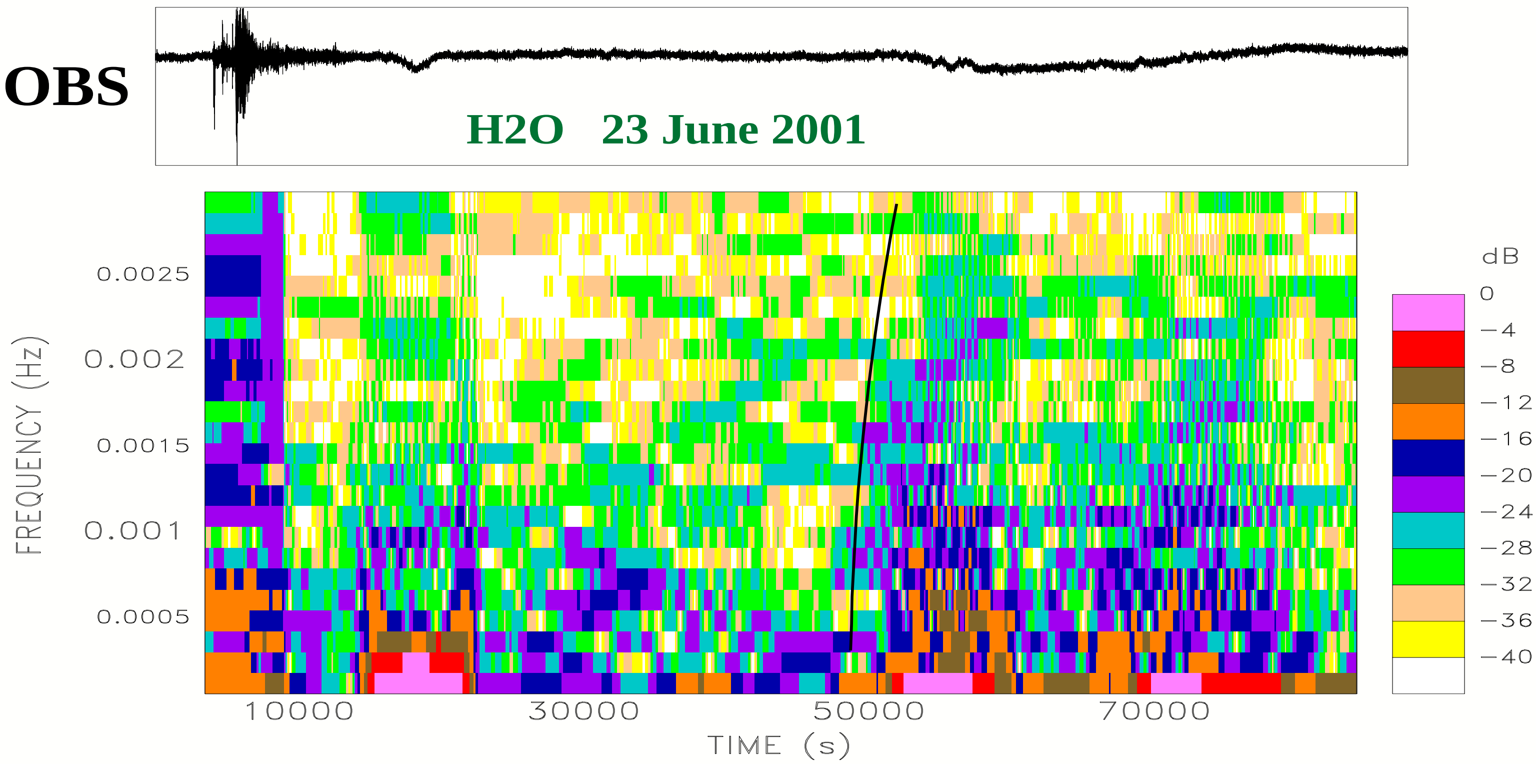
b. 13 october 2010

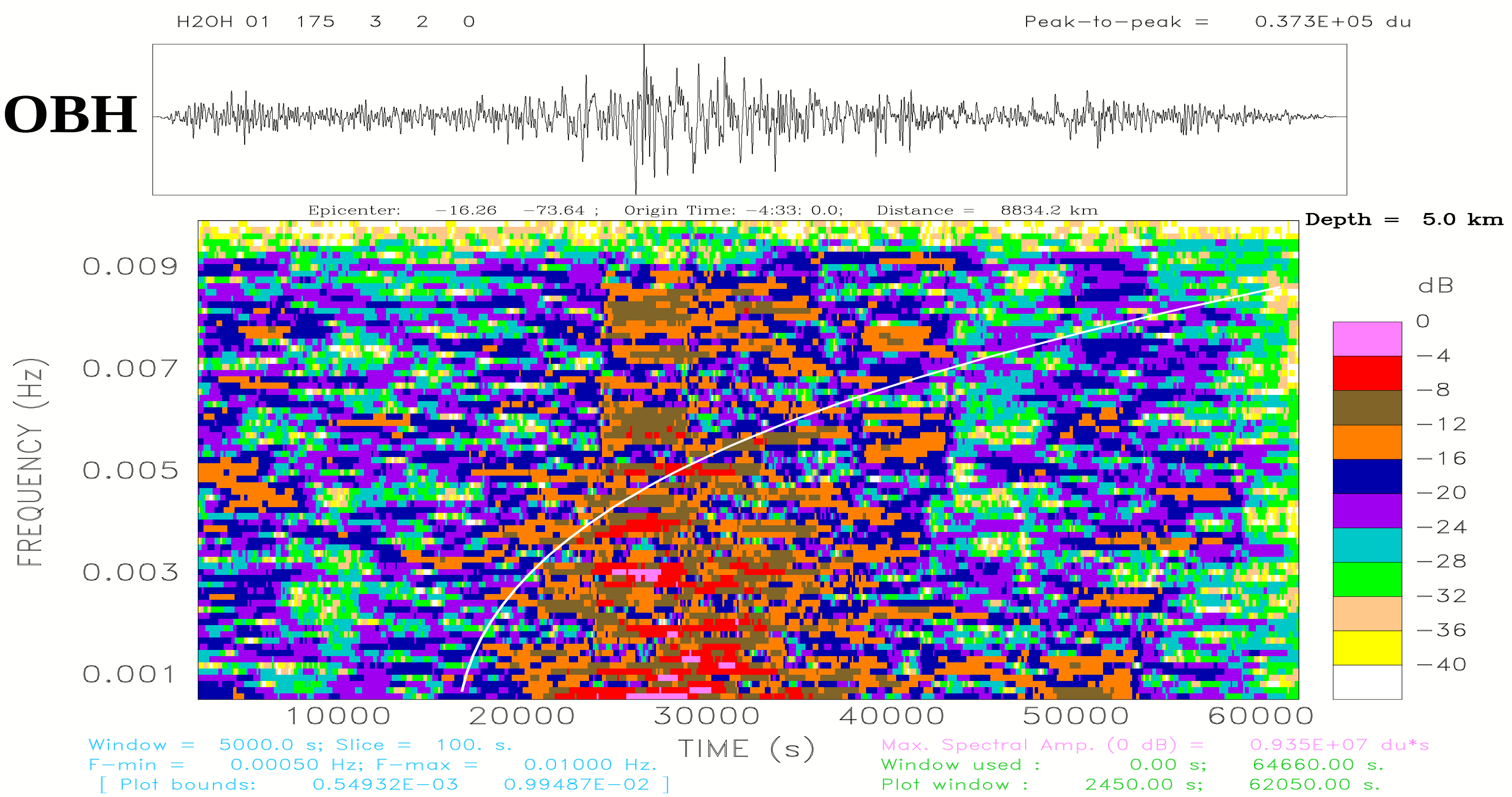


H2O: THE LONE TSUNAMI

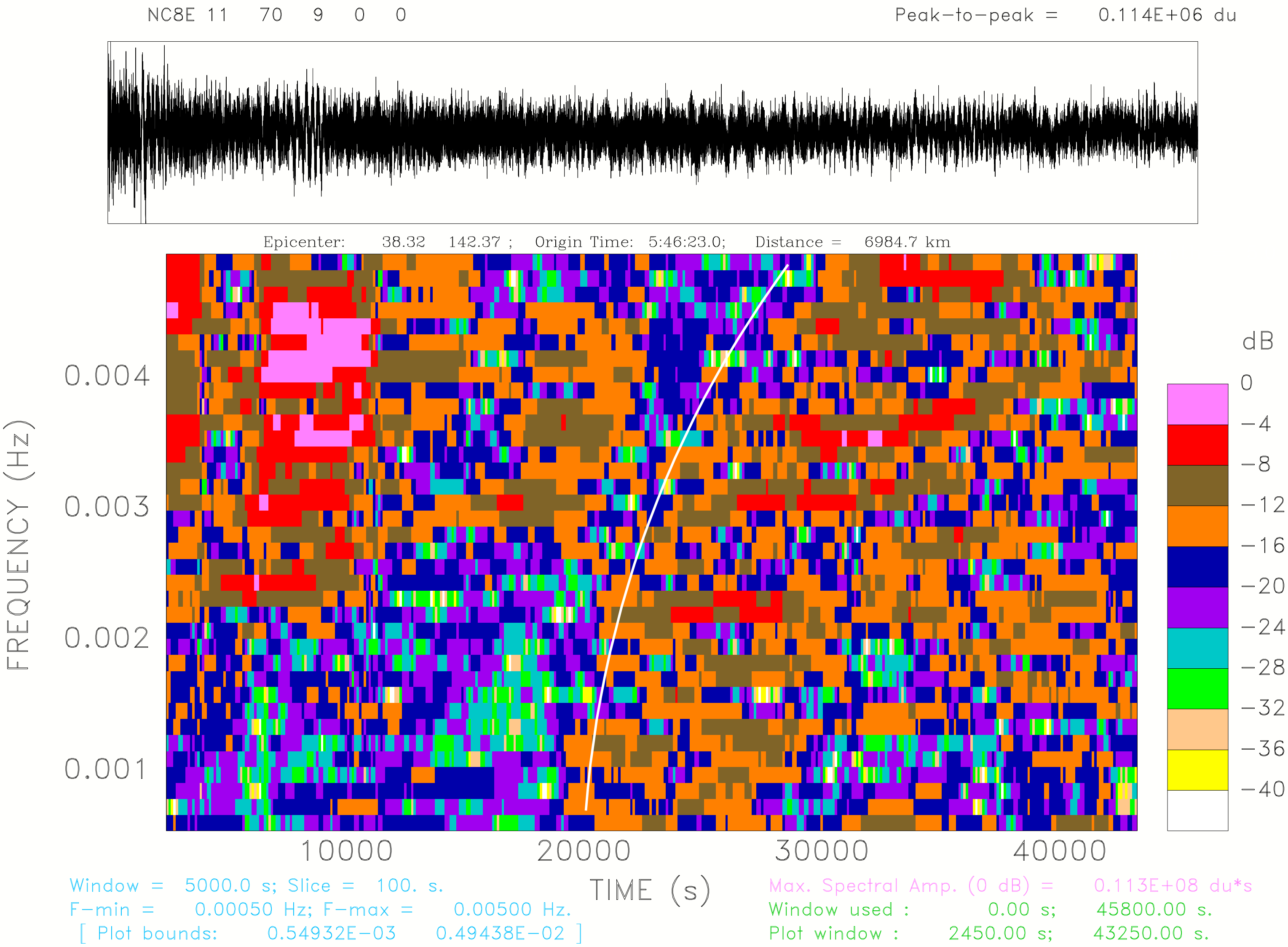
During its short operation, H2O recorded one significant tsunami: the Peruvian event of 23 June 2001.

While the event is clearly detected, both by the horizontal OBS and by the hydrophone, the recording characteristics are strongly non-linear, possibly raising doubt about the coupling of the instrument to the ocean bottom. At any rate, such signals cannot be quantified.





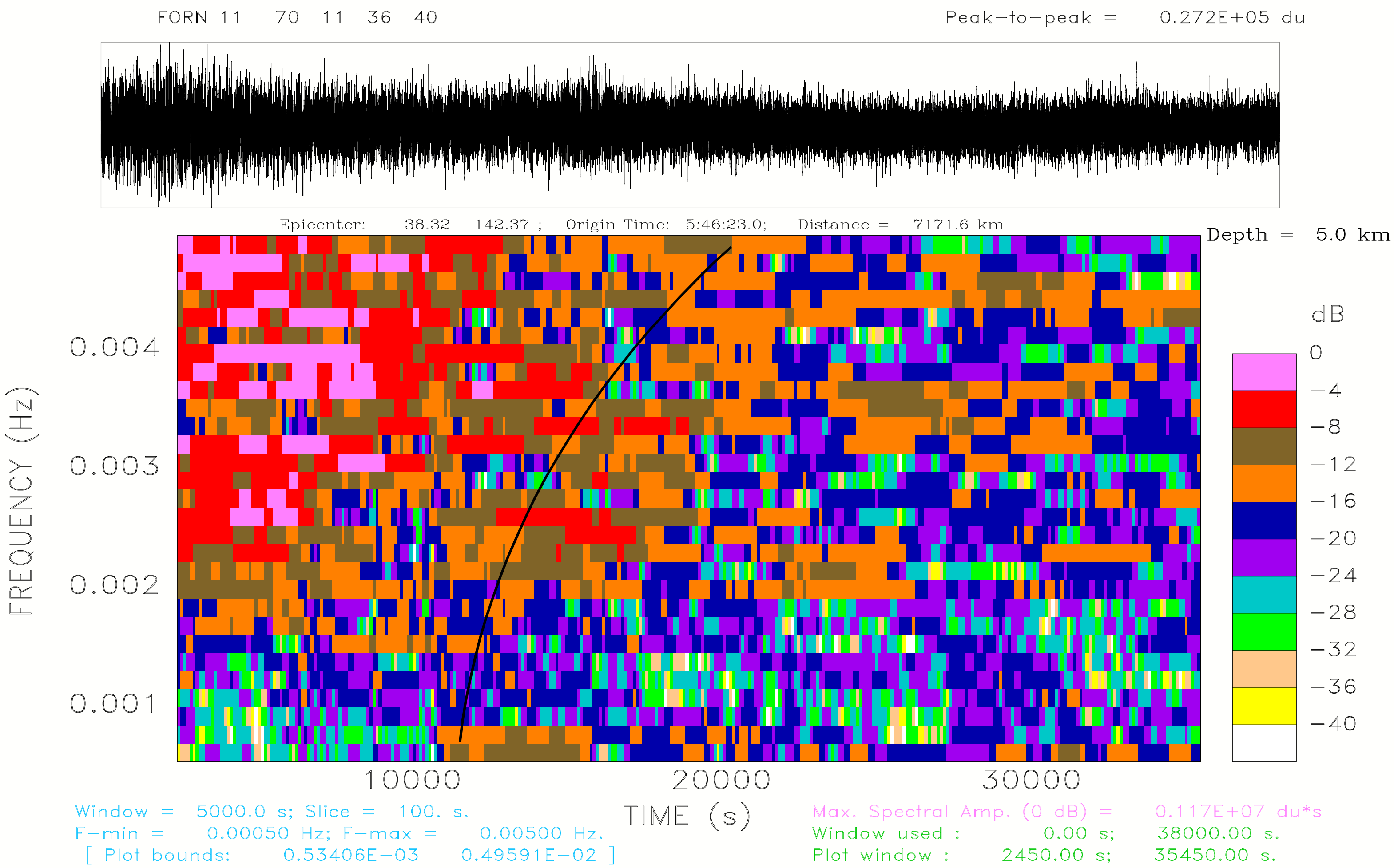
2011 TOHOKU TSUNAMI RECORDED at NEPTUNE



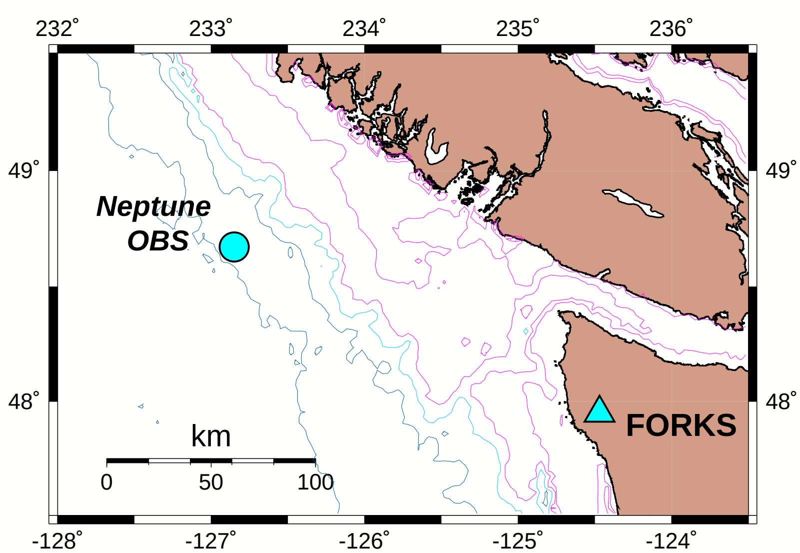
Inferred Moment: $M_0 = 3.3 \times 10^{29}$ dyn*cm at 1130 s

[NEIC *W* phase moment = 4.0×10^{29} dyn*cm]

2011 ONLAND RECORD at FORKS, Washington



Inferrred Moment: $M_0 = 5.3 \times 10^{29}$ dyn*cm at 1400 s



Note: "Least Bad station"

with relatively poor site:

5 km inland

35−km wide continental shelf

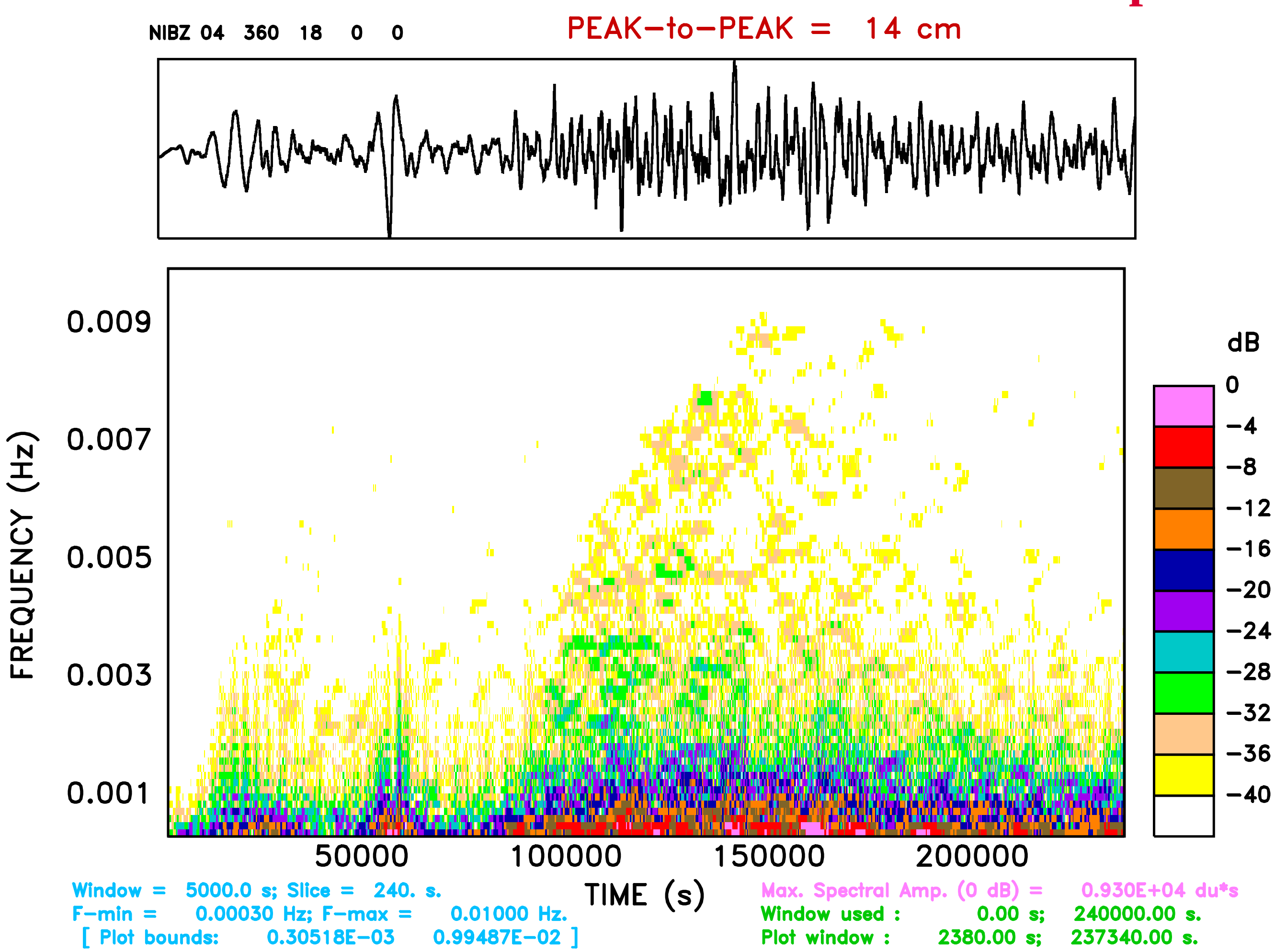
***YET,* General order-of-magnitude agreement with OBS value (and NEIC moment) validates argument that onland station functions as *de facto* O.B.S.**

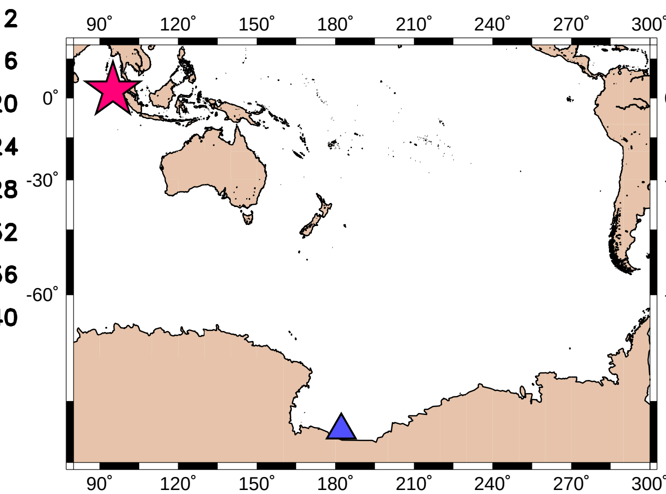
THE FLOATING SEISMOMETER

2004 TSUNAMI RECORDED on ICEBERGS

Since 2003, we had been operating seismic stations on detached and nascent icebergs adjoining the Ross Sea.

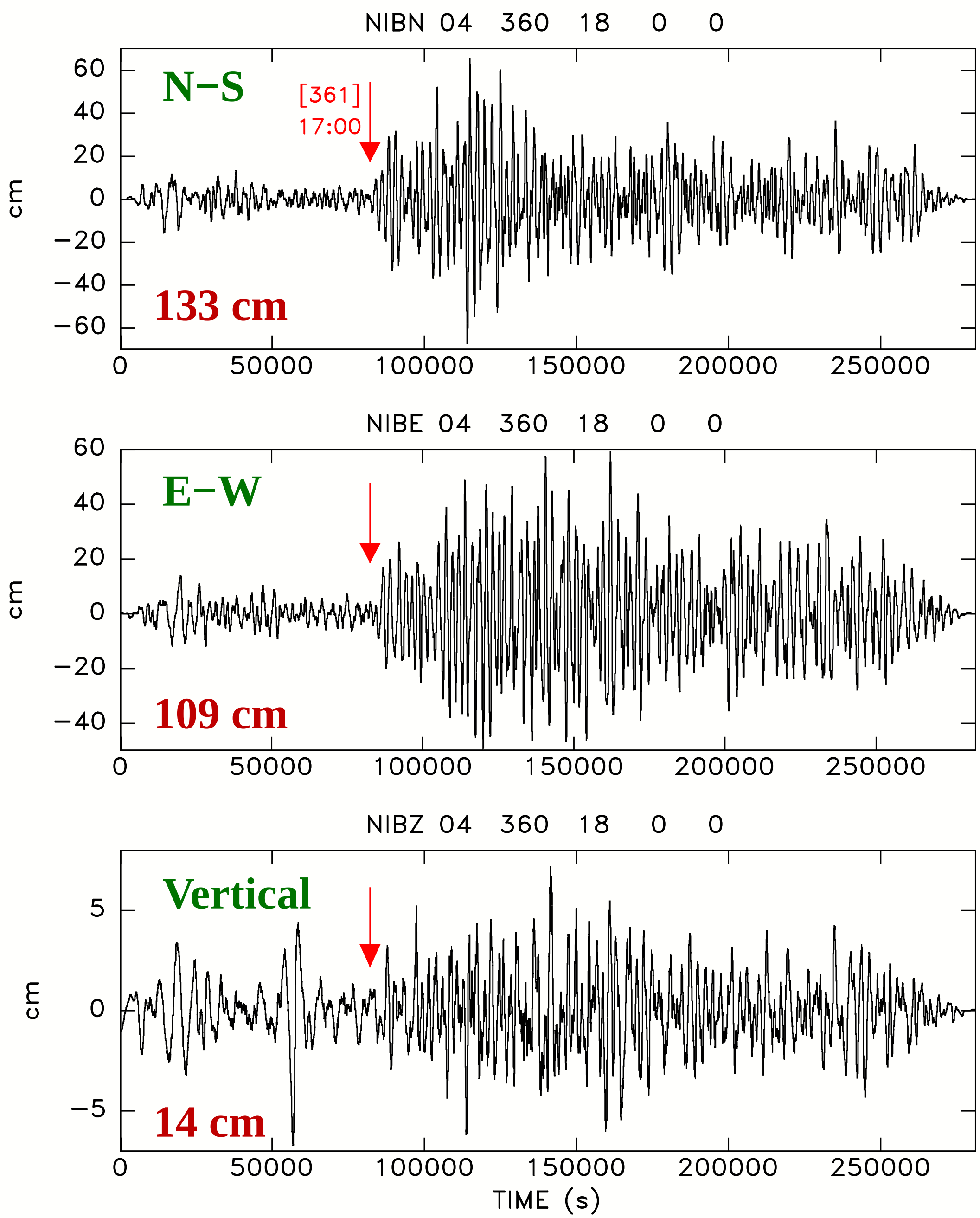
The tsunami was recorded by our 3 seismic stations, on all 3 components, with amplitudes of 10−20 cm.





Seismic recordings of 2004 Sumatra Tsunami on Iceberg

Nascent (NIB); *26 DECEMBER 2004*



This time, the iceberg (and the seismometer) float like a raft on the sea and **record directly the 3-dimensional displacement of the tsunami.**

In the Shallow-Water Approximation,

$$AR = \frac{u_x}{u_z} = \frac{1}{\omega}\sqrt{\frac{g}{h}}$$

Iceberg:

$T = 500$ s; $h = 500$ m $\quad AR \approx 11$

FIRST DIRECT MEASUREMENT OF HORIZONTAL COMPONENT OF TSUNAMI ON THE HIGH SEAS

ELLIPTICITY of TSUNAMI SURFACE MOTION

(Shallow Water Approximation)

$$AR = \frac{u_x}{u_z} = \frac{1}{\omega}\sqrt{\frac{g}{h}}$$

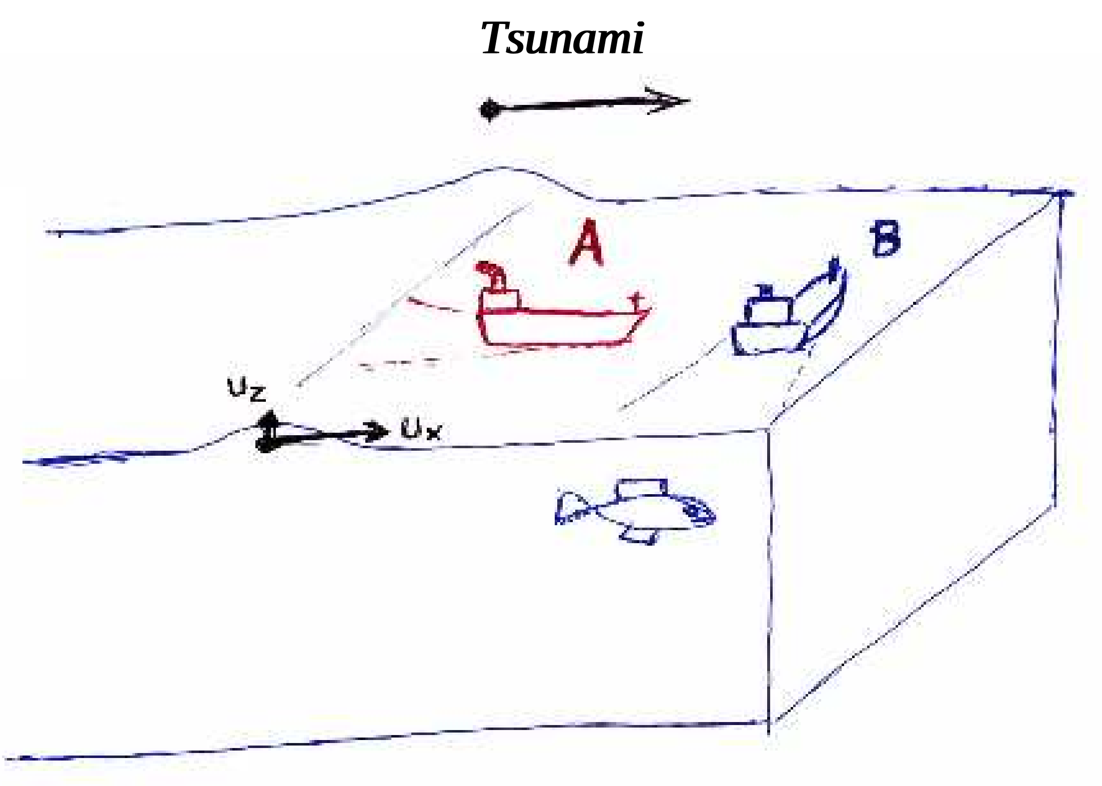
On the high seas (T = 1000−2000 s; h = 2000 – 5000 m),

AR can be typically between **10 and 25.**

Sumatra 2004: $u_z \approx 1$ m (JASON; seismic stations)

$u_x \approx 15$ meters ?

Conceivable to use GPS-equipped ships to detect tsunami.



Ship A should see a perturbation in speed

Ship B would show a zig-zag in trajectory

CTBT HYDROPHONES

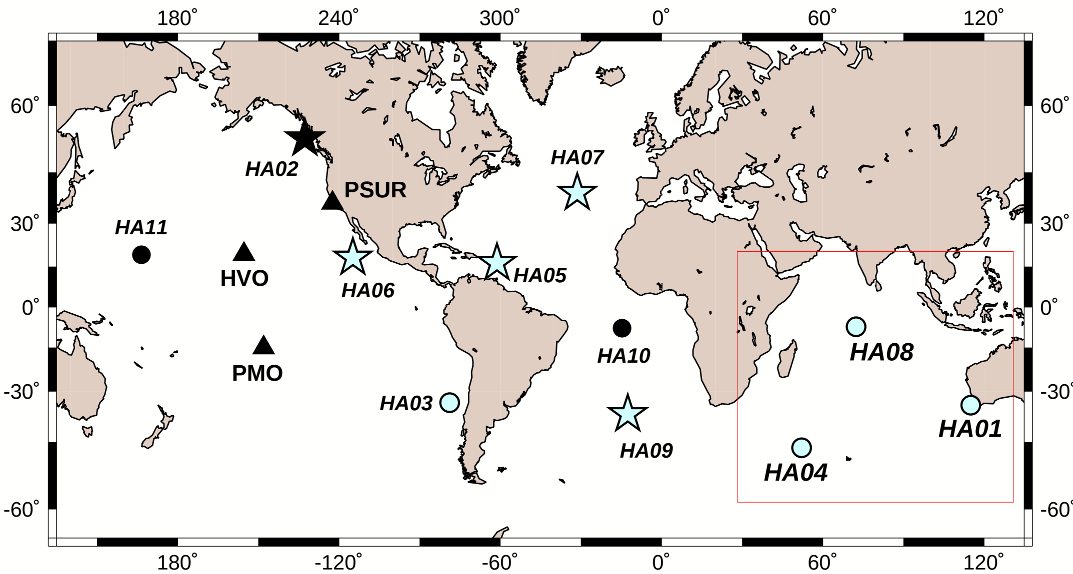
DETECT TSUNAMI

or

One Filter Too Many !

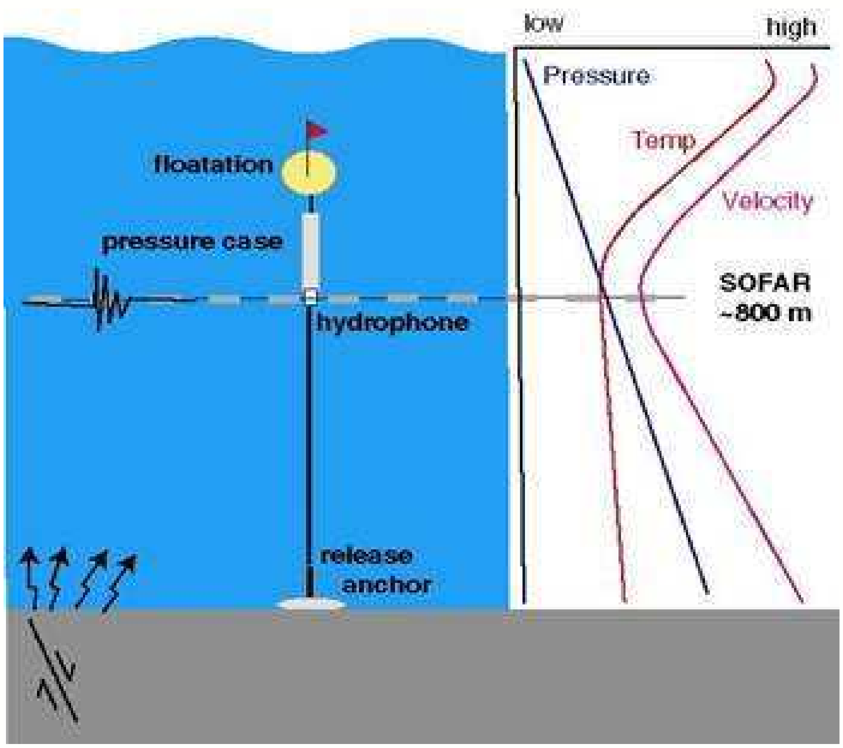
CTBT HYDROPHONE RECORDS

In the context of the CTBTO ("Test-Ban Treaty Organization"), the **I**nternational **M**onitoring **S**ystem comprises six hydrophone stations deployed in the SOFAR channel, including three in the Indian Ocean.



Diego Garcia, BIOT

Each station features several (3−6) sensors, allowing *beaming* of the array



[*M. Tolstoy*, Columbia University]

These instruments recorded not only the hydroacoustic ("***T***") waves generated by the earthquake, but also its conventional seismic waves (Rayleigh), and most remarkably,

the tsunami itself.

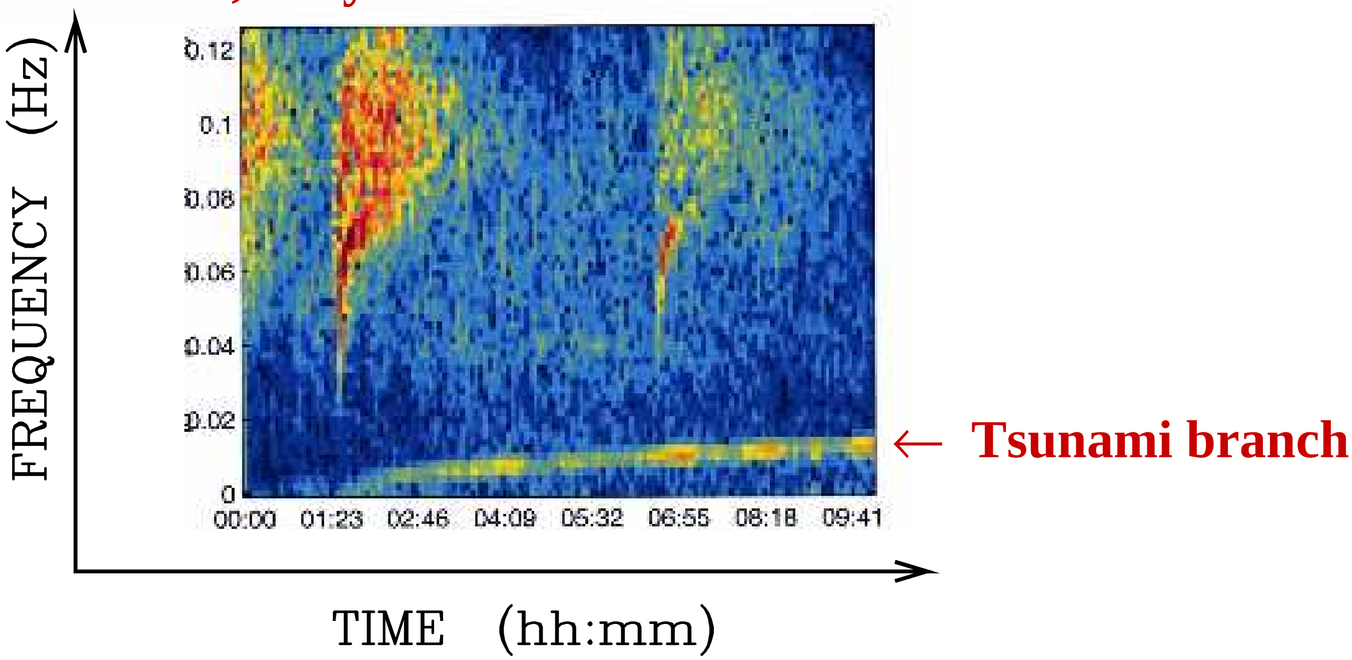
[*Okal et al.*, 2006]

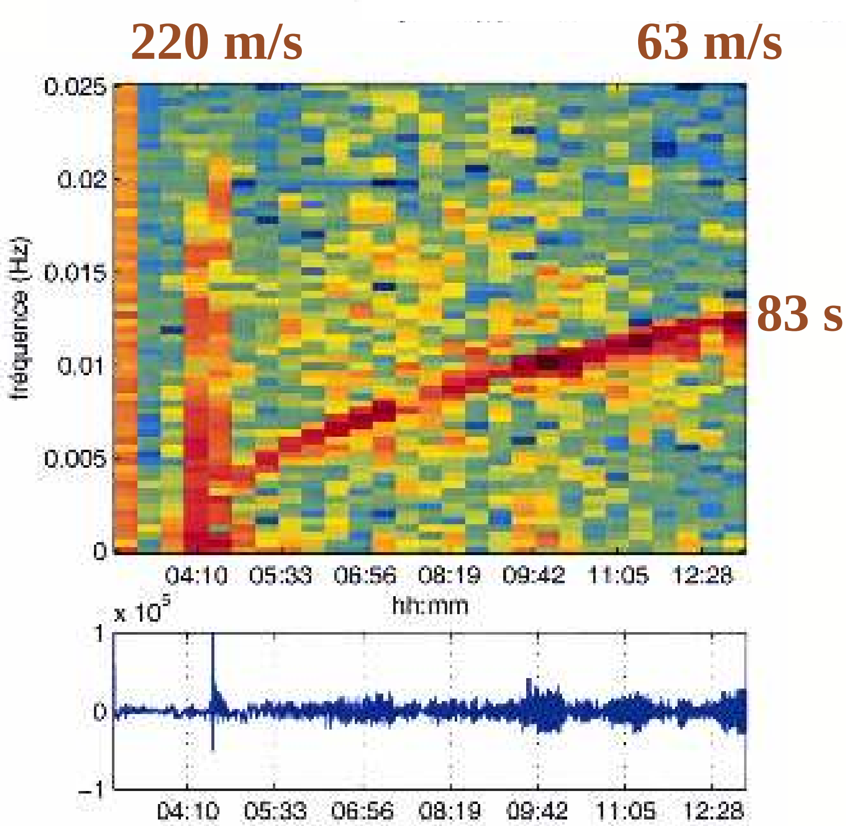
TSUNAMI recorded by HYDROPHONES of the CTBTO

(hanging in ocean at 1300 m depth off Diego Garcia)

→ Instruments are severely filtered at infra-acoustic frequencies.

YET, they recorded the TSUNAMI!





Note first ever observation of *DISPERSION* of tsunami branch at *VERY HIGH* [tsunami] frequencies in the far field

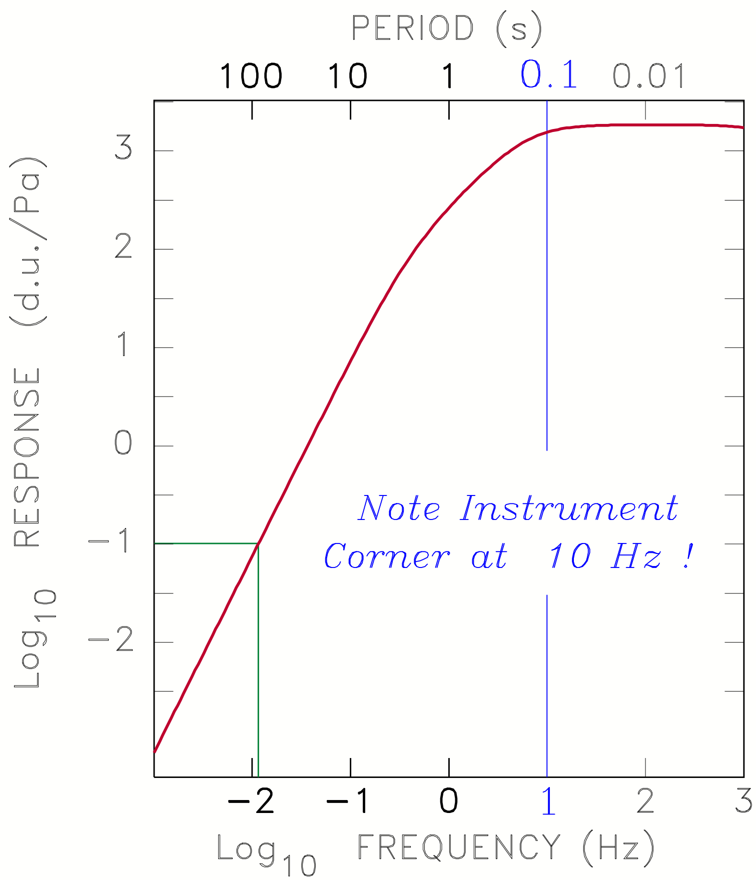
$$\omega^2 = g\,k \cdot \tanh(k\,h)$$

All of this on the high seas, unaffected by coastal response.

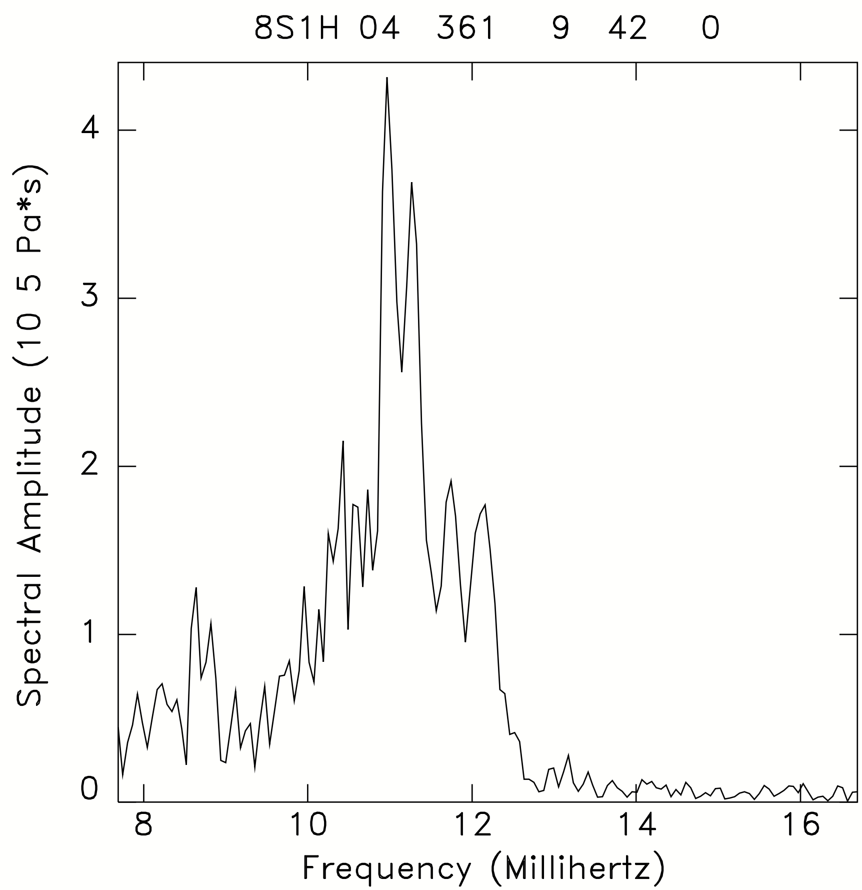
NOTE STRONG HIGH-FREQUENCY TSUNAMI COMPONENTS

Retrieving Seismic Moment from High-Frequency Tsunami Branch

- Use Hydrophone H08S1 from IMS at Diego-Garcia (BIOT)
- Deconvolve instrument and retrieve pressure spectrum



Note Instrument Response Down
by Factor 17,800 at 87 s.

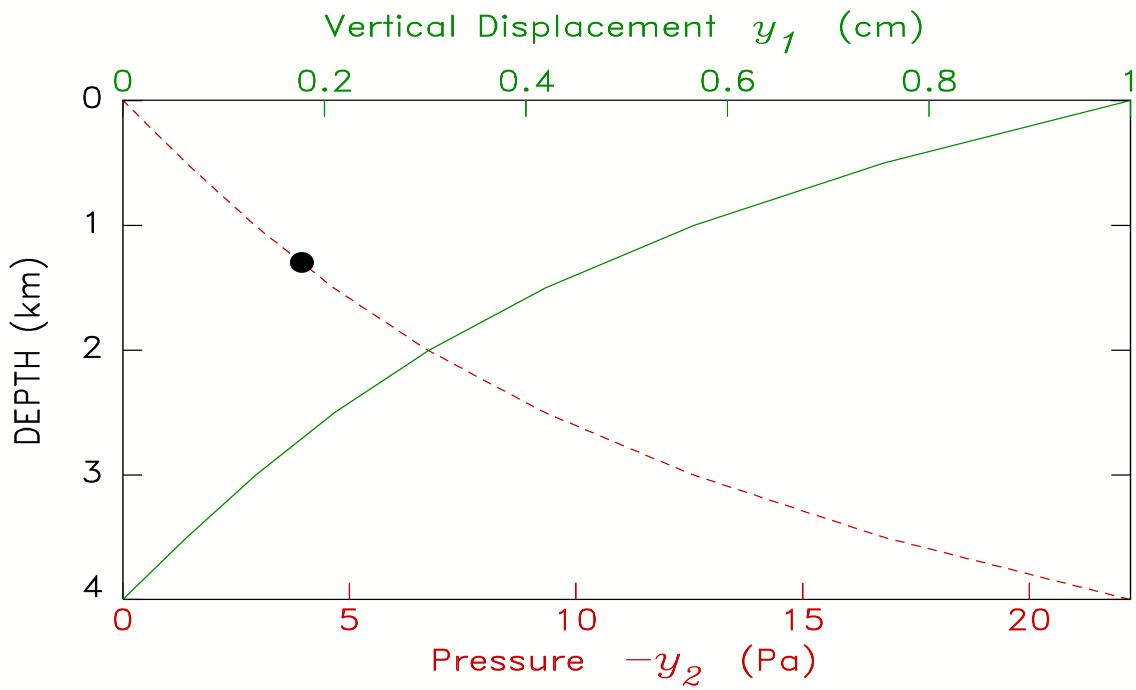


$$P(\omega) = 0.35\ \mathrm{MPa} * \mathrm{s}\ \ \text{at}\ \ 87\ \mathrm{s}$$

Retrieving Seismic Moment from High-Frequency Tsunami Branch (ctd.)

- Use *Okal* [1982; 2003; 2006] to convert overpressure at 1300 m depth (0.35 MPa*s) to surface amplitude η ,

outside classical Shallow-Water Approximation.



Structure of tsunami eigenfunction through water column outside S.W.A.

Find $\eta(\omega) = 78000$ cm*s at $T = 87$ s.

- Use *Haskell* [1952], *Kanamori and Cipar* [1974], *Ward* [1980], *Okal* [1988; 2003] in normal mode formalism to compute excitation coefficients.

Find $\boldsymbol{M_0 = 8 \times 10^{29}}$ **dyn – cm**

ACCEPTABLE !

(Moment from Earth's free oscillations: $\mathbf{1}$ **to** $\mathbf{1.2 \times 10^{30}}$ **dyn-cm**)

[*Stein and Okal*, 2005; *Nettles et al.*, 2005]

CONCLUSION: We understand ***QUANTITATIVELY*** the **excitation of the high-frequency components of the tsunami...**

TOAMASINA, Madagascar 26-DEC-2004

(a)

(b)

(c)

Figure 5. *(a):* The 50−m freighter *Soavina III* photographed on 2 August 2005 in the port of Toamasina. *(b):* Sketch of the port of Toamasina showing its complex geometry. *(c):* Captain Injona uses a wall map of the port (ESE at top) to describe the path of *Soavina III* from her berth in Channel 3B (pointed on map), where she broke her moorings around 7 p.m., wandering in the channels up to the location of the red dot (also shown on Frame *b*), before eventually grounding in front of the Water-Sports Club Beach (white dot; Site 17).

50−m SHIP BROKE MOORINGS around 19:00 (GMT+3), FOUR HOURS AFTER MAXIMUM WAVES

Preliminary modeling for Toamasina [Tamatave], Madagascar

[*D.R. MacAyeal*, pers. comm., 2006]

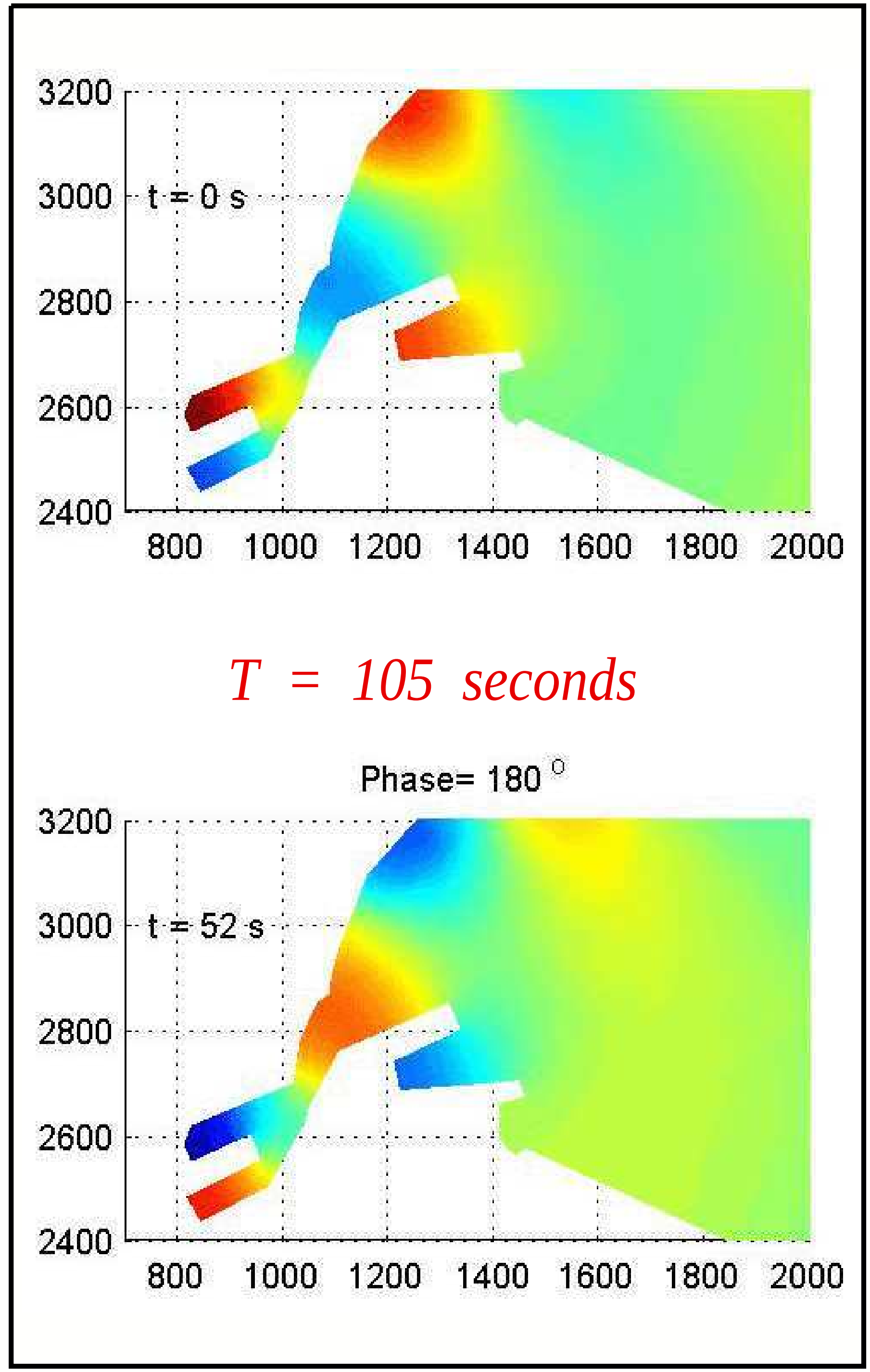
- Finite element modeling of the oscillations of the port of Toamasina reveals a fundamental mode of oscillation at T = 105 s, characterized by sloshing back and forth of water into the interior of the harbor, thus creating strong *currents* at the berth of *Soavina III*.

- At this period, the group velocity of the tsunami wave is found to be **97 m/s** for an average ocean depth of 4 km.

- This would correspond to an arrival at **16:55 GMT, or 19:55 Local Time**.

- This is in good agreement with the Port Captain's testimony

  "After 7 p.m. and lasting several hours"



DELAYED HARBOR OSCILLATIONS ARE INDEED SYSTEMATIC

- **2004 Sumatra:**
  Le Port, Réunion
  Salalah, Oman
  Dar-es-Salaam, Tanzania

- **2006 Kuriles:**
  Crescent City, Calif.

- **2010 Chile:**
  Puamau, Marquesas
  Los Angeles, Calif.

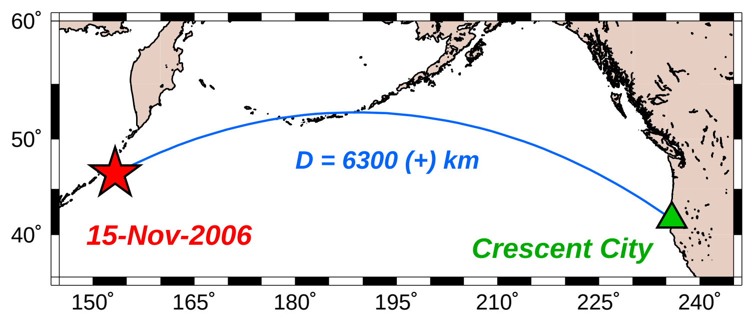
They emphasize the subtle character of sounding an all-clear...

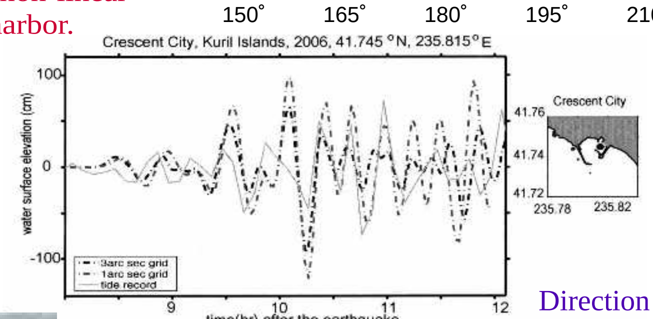
CHILE 2010: PUAMAU, Hiva Oa, Marquesas

- In the village of Puamau, the Mayor reported that a launch from the supply ship *Ara Nui* was flung onto the wharf (*Arrow*; altitude 1.5 m) **around 2 p.m. local time,** *i.e.,* 2.5 hours after the all-clear, and 7 hours after the first arrivals.

2006 KURIL TSUNAMI DID SIGNIFICANT DAMAGE in CRESCENT CITY, California, TWO HOURS AFTER THE "MAIN" WAVES

- Harbor struck 8.5 hours after seismic O.T.
- Damage reached US$ 15 million
- Wave height reached 1.7 m (pk-to-pk) on local tide gauge
- Damage resulting from (i) beaming of some tsunami energy towards Northern California; (ii) non-linear amplification by bay and harbor.





Tidal gauge record

[*Uslu*, 2007]

Damage to docks in harbor

Direction of flow into harbor

Docks H, G, F severely damaged

FROM GROUND UP ...

or

Could Ionospheric Seismology

Help Tsunami Warning ?

IONOSPHERIC RADAR DETECTS SEISMIC RAYLEIGH WAVE 150 km UP !

Tokachi Oki — 16 May 1968

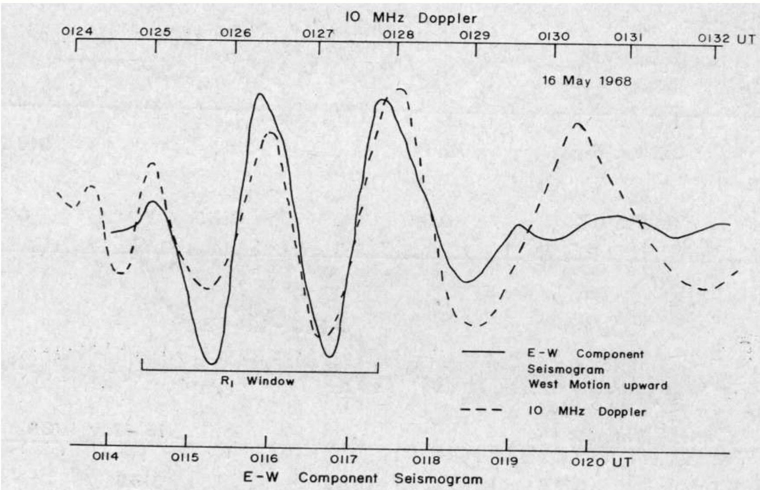


Fig. 5. Comparison of Doppler and longer-period seismogram data.

JOURNAL OF GEOPHYSICAL RESEARCH, SPACE PHYSICS VOL. 74, NO. 9, MAY 1, 1969

Continuous, Traveling Coupling between Seismic Waves and the Ionosphere Evident in May 1968 Japan Earthquake Data

P. C. YUEN, P. F. WEAVER, AND R. K. SUZUKI

Department of Electrical Engineering
University of Hawaii, Honolulu, Hawaii 96822

A. S. FUROMOTO

Hawaii Institute of Geophysics
University of Hawaii, Honolulu, Hawaii 96822

Kuriles, 11 August 1969 *Detected in Hawaii*

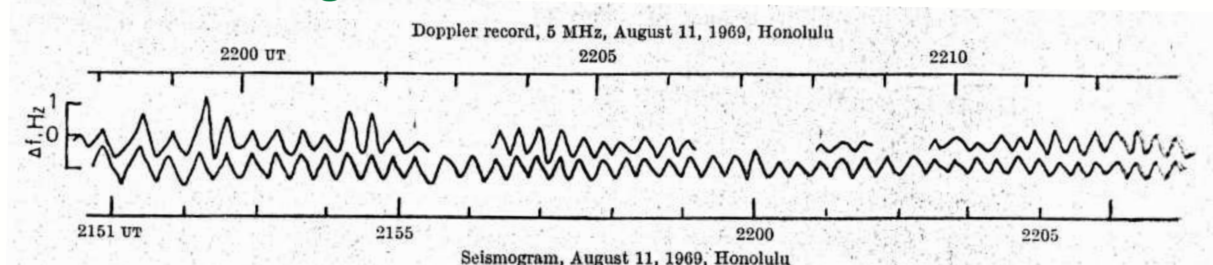


Fig. 2. Comparison of 5 MHz Doppler record and seismogram with expanded time scale.

WHY and HOW ?

- Atmosphere is not vacuum... and so, Rayleigh waves do not stop at a free boundary, but rather are continued upwards in the form of an pseudo-gravity wave, whose phase velocity is forced to that of the main Rayleigh wave.

- Energy density decays exponentially upwards, but since *material density decays faster,* wave amplitude can actually **increase with height !** Radar detects variation in TEC due to perturbation of ionosphere.

WHAT ABOUT TSUNAMIS ?

- *Hines* [1972] speculates that the concept could be extended to tsunamis.

But a tsunami must displace the atmosphere as it propagates and the displaced atmosphere must respond by generating a gravity wave. The parameters are such that these waves will be of the internal type, and so will grow exponentially with height. A rise of a few metres at the surface of the water might well amplify to a few km at ionospheric heights, and that sort of amplitude could hardly escape detection if it were sought. We arrive, then, at this speculative question: If we wish to keep track of the progress of a tsunami, and so predict with some assurance the onslaught of its destructive force, might we not serve our interests best by keeping watch on the ionosphere?

Peltier and Hines [1976] elaborated on the subject, but

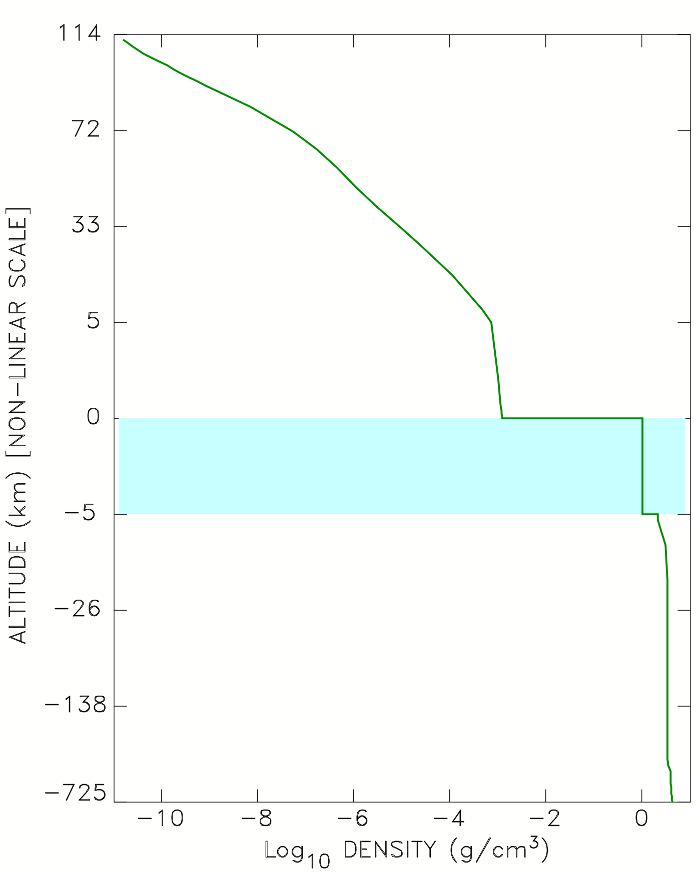
IT TOOK CLOSE TO 30 YEARS TO OBSERVE...

STRUCTURE of the TSUNAMI WAVE in the ATMOSPHERE

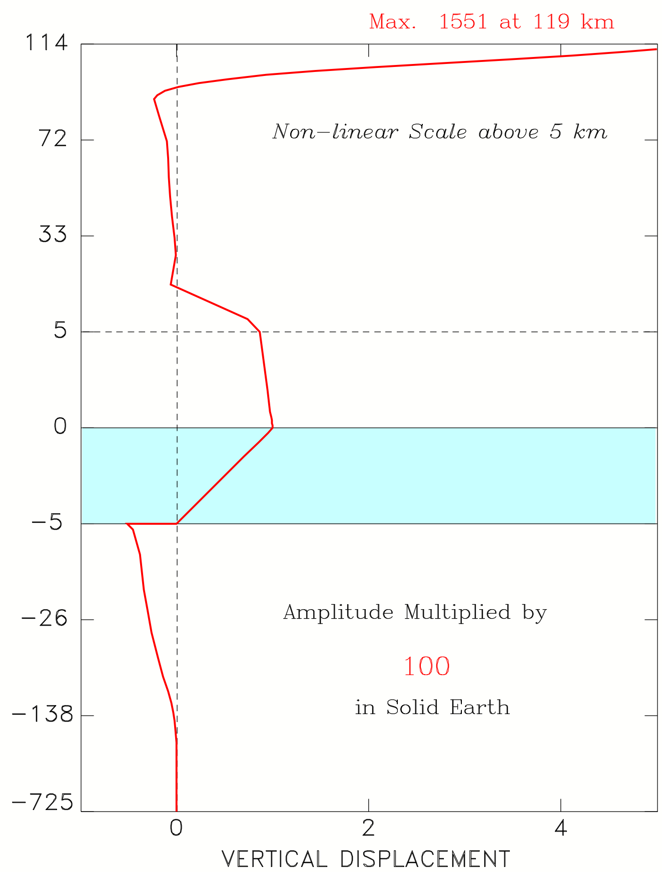
→ We compute the continuation of the tsunami wave both in the solid Earth and in the atmosphere using the generalized code "***HASH***" by *Harkrider et al.* [1974].

- Flat-layered model
- 5−km deep ocean
- Period ≈ 1000 seconds

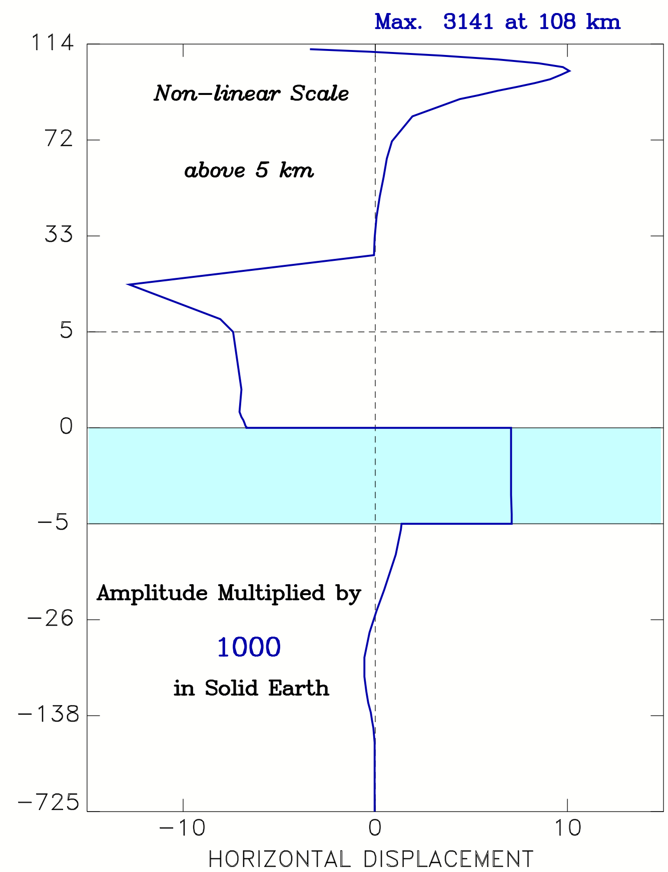
Density ρ



Vertical Amplitude



Horizontal Amplitude

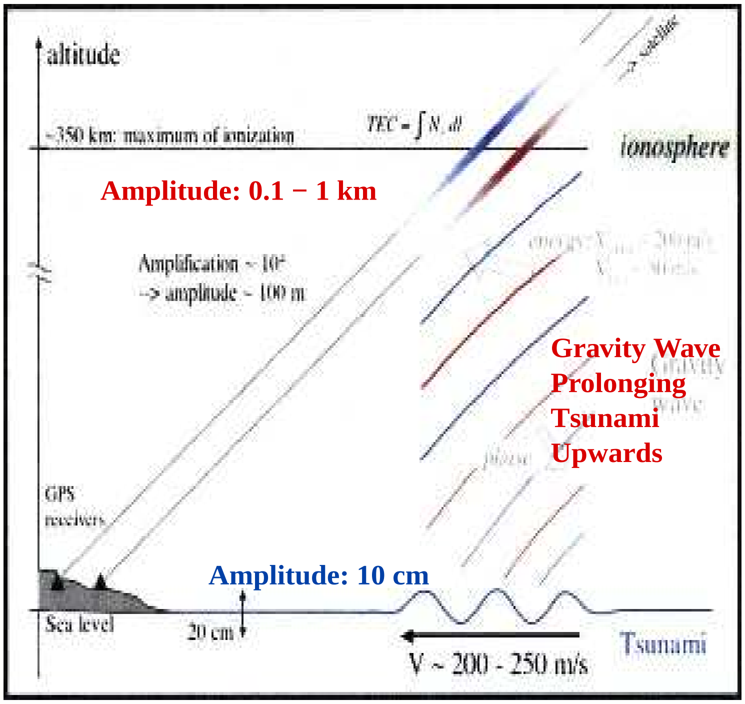


TOWARDS DIRECT DETECTION of a TSUNAMI on the HIGH SEAS

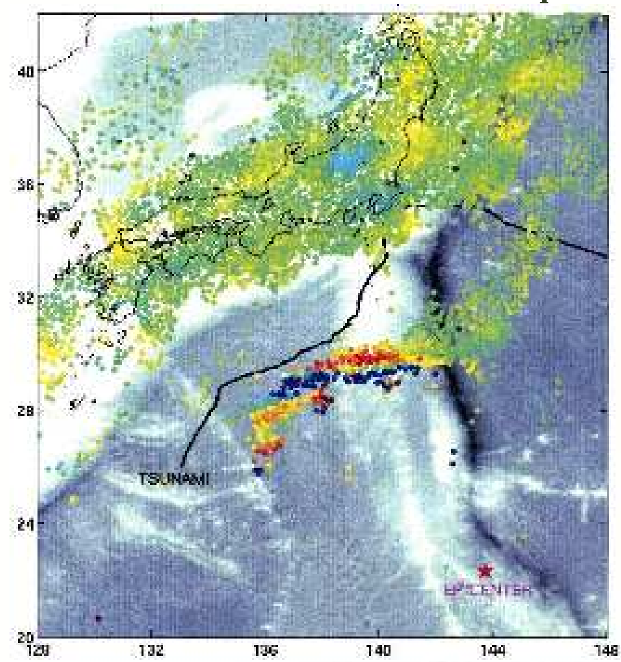
3. TSUNAMI DETECTION by GPS IONOSPHERIC MONITORING

J. Artru, H. Kanamori (Caltech); M. Murakami (Tsukuba); P. Lognonné, V. Dučić (IPG Paris) -- (2002)

- Ocean surface is not free boundary — Atmosphere has finite density
- Tsunami wave *prolonged* into atmosphere; *amplitude increases* with height.
- Perturbation in ionosphere (h = 150−350 km) detectable by GPS.



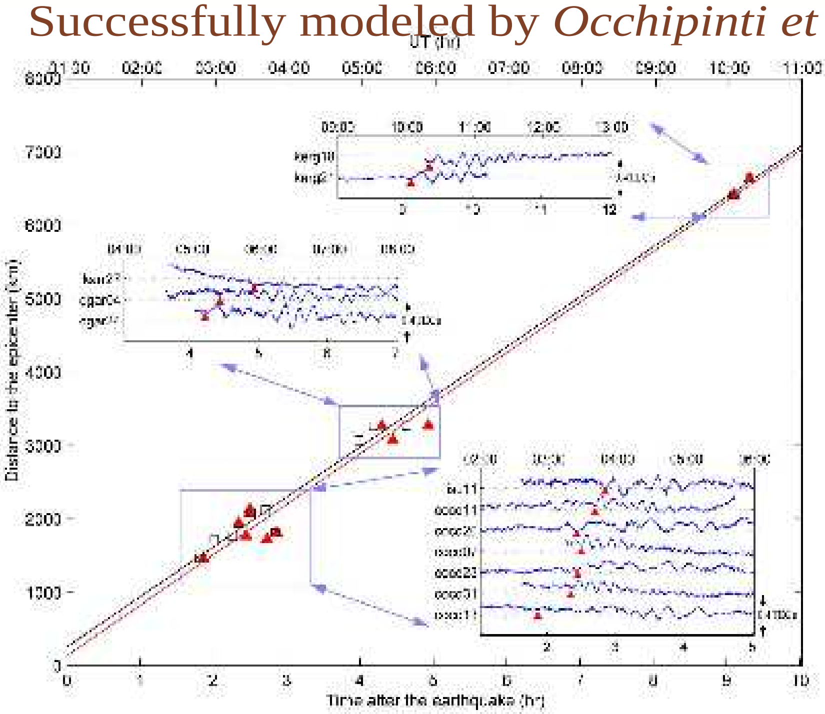
28 MAR 2000 -- 90 mn after earthquake

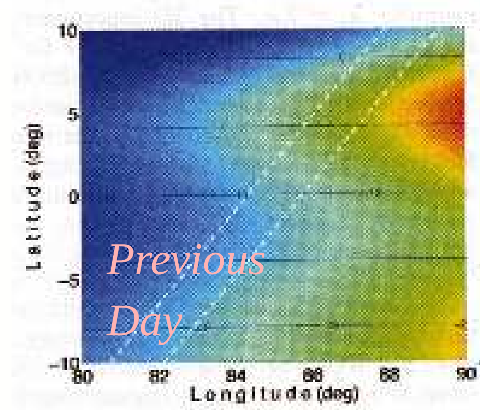


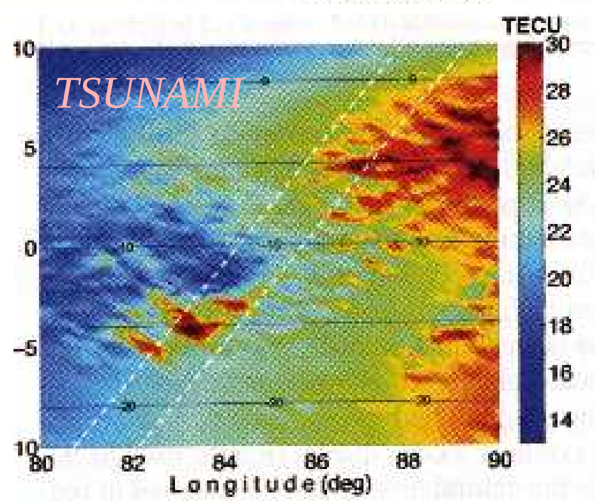
SUMATRA 2004

Perturbations detected in ionospheric **T**otal **E**lectron **C**ontent [*Liu et al.*, 2006]

Successfully modeled by *Occhipinti et al.* [2006].

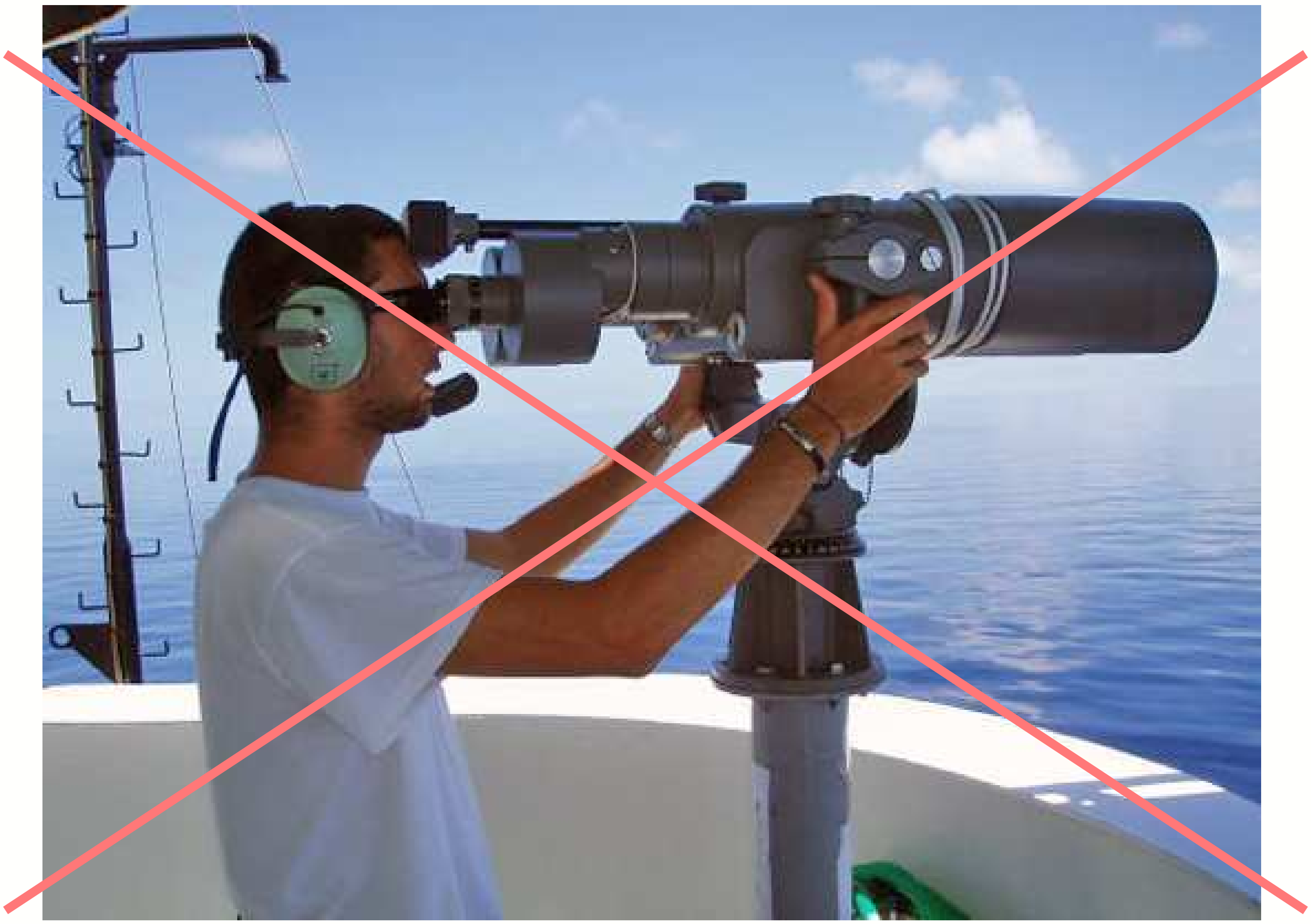






DIRECT "VISUAL" DETECTION of TSUNAMI on HIGH SEAS ??

- *In principle, should be impossible*



(Amplitudes too small; wavelengths too large)

YET **· · · ?**

TSUNAMI SHADOWS — *Can we "SEE" Tsunamis, after all ?*

There exist a number of somewhat anecdotal reports of tsunamis accompanied by a *"shadow"* on the ocean surface.

- *Walker* [1996] has published a shot from a video lending support to this idea.

11

Figure 1. The tsunami "shadow" can be seen just below the horizon and extends across the entire field of view of the camera. Approximately 12 minutes has to be added to the time indicated based on simultaneously recorded audio of a local radio station. The video was taken at an elevation of about 50 meters above sea-level.

Godin [2003] explains this phenomenon theoretically as follows:

- Tsunami wave creates steep *gradient* in sea surface.
- This gradient affects boundary condition of lower atmosphere **wind** near surface, making it ***turbulent***.
- In turn, this turbulence creates *roughness* in Sea Surface, perceived as *Tsunami Shadow*.

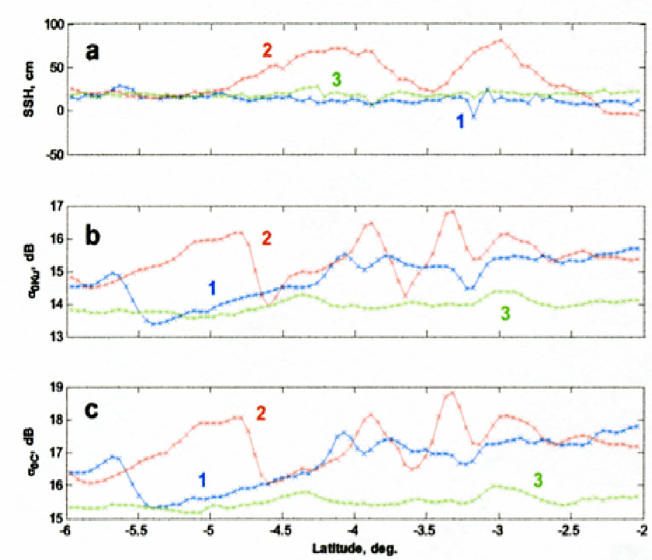


Fig. 3. Jason-1 data for pass 129 from 6° S to 2° S obtained days before (Cycle 108) (1), coincident with (Cycle 109) (2), and 10 days after (Cycle 110) (3) the Sumatra-Andaman tsunami.**(a)** Sea surface height. **(b)** Ku-band radar backscattering strength. **(c)** C-band radar backscattering strength.

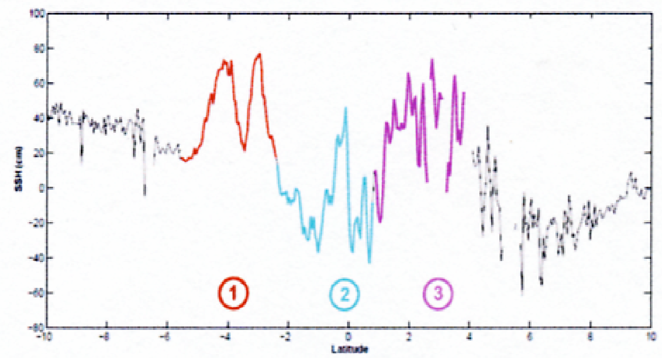


Fig. 4. Sea surface height data from Jason-1 ascending path 129 for cycle 109. Data segments 1, 2, and 3 chosen for detailed analysis of tsunami manifestations are shown in color. Breaks in the graph reflect gaps in the available SSH data.

At present, there is no universally accepted model of air flow over fast, as compared to the background wind, sea waves. Under assumptions made in (Godin, 2005), in the presence of a monochromatic tsunami wave, the wind speed relative to the ocean surface retains a logarithmic profile up

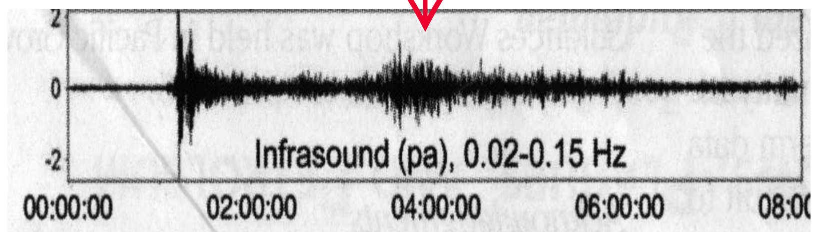
Godin et al. [2009] detect roughness in JASON altimeter records of 2004 Sumatra tsunami.

LOUD TSUNAMI ??

TSUNAMI DETECTED by INFRA SOUND ARRAYS (CTBT)

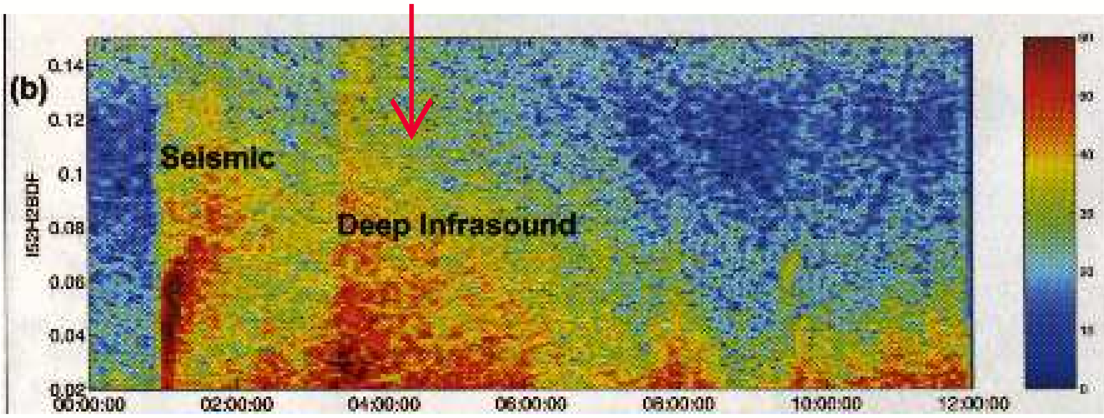
Arrays of barographs monitoring pressure disturbances carried by atmosphere.

(Deployed as part of International Monitoring System of CTBT.)



Diego Garcia, BIOT, 26 Dec. 2004

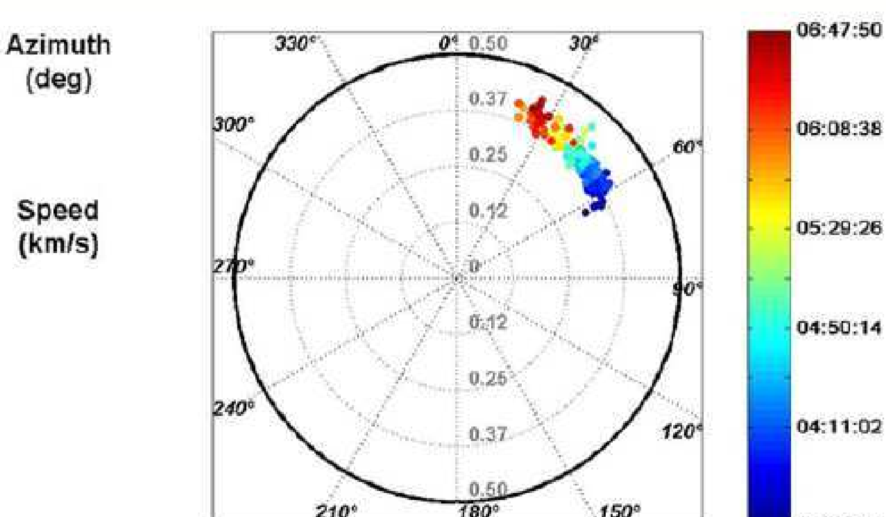
[*Le Pichon et al.*, 2005]



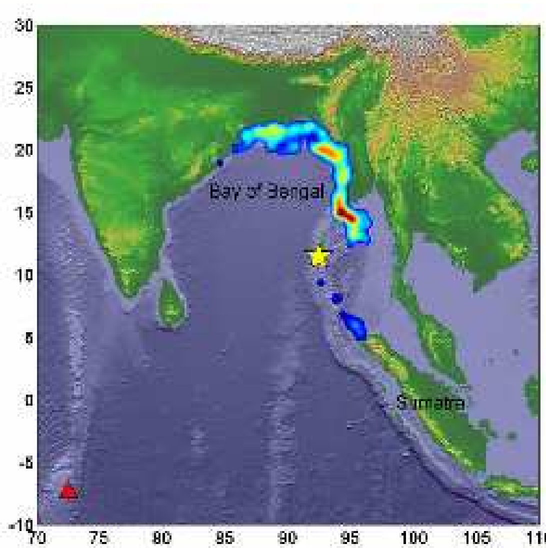
Detects signal in DEEP INFRASOUND about 3 hours after source time

BEAM ARRAY to determine azimuth of arrival and velocity of air wave.

USE TIMING of arrival to infer source of disturbance as *TSUNAMI HITTING CONTINENT* then continent shaking atmosphere.



Time



TSUNAMI DETECTED IN GEOMAGNETIC FIELD

A SENSIBLE IDEA...

- Tsunami moves water, a conducting fluid, inside the magnetic field of the Earth.
- Should create a current, which in turn, perturbs the Earth's magnetic field **B**.
- Indeed, tidal signals have been detected in daily fluctuations of **B** [*e.g.*, *McKnight*, 1995].

→ *Tyler* [2005] showed that the perturbation b_z of the vertical component of **B** should be linked to the tsunami's amplitude η through

$$\frac{b_z}{\eta} = \frac{F_z\, c}{h\, c_s} \cdot e^{-\kappa z}$$

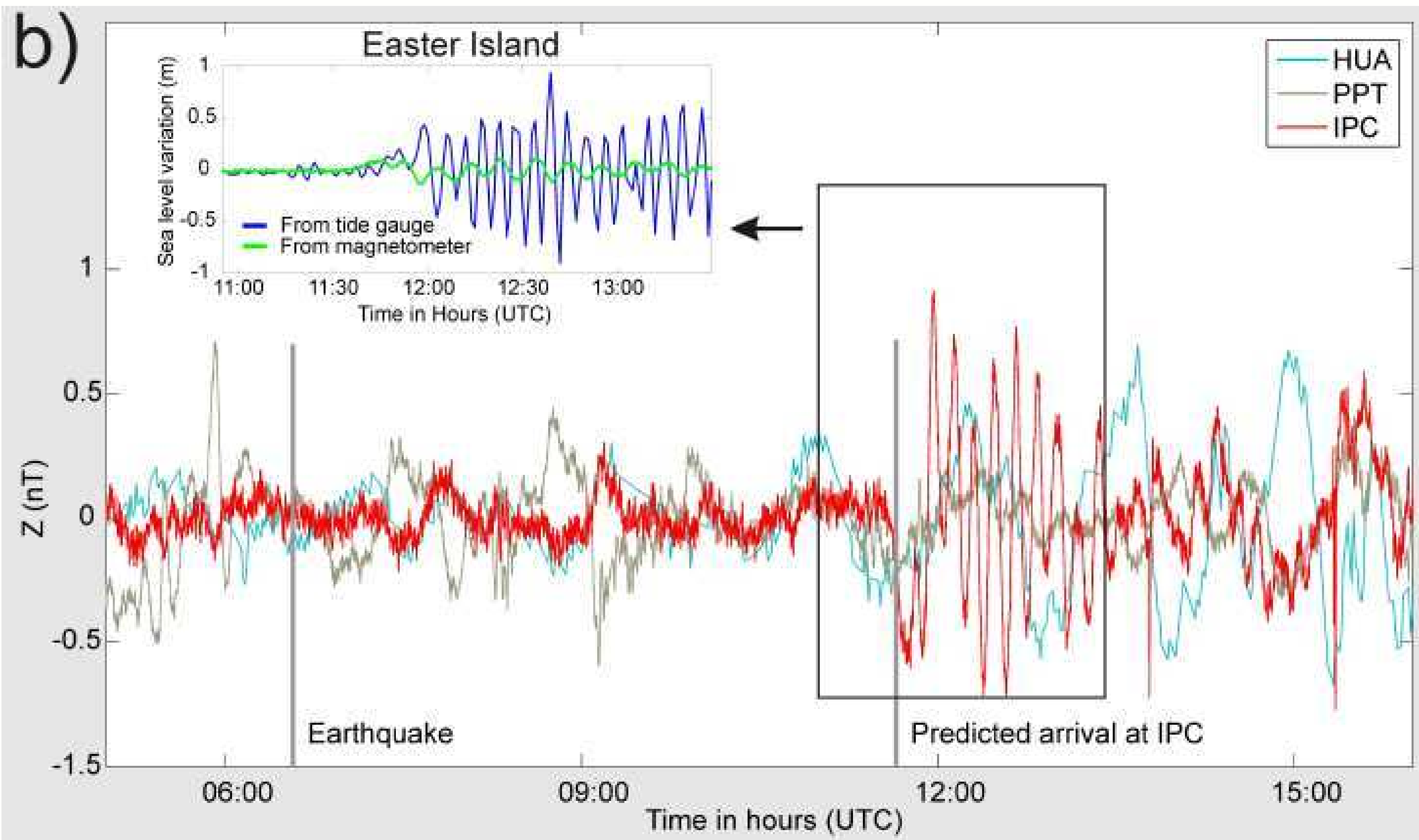
where F_z is the unperturbed vertical field, $c = \sqrt{g\, h}$ the tsunami's phase velocity, $c_s = c + i\, c_d$ with $c_d = 2\, K\, /\, h$ and K the magnetic diffusivity ($K = 1/\mu\sigma$).

- Unfortunately, in the case of the 2004 Sumatra tsunami, the areas with maximum η are at the magnetic Equator, and no signal was detected...

→ Otherwise, one would expect about **10 to 20 nT per meter** of vertical sea surface displacement...

DETECTION DURING THE 2010 CHILEAN TSUNAMI

- *Manoj et al.* [2011] detected this effect during the 2010 Chilean tsunami using the geomagnetic station at Easter Island (IPC -- below, **red**)



→ *The amplitude detected, ≈ 1 nT, is in good agreement with that of the tsunami on the high seas (15 to 20 cm),* **as recorded on DART buoys.**

- They should **NOT** be comparing to a tide gauge record, which is strongly affected by harbor response.

CONCLUSIONS

- The exceptional size of the 2004 tsunami emphasizes the detailed structure of its tsunami.

→ The tsunami includes significant high-frequency components (3−10 mHz), propagating outside the SWA and which are relevant to harbor response.

→ The tsunami does not stop at water interfaces, but is prolonged into both the solid Earth and the atmosphere.

→ This remark enables the interpetation of the tsunami as a particular case of the Earth's free oscillations; this approach allows the quantification of many secondary properties of the tsunami, as excited by a dislocation source.

→ Because of the complex nature of the tsunami eigenfunction (consisting not only of a displacement field, but also of pressure, changes in gravity, tilt, etc.), many technologies can be used to detect the tsunami, using equipment already deployed.

→ More work would be warranted to understand the generation of deep infrasound signals, as detected in Diego Garcia.