INDISCREET APPLICATIONS OF DISCRETE MATHEMATICS

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THE SIGN OF FOUR

1) SELF-REPRODUCTION (Gödel, Watson & Crick)
2) APPORTIONMENT (Balinsky & Young)
3) SOCIAL CHOICES (Sen, Arrow)
4) REALITY (Bell)

1) SELF-REPRODUCTION

Problem:

How to build self-reproducing objects?

First trial

A = universal constructor

x = description of X

(A,x) produces X

In particular: (A,a) produces A

Not yet self-reproducing!

Second trial

B = copying machine (B,x) produces x

C = coupling of A plus B

1st diagonalization (on X): (C,x) produces (X,x)

2nd diagonalization (on C): (C,c) produces (C,c)

Now self-reproducing!

Applications

1) Self-referential sentences

"I have the property P"

Gödel 1931: P(x) = x is (not) provable

Tarski 1936: P(x) = x is (not) true

2) Fixed-point theorem

Kleene 1938: recursive programs

Applications

3) Self-reproduction

Von Neumann 1948: cellular automata

Watson & Crick 1953: biological cells

A = ribosome building proteins

x = gene (DNA)

B = enzyme (RNA polymerase)

C = self-reproducing cell

2) APPORTIONMENT

Problem:

How to assign seats to parties or districts according to their votes or population?

Axioms

1) Proportionality

Use excess or defect approximations

E.g. a proportion of 10/3 produces 3 or 4 seats

2) Monotonicity

More votes = more seats

Balinsky & Young, 1982

No apportionment method satisfies both

1) proportionality

and

2) monotonicity

Proof

Parties	First election	Second election
A	$5 + \varepsilon$ (≥ 5)	$4 - \varepsilon$ (≤ 4)
B	$2/3$	2 (2)
C	$2/3$	$1/2$
D	$2/3 - \varepsilon$ (0)	$1/2 + \varepsilon$ (≥ 1)
Total	7	7

A loses one seat and D gains one

this is against relative monotonicity, if

$$(4-\varepsilon)/(1/2 + \varepsilon) > (5+\varepsilon)/(2/3 - \varepsilon)$$

Example

Parties	First election	Second election
A	5.01 (5)	3.99 (4)
B	0.67 (1)	2.00 (2)
C	0.67 (1)	0.50 (0)
D	0.65 (0)	0.51 (1)

A loses one seat and D gains one

but $3.99/0.51 \quad > \quad 5.01/0.65$

i.e. $\quad 8 \quad > \quad 7.5$

3) SOCIAL CHOICES

Problem:

How to amalgamate the
individual orders of preferences
into a social order?

Axioms

1) Totality

Either A is preferred to B,
or B is preferred to A,
or they are indifferent

2) Transitivity

If A is preferred to B,
and B to C,
then A is preferred to C

Axioms

3) Unanimity (Pareto's principle)

If every individual prefers A to B, then society does not prefer B to A

4) Freedom of choice

Any individual order of preference is acceptable

Definition

An individual has a right over the alternatives A and B if, whenever he prefers one over the other, so does society

Amartya Sen, 1970

Unanimity and freedom of choice

imply that, in a society,

at most one individual

can have rights!

Proof

Suppose 1 has a right over A and B, and prefers

$$D < A < B < C.$$

Suppose 2 has a right over C and D, and prefers

$$B < C < D < A.$$

Then society must have the order

$$A < B \leq C < D \leq A,$$

contradiction.

Definition

A system is vote-dependent if social choices are made solely on the basis of individual preferences

Arrow, 1951

Unanimity, freedom of choice and vote-dependence

imply that, in a society,

exactly one individual

has rights!

In other words, there must be a dictator!

Proof

If A and B are not socially indifferent, some individual must have a right over them.

Otherwise, all individuals would prefer one over the other, and society would prefer the other, against unanimity.

But only one individual can have rights, so it must always be the same.

4) REALITY

Problem:

Does classical metaphysics accord with quantum mechanics?

Classical metaphysics

The universe consists of systems that are:

1) Real

   Their properties are independent of observation

2) Separated

   In space-time

3) Local

   There is no action-at-a-distance,

   or faster-than-light

Einstein, 1935

Realism, separation and locality

imply

Incompleteness of quantum mechanics

A thought experiment

Suppose two observers 1 and 2

receive envelopes A, B and C

which contain blue or green sheets of paper.

1) If they open the same envelope,

they always observe the same colour

2) If they open random envelopes,

they observe the same colour at least 5/9 of the times

Bell, 1964

Realism, separation and locality

Imply that it is impossible to find the same colour:

1) always on the same envelope

2) 1/2 of the times on random envelopes

Proof. 5/9 is greater than 1/2

Aspect, 1982

Reality, separation and locality

are in contrast with experience!

Proof. Experiments with polarizing filters show that correlation is exactly 1/2

(in accordance with quantum mechanics)

A modern metaphysics

Realism, separation and locality cannot stand together.
Since nobody really doubts separation,

1) either the universe is not real
2) or it is holistic, i.e. not local

CONCLUSION

A little logic and
some discrete mathematics
show that common conceptions of
life, democracy and reality
are naive and wrong.