Ramsey Degrees of Boron Tree Structures

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- We call $t(\mathbf{A}, \mathcal{K})$ the *Ramsey degree* of $\mathbf{A}$ (in $\mathcal{K}$).

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Theorem (Fouché, 1999)

If $\mathbf{A} \in \mathcal{T}_h$, then $t(\mathbf{A}, \mathcal{T}_h)$ equals the number of possible "orientations" of $\mathbf{A}$. In particular, $t(\mathbf{A}, \mathcal{T}_h) = 1$ iff for all $h' \leq h$, given two vertexes in A of height h', their valence is the same.

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- Suppose for all $h' \leq h$, given two vertexes in A of height h', their valence is the same. Then the only possible orientation of **A** is
$$(m_0), (m_1, \ldots, m_1), \ldots, (1, \ldots, 1)$$
where $m_{h'}$ is the valence of elements of height h'.

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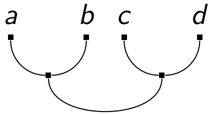
- $(T) \models R(a, b, c, d)$ iff we have the following picture



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Fact

Let T_0, T_1 be boron trees. Let $\mathbf{A}_0, \mathbf{A}_1 \in \mathcal{B}_0$ be such that $\mathbf{A}_i = (T_i)$. If $\varphi : A_0 \to A_1$ is an isomorphism of structures, then there is a graph isomorphism $\lambda : T_0 \to T_1$ such that $\lambda \restriction A_0 = \varphi$.

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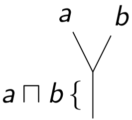

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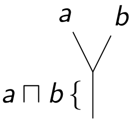

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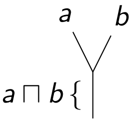

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- The *height* of $\mathcal{O}$ is defined to be the largest of the heights of $\mathcal{O}_l$ and $\mathcal{O}_r$.
- The *standard* orientation $\mathcal{O}(n)$ is defined to be $\langle \mathcal{O}(n-1), \mathcal{O}(n-1) \rangle$

Orientations III (examples)

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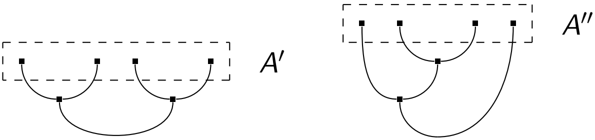
- A' below represents a copy of $\mathbf{B}(2)$ with the standard orientation $\mathcal{O}(2)$, while A'' is also a copy of $\mathbf{B}(2)$, but has orientation

$$\langle\langle\emptyset, \langle\emptyset, \emptyset\rangle\rangle\,, \emptyset\rangle\,.$$



Upper Bound for the Degree I

- $\binom{\mathbf{B}(k)}{\mathbf{A}}_{\mathcal{O}}$ refers to the set of substructures of $\mathbf{B}(k)$, isomorphic to $\mathbf{A}$, with orientation $\mathcal{O}$ (computed using the identity embedding).

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Theorem

Let $n, p, r \in \omega$. There exists $N = N_0(n, p, r)$ that satisfies the following: Suppose $\mathcal{O}$ is an orientation of $\mathbf{A}$ with height p. Then, given a colouring $c : \binom{\mathbf{B}(N)}{\mathbf{A}}_{\mathcal{O}} \to r$, there exists a copy $\mathbf{B}' \in \binom{\mathbf{B}(N)}{\mathbf{B}(n)}_{\mathcal{O}(n)}$, such that $\left| c'' \left(\binom{\mathbf{B}(N)}{\mathbf{A}}_{\mathcal{O}} \cap \binom{\mathbf{B}'}{\mathbf{A}} \right) \right| = 1$. We represent this latter statement by

$$\mathbf{B}(N) \to (\mathbf{B}(n))_r^{(\mathbf{A},\mathcal{O})}.$$

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- Define $[\Lambda]^*\binom{n}{m}$ to be the set of functions $f : n \longrightarrow \Lambda \cup \{*\} \cup V$ that satisfy the following conditions:

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- Define $[\Lambda]^*\binom{n}{m}$ to be the set of functions $f : n \to \Lambda \cup \{*\} \cup V$ that satisfy the following conditions:
  1. $f^{-1}(v_i) \neq \emptyset$ for all $i \in m$.
  2. $\min f^{-1}(v_i) < \min f^{-1}(v_j)$ for all $i < j \in m$.
  3. If $f(i) = *$ for some $i \in n$, then $f(j) = *$ for all $j > i$.

Truncated Hales-Jewett Theorem II

$$(f \cdot g)(i) = \begin{cases} g(j) & \text{if } f(i) = v_j \\ * & \text{if } (f \cdot g)(j) = * \text{ for some } j < i \\ f(i) & \text{otherwise} \end{cases}$$

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Theorem (Voigt, 1980)

For every finite Λ and every $r, m \in \mathbb{N}$, there exists $N = N^*(r, m) \in \mathbb{N}$ such that for every colouring $c : [\Lambda]^* \binom{N}{0} \to r$ there exists $f \in [\Lambda]^* \binom{N}{m}$ such that $\left| \hat{c}'' \left([A] \binom{m}{0} \right) \right| = 1$, with $\hat{c}(g) = c(f \cdot g)$.

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2. There exists $N = N_2(n, p, r, m)$ that satisfies the following: Given a colouring of $\binom{\mathbf{B}(N)}{\mathbf{A}}_{\mathcal{O}}$, there exists a copy $\mathbf{B}' \in \binom{\mathbf{B}(n)}{\mathbf{B}}_{\mathcal{O}(n)}$ such that every copy of $\mathbf{A}$ with orientation $\mathcal{O}$ and the same beginning of height j (w.r.t. $\mathbf{B}'$) is assigned the same colour, for $j \leq m$.

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3. Combine step 2 and the Truncated Hales-Jewett Theorem.

Lower Bound for the Degree

Let n_0 be such that every orientation of $\mathbf{A}$ is witnessed in $\mathbf{B}(n_0)$ and $n_0 \geq 2$. Set $N = n_0 + 1$. Let $f : B(N) \hookrightarrow B(M)$ be an embedding. It suffices to establish the following:

Lemma

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- $C_3 = 5 = t(\mathbf{B}(2), \mathcal{B}_0)$.
- In general, $t(\mathbf{A}, \mathcal{B}_0) \leq C_{n-1}$.

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- Suppose $\mathbf{A}$ is a boron tree structure with a boron tree structure embedding $\varphi : A \to B(n)$, for some $n \in \omega$.
- Define $o(\mathbf{A}, \varphi)$ to be the (extending) structure $\mathbf{B}$ with signature $\mathcal{L}$ and universe A such that
  1. $R^{\mathbf{B}} = R^{\mathbf{A}}$.
  2. For all $a, b \in A$, $\mathbf{B} \models \prec (a, b)$ if and only if $\varphi(a) <_{\mathrm{lex}} \varphi(b)$.
  3. For all $a, b, c \in A$, $\mathbf{B} \models S(a, b, c)$ if and only if

$$\varphi(a) <_{\mathrm{lex}} \varphi(b) <_{\mathrm{lex}} \varphi(c)$$

and

$$|\varphi(a) \sqcap \varphi(b)| > |\varphi(b) \sqcap \varphi(c)|.$$

Extensions II

- Define $\mathcal{OB}_0$ to be the set of $o(\mathbf{A}, \varphi)$ for all pairs of boron tree structures $\mathbf{A}$ and boron tree structure embeddings $\varphi : A \longrightarrow B(n)$ $(n \in \omega)$.

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