On the Optimization of Condition-Based Maintenance Decisions

Andrew Jardine, Dragan Banjevic & Neil Montgomery

C-MORE Centre for Maintenance Optimization & Reliability Engineering

University of Toronto | Department of Mechanical and Industrial Engineering

The Fields Institute
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Part I - Theory

Problem Definition

- To optimize the replacement policy for a critical item:
  - Diesel engine
  - Gearbox
  - Pump
- that is subject to condition monitoring:
  - Oil Analysis
  - Vibration analysis
  - Infrared thermography…
- Goal: calculate conditional reliability function, remaining expected life (RUL), and …

  produce a decision rule to minimize the average long-run cost per unit time

Data Required

- Lifetime "Events" data:
  - Installations
  - Failures
  - Suspensions
  - Minor maintenance actions (e.g. oil changes)
- Condition monitoring "Inspections" data.
- Assumption: replacements (planned, or due to failure) are "as-good-as-new"

Data Example
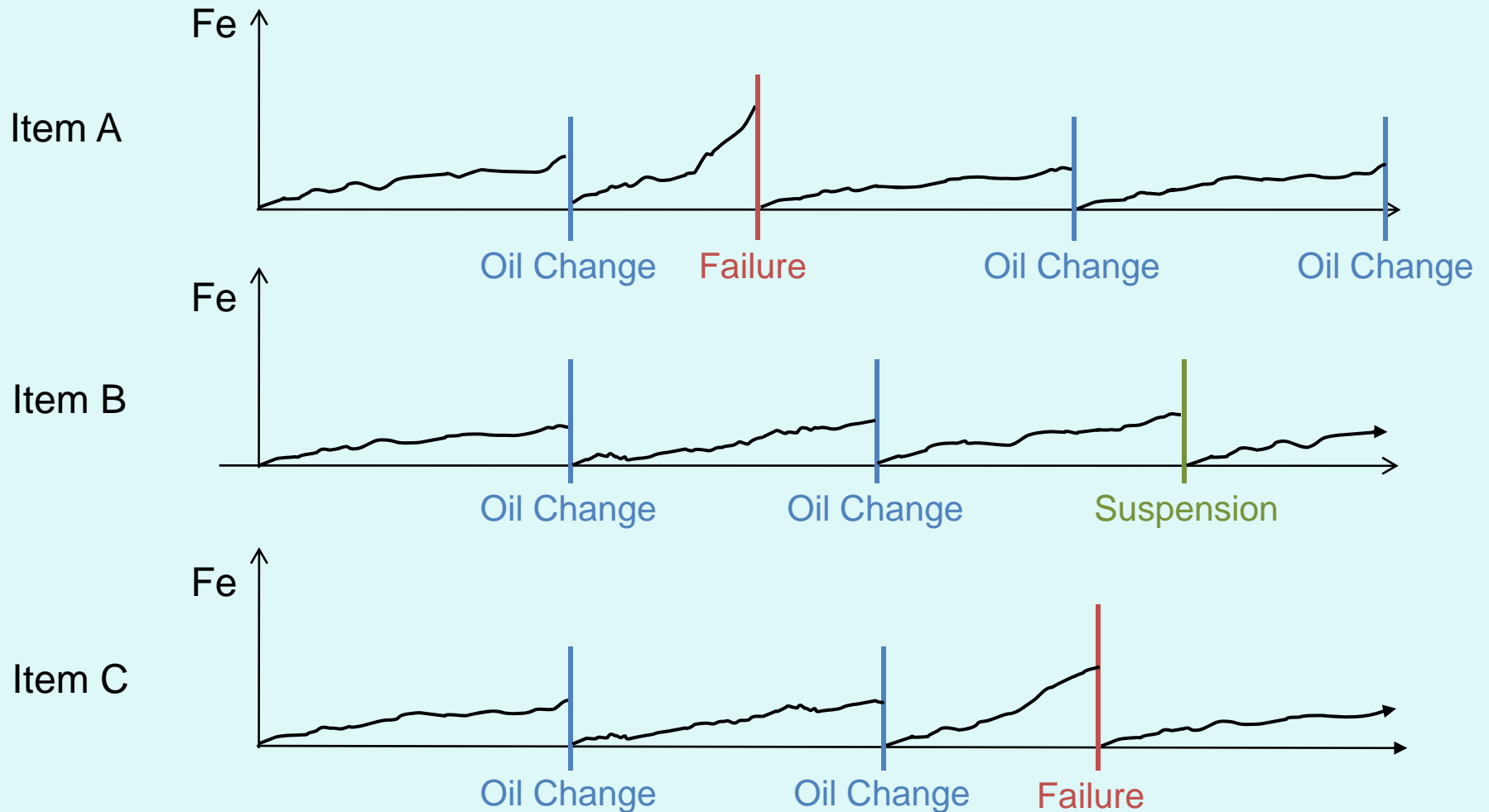
Model Definition

T – time to failure

$Z(t)$ – covariate process with finite number of states

A joint model for failure time and covariates:

$$(N(t), Z(t)), \text{ where } N(t) = I(T > t),$$

a non-homogeneous Markov process.

What describes a Markov process?

Transition probabilities:

$$P_{ij}(s,t) = P(T > t, Z(t) = j | T > s, Z(s) = i)$$

$$= P(T > t | T > s, Z(s) = i)P(Z(t) = j | T > t, Z(s) = i)$$

Consider: $P_{ij}(t, t + \Delta t)$.

"Survival Model" (Hazard Rate Model)

$$P(t < T \leq t + \Delta t | T > t, Z(t) = i) = h(t, i)\Delta t$$

"Covariate Process Model" (Transition Rate Model)

$$P(Z(t + \Delta t) = j | T > t + \Delta t, Z(t) = i) = \lambda_{ij}(t)\Delta t$$

Conditional Reliability Function

$$R(s, x, i) = P(T > s + x | T > s, Z(s) = i)$$

$$= R(s, \delta, i) \sum_k p_{ik}(s, \delta) R(s + \delta, x - \delta, k)$$

$$p_{ik}(s, \delta) = \lambda_{ij}(t)\delta \qquad \delta \text{ - small}$$

$$R(s, \delta, i) = \exp\left\{-\int_s^{s+\delta} h(u, i)du\right\} = 1 - h(s, i)\delta$$

Remaining Expected Life:

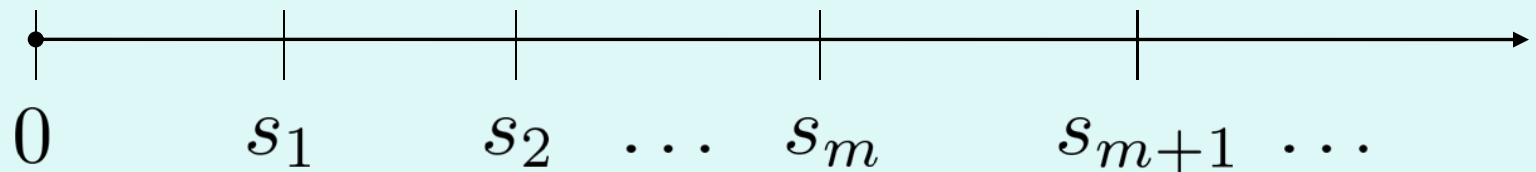$$E(T - s | T > s, Z(s) = i) = \int_0^\infty R(s, x, i)dx$$

Hazard Rate Model

Weibull PHM:

$$h(t, Z(t)) = \underbrace{\frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1}}_{\substack{\text{Contribution of age} \\ \text{to hazard}}} \underbrace{e^{\gamma_1 Z_1(t) + \cdots + \gamma_n Z_n(t)}}_{\substack{\text{Contribution of condition} \\ \text{information to hazard}}}$$

Parameters estimated using maximum likelihood.

Transition Rate Model

Time range split into intervals:

$$\lambda_{ij}(t) = \lambda_{ij}^{(m)} \quad \text{for} \quad s_m \leq t \leq s_{m+1}$$

The $\lambda_{ij}^{(m)}$ are estimated using maximum likelihood:

$$\hat{\lambda}_{ij}^{(m)} = \frac{n_{ij}^{(m)}}{A_i^{(m)}}, \quad i \neq j \quad - \quad \text{occurrence/exposure rate}$$

Decision Model

Decision rule:

- Always replace at failure time T
- At time t, replace preventively if:

$$h(t, Z(t)) \geq d$$

d – decision risk level

$$T_d = \min\{t \ : \ h(t, Z(t)) \geq d\}$$

- Stopping time: $\min\{T, T_d\}$

Optimal Decision Risk Level

– Average cost per unit time:

$$\Phi(d) = \frac{C_f P(T \leq T_d) + C_p P(T > T_d)}{E(\min\{T_d, T\})}$$

C_f and C_p are the failure and preventive replacement costs.

– Optimal decision risk level d_{opt}:

$$\Phi(d_{\mathrm{opt}}) = \min_d \Phi(d)$$

Optimal Policy Display

Replace, if at time t:

$$\boldsymbol{Z_c}(t) \geq \delta_{\text{opt}} - (\beta - 1) \log t$$

$$\boldsymbol{Z_c}(t) = \gamma_1 Z_1(t) + \cdots + \gamma_n Z_n(t)$$

$$\delta_{\text{opt}} = \log \left(\frac{d_{\text{opt}} \eta^{\beta}}{\beta} \right)$$
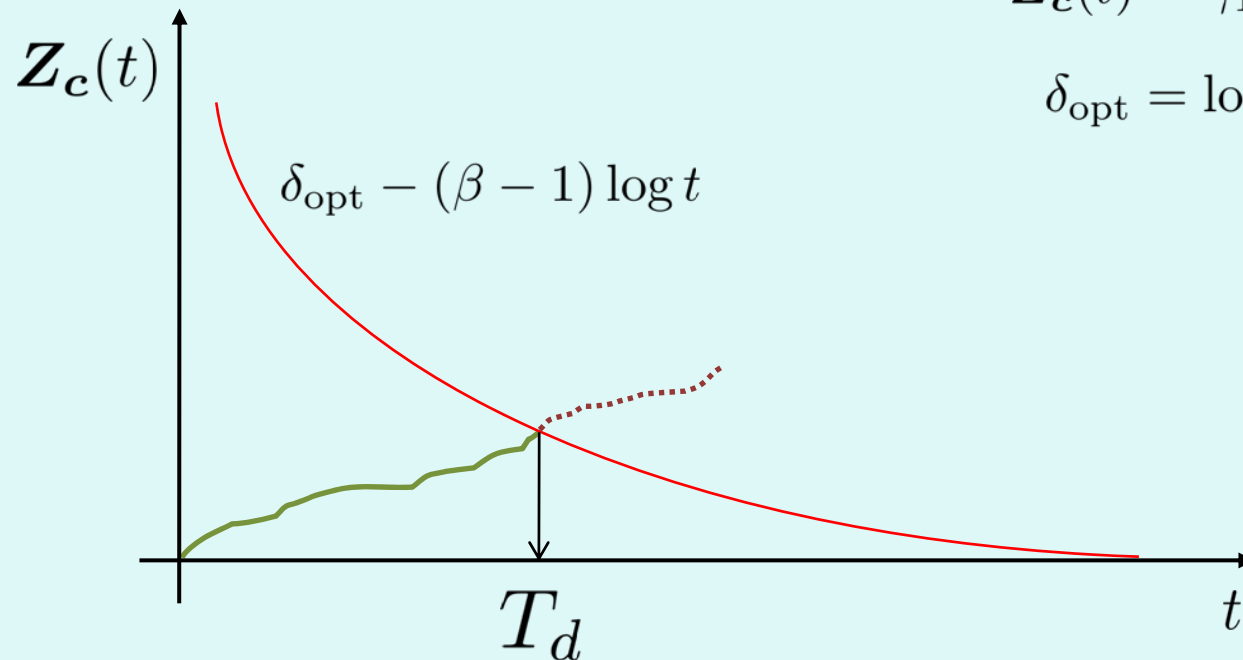
Part II - Applications

Where it started - 1982

Estimated Hazard Rate at Removal

Number	Flight Hours	Fe	Cr	Hazard Rate
1	11770	5	6	0.043
2	11660	2	6	0.012
3	8460	12	2.4	0.0071
4*	12630	8	1	0.0014
5	7710	8	0	0.00094
6*	9240	2	3	0.00029
7*	5660	10	1	0.00020
8*	7190	2	2.5	0.000073

* Doubtful Removal

$$h(t, Z(t)) = \frac{4.47}{24100} \left(\frac{t}{24100} \right)^{3.47} e^{0.41Z_1 + 0.98Z_2}$$

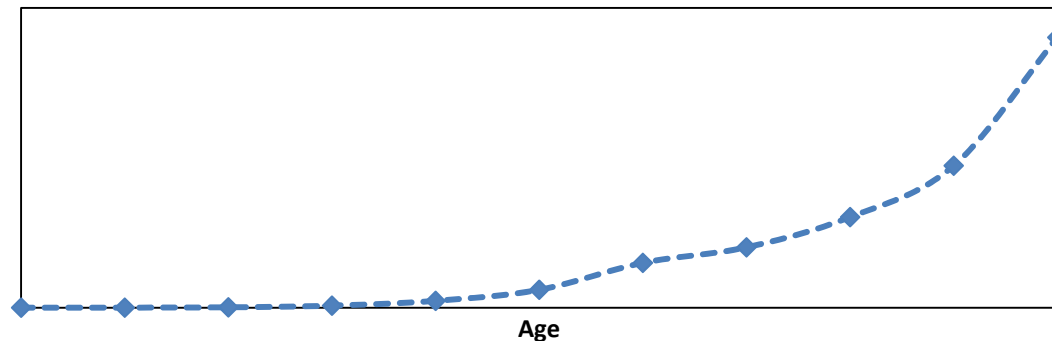$Z_1 = \text{Fe concentration}$
$Z_2 = \text{Cr concentration}$

From Data to Hazard
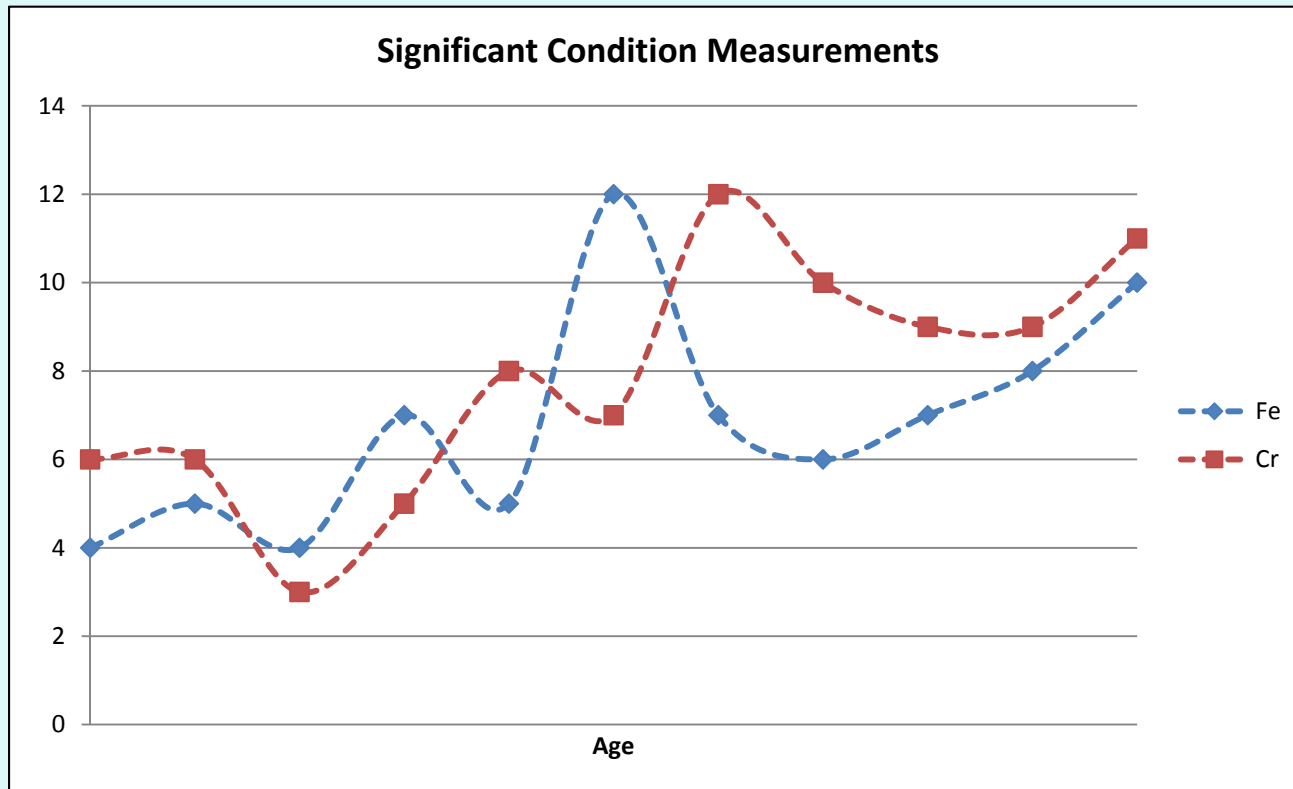
Significant Condition Measurements

From Data to Hazard

PHM
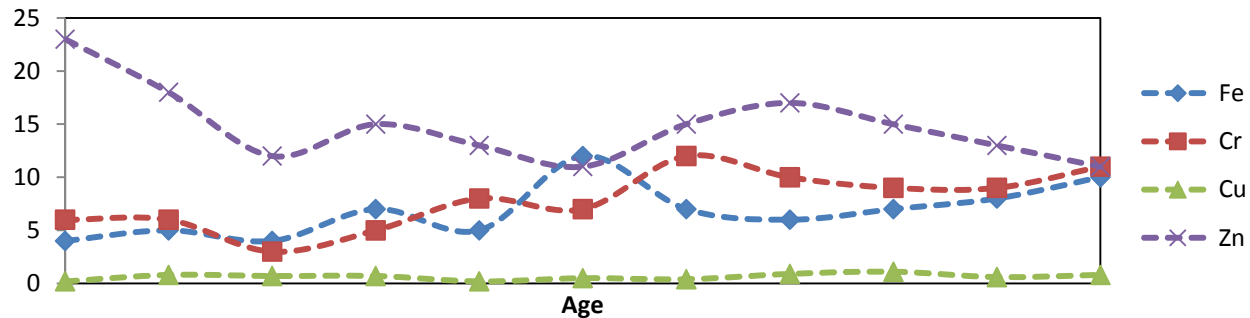
Optimal Policy: Optimal Hazard Level

Hazard Plot

Hazard

Ignore Hazard

Optimal Hazard level

Age

Cost Plot

Cost/unit time

Replace at failure only

minimal cost

optimal hazard

Hazard
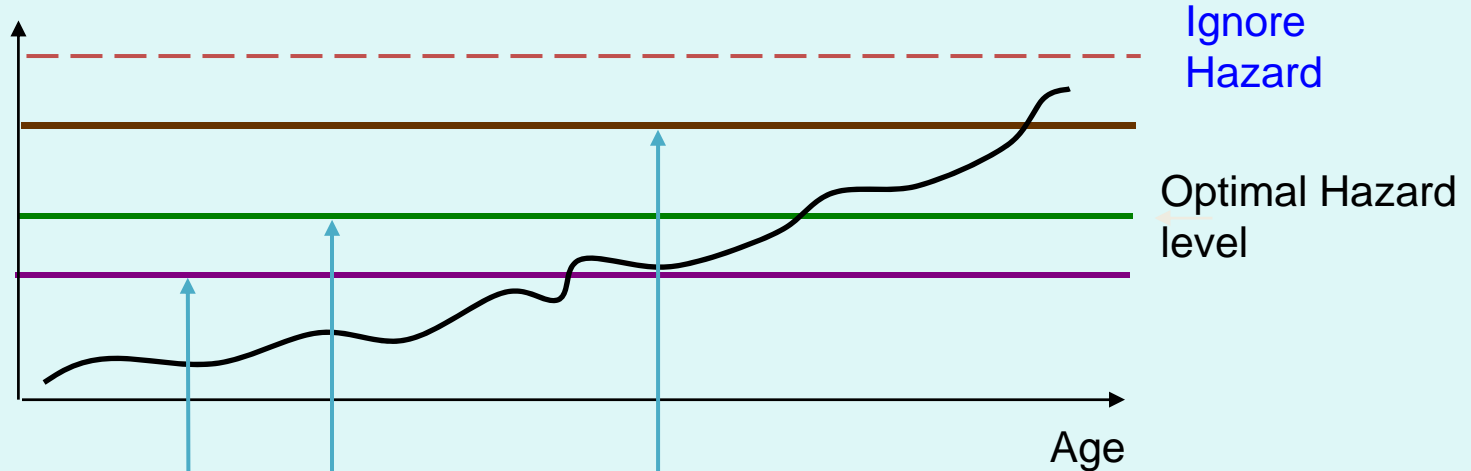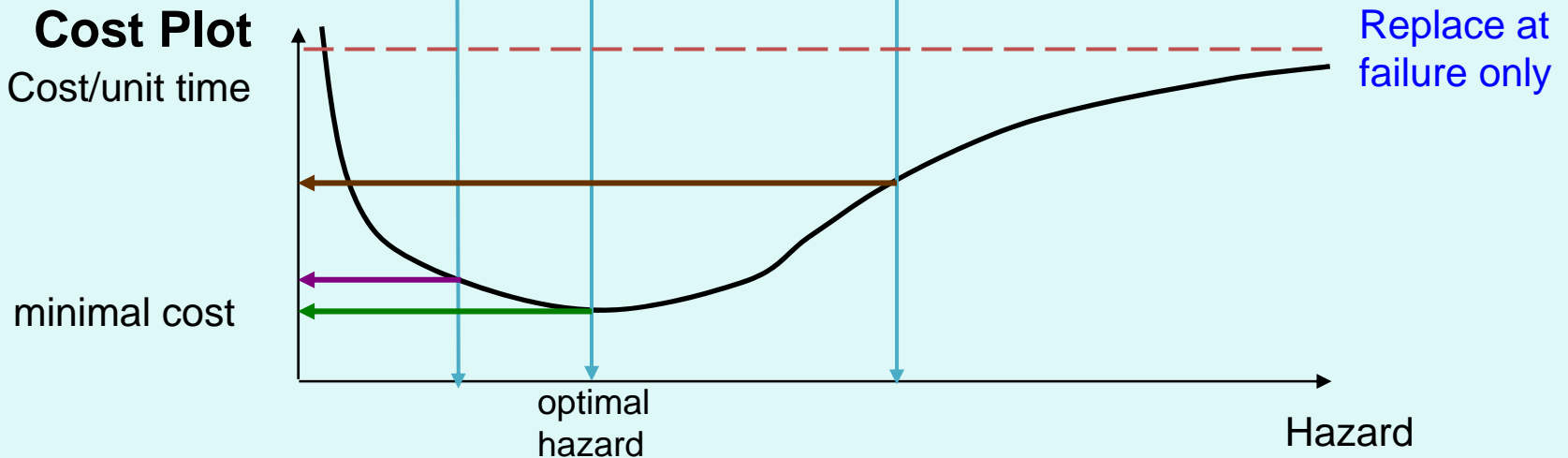
Optimal Decision Chart

Fields Institute
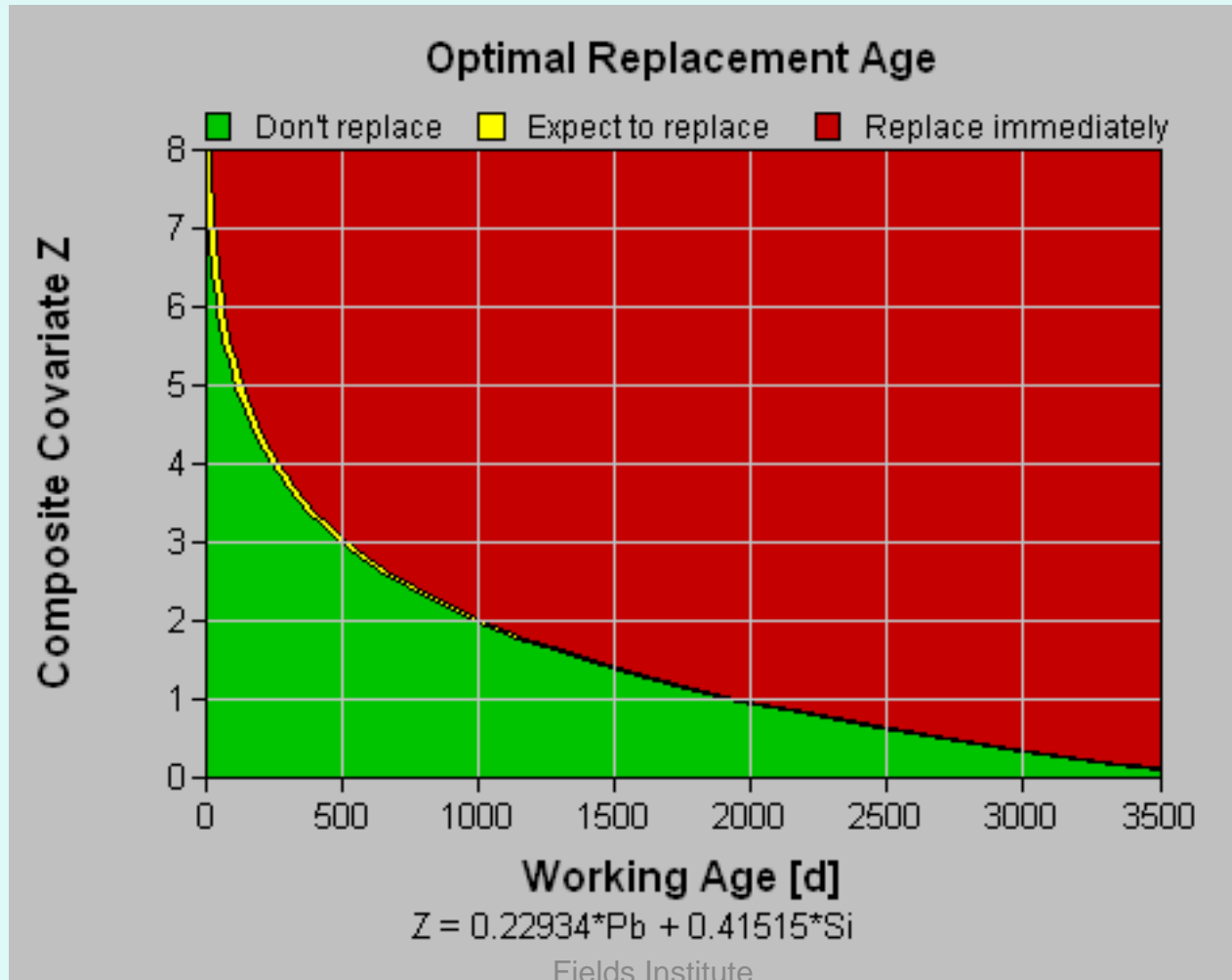
Condition-Based Maintenance

Irving Pulp and Paper: Executive Summary

Analysis of Goulds 3175L Pumps
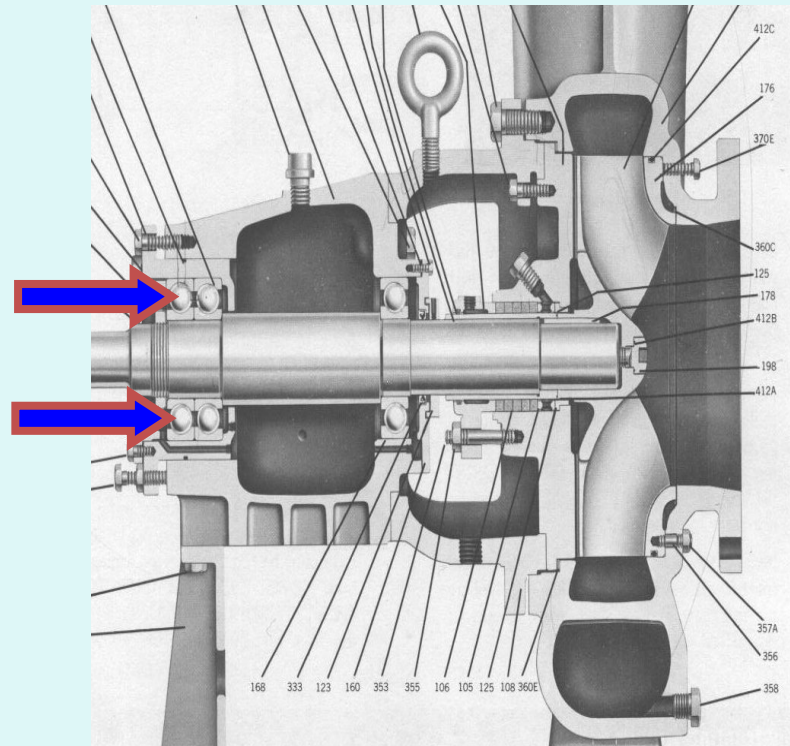
Bearings Vibration Data

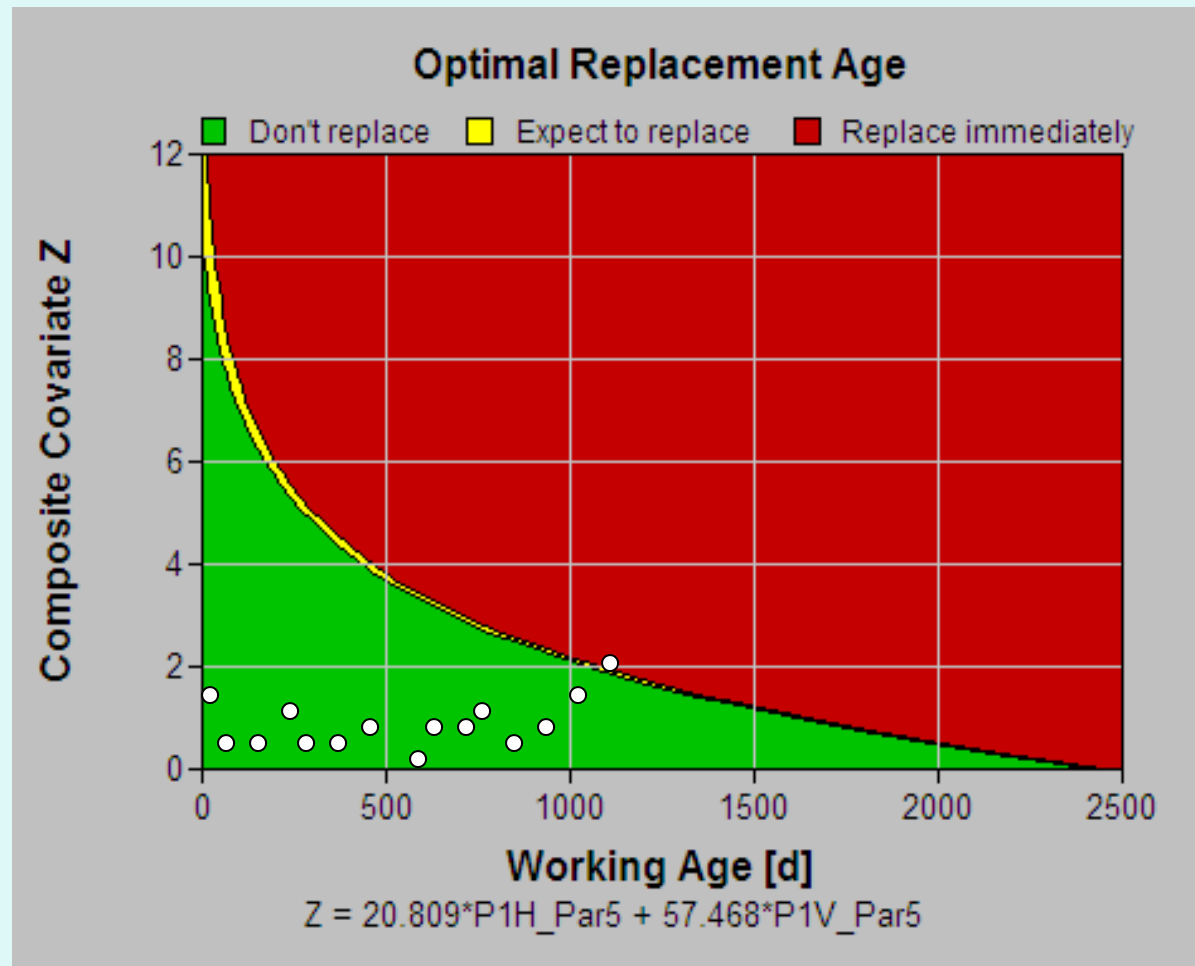– 56 vibration measurements provided by accelerometer

Using <EXAKT>:

– 2 measurements significant

A Check:

– Had <EXAKT> model been applied to previous histories

– Savings obtained = 33 %

Using EXAKT Decision Chart

EXAKT at Irving Pulp & Paper

- Irving P&P have installed the software for everyday use, and have developed a tool to link the relevant databases.

- Steps were taken to make small modifications to the pumps to improve reliability.

- No pump failures for several years, after 15 failures in previous 5 years for 12 pumps.

Modeling of Diesel Engines

Diesel Engines: Failure Modes

	Type of Failure	Count	
0	Not Known	3	
1	Cooling System	13	**Unrelated to oil readings**
2	Fuel System	6	
3	Generator	9	
4	Accessories	9	
5	Cylinder Liners and Rings	24	**Possibly related to oil readings**
6	Valves and Running Gear	20	
7	Pistons, Articulations and Bearings	15	
8	Cylinder Heads	14	
9	Misc, including cylinder block failures.	12	

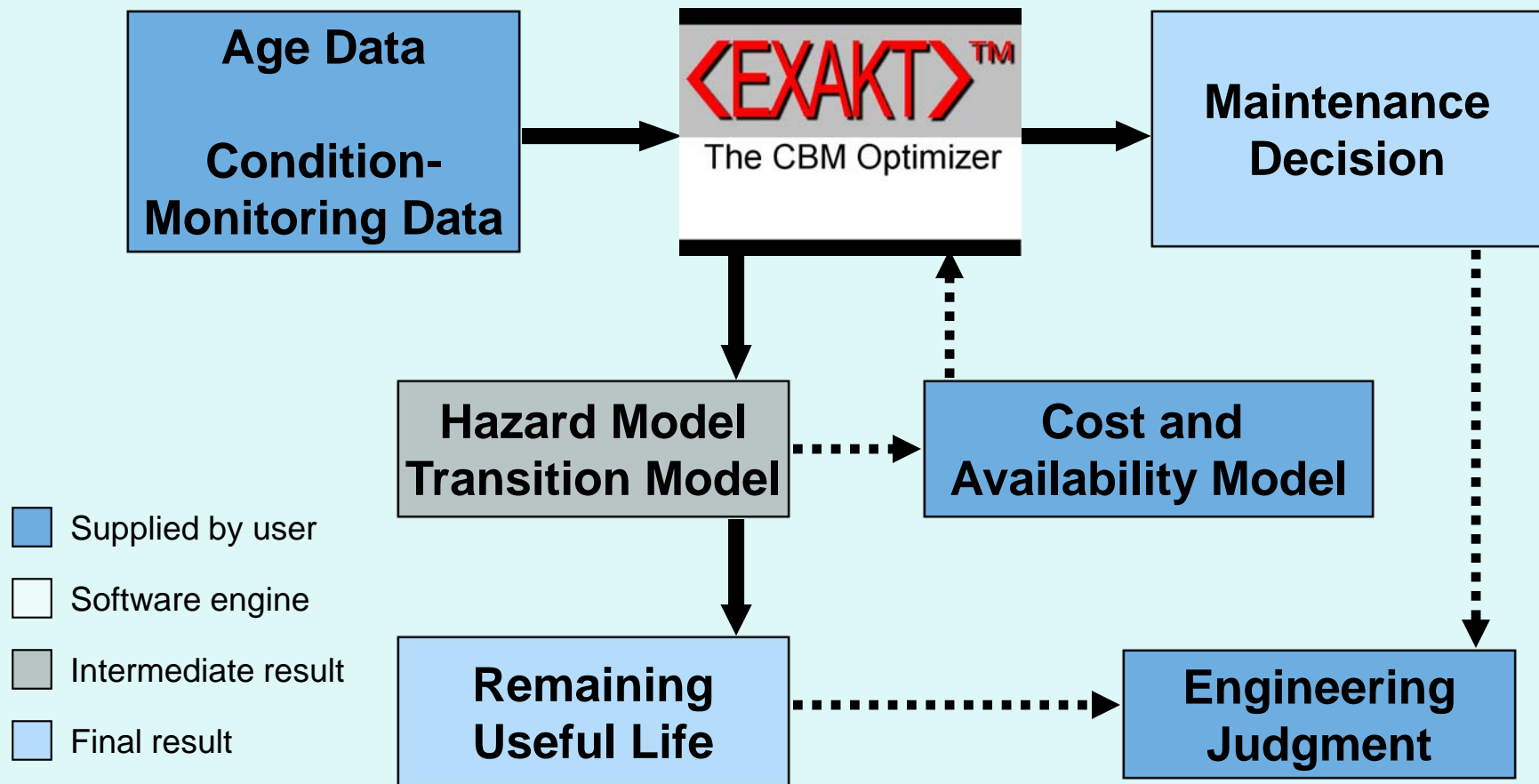# Simultaneous decisions for each failure mode of a repairable system
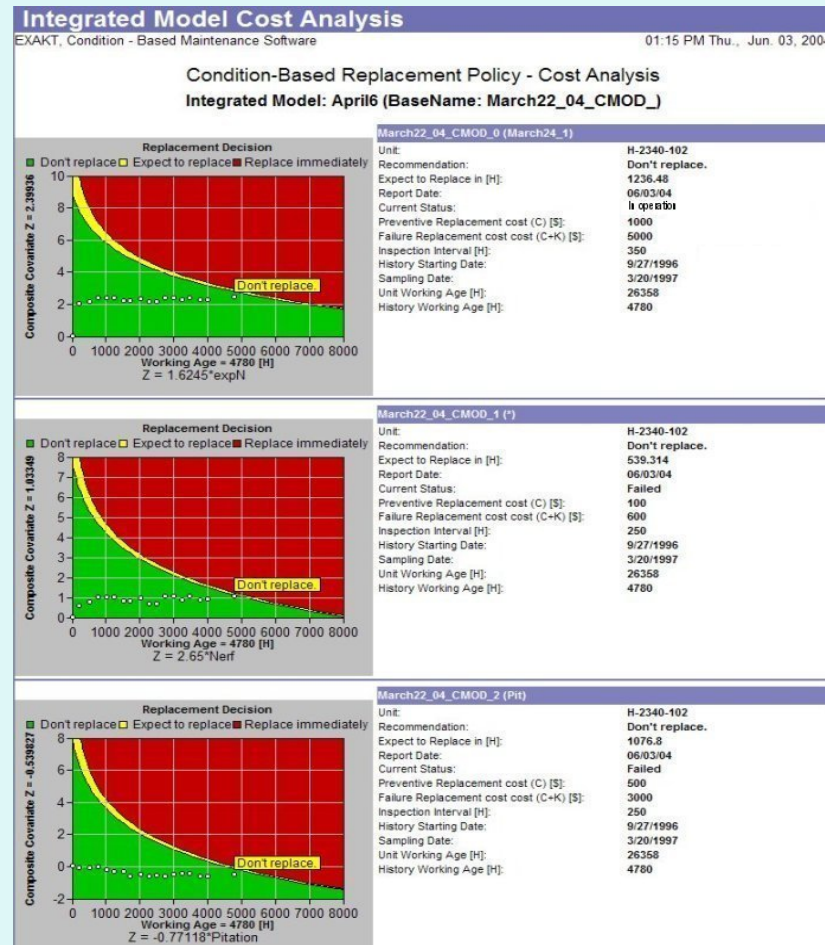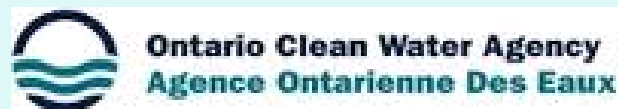
Conditional Density Function & Remaining Useful Life

- Shows the shape of the distribution of the time to failure given current conditions

- Expected time to failure (Remaining Useful Life, or RUL)

Conditional Density Function

Working Age = 9960.00 [h]

Probability Intervals
RUL=12425.554043, StdDev=10724.113435
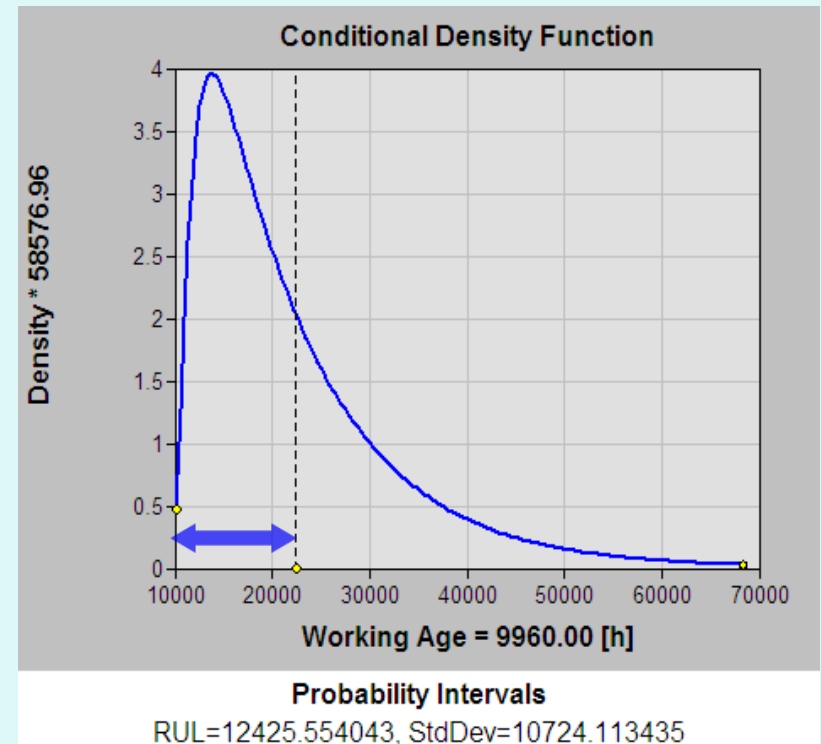
Summary: Principle of CBM Optimization

The MORE Consortium Members

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Thank you

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