String Theory and Geometry of the Universe's Hidden Dimensions

Shing-Tung Yau

Harvard University

Fields Institute

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Introduction

This is the second part of my talk, which relates to THE SHAPE OF INNER SPACE, a new book I've written with the science writer Steve Nadis. At the heart of this book is a mathematical conjecture, raised by the geometer Eugenio Calabi, which ties topology to geometry in ways that many mathematicians considered hard to believe. I was among them. My colleagues and I believed the conjecture was "too good to be true," and, for several years, I tried very hard to prove it was wrong. In my abject failure to do so, I realized that Calabi must have been right after all. I then spent another several years amassing the tools I would need to prove the conjecture, just as he stated it.

VI. A Proof at Long Last

I felt I was close to that point in May 1976. I had all the ducks lined up, as they say. Perhaps my confidence in this problem had something to do with the fact that my girlfriend and I got engaged at that time, while I was visiting her in Princeton. In June, I drove cross-country with my fiance and her parents from Princeton to Los Angeles. It was a very enjoyable trip. But for me, it wasn't strictly for pleasure. Along the way, I was working behind the scenes.

As I drove and sightseed, I was thinking long and hard about solving both the Poincare conjecture and the Calabi conjecture-two of the biggest problems of the day. For the Poincare conjecture,I was hoping to use the theory of minimal surfaces. My original ideas did not quite work, but I could see that the potential was there. I just needed some time and, hopefully, some inspiration.

As for the Calabi conjecture, I thought through the estimates that were needed to solve the nonlinear differential equations-all this while I was out enjoying the American countryside. (As a young man, I didn't know much in those days. But I did know enough not to tell my future wife what I was thinking about at the time.) When we arrived in Los Angeles, my friends at UCLA were very friendly. We found a temporary apartment, and I then went out to buy my first house with my future wife.

We got married in early September and moved to a house in the San Fernando Valley. I was given an office right next to Professor Robert Greene. It was a small office but still very nice. Best of all, I could talk with Robert and other faculty members about subjects of mutual interest-of which there were many. Marriage proved to be truly enjoyable, so much so that within a couple of weeks in this new setting, I was able to put all my ideas together to assemble a proof of the Calabi conjecture.

Life was good. The proof of the Calabi conjecture looked beautiful to me, especially after such a long struggle. It was extremely satisfying to be the first person to understand the argument I had concocted, and I felt certain that it would eventually be important in physics. There is a poem that conveys some of what I was feeling:

> *In the spring, the flowers are falling while I was watching alone. The pair of birds (swallows) were flying together in the light rain.*

I felt that I was truly mingled with nature.

But then I got practical. I remembered all of my earlier efforts to disprove the Calabi conjecture. Each of the supposed counterexamples I had gathered turned out to be actual theorems for which I now had a proof. What's more, many of these statements turned out to be important.

In September of 1976, David Mumford gave a seminar talk at UCLA on solitions. I attended that lecture and another lecture he gave at UC Irvine. There he discussed a conjecture related to the work of Bogomolov about some inequalities between topological numbers of algebraic surfaces. After staring at it, I realized it was an exact consequence of the Calabi conjecture. I had used that same inequality about three years ago in my attempt to disprove the conjecture. (This particular idea was inspired by works of Hitchin and Grey.) So I told Mumford about it.

I double checked it at home and sent the details to Mumford a week later. I was gratified that the expected inequality turned out to be true. But I was also able to prove a further result that led to a solution of the famous Severi conjecture, which concerns the algebraic structure of the so-called “projective space.” This conjecture can be viewed as the Poincare conjecture in an algebraic setting.

The math department at UCLA provided me with a comfortable space to develop my thought. Within a month or so, I met Bill Meeks, whom I'd known from graduate school. Meeks and I immediately got involved in a major development on minimal surfaces, which related geometry with topology. It was used to solve the Smith conjecture later.

Hence, in the period of less than a year, I managed to solve several major mathematical problems. Needless to say, it was the most fruitful year in my career, both personally and professionally.

VII. Enter Physics

Upon solving the Calabi conjecture, I had a strong sense that I had hit upon a beautiful piece of mathematics. And as such, I felt it must be relevant to physics and to our deepest understanding of nature. However, I did not know exactly where these ideas might fit in, as I didn't know much physics at the time.

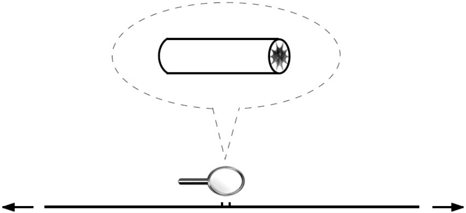
Which isn't to say that I knew nothing about physics. For example, I had been interested in general relativity for a while. In 1973, I was exposed to a problem in general relativity called the positive mass conjecture, which the physicist Robert Geroch discussed at a conference in Stanford-the same conference at which I had tried to disprove the Calabi conjecture.

I started working on this problem with my friend (and former student) Richard Schoen. Expressed in simple terms, the conjecture says that the mass or energy of our universe-or any other isolated physical system-must be positive. Our proof made use of the Plateau problem that I mentioned in the first part of this talk. This work, moreover, brought me closer to my colleagues in physics.

I ran a special year of geometry seminars at the Institute for Advanced Study in Princeton in 1979, where quite a few physicists participated. Subsequently I moved to the Institute for Advanced Study in Princeton as a faculty member. There were many young postdoctoral fellows at the institute. In 1981, I decided to offer Gary Horowitz a postdoctoral fellowship with the intention of studying questions with him in classical relativity.

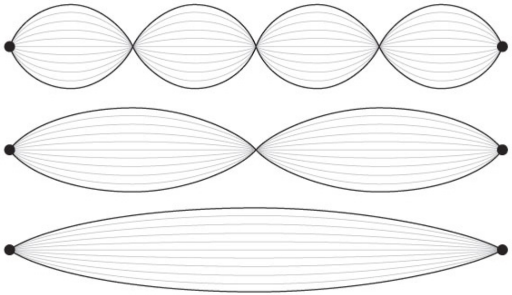
VIII. Close Encounters with String Theory

A couple of years later, in 1984 to be exact, I got several phone calls. Horowitz and his colleague Andy Strominger said that they were very excited about a model for describing the vacuum state of the universe, based on a new theory called string theory.



Kaluza-Klein garden hose

String theory is built on the assumption that particles, at their most basic level, are made of vibrating bits of tiny strings. In order for the theory to be consistent with quantum theory, spacetime has a certain symmetry built into it called supersymmetry. Spacetime is also assumed to be ten dimensional.



Vibrating strings

Horowitz and Strominger were interested in the multidimensional spaces whose existence I proved, mathematically, in my confirmation of the Calabi conjecture. They believed that these spaces could play an important role in string theory, as they seemed to be endowed with the right kind of supersymmetry — a property deemed essential to their theory. They asked me if their assessment of the situation was correct and, to their delight, I told them that it was.

Then I got a phone call from Ed Witten whom I'd met in Princeton the year before. Witten told me that this was the one of the most exciting eras in theoretical physics. It was just like the time when quantum mechanics was being developed.

Witten

He told me that everyone who made contributions to quantum mechanics in early days left their name in the history of physics. He said that the important discoveries of early string theorists, such as Michael Green and John Schwarz, could lead to the grand unification of all forces — the goal that Einstein had spent the last 30 years of his life working toward.

Witten was now collaborating with Candelas, Horowitz, and Strominger, trying to figure out the shape, or geometry, of the six "extra" dimensions of string theory. The physicists believed these six dimensions were curled up in a tiny space, which they called Calabi-Yau space — the same family of spaces originally proposed by Calabi and later proved by me.

With Candelas, 2001

String theory, again, assumes that spacetime has 10 dimensions overall. The three large spatial dimensions that we're familiar with, plus time, make up the four-dimensional spacetime of Einstein's theory. But there are also six additional dimensions hidden away in Calabi-Yau space, and this invisible space exists at every point in "real space," according to the theory, even though we can't see it.

The existence of this extra-dimensional space is fantastic on its own, but string theory goes much farther. It says that the exact shape, or geometry, of Calabi-Yau space dictates the properties of our universe and the kind of physics we see. The shape of Calabi-Yau space — or the "shape of inner space," as we put it in our book — determines the kinds of particles that exist, their masses, the ways in which they interact, and maybe even the constants of nature.

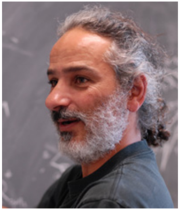
While Einstein had said the phenomenon of gravity is really a manifestation of geometry, string theorists boldly proclaimed that the physics of our universe is a consequence of the geometry of Calabi-Yau space. That's why string theorists were so anxious to figure out the precise shape of this six — dimensional space — a problem we're still working on today.

Witten was eager to learn more about Calabi-Yau spaces. He flew from Princeton to San Diego to talk with me about how to construct them. He also wanted to know how many Calabi-Yau spaces there were for physicists to choose among. Initially, physicists thought there might only be a few examples — a few basic topologies — which made the goal of determining the shape that corresponds to our universe seem a lot more manageable. But we soon realized there were many more examples of Calabi-Yau spaces-many more possible topologies — than were originally anticipated.

The task of figuring out the shape of inner space suddenly seemed more daunting, and perhaps even hopeless if the number of possibilities turned out to be infinite. The latter question has yet to be settled, although I have always thought that the number of Calabi-Yau's is finite. That number is certain to be big, but I believe it is bounded.

The great excitement over Calabi-Yau space started in 1984, when physicists first found out about them. That enthusiasm kept up for a couple years, before waning. But the excitement picked up again in the late 1980s, when Brian Greene, Ronen Plesser, Philip Candelas, and others began exploring the notion of "mirror symmetry."

Greene



Plesser

The basic idea here was that two different Calabi-Yau spaces, which had different topologies and seemed to have nothing in common, nevertheless gave rise to the same physics. This established a previously unknown kinship between so-called mirror pairs of Calabi-Yau's.

The connection uncovered through physics proved to be extremely powerful in the hands of mathematicians. When they were stumped trying to solve a problem involving one Calabi-Yau space, they could try solving the same problem on its mirror pair. On many occasions, this approach was successful. As a result, problems that had defied resolution, sometimes for as long as a century, were now being solved. And a branch of mathematics called enumerative geometry was suddenly rejuvenated. These advances gave mathematicians greater respect for physicists, as well as greater respect for string theory itself.

IX. Conclusion

Before we get too carried away, we should bear in mind that string theory, as the name suggests, is just a theory. It has not been confirmed by physical experiments, nor have any experiments yet been designed that could put that theory to a definitive test. So the jury is still out on the question of whether string theory actually describes nature, which was the original intent.

On the positive side of the ledger, some extremely intriguing, as well as powerful, mathematics has been inspired by string theory. Mathematical formulae developed through this connection have proved to be correct independent of the scientific validity of string theory. So far it stands as the only consistent theory that unifies the different forces. And it is beautiful. Moreover, the effort to unify the different forces of nature has unexpectedly led to the unification of different areas mathematics that at one time seemed unrelated.

We still don't know what the final word will be. In the past two thousand years, the concept of geometry has evolved over several important stages to the current state of modern geometry. Each time geometry has been transformed in a major way, the new version has incorporated our improved understanding of nature arrived at through advances in theoretical physics. It seems likely that we shall witness another major development in the 21st century, the advent of quantum geometry — a geometry that can incorporate quantum physics in the small and general relativity in the large.

The fact that abstract mathematics can reveal so much about nature is something I find both mysterious and fascinating. This is one of the ideas that my coauthor and I have tried to get across in our book, The Shape of Inner Space. We also hope that the book gives you a description of how mathematicians work. They are not necessarily weird people, such as a janitor who solves centuries-old math problems on the side while mopping and dusting floors, as described in the movie "Good Will Hunting". Nor does a brilliant mathematician have to be mentally ill, or exhibit otherwise bizarre behavior, as depicted in another popular movie and book.

Mathematicians are just scientists who look at nature from a different, more abstract point of view than the empiricists. But the work mathematicians do is still based on the truth and beauty of nature, the same as it is in physics. Our book tries to convey the thrill of working at the interface between mathematics and physics, showing how important ideas flow through different disciplines, with the result being the birth of new and important subjects.

In the case of string theory, geometry and physics have come together to produce some beautiful mathematics, as well as some very intriguing physics. The mathematics is so beautiful, in fact, and it has branched out into so many different areas, that it makes you wonder whether the physicists might be onto something after all.

The story is still unfolding, to be sure, and I consider myself lucky to have been part of it.