Multi-fluid Poro-elastic Modelling of the CSF Infusion Test

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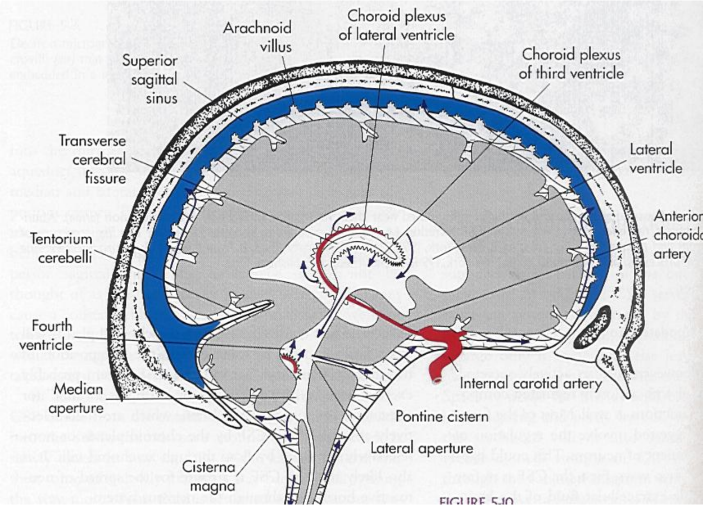
Outline

Cerebrospinal Fluid Flow Path



From: Nolte, The human brain: an introduction to its functional anatomy, 5th ed., p. 108

Infusion Test

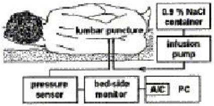
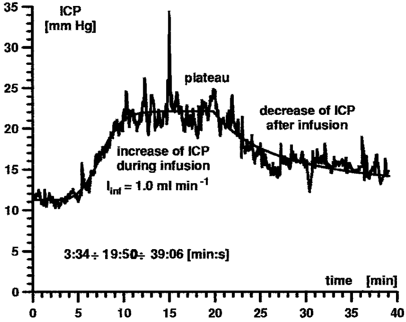
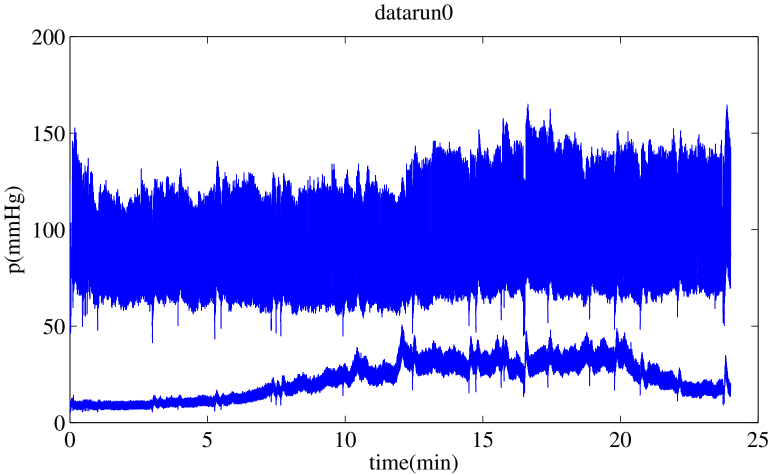


Figure 1. ICP monitoring system used during infusion test.



From Juniewicz et al.: Physiological Measurement 2005, pp1039–1048

Clinical Data



Arterial blood pressure (upper) and CSF pressure (lower)

Single Compartment Model (standard)

ODE for CSF pressure

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}p}\frac{\mathrm{d}p}{\mathrm{d}t} = C(p)\frac{\mathrm{d}p}{\mathrm{d}t} = \hat{Q} - \frac{p - p_\mathrm{r}}{R}$$

with compliance

$$C(p) = \frac{1}{E(p - p_\mathrm{r})},$$

resistance to outflow R, elasticity E, reference pressure p_r,

and given step function for the inflow rate

$$\hat{Q} = \begin{cases} Q_\mathrm{prod} & \text{before and after infusion} \\ Q_\mathrm{prod} + Q_\mathrm{inf} & \text{during infusion} \end{cases}$$

Single Compartment Model (standard)

ODE for CSF pressure

$$C(p)\frac{\mathrm{d}p}{\mathrm{d}t} = Q - \frac{p - p_\mathrm{b}}{R}, \qquad p(0) = p_b$$

with compliance

$$C(p) = \frac{1}{E(p - p_\mathrm{r})},$$

resistance to outflow R, elasticity E, reference pressure p_r,
baseline pressure $p_\mathrm{b} = RQ_\mathrm{prod} + p_\mathrm{r}$,
and given step function for the infusion rate

$$Q = \begin{cases} 0 & \text{before and after infusion} \\ Q_\mathrm{inf} & \text{during infusion} \end{cases}$$

Single Compartment Model (extended)

ODE for CSF pressure

$$C(p)\frac{\mathrm{d}p}{\mathrm{d}t} = Q - \frac{p - p_{\mathrm{b}}}{R}, \qquad p(0) = p_b$$

with compliance

$$C(p) = \frac{1}{\tilde{e}(p - p_{\mathrm{r}})^n} = \frac{1}{\tilde{e}(p - p_{\mathrm{r}})^{n-1}}\frac{1}{(p - p_{\mathrm{r}})} = \frac{1}{E(p)}\frac{1}{(p - p_{\mathrm{r}})},$$

resistance to outflow R, elasticity paramater $\tilde{e}$, reference pressure p_{r},
baseline pressure $p_{\mathrm{b}} = RQ_{\mathrm{prod}} + p_{\mathrm{r}}$,
and given step function for the infusion rate

$$Q = \begin{cases} 0 & \text{before and after infusion} \\ Q_{\mathrm{inf}} & \text{during infusion} \end{cases}$$

Solutions and Least Squares

Standard model ($n = 1$): Analytic solution

$$p(t) = p_r + \frac{(p_b - p_r)(RQ_{inf} + p_b - p_r)}{(p_b - p_r) + RQ_{inf} \exp\left(-\frac{E}{R}(RQ_{inf} + p_b - p_r)(t - t_0)\right)}$$

Extended model: Solve numerically (`ode45`)
Use scaled parameters

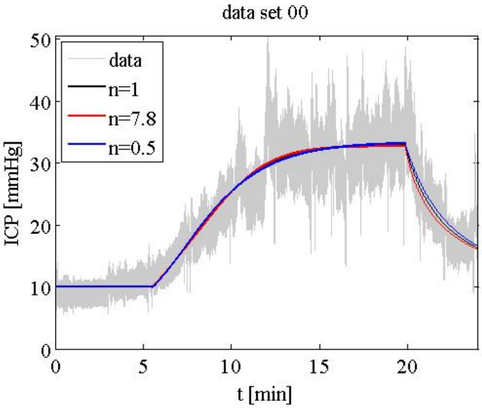
$$x_1 = \frac{p_b}{p_b^g}, \quad x_2 = \frac{\tilde{e}}{R}\frac{R^g}{\tilde{e}^g}, \quad x_3 = \frac{p_r}{p_r^g}, \quad x_4 = \frac{RQ_{inf} + p_b}{R^g Q_{inf}^g + p_b^g}, \quad x_5 = n.$$

Least squares fit (`fminsearch`)

$$\min_{x \in \mathbb{R}^5} F(x) = \sum_{t \in \mathcal{T}} (p_x(t) - p_{meas}(t))^2$$

with $|\mathcal{T}| \sim 50.000$

Results

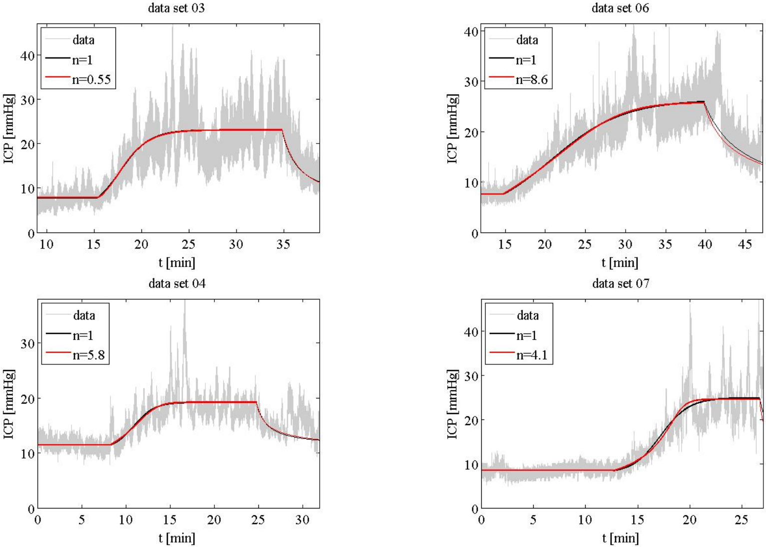


Parameter values:

n_{giv}	1	–	0.5
n_{fit}	–	7.8	–
$\tilde{e}$	–	≈ 0	1.2
$E\vert_{p_b}$	0.24	0.02	0.91
R	15.4	15.2	15.6
p_b	9.97	9.99	9.95
p_r	2.1	-90	8.1
Q_{pr}	0.51	6.63	0.12
ΔV	5.7	5.5	5.8

$[\tilde{e}] = \frac{\text{mmHg}^{1-n}}{\text{ml}}$, $[E] = \frac{1}{\text{ml}}$, $[R] = \frac{\text{mmHg min}}{\text{ml}}$,
$[p] = \text{mmHg}$, $[Q_{pr}] = \frac{\text{ml}}{\text{min}}$, $[\Delta V] = \text{ml}$

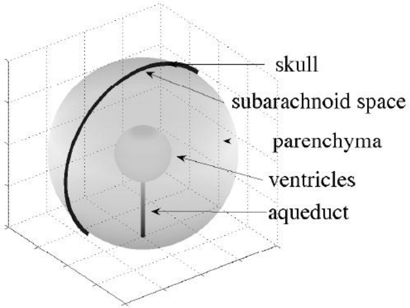
Results (2)


data set 03
data
n=1
n=0.55
ICP [mmHg]
t [min]
data set 06
data
n=1
n=8.6
ICP [mmHg]
t [min]
data set 04
data
n=1
n=5.8
ICP [mmHg]
t [min]
data set 07
data
n=1
n=4.1
ICP [mmHg]
t [min]

Poroelastic Model (single fluid)

Thick spherical shell of porous elastic solid filled with incompressible fluid

u – radial displacement of solid
p – fluid pressure



Boundary conditions:

- Given fluid flux Q_{in} into the ventricle
- Stress continuity at ventricle wall
- Given resistance to outflow $Q_{out} = \frac{p - p_{venous}}{R}$ at skull
- Porous tissue (solid) fixed to skull: $u(r_{out}, t) = 0$
- Poiseuille flow through aqueduct included in boundary conditions

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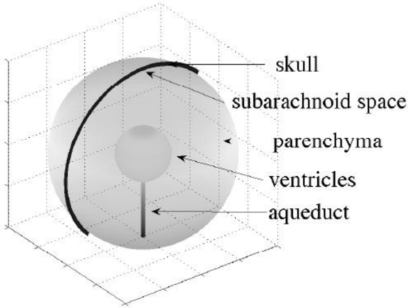

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Governing equations (single fluid)

(Linearised) strain of the solid

$$\epsilon = \mathrm{tr}(\varepsilon) = \frac{\partial u}{\partial r} + 2\frac{u}{r}$$

Combined stress

$$\sigma = \sigma_{\text{solid}} + \alpha\sigma_{\text{fluid}} = K\varepsilon - \alpha p I$$

Conservation of momentum

$$\underbrace{\nabla\cdot\sigma}_{\sim 10^4} = \underbrace{\rho^{\mathrm{s}}\frac{V_{\mathrm{s}}}{V_0}\frac{\partial^2 u}{\partial t^2}}_{\sim 10^{-3}} + \underbrace{\rho^{\mathrm{f}}\frac{V_{\mathrm{f}}}{V_0}\frac{\partial q}{\partial t}}_{\sim 10^{-3}},$$

thus, neglecting inertial terms,

$$K\frac{\partial \epsilon}{\partial r} - \alpha\frac{\partial p}{\partial r} = 0$$

Increase of fluid content in volume V_0

$$\zeta := \frac{V_{\mathrm{f}} - V_{\mathrm{f},0}}{V_0} = \alpha\epsilon + sp$$

Darcy flow through porous medium

$$q = -\frac{k(\epsilon)}{\mu}\frac{\partial p}{\partial r}$$

Fluid volume balance

$$\frac{\partial \zeta}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 q\right)$$

yields

$$\alpha\frac{\partial \epsilon}{\partial t} + s\frac{\partial p}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{k(\epsilon)}{\mu}\frac{\partial p}{\partial r}\right)$$

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Extension to two fluid model

Include measured arterial blood pressure $p_{\text{ab}}(t)$ into stress

$$\sigma = \sigma_{\text{solid}} + \alpha\sigma_{\text{CSF}} + \alpha_{\text{ab}}\sigma_{\text{ab}} = K\varepsilon - \alpha pI - \alpha_{\text{ab}}p_{\text{ab}}I$$

and increase in CSF content

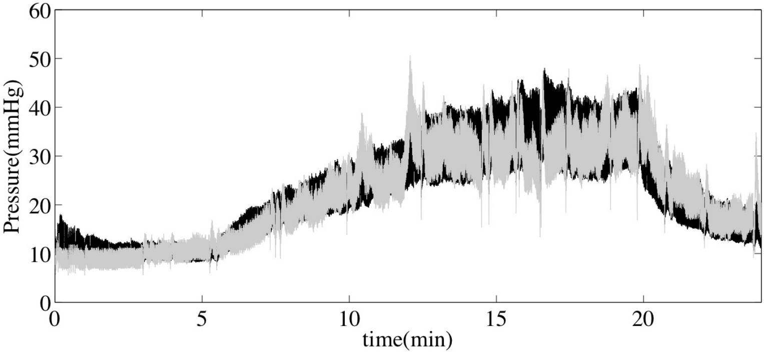
$$\zeta = \alpha\epsilon + sp - s_{\text{ab}}p_{\text{ab}}$$

yielding the equations

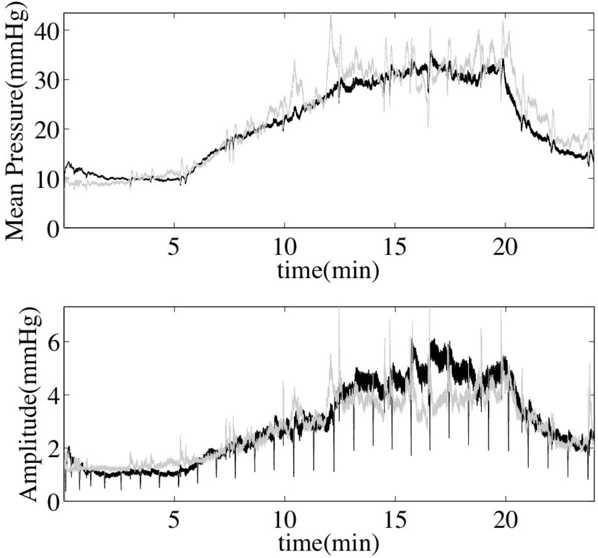
$$K\frac{\partial\epsilon}{\partial r} - \alpha\frac{\partial p}{\partial r} = \alpha_{\text{ab}}\frac{\partial p_{\text{ab}}}{\partial r},$$
$$\alpha\frac{\partial\epsilon}{\partial t} + s\frac{\partial p}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{k(\epsilon)}{\mu}\frac{\partial p}{\partial r}\right) + s_{\text{ab}}\frac{\partial p_{\text{ab}}}{\partial t}$$

Results

Finite Difference simulation of data set 00:



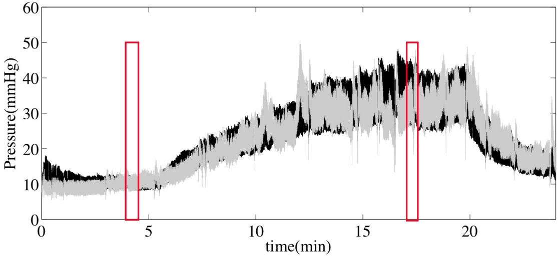
Results (Details)



Simulation Movies

Before infusion: 4.0–4.5 min

During infusion: 17.0–17.5 min



Conclusions & Further work

Conclusions

- Highly nonlinear relation between model parameters and form of pressure curve in extended compartment model
- Only some parameters can be fitted reliably to clinical data
- No spatial variation in compliance model
- Poroelastic model predicts reasonable values
- Physical parameters may not be known

Further work

- Finite Element simulation (time dependent)
- Improve multifluid poroelastic model
- Further analysis of clinical data

Thank you!