Human Capital as Age-Dependent Asset Mix and Optimal Life-Cycle Portfolios

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Introduction

- The largest component of household wealth consists of nontraded human capital. For a nation as a whole, returns to human capital (i.e., labor's share) is about 60% in national income. Therefore, labor income should play a central role in structuring household financial decisions.

- Human capital should be viewed as the core wealth, and households' financial wealth should be structured as a supplementary portfolio to make up the most desirable form of the total wealth portfolio.

- Investments in financial assets should act as a powerful tool for hedging the risk associated with nontraded human capital.

- Therefore, a key in understanding life-cycle pattern of household finance is the age-dependence of human capital.

Samuelson (1969) and Merton (1969)

- Economic inquiries into agents' consumption and portfolio decisions originates in Samuelson (1969) and Merton (1969).

- Based on three basic assumptions:
  (1) asset returns are independently and identically distributed over time (i.e., asset prices follow geometric Brownian motions),
  (2) agents have utility functions of the CRRA (constant relative risk aversion) class that are time-invariant and additively separable over time,
  (3) markets are frictionless and complete,

  Samuelson and Merton showed that optimal portfolio weights are constant over the life cycle; in other words, optimal portfolio weights are neutral to investment time horizon.

Including labor income

- Including labor income in the portfolio choice problem, several authors have shown that the Samuelson-Merton neutrality theorem is converted to a prediction that the share of bonds tend to rise with age up until retirement.
- Merton (1971) was the first in this line of argument.
- Assuming that labor income is certain, he showed that households will alter their holdings of riskless asset in order to maintain constant portfolio weights on risky and riskless asset on overall wealth.
- More specifically, he showed that the optimal portfolio rule is still a constant-mix of the overall wealth if lifetime flow of labor income is capitalized at the risk-free rate and added to the financial wealth.

Target Date Funds

- Since the human capital thus defined is risk-free and very large for young workers, they tend to have less bonds and more stocks in their financial wealth. While the goal of this activity is to ensure that the share of risky assets in the overall portfolio remains constant, the share of stocks in the portfolio of financial assets tend to decline toward retirement.

- This view has laid a theoretical foundation for the typical financial advise that as workers age their portfolio's allocation should shift from primarily stocks, to a balanced portfolio, and then to a primarily bond portfolio.

- The fast grown financial product of the mutual fund industry, *life cycle funds* (also called *target date funds*), is based on this exact premise.

- Indeed, one often-quoted strategy suggested by financial advisors is that investors should place (100 − age)% of their wealth in a well-diversified equity portfolio.

Target Date Funds

Bodie and Treussard [2006]

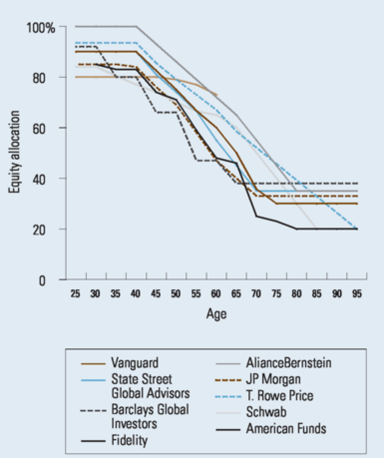

Fig. 1 Target Date Funds

Human Capital over the life Cycle

- Other authors considered the effect of uncertain labor income streams and concluded that the theoretical finding of Merton (1971) with non-random labor income should not be altered.
- These authors include include Bodie, Merton, and Samuelson (1992), Jagannathan and Kocherlakota (1996), Heaton and Lucas (1997), Davis and Willen (2000), Viceira (2001), Campbell et al. (2001), Campbell and Viceira (2002), Haliassos and Michaelides (2003), Cocco, Gomes, and Maenhout (2005), and so on.

Human Capital over the life Cycle

- This result is founded on the observation that the correlation between labor income risk and stock market risk is small and therefore human capital should act more like riskless bonds rather than stocks.

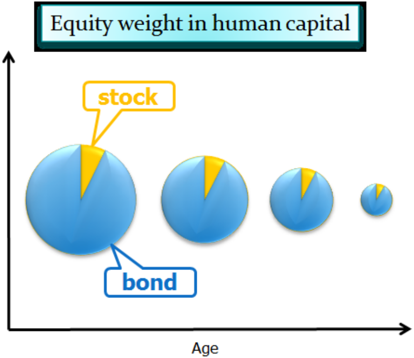

Fig. 2 Human Asset as a constant Asset Mix over the LIfe Cycle

Correlation between Labor Income and Stock Market in the Short- and Long Horizon

- We question whether the low correlation between labor income risk and stock market risk should automatically imply that human capital is bond-like.

- We think the answer depends critically on the stochastic nature of labor income process.

- In this regard much of the previous human capital literature adopts the assumption that labor income and stock prices follow correlated AR(1) processes. We will show in a continuous time setting that this assumption generates an endogenous property of human capital that its proportional exposure to the stock market return is constant over the life cycle. If human capital is 70% bond and 30 % stock at a worker's age 30, it is of the same mix at age 55.

Our Model of Labor Income Flow

- We propose an alternative model of stochastic labor income with the following basic insight.

- Unlike liquid asset prices, movements of labor income are sluggish and are more or less foreseeable in the short run. Large uncertainty of a worker's labor income in a longer horizon is a result of accumulation of short-run fluctuations of economic conditions. A young worker has many years to go and his future labor income picks up the impacts of temporal economic shocks and accumulates them over a long horizon.

- Then, the fluctuations of labor income over a horizon is very similar to the fluctuations of balance of a money market account over time, except that the temporal fluctuations of labor income are driven by equity-related shocks while the temporal fluctuations of money market balance are driven by shocks in short-term interest rate.

Our Model of Labor Income Flow

- Based on this motivation we model the labor income as a finite variation process
$$dw_t = \mu_t^w w_t dt,$$
with μ_t^w the instantaneous growth rate of labor income at time t, which we assume to be observable at time t.

- This formulation suggests that to express the evolution of μ_t^w one can adopt various class of term structure models which represent movements of the short-term interest rate.

Our Model of Labor Income Flow

- We specifically assume that the instantaneous growth rate μ_t^w evolves according to a generalized Vasicek process, a stationary Gaussian process that reverts toward the mean $\alpha_\mu(t)$

$$d\mu_t^w = \kappa\left(\alpha_\mu(t) - \mu_t^w\right) dt + \sigma^w dW_t.$$

  Here, $W_t = (W_{1t}, \ldots, W_{nt})$ denotes multiple sources of independent Brownian uncertainty, and $\sigma^w = (\sigma_1^w, \ldots, \sigma_n^w)$ is the vector of volatility coefficients. The mean $\alpha_\mu(t)$ is a deterministic function of time which embeds the time-path of expected labor income over the life cycle. A subset of the Brownian motion W_t are also factors which drive returns of individual stocks.

- This is the simplest model that captures the essence of our insight. Various correlation structure between labor income and stock returns can be represented by the vector σ^w, and various forms of incomplete and imperfect market assumptions can be imposed.

Cointegration

- "Cointegration school" propose a model in which labor income and stock market returns are cointegrated.
- For young workers cointegration has numerous opportunities to act over business cycles until retirement, and so young worker's human capital effectively becomes stock-like. In contrast, for older workers with shorter times to retirement, cointegration does not have sufficient time to act, and thus their human capital becomes more bond-like.

Cointegration

- Baxter and Jermann test for the existence of cointegration by using data on aggregate employee compensation and GDP growth and theoretically examined the impact of human capital on international diversification. They argue that because of the stock-like nature of human capital a substantial short position in domestic stocks is required to hedge human capital risk, which makes the home country bias a more serious challenge.

- Lucas and Zeldes ask how a defined benefit pension plans should hedge their projected benefit obligation (PBO) liabilities. Since pension payments are linked by formula to the number of years of employment and wage earnings in the final year(s) of employment, a long-run cointegration between labor income and stock returns implies that PBO liabilities are more stock-like for firms with greater percentage of active workers than for firms with high proportion of retirees and separated workers. This in turn implies that a large share of the portfolio should be invested in stocks for young firms, with the share in stocks declining as the employees age.

Cointegration

- Benzoni, Collin-Dufresne, and Goldstein (2007) consider how workers' portfolio decisions should reflect human capital, in a domestic portfolio context. They assume that labor income is the sum of a component linked to the aggregate economy and a component linked to idiosyncratic shocks. T*hey specify aggregate labor income to be cointegrated with dividends, instead of stock market returns*. They then calibrate their labor income model to the Panel Study of Income Dynamics (PSID) micro data.
- They find that young workers should take substantial short positions in the stock market because their human capital is stock-like and large in size. For older generation with shorter times to retirement, their human capital becomes more bond-like. Combined with how the size of human capital changes over the life cycle, cointegration in effect creates hump-shaped life-cycle portfolio holdings. Among the papers cited above, BCG proposes a most comprehensive model of cointegrated labor income and offers a complete dynamic analysis.

Cointegration *vs* our Model

- Our economic insight is different from the cointegration school, since mean reversion is not the mechanism that creates age-dependence of stock market exposure of human capital.
- In contrast, without mean reversion the cointegration models reduce to the more traditional models in which stock market exposure of human capital is age-independent. This is the reason that the authors of the cointegration school repeatedly defend themselves against the alleged weak rejection of the unit-root hypothesis.

Our Mathematical Approach

- We use the martingale method developed by Karatzas et al. (1987) and Cox and Huang (1989) to represent the optimality condition. We then derive a closed-form formula for the optimal consumption and portfolio policy with the help of *Malliavin calculus* following a procedure developed by Ocone and Karatzas (1991) and Detemple et al. (2003, 2005).
- The Malliavin derivative is particularly useful to analyze the exposure of human capital to various risk factors and to investigate how the exposure changes over the life cycle. The age-dependence of the risk exposure combined with the age profile of the magnitude of the value of human capital generates the optimal life-cycle portfolio policy.

Our Mathematical Approach

- Our main focus is to investigate the factors which determine portfolio rules over the life cycle, whether the optimal stock holdings should decline as workers age as more traditional models predict, or they exhibit hump-shaped life-cycle stock holdings as the cointegration literature predict.
- We assume a constant investment opportunity set in the sense that stock prices evolve according to geometric Brownian motions.
- We also assume that the short-term rate of interest is constant.
- A setback is that constant interest rate makes the distinction of bond and cash meaningless, but we leave consideration of stochastic interest rate for a future task as it add another dimension to the nature of the optimal life-cycle portfolio problem.

Our Mathematical Approach

- Another key assumption is that households have time-additive constant relative risk aversion (CRRA) utility functions. We will see that the CRRA assumption generates the simple but perspicacious property that the optimal portfolio policy is to maintain constant portfolio weights on risky and riskless asset on overall wealth which includes human capital, and that this property does not depend on the market completeness. We will carry out the investigation both in a complete and incomplete market settings.
  - Following the tradition we use the word "incomplete market" to refer to missing markets for hedging some components of investment risks. In contrast, by "imperfect market" we mean that there are some trading restrictions, such as short selling and borrowing restrictions. In this paper we do not consider the impact of market imperfectness.

Our Findings

- We find that both of the two regimes, the declining stock holdings and the hump-shaped stock holdings, can emerge when labor income risk has a component which cannot be hedged by any set of stock portfolios.

- When the unhedgeable component is a relatively small fraction of the overall labor income risk, the optimal fraction of stock holdings depicts a hump shape over the life cycle. In contrast, when the unhedgeable component dominates the overall labor income risk, the optimal fraction of stock holdings becomes a decreasing function of age.

- Our finding suggests that which life-cycle portfolio policy contributes to the maximum household welfare rests on the extent of market completeness, or the breadth of opportunities that the financial market offers to hedge the labor income risk.

General Optimal Portfolio Policy

- Define

$$\Xi_{t,s} \equiv \int_t^s (\mathcal{D}_t r_u + \theta_u' \mathcal{D}_t \theta_u) du + \int_t^s dW_u' \cdot \mathcal{D}_t \theta_u \qquad (1)$$

As we obtain $\mathcal{D}_t \xi_{t,s} = -\xi_{t,s} \Xi_{t,s}$, the quantity $\Xi_{t,s}$ measures the magnitude of Malliavin derivative at date t of conditional state price density $\xi_{t,s}$. If the interest rate and the market price of risk have no stochastic component, this magnitude is zero.

Proposition (3)

The optimal portfolio policy $\pi^* = \{\pi_t : 0 \leq t \leq T\}$ is given by

$$\pi_t^* = \frac{1}{\gamma} (X_t + H_t) \sigma_t'^{-1} \theta_t + \left(1 - \frac{1}{\gamma}\right) (X_t + H_t) \sigma_t'^{-1} \psi_t - \sigma_t'^{-1} (\mathcal{D}_t H_t)', \qquad (2)$$

where ψ_t is a $n \times 1$ vector defined by

$$\psi_t \equiv -\frac{E_t \left[\int_t^T \xi_{t,s}^{1-1/\gamma} \beta_{t,s}^{1/\gamma} \Xi_{t,s}' ds + \xi_{t,T}^{1-1/\gamma} \beta_{t,T}^{1/\gamma} \Xi_{t,T}' \right]}{E_t \left[\int_t^T \xi_{t,s}^{1-1/\gamma} \beta_{t,s}^{1/\gamma} ds + \xi_{t,T}^{1-1/\gamma} \beta_{t,T}^{1/\gamma} \right]} \qquad (3)$$

Structure of the Optimal Portfolio Policy: Constant Investment Opportunuty

- A well-known property is that if the investment opportunity set is constant the optimal portfolio policy of a CRRA agent is to follow a *constant mix strategy*; a strategy of determining the baseline portfolio and maintaining the portfolio weights by rebalancing the portfolio instantly as the security prices move.

- In fact, without labor income Equation (40) reduces to $\pi_t^{*\prime}\sigma = X_t(1/\gamma)\theta'$ since the hedging term vanishes. This yields a portfolio policy $\pi_t^* = X_t(1/\gamma)\sigma'^{-1}\theta$, or

$$\pi_t^*/X_t = (1/\gamma)\sigma'^{-1}\theta. \tag{4}$$

  The instruction of this portfolio policy is simply to form a mean-variance efficient portfolio reflecting the level of agent's relative risk aversion γ. The remaining balance should be invested in the cash account.

- Note that the relative weight on the portfolio of risky assets and cash is time invariant. Its agenda is to maintain a constant mix portfolio of all securities including the riskless asset.

Structure of the Optimal Portfolio Policy: Stochastic Investment Opportunity

- When the investment opportunity is stochastic (40) shows that there is an additional term which hedges against fluctuations in the interest rate and the market price of risk. The composition of this hedging portfolio is given by $\sigma_t'^{-1}\psi_t$.
- Three observations must be made regarding this hedging component.

(1) Relative weight between the mean-variance portfolio $\sigma_t'^{-1}\theta_t$ and the hedging portfolio $\sigma_t'^{-1}\psi_t$ is $1/\gamma$ to $1-1/\gamma$. For a more risk-averse household a higher weight should be placed on the hedging portfolio and less weight on the mean-variance portfolio. When $\gamma = 1$ (logarithmic utility) the hedging term vanishes, which has been known for a long time.

Structure of the Optimal Portfolio Policy: Stochastic Investment Opportunity

(2) Like the mean-variance portfolio the proportion of the hedging portfolio is independent of wealth. The hedging demands can be synthesized using combinations of financial securities. If one of the assets available is a mutual fund with return volatility collinear with ψ'_t, then this mutual fund alone will suffice to synthesize the hedge.

(3) A desirable property would be that these portfolios fit to all households. Unfortunately the assumption of CRRA utility functions is not sufficient to guarantee this separation property. Additionally it is required that all households have an identical relative risk-aversion γ and an identical time-discount factor β, since these parameters play a role in (41). Otherwise, these hedging portfolios depend on preferences and are therefore investor-specific.

Structure of the Optimal Portfolio Policy: Human Capital

- If the problem involves labor income, Equation (40) provides

$$\pi_t^* = \frac{1}{\gamma} \left(X_t + H_t \right) \sigma'^{-1} \theta - \sigma'^{-1} \left(\mathcal{D}_t H_t \right)' . \tag{5}$$

- Consider the simplest case where labor income is certain. The value of human capital defined in (31) becomes

$$H_t \equiv \int_t^T E_t \left[\xi_{t,s} \right] w_s ds = \int_t^T \exp\left(-r \left(s - t \right) \right) w_s ds, \tag{6}$$

since w_s is nonrandom in this case. It means that the lifetime flow of labor income should be capitalized at the risk-free rate and added to the financial wealth. If human capital H_t is riskless, $\mathcal{D}_t H_t = 0$ and we get

$$\pi_t^* = \frac{1}{\gamma} \left(X_t + H_t \right) \sigma'^{-1} \theta. \tag{7}$$

- With certain labor income stream the optimal portfolio policy is still a constant-mix strategy. But it should be a constant-mix portfolio of total wealth, and not a constant mix portfolio of her financial wealth.

Structure of the Optimal Portfolio Policy: Human Capital

- This basic property extends to the case of uncertain labor income. The second term comprising the optimal policy in (43) is the weight vector of a portfolio of risky securities which replicates the human capital. When the market is complete, human capital can be monetized as a completely liquid asset. The optimal consumption-terminal wealth plan is unchanged given the household's total initial wealth, regardless of its composition between financial assets and human capital.

- This leads to the observation that the optimal portfolio of financial assets heavily depends on the magnitude of human capital and the nature of stochastic fluctuations of labor income.

- One-size-fits-all portfolio is far from optimal since the portfolio which replicates the human capital should depend on such factors as occupation and industry sectors that the household work. It is also important to recognize that the portfolio replicating the human capital should depend on the life stage of the household. This is the theoretical basis for life-cycle funds.

How the Asset-Mix of Human Capital Depends on Age?

Labor Income as a Geometric Brownian Motion

- A majority of authors studying optimal life-cycle portfolio decisions assume that a household's labor income over time has a stochastic component which is instantaneously correlated with the risky financial securities.

- Assume that the labor income evolves according to a geometric Brownian motion, that is,

$$dw_t = w_t \left[\mu^w dt + \sigma^w dW_t \right] . \tag{8}$$

- The security prices including their accumulated dividends, $S_t = (S_{1t}, \ldots, S_{nt})$, are also geometric Brownian motions

$$dS_{it} = S_{it}[\mu_i dt + \sigma_i dW_t]; \qquad 1 \leq i \leq n.$$

How the Asset-Mix of Human Capital Depends on Age?

Labor Income as a Geometric Brownian Motion

- In this case the value of human capital is given by

$$\begin{aligned} H_t &= w_t \int_t^T e^{(\mu^w - (r + \sigma^{w\prime}\theta))(s-t)} ds \\ &= w_t \frac{1}{r + \sigma^{w\prime}\theta - \mu^w} \left\{ 1 - e^{-[(r+\sigma^{w\prime}\theta)+\mu^w](T-t)} \right\}. \end{aligned} \tag{9}$$

- The first equality of Equation (9) shows that the value of human capital is given by the sum of expected future labor income discounted at the rate $r + \sigma^{w\prime}\theta$.

- Note that θ_i is the market price of risk associated with each Brownian motion W_{it}. One finds that this valuation formula is consistent with the Arbitrage Pricing Theory (APT) of Ross (1976). We note that the value of human capital exponentially declines toward zero if $(r + \sigma^{w\prime}\theta) > \mu^w$.

How the Asset-Mix of Human Capital Depends on Age?

Labor Income as a Geometric Brownian Motion

- Write (9) as $H_t = k(t)\, w_t$. Since $k(t)$ is a deterministic function of time, the basic rule of Malliavin calculus yields $\mathcal{D}_t H_t = k(t)\, \mathcal{D}_t w_t$, and since $\mathcal{D}_t w_t = w_t \sigma^w$ from (8) we get

$$\mathcal{D}_t H_t = H_t \sigma^w. \tag{10}$$

- Equation (10) implies that the volatility term of human capital growth is given by $\sigma^w dW_t$, which is identical to the volatility term of labor income growth. Therefore, the instantaneous correlation of the return on an arbitrary financial portfolio with labor income growth must be the same as its instantaneous correlation with human capital growth.

- This is in sharp conflict with the observation by Baxter and Jermann (1997) and BCG (2007) that while growth rates of labor and capital income are not highly correlated, the returns to human capital and physical capital are very highly correlated. Lucas and Zeldes (2006) posits the same observation by contrasting low correlation between labor income growth and stock returns at an annual frequency with high correlations over longer horizons.

How the Asset-Mix of Human Capital Depends on Age?

Labor Income as a Geometric Brownian Motion

- Substituting in (43) we obtain the optimal portfolio policy

$$\pi_t^* = \frac{1}{\gamma}(X_t + H_t)\,\sigma'^{-1}\theta - H_t\left(\sigma^w \sigma^{-1}\right)'. \tag{11}$$

  The term $\sigma^w \sigma^{-1}$ provides weights of the risky securities in the portfolio which perfectly replicates the labor income risk. The portfolio's weight on the riskless asset is $1 - \mathbf{1}'\sigma^w \sigma^{-1}$.

- Equation (11) shows that the replicating portfolio is independent of the household's life stage. It says that if human capital is 70% bond and 30 % stock at a worker's age 30, it is of the same mix at age 55 (see Fig. 2).

- We think that this age-independence of human capital as an asset mix constitutes the fundamental flaw of foregoing theoretical models of human capital.

How the Asset-Mix of Human Capital Depends on Age?

Labor Income with Short-run Certainty

- We model labor income as a stochastic process with finite variation. We assume that the growth rate of labor income in the very short-run is known with certainty but is driven by the set of Brownian motions which drives the economy. Specifically,

$$dw_t = \mu_t^w w_t dt, \tag{12}$$

$$d\mu_t^w = \kappa \left(\alpha_\mu (t) - \mu_t^w\right) dt + \sigma_\mu dW_t. \tag{13}$$

- The instantaneous growth rate of labor income, μ_t^w in (12), is observable at date t. This rate evolves according to a generalized Vasicek process, a stationary process that reverts toward the mean $\alpha_\mu (t)$. The mean $\alpha_\mu (t)$ is a deterministic function of time which embeds the time-path of expected labor income over the life cycle. The parameter κ (> 0) denotes the speed of mean reversion and σ_μ is a $1 \times M$ vector of volatility coefficients. Both κ and σ_μ are constants. The critical feature of this model is that dw_t in (12) contains no local martingale component.

How the Asset-Mix of Human Capital Depends on Age?

Labor Income with Short-run Certainty

- This model of labor income has two state variables w_t and μ_t^w. We find that the value of human capital defined by (31) is proportional to w_t, given by

$$H_t = R\left(\mu_t^w, t, T\right) w_t. \tag{14}$$

where

$$R\left(\mu_t^w, t, T\right) \equiv \int_t^T \exp\left\{a\left(t,s\right) + \left(\frac{1-e^{-\kappa(s-t)}}{\kappa}\right)\mu_t^w\right\} ds, \tag{15}$$

$$\begin{aligned} a\left(t,s\right) \equiv & \int_t^s \left\{-\left(r+||\theta||^2/2\right) + \left[1-e^{-\kappa(s-v)}\right]\alpha_\mu\left(v\right)\right. \\ & \left. +\frac{1}{2}\left\|\theta' - \frac{\sigma_\mu}{\kappa}\left[1-e^{-\kappa(s-v)}\right]\right\|^2\right\} dv. \end{aligned} \tag{16}$$

- Note that $a\left(t,s\right)$ is a deterministic function, whose value depends on $\left(r,\theta,\kappa,\sigma_\mu\right)$ and the function $\alpha_\mu\left(\cdot\right)$ showing the average rate of labor income growth.

How the Asset-Mix of Human Capital Depends on Age?

Labor Income with Short-run Certainty

- The Malliavin derivative $\mathcal{D}_t H_t$ is linear in H_t and given by

$$\mathcal{D}_t H_t = \Lambda\left(\mu_t^w, t, T\right) H_t \sigma_\mu, \tag{17}$$

where $\Lambda\left(\mu^w, t, T\right)$ is a function defined by

$$\Lambda\left(\mu_t^w, t, T\right) \equiv \frac{\partial}{\partial \mu_t^w} \log R\left(\mu_t^w, t, T\right). \tag{18}$$

- Substituting in (43) we find that the optimal portfolio policy is given by

$$\pi_t^* = \frac{1}{\gamma}\left(X_t + H_t\right)\sigma'^{-1}\theta - \Lambda\left(\mu_t^w, t, T\right) H_t \left(\sigma_\mu \sigma^{-1}\right)'. \tag{19}$$

How the Asset-Mix of Human Capital Depends on Age?

Labor Income with Short-run Certainty

- Figure 3 shows the value of human capital as a function of the worker's age[1]. It starts with \$400,000 at age 20, peaks with \$580,000 at age around 40, and then declines toward zero at retirement. The hump shape of the value of human capital reflects the hump shape of labor income over the life cycle. The value of human capital is about 26 times the initial annual labor income at age 20. This roughly corresponds to the report in Bodie and Treussard (2006).

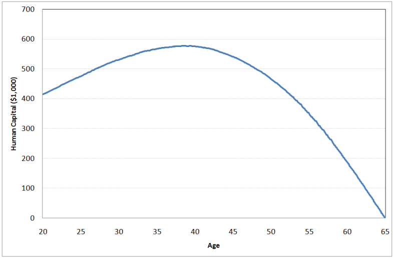

Figure 3 Age profile of the value of human capital

[1] Our simulation consists of 10,000 runs.

How the Asset-Mix of Human Capital Depends on Age?

Labor Income with Short-run Certainty

- Figure 4 depicts the asset composition of human capital over the life-cycle when $\rho = 1$. The fraction of the worker's human capital tied up to the stock market portfolio is more than 90% at age 20. The fraction decreases at a slow pace until age 45, and then rapidly drops toward 0%. Since we are assuming a one-factor economy ($\rho = 1$) in the baseline case, the residual tranche of human capital is cash, which is drawn in dotted line.

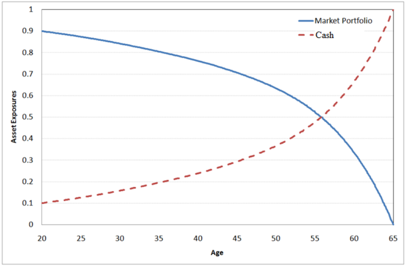

Figure 4 Stock market exposure of Human Capital

How the Asset-Mix of Human Capital Depends on Age?

Labor Income with Short-run Certainty

- Similar to the cointegrfation model, the baseline case of our model produces a hump shape in the optimal proportion of financial wealth invested in stocks. The worker's stock holding peaks at around age 60 and then she starts reducing her stock holdings until retirement. Possibility of this hump can be illustrated by a simple algebra.

- The optimal portfolio policy (19) can be expressed as

$$\pi_t^* = m^* \left(X_t + H_t\right) - E_t H_t,$$

where $m^* \equiv$ the target fraction of stock in total wealth and $E_t \equiv$ fraction of stock in human capital at t. Remember that m^* is the Merton's solution (without labor income), which is 0.47 in our baseline case. Dividing both sides by X_t and defining $h_t \equiv H_t / X_t$, we obtain

$$\frac{\pi_t^*}{X_t} = m^* + \left(m^* - E_t\right) h_t. \tag{20}$$

How the Asset-Mix of Human Capital Depends on Age?

Labor Income with Short-run Certainty

- In our model E_t decreases in t (Fig. 4) and when the worker is near retirement h_t decreases in t (Fig. 3). If $E_t < m^*$ for some t, π_t^* / X_t exceeds m^*. This is the case of Fig. 5. If there is an age zone where optimal stock holdings exceeds m^*, it should decline to m^* since the latter is the target ratio at retirement. As t increases, both E_t and h_t decrease. These two forces offset each other, but the impact of the latter overwhelms.

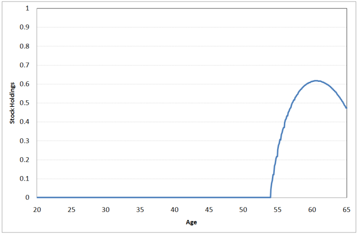

Figure 5 Optimal Stock holdings

- If the labor income is a Brownian motion, E_t is a constant. If it is smaller than m^* as most of the previous literature assume, the optimal stock holding π_t^* / X_t monotonically decreases in t.

Nontradable Human Capital

- Given thousands of liquid financial assets whose prices are driven by many economic and market factors, it should not be impossible to find a set of portfolios of financial assets which provides an almost perfect hedge against the labor income risk.

- However it is of interest to examine the impact of nontradability of human capital on the optimal life-cycle portfolio policy. We again impose no short-selling restriction on the stock market portfolio.

Nontradable Human Capital

- When the market is incomplete, there exist infinitely many state price densities that are compatible with no arbitrage conditions. The challenge is to find an analytically tractable way to pin down the state price density which really provides the optimal solution. While general methods have been proposed by such papers as Cvitanic and Karatzas (1992), Cuoco (1997), Schroder and Skiadas (2003), the route to finding the state price density is not obvious.

- The methodology proposed by Detemple and Rindisbacher (2005) seems promising in this regard, but the problem they solve does not include stochastic labor income. It is not straightforward to extend their analysis to optimal portfolio problem with stochastic labor income.

Nontradable Human Capital

- The martingale method is still useful in finding the basic structure of the optimal portfolio policy. More specifically, within our model with time-additive CRRA utility functions the constant-mix strategy relative to total wealth is still the optimal policy for incomplete market settings.

- Furthermore, the unique state price density is the one that creates zero net demand for fictitious financial instruments which would complete the market with additional hedging opportunities. The difficulty is a purely numerical one of finding the correct progressively measurable (implicit) market premium processes for each unhedgeable Brownian motion factors[2].

[2] In Detemple and Rindisbacher (2005) the implicit price of interest rate risk turns out to be a deterministic process. In our model the implied price of labor income risk becomes state-dependent.

Nontradable Human Capital

- The results are presented in Figure 6. We first note that imposing the short-sale restriction induces gradual increase of stock holdings in early life stage.

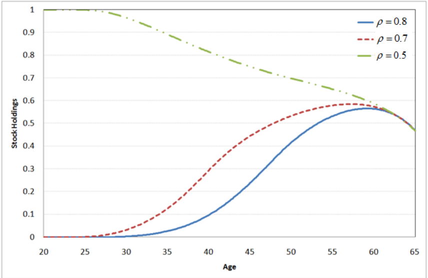

Figure 6 Optimal stock holdings in an incomplete market setting

- When the coefficient ρ, which captures the instantaneous correlation between labor income growth and the stock market return, is high, human capital has a strong stock market exposure. Therefore, we obtain a life-cycle profile of optimal stock holdings which is similar to the complete market case.

Nontradable Human Capital

- As the correlation decreases, a larger portion of labor income risk becomes unhedgable. This means that the stock market exposure of human capital gets smaller in magnitude. Therefore, workers tend to start investing in the stock market earlier and the optimal fraction of stock holdings increases over the life cycle. On the contrary, when the value of ρ becomes as low as 0.5, human capital gets sufficiently bond-like in early life stage. This implies that the conventional strategy embedded in target date funds becomes the rule.

Summary

- A majority of life cycle literature, including BMS(1992), and CGM(2005), claim that if the correlation between labor income risk and stock market risk is small human capital acts more like risk-free bonds rather than stocks.

- In these models only counterfactually high correlations between shocks to labor income and stock returns, or the possibility of disastrous labor income shocks (see, for example, CGM), can explain young investors' low holdings of the risky asset.

- Baxter and Jermann (1997), Lucas and Zeldes (2006), and BCG(2007) counterargue them with the empirical evidence of cointegration between the labor income and dividends. They argue that although labor income risk and stock market risk are not correlated significantly over a short horizon their correlation increases as time span expands due to cointegration. They show that this cointegration can explain why younger wage earners do not hold stocks and also why the stock holdings exhibit hump shape over the life cycle.

Summary

- Our model assumes no contemporaneous correlation between labor income shock and stock return and yet contains both of the above regimes.
- If the shocks in labor income growth and stock market return is less correlated and the uncorrelated component of labor income growth is unhedgeable, households' stock holdings should decline with age as BMS, BCG, and others suggest.
- In contrast, if the correlation is high, the optimal profile belongs to the cointegration school.
- In the latter regime, the coefficient κ, which indicates the speed of mean reversion, plays an exactly opposite role in our model and in the cointegration model. In our model a larger κ dampens the equity-like nature of human capital, while in the cointegration model a larger κ strengthens the equity-like nature of human capital.

Summary

- If longer horizon correlation between labor income shock and stock market return is not sufficiently high and the financial market does not provide a tool to hedge the uncorrelated component of labor income growth, households' stock holdings should decline with age consistent with the typical way that target date funds are structured. Which school of thoughts on life cycle portfolio rule enhances households' welfare more still rests on the empirical magnitude of hedgeability of the labor income growth over a longer time horizon.

Topics for Future Study

- Since our model of stochastic labor income is highly tractable to analyze life cycle effects on optimal portfolio decisions, a general equilibrium resolution of the equity premium puzzle with heterogeneous population commands a high priority. It would also be interesting to investigate how the equity premium is related to the age structure of the society.
- Mathematically, there still is a strong need for some powerful method to pin down the state price density for the optimal consumption and portfolio problem with unhedgeable labor income risk. With such a mathematical development we will be able to understand the impact of short sales restrictions, borrowing restrictions, borrowing rate which is higher than the riskless rate and collateral borrowing with housing assets in a more depth.

References

Baxter, Marianne, and Urban J. Jermann, 1997. The international diversification puzzle is worse than you think. *American Economic Review*, 87 (1), 170–180.

Benzoni, Luca, Pierre Collin-Dufresne, and Robert S. Goldstein, 2007. Portfolio Choice over the Life-Cycle when the Stock and Labor Markets are Cointegrated. *Journal of Finance*, 62 (5), 2123–2167.

Bodie, Zvi., Robert Merton, and William Samuelson, 1992. Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model. *Journal of Economic Dynamics and Control*, 16 (3–4), 427–44.

Bodie, Zvi, and Jonathan Treussard, 2006. Making Investment Choices as Simple as Possible, but Not Simpler. *Financial Analysts Journal*, 63 (3), 42–47.

Cvitanic, Jaksa., and Ioannis Karatzas, 1992. Convex Duality in Constrained Portfolio Optimization. *Annals of Applied Probability*, 2 (4), 767–818.

Cocco, João, Francisco J. Gomes, and Pascal J. Maenhout, 2005. Consumption and Portfolio Choice over the Life-Cycle. *Review of Financial Studies*, 18 (2), 491–533.

Cox John C., and Chi-Fu Huang, 1989. Optimal consumption and portfolio policies when asset prices follow a diffusion process. *Journal of Economic Theory*, 49 (1), 33–83.

References

- Cuoco, Domenico, 1997. Optimal Consumption and Equilibrium Prices with Portfolio Constraints and Stochastic Income. *Journal of Economic Theory*, 72 (1), 33–73.
- Davis, Steven J., and Paul Willen, 2000. Occupation-level income shocks and asset returns: Their covariance and implications for portfolio choice. NBER Working paper number 7905.
- Detemple, Jérôme B., René Garcia, and Marcel Rindisbacher, 2003. A Monte Carlo method for optimal portfolios. *Journal of Finance*, 58 (1), 401–46.
- Detemple, Jérôme B., and Marcel Rindisbacher, 2005. Closed Form Solutions for Optimal Portfolio Selection with Stochastic Interest Rate and Investment Constraints. *Mathematical Finance*, 15 (4), 539–68.
- Haliassos, Michael, and Alexander Michaelides, 2003, Portfolio choice and liquidity constraints. *International Economic Review*, 44 (1), 143–177.
- Heaton, John, and Deborah J. Lucas, 1997. Market Frictions, Savings Behavior, and Portfolio Choice. *Macroeconomic Dynamics*, 1 (1), 76–101.
- Jagannathan, Ravi, and Narayana R. Kocherlakota, 1996. Why should older people invest less in stocks than younger people? *Federal Reserve Bank of Minneapolis Quarterly Review* 20, 11–23.

References

Karatzas, Ioannis, John P. Lehoczky, and Steven E. Shreve, 1987. Optimal portfolio and consumption decisions for a “small investor” on a finite horizon. *SIAM Journal on Control and Optimization* 25 (6), 1557–86.

Lucas, Deborah J., and Stephen P. Zeldes, 2006. Valuing and hedging defined benefit pension obligations — The role of stocks revisited. Working paper, Northwestern University.

Merton, Robert C., 1969. Lifetime Portfolio Selection Under Uncertainty: The Continuous-Time Case. *Review of Economics and Statistics*, 51 (3), 247–257.

Merton, Robert C., 1971. Optimum Consumption and Portfolio Rules in a Continuous Time Model. *Journal of Economic Theory*, 3 (4), 373–413.

Ocone, Daniel L., and Ioannis Karatzas I. 1991. A generalized Clark representation formula, with application to optimal portfolios. *Stochastics and Stochastics Reports*, 34 (3–4), 187–220.

Ross, Stephen A., 1976. The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory*, 13 (3), 341–360.

Samuelson, Paul A., 1969. Lifetime Portfolio Selection by Dynamic Stochastic Programming. *Review of Economics and Statistics*, 51 (3), 239–246.

References

Schroder, M., and C. Skiadas, 2003. Optimal lifetime consumption-portfolio strategies under trading constraints and generalized recursive utility. *Stochastic Processes and their Applications*, 108, 155–202.

Viceira, Luis M., 2001. Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income. *Journal of Finance*, 56 (2), 433–470.