Positive versus Normative Approaches to Lifecycle Saving and Spending Decisions

- Sherman D. Hanna
- Professor, Consumer Sciences Department
- Ohio State University, Columbus, Ohio

Household Preference Parameters

- In order to prescribe actions or policies for individuals
- or
- evaluate whether households are making mistakes
- necessary to use either analysis of
- actual choices
- or of
- choices between hypothetical alternatives
- to rigorously derive household preference parameters

Household Preference Parameters

- challenging to infer preference parameters from household decisions,
- e.g., from the equity premium,

- I will focus on use of simple hypothetical examples to try to estimate plausible values of preference parameters for prescriptions for household financial decisions.

Relative Risk Aversion Level

- Barsky et al. (1997) provided some insights into relative risk aversion levels based on
- analyses of the hypothetical income gamble questions in the Health and Retirement Study.
- Barsky et al. also presented some hypothetical questions to elicit intertemporal preference parameters
- I will focus on intertemporal parameters, but will also discuss the income gamble questions designed to elicit relative risk aversion for quasi-static risky choices

Simple Lifecycle Model, No Uncertainty

- Lifetime budget constraint

$$\sum_{t=1}^{T} C_t (1+r)^{-t} \leq A_0 + \sum_{t=1}^{T} Y_t (1+r)^{-t}$$

Simple Lifecycle Model , No Uncertainty

- Maximization problem:

$$\max_{\{C_1 \ldots C_T\}} \sum_{t=1}^{T} (1+\rho)^{-t} U(C_t)$$

Simple Lifecycle Model , No Uncertainty

- Optimal condition for spending growth rate

$$\frac{U'(C_{t+1})}{U'(C_t)} = \left(\frac{1+\rho}{1+r}\right)$$

Simple Lifecycle Model , No Uncertainty

- Common to assume that the utility function has constant elasticity
- Some authors refer to the elasticity of marginal utility with respect to consumption, which we will denote as elasticity, ε
- other authors refer to the intertemporal substitution elasticity, which we will denote as θ. The relationship between θ and ε is:
- $\varepsilon = -1 / \theta$

Simple Lifecycle Model , No Uncertainty

- If the intertemporal utility function has constant elasticity, ε,
- the optimal growth rate in spending in the case of certainty about future income can be approximated by

$$G_c \approx \frac{r - \rho}{-\varepsilon}$$

Household Parameters: Interest Rate

- What real rate does a household face?
- Risk-free investment opportunities?
- Credit card rate?

Household Parameters: personal discounting

- What are plausible values of the personal discount rate, ρ?
- Does it change over time for an individual or household
- Inferences based on behavior and hypothetical scenarios are mixed –
- from very high based on reluctance to purchase energy saving features
- to negative

Household Parameters: personal discounting

- Consider retirement adequacy research
- If we assume very low discounting
- Then 50% or more of U.S. households are not saving adequately for retirement
- If we assume high discounting, then perhaps only 20% are not saving adequately for retirement

Household Parameters: personal discounting

- Note, however, that attempts to
- infer the personal discount rate from behavior are complicated by the fact that the simple theory predicts that
- the effect on behavior depends on the elasticity parameter
- huge effect for $\varepsilon=-1$,
- small effect for $\varepsilon=-20$

Household Parameters: personal discounting

- My take – for normative personal finance applications, calibrate based on death rates and planned changes in household size
- So, e.g., a 25 year old is 99.9% likely to be alive next year, so very low discounting
- However, the cumulative effect of discounting for comparison of utility of age 25 consumption to utility of age 90 consumption is very high

Estimating Elasticity

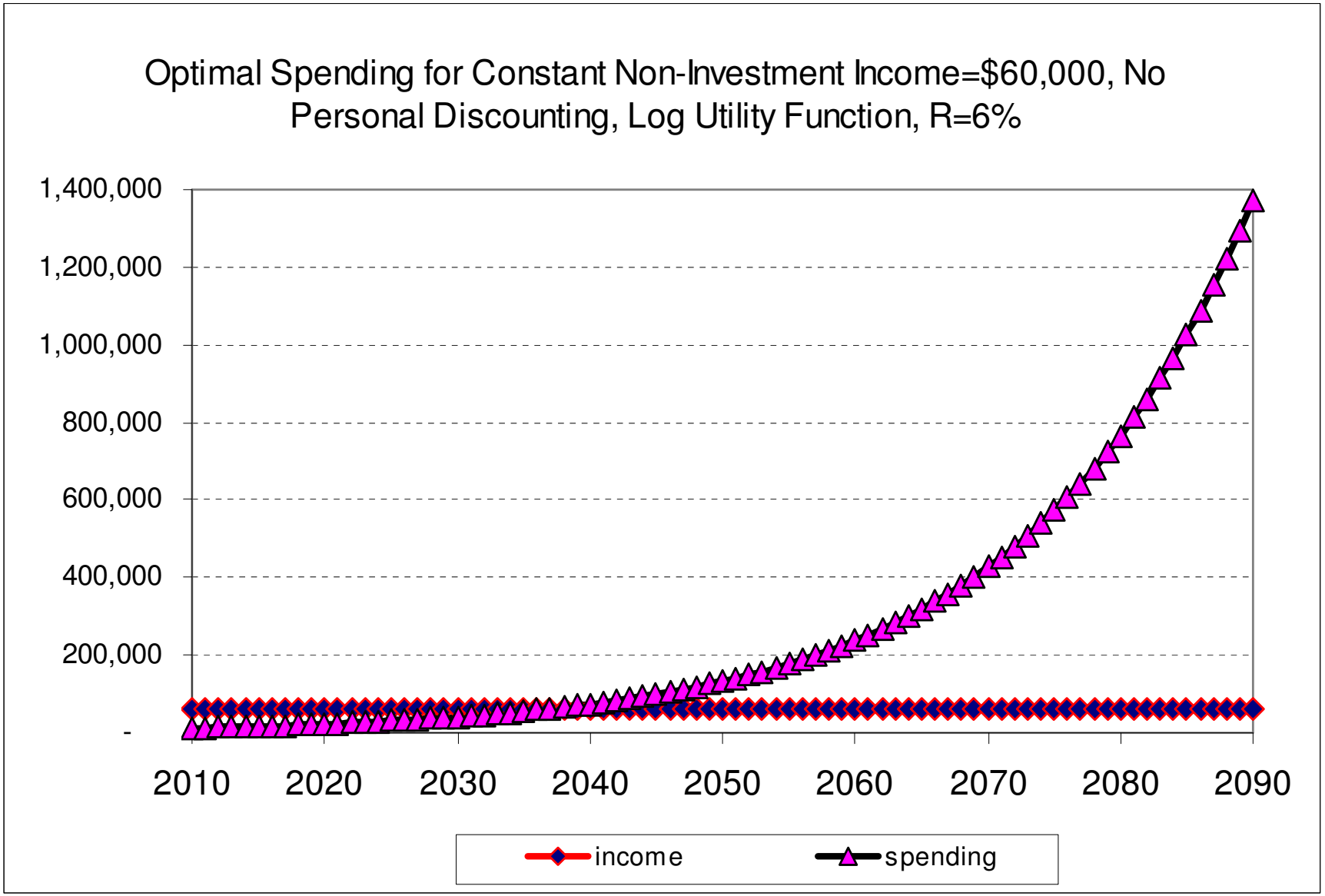
- My hypothetical example:
- You are 20 years old, and know with certainty that you will live to be 100 in good health.
- Everything about your personal situation will remain the same for the next 80 years.
- Assume no discounting of utility of future consumption (can you imagine enjoying a vacation when you are 99?)
- You want to spend all of your wealth by the day of your death.

Estimating Elasticity

- Your non-asset income will be $60,000 per year in real terms.
- You can obtain a return of 6% per year after inflation and taxes on investments.
- None of the usual reasons for saving exist (e.g., retirement and uncertainty)
- so that the only reason to save is to take advantage of compound interest with a positive real rate of return.
- What consumption path would you prefer?

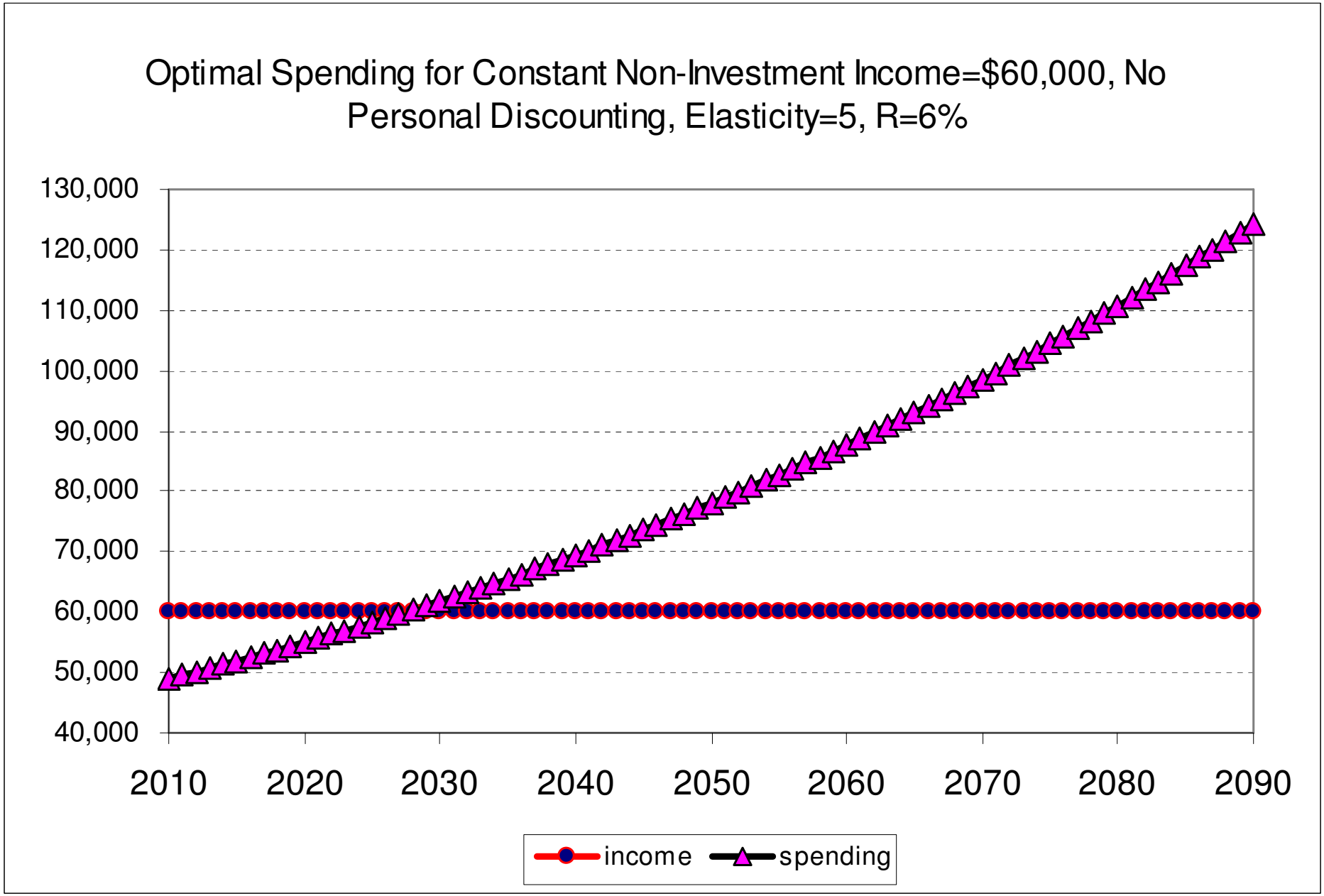
Estimating Elasticity

- Given the assumptions
- If you had a natural log utility function
- And no discounting of the utility from future consumption
- You should save 78% of your income in year 1
- And only consume $12,970 this year out of your $60,000 aftertax income
- In your last year of life you would be able to spend
- $1,372,145


Optimal Spending for Constant Non-Investment Income=$60,000, No Personal Discounting, Log Utility Function, R=6%
1,400,000
1,200,000
1,000,000
800,000
600,000
400,000
200,000
-
2010
2020
2030
2040
2050
2060
2070
2080
2090
income
spending

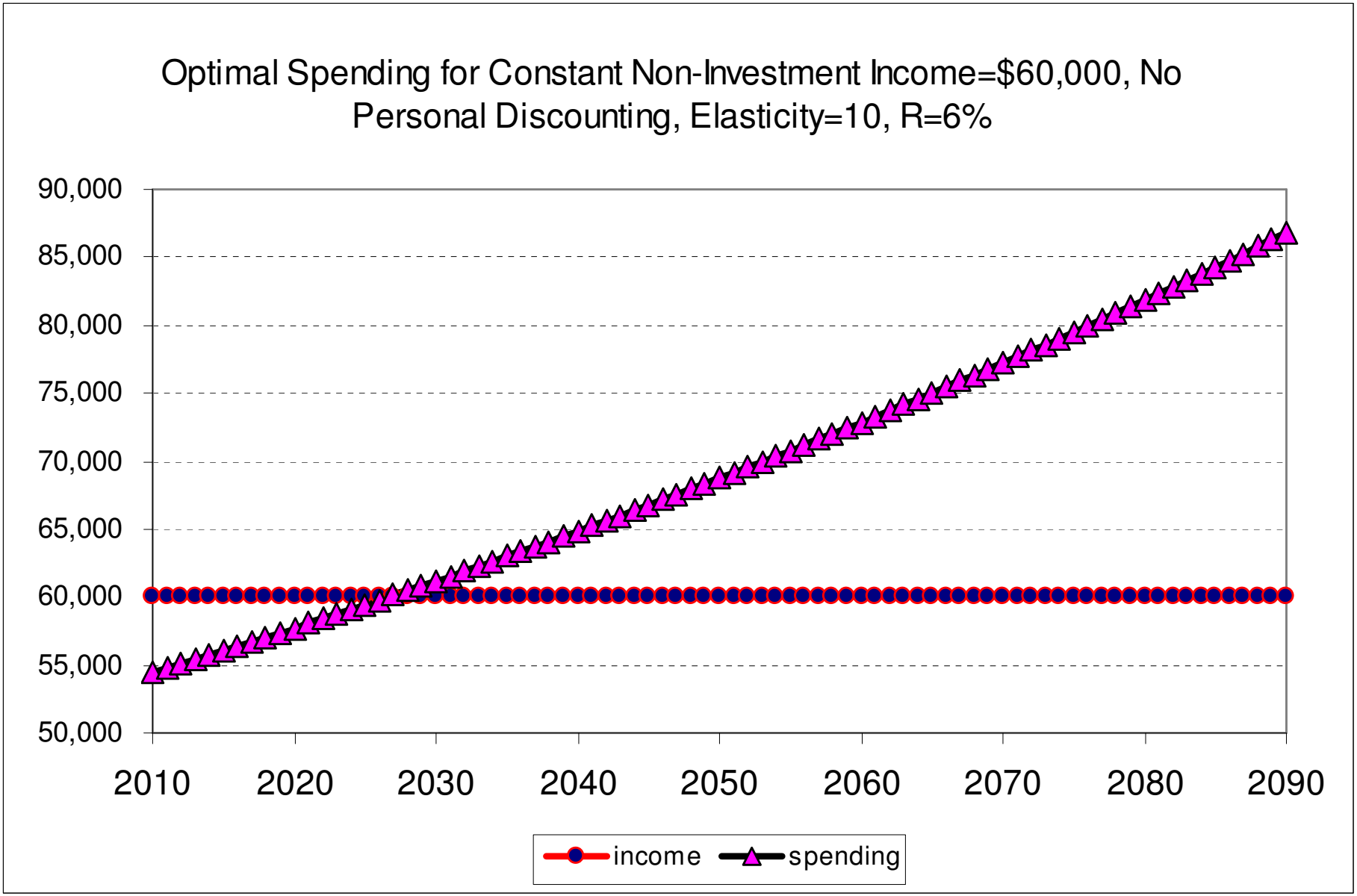
Estimating Elasticity

- Given the assumptions
- If you had elasticity = -5
- And no discounting of the utility from future consumption
- You should save 13% of your income in year 1
- And consume $48,970 this year out of your $60,000 aftertax income
- In your last year of life you would be able to spend
- $124,477


Optimal Spending for Constant Non-Investment Income=$60,000, No Personal Discounting, Elasticity=5, R=6%
130,000
120,000
110,000
100,000
90,000
80,000
70,000
60,000
50,000
40,000
2010
2020
2030
2040
2050
2060
2070
2080
2090
income
spending

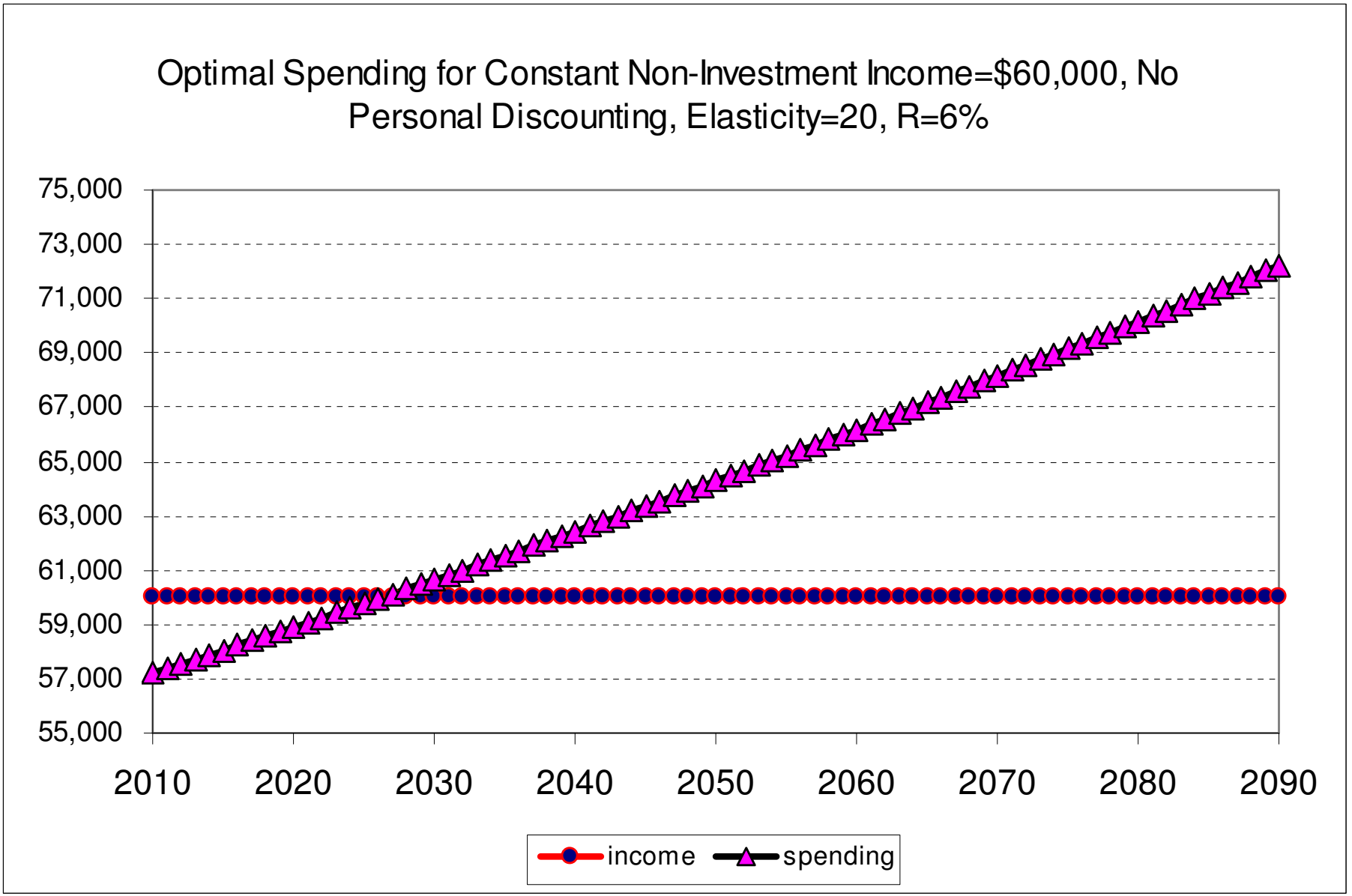
Estimating Elasticity

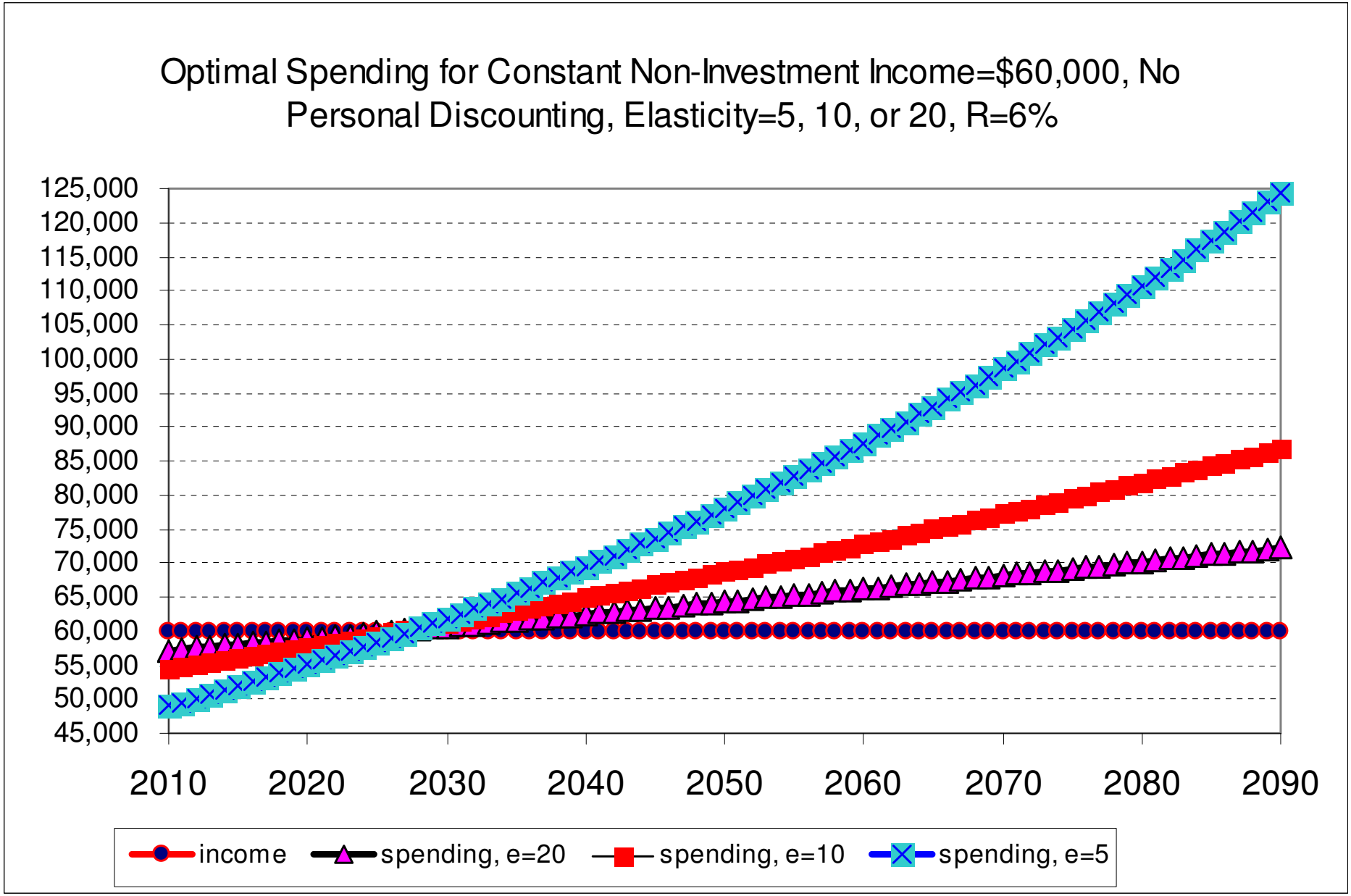
- Given the assumptions
- If you had elasticity = -10
- And no discounting of the utility from future consumption
- You should save 8% of your income in year 1
- And consume $54,452 this year out of your $60,000 aftertax income
- In your last year of life you would be able to spend
- $86,788


Optimal Spending for Constant Non-Investment Income=$60,000, No Personal Discounting, Elasticity=10, R=6%
90,000
85,000
80,000
75,000
70,000
65,000
60,000
55,000
50,000
2010
2020
2030
2040
2050
2060
2070
2080
2090
income
spending

Estimating Elasticity

- Given the assumptions
- If you had elasticity = -20
- And no discounting of the utility from future consumption
- You should save 5% of your income in year 1
- And consume $53,869 this year out of your $60,000 aftertax income
- In your last year of life you would be able to spend
- $72,238


Optimal Spending for Constant Non-Investment Income=$60,000, No Personal Discounting, Elasticity=20, R=6%
75,000
73,000
71,000
69,000
67,000
65,000
63,000
61,000
59,000
57,000
55,000
2010
2020
2030
2040
2050
2060
2070
2080
2090
income
spending


Optimal Spending for Constant Non-Investment Income=$60,000, No Personal Discounting, Elasticity=5, 10, or 20, R=6%
125,000
120,000
115,000
110,000
105,000
100,000
95,000
90,000
85,000
80,000
75,000
70,000
65,000
60,000
55,000
50,000
45,000
2010
2020
2030
2040
2050
2060
2070
2080
2090
income
spending, e=20
spending, e=10
spending, e=5

Estimating Elasticity

- So, what is a plausible value?
- With undergraduate students, I have obtained a range of choices, but the median value of ε was -4. Barsky et al. (1997) with a small sample of older adults reported a midpoint value equivalent to -5.6.
- For the general population, -10 to -20 seems plausible to me for many people
- However, for normative purposes, -5 seems to me a plausible level
- Because many people are overly optimistic in their income projections

Elasticity versus Relative Risk Aversion

- Are these estimates of elasticity, -4 to -6, plausible?
- Consider that the constant elasticity intertemporal utility function for each year's consumption is
- structurally the same as the assumption of a constant relative risk aversion utility function for states of the world.
- If we assume wealth levels for discrete states of the world,
- then each state of the world is in some sense like different years in a lifecycle model. Instead of consumption smoothing in the lifecycle model,
- maximization of expected wealth in a model with positive relative risk aversion assumes some "smoothing" of wealth across states of the world.

Elasticity versus Relative Risk Aversion

- In the lifecycle model the cost of consumption smoothing is related to
- the interest rate faced by the household, and for a given interest rate,
- the degree of smoothing depends on the elasticity parameter of the household.

Elasticity versus Relative Risk Aversion

- In the static risky choice model,
- the set of investment opportunities between riskier and less risky investments will affect the degree of wealth smoothing that will be done,
- but for a given set of investment opportunities,
- the degree of wealth smoothing will depend on the relative risk aversion level of the household.

Elasticity versus Relative Risk Aversion

- Major difference in usual assumptions in simple versions of each approach is
- that for the lifecycle model,
- we usually assume that the household discounts the utility from consumption in future periods.
- Obviously it is **not necessary** to assume that relative risk aversion is the same as the intertemporal elasticity parameter (e.g., Epstein & Zin, 1989) but let us assume that for simplicity.

Elasticity versus Relative Risk Aversion

- Ignoring many complexities, what are plausible estimates of relative risk aversion?

Elasticity versus Relative Risk Aversion

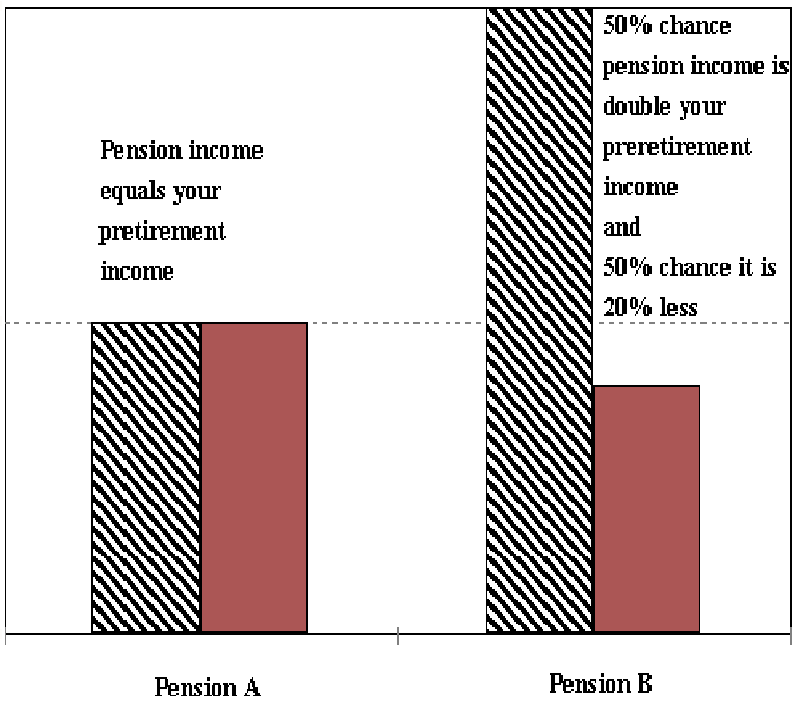
- Barsky, et al., QJE, 1997
- reported the distribution of relative risk aversion levels based on
- a series of income gamble questions presented to older adult respondents in the Health and Retirement Study,
- and estimated that 65% of respondents had a relative risk aversion level of 3.8 or above.

My version of Barsky et al. Income Gamble Questions

- Suppose that you are about to retire, and have two choices for a pension
  - Pension A gives you an income equal to your preretirement income.
  - Pension B has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be 20% less than your preretirement income.
- You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future.
- All incomes are aftertax.

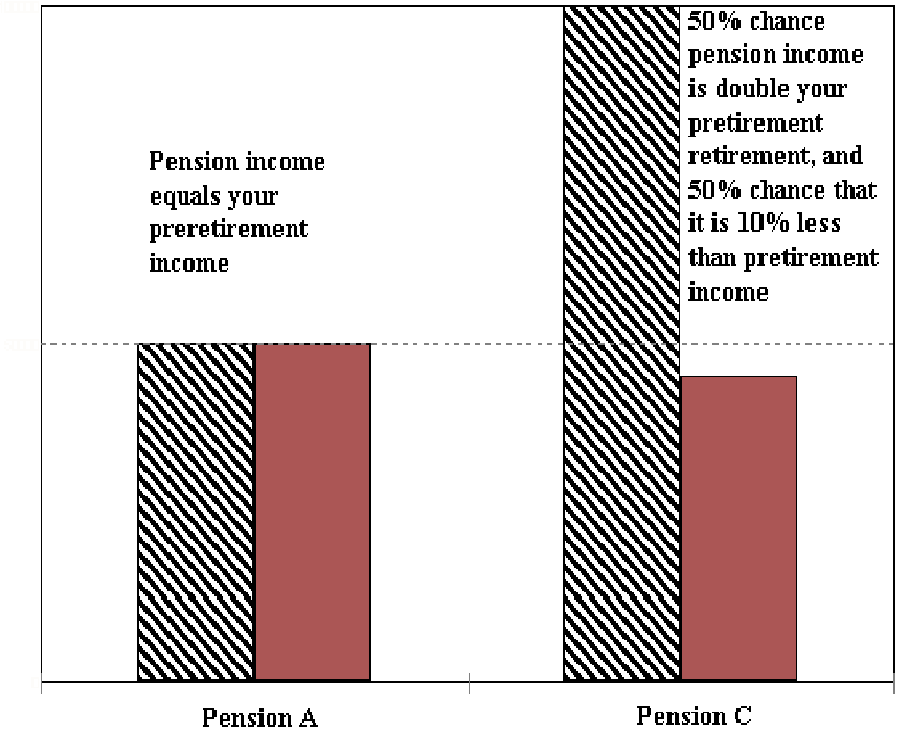
Pension Gamble Questions screen 1

- Suppose that you are about to retire, and have two choices for a pension
  - Pension A gives you an income equal to your preretirement income. (go to #2)
  - Pension B has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be **20% less** than your preretirement income. (go to #5)
- You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are aftertax.



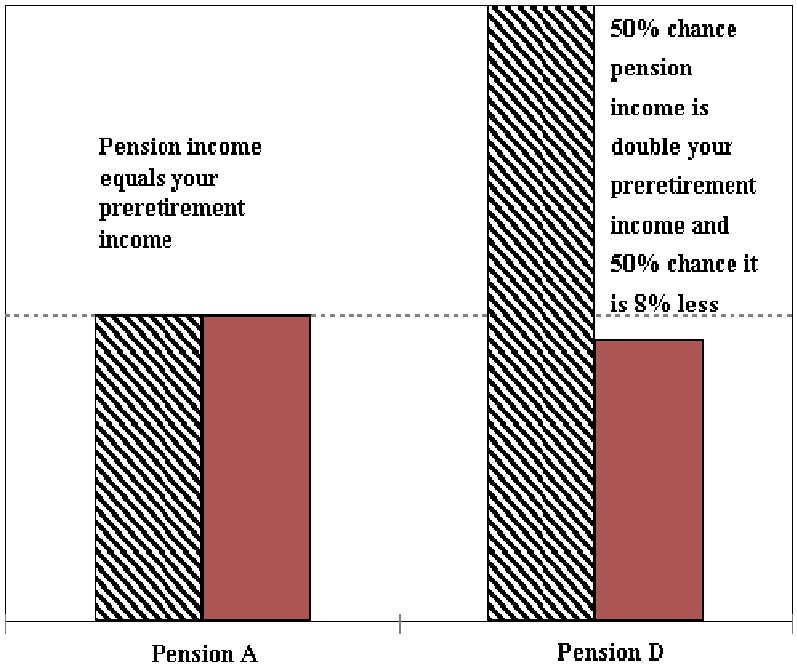
Pension Gamble Questions-2

- Suppose that you are about to retire, and have two choices for a pension
  - Pension A gives you an income equal to your preretirement income. (go to #3)
  - Pension C has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be **10% less** than your preretirement income. {**Moderate subjective risk tolerance**}
- You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are aftertax.



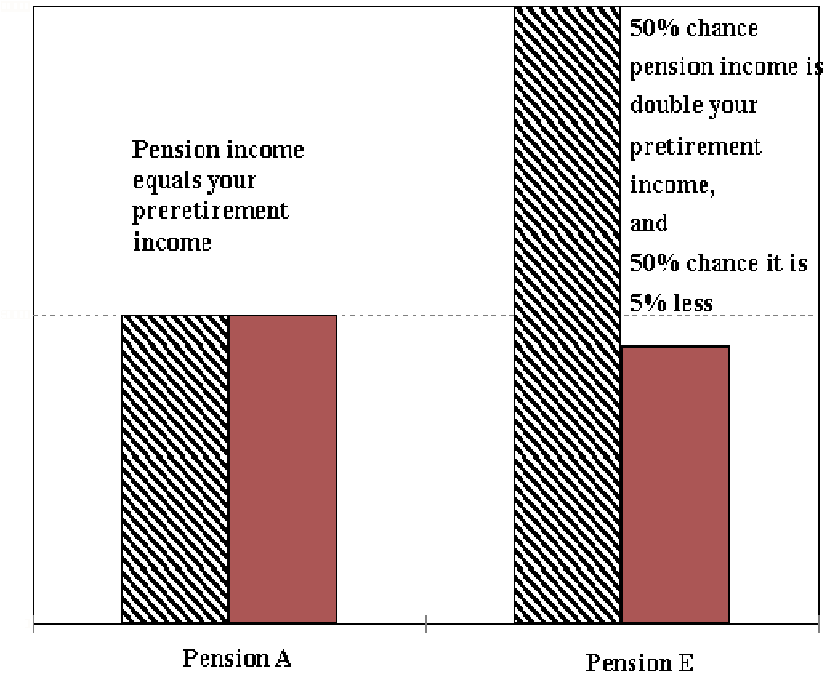
Pension Gamble Questions- 3

- Suppose that you are about to retire, and have two choices for a pension
  - Pension A gives you an income equal to your preretirement income. (go to #4)
  - Pension D has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be **8% less** than your preretirement income. {**Low subjective risk tolerance**}
  - You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are aftertax.



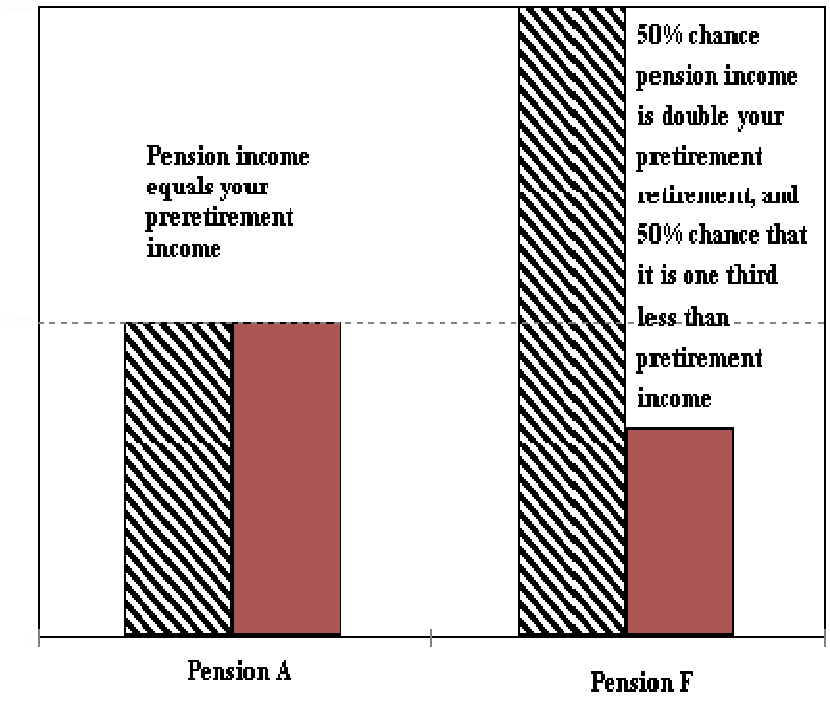
Pension Gamble Questions-4

- Suppose that you are about to retire, and have two choices for a pension
  - Pension A gives you an income equal to your preretirement income. {**Extremely Low subjective risk tolerance**}
  - Pension E has a 50% chanc your income will be double your preretirement income, and a 50% chance that your income will be **5% less** than your preretirement income. {**Very Low subjective risk tolerance**}
- You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are aftertax.



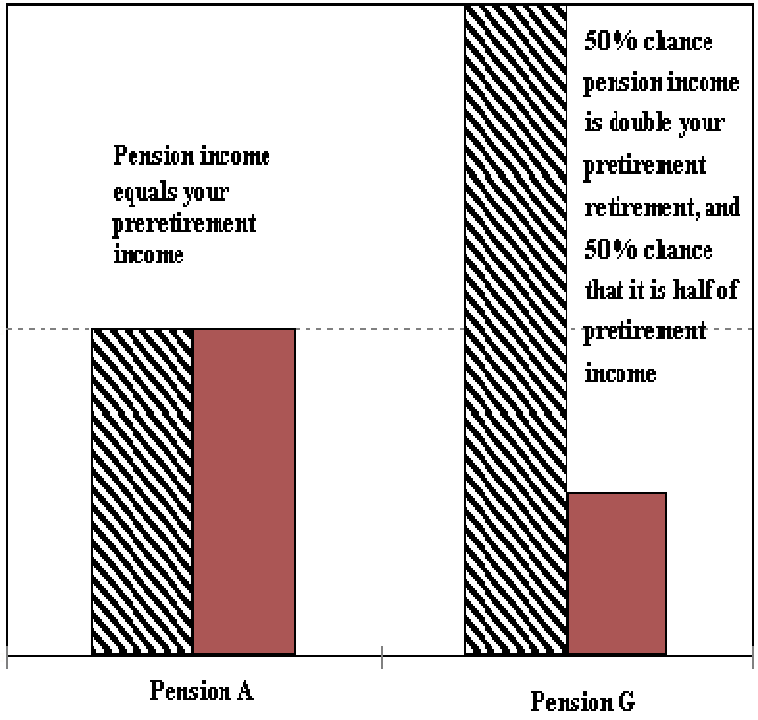
Pension Gamble Questions-5

- Suppose that you are about to retire, and have two choices for a pension
  - Pension A gives you an income equal to your preretirement income. {**Moderately High subjective risk tolerance**}
  - Pension F has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be **a third less** than your preretirement income. (Go to #6)
- You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are aftertax.



Pension Gamble Questions-6

- Suppose that you are about to retire, and have two choices for a pension
  - Pension A gives you an income equal to your preretirement income. {**Very High subjective risk tolerance**}
  - Pension G has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be **half** of your preretirement income. {**Extremely High subjective risk tolerance**}
- You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are aftertax.



Student Responses

- 46% of students had a relative risk aversion level of at least 3.8
- We also presented the students with the Survey of Consumer Finances risk tolerance question, then used a regression to extrapolate to U.S. adults
- Mean level of relative risk aversion = 5.5

Implications of Assumptions

- How should we use estimates of utility function parameters for normative recommendations?
- There are various complex issues that could be considered and a variety of applications, such as whether to annuitize one's assets.
- I will present a very simple example, based on an Excel file I use in my classes for life cycle consumption smoothing.

Illustration of Impact of Parameter Estimates

- My spreadsheet has a default for personal discounting of future utility based on actuarial values of the annual risk of death by age, and
- on planned changes of household size.

Illustration of Impact of Parameter Estimates

- The default value of the intertemporal elasticity is assumed to be -5.
- The user is prompted to input pessimistic projections of inflation-adjusted aftertax non-investment income.

Application of simple lifecycle model

- Each year, r is set, whether net financial assets > or < 0
- E.g., r=4% or r=12%
- Discount rate changes based on death rate for age
- Solve for initial consumption resulting in target net financial assets if live to be 100

$$G_c \approx \frac{r - \rho}{-\varepsilon}$$

Illustration of Impact of Parameter Estimates

- In the following example
- assumed pattern of aftertax non-investment income increasing from $60,000 per year at age 30 to $80,000 just before retirement,
- then a Social Security pension of $24,000 per year starting at the 67th birthday.

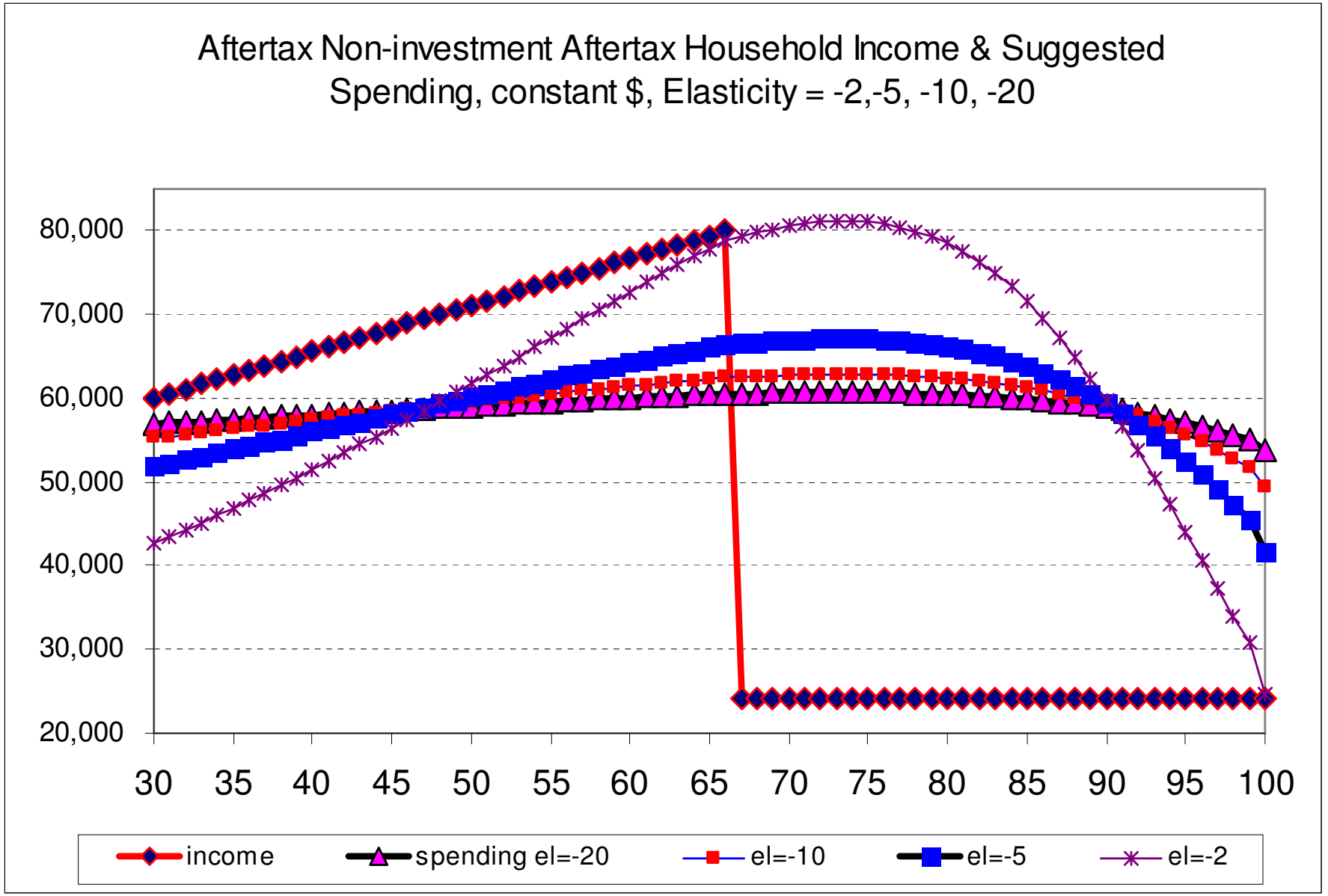

Aftertax Non-investment Aftertax Household Income & Suggested Spending, constant $, Elasticity = -2,-5, -10, -20
80,000
70,000
60,000
50,000
40,000
30,000
20,000
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
income
spending el=-20
el=-10
el=-5
el=-2

Illustration of Impact of Parameter Estimates

- If the elasticity is **-5**, the optimal consumption path is fairly flat, increasing from $51,917 at age 30 (**saving 13%** of income) to $66,384 at age 66, saving 29% of income.
- If the elasticity is **-20**, the optimal consumption path is almost completely flat, increasing from $56,923 at age 30 (**saving 5%** of income) to $60,531 at age 66, saving 30% of income.
- If the elasticity is **-2**, the optimal consumption path increases much more, increasing from $42,628 at age 30 (**saving 29%** of income) to $78,808 at age 66, saving 1% of income.

Implications

- Parameters chosen for normative applications can make a big difference,
- e.g., initial saving of 29% of income (-2)
- To
- 5% of income (-20)
- Already we are seeing Nudge type policies, e.g., default contribution rates to retirement accounts
- So it is important to have an explicit discussion of the assumptions