## Some Transport Mechanisms in the Solar System

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## Outline

1. Distribution of low-energy lunar impact craters Elisa Maria Alessi (U. Pisa), Josep J. Masdemont (UPC)
2. Two kinds of Mars-Earth transport Josep J. Masdemont (UPC), Yuan Ren (CNSA)
3. From the outer to the inner Solar System: preliminary results Esther Barrabés (UdG), Josep M. Mondelo (UAB), Mercè Ollé (UPC)

## 1. Distribution of low-energy lunar impact craters

E.M. Alessi, G. Gómez, J.J. Masdemont: A motivating exploration on lunar craters and low-energy dynamics in the Earth-Moon system, Cel. Mech. \& Dyn. Astron. 107 (1-2)

## The surface of the Moon



Nearside


Farside

## Some questions

1. Is there a reason for the alleged asymmetry of impact between nearside and farside?
2. Does the Moon act as a shield for the Earth?
3. Which is the role of the Sun in this process ?

## What we know

1. Intense lunar bombardment took place between 3.8 and 4 Gy ago. ( 1 Gy $=10^{9}$ years)
2. Moon is receding from the Earth. The rate of recession has not been constant in the past and it did not behave linearly either
3. If $v_{\text {impacte }} \approx 2.4 \mathrm{~km} / \mathrm{s}$, then diameter of the crater is $D_{\text {crater }} \leq 60$ km

## The basic model: The 3D restricted three body problem (CR3BP) and the effective potential




## Our approach

1. Main channel to get to the Moon $\mathcal{W}^{s}\left(\mathcal{W}_{L_{2}}^{c}\right)$
2. Use transit trajectories inside $\mathcal{W}^{s}\left(\mathcal{W}_{L_{2}}^{c}\right)$ for $C_{3}<C_{J}<C_{2}$
3. Use different distributions of initial conditions
4. Use different values of the Earth - Moon distance: $d_{E M}$

## $W^{s}\left(W_{L_{2}}^{c}\right)$ : The main channel




## Transit trajectories



Are orbits associated to the $\mathcal{W}^{s / u}\left(\mathcal{W}_{L_{2}}^{c}\right)$ that cross the region around the point
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## Transit orbits

For a given value of the energy $C_{J}$, transit trajectories are determined by the hyperbolic manifolds of the planar and vertical Lyapunov periodic orbits with the same value of $C_{J}$



Random points inside these curves give initial conditions for $x=0$ of transit orbits around the $L_{2}$ point
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## Initial conditions

1. For each value of $C_{J}$, uniformly distributed in $\mathcal{W}^{s}\left(W_{L_{2}}^{c}\right)$
2. Uniformly distributed for each energy level, for different values of $C_{J}$
3. To belong to a specific range of inclinations w.r.t. the Earth Moon orbital plane
4. To be in resonance w.r.t. the Moon
5. ....

For 1. i 2. we have explored 20 values of $C_{J}$ and, at least, $10^{6}$ initial conditions for each value of $C_{J}$

## Distribution of impacts for i.c. uniformly distributed in $\mathcal{W}^{s}\left(\mathcal{W}_{L_{2}}^{c}\right)$



$$
d_{E M}=232400 \mathrm{~km}
$$


$d_{E M}=308400 \mathrm{~km}$

## Outcome and question

1. The largest probability of impact takes place at the apex of the lunar surface $\left(90^{\circ} W, 0^{\circ}\right)$
2. Most of the impacts take place within the first 20 years
3. The smaller $d_{E M}$, the higher the percentage of lunar impacts
4. The amount of particles that still wanders around the Earth inside the zone bounded by the zero-velocity surface after 60 years is $0.1 \%$

Do there exist other sources of low-energy lunar impacts ?

1. Take a uniform distribution of initial conditions on the lunar sphere
2. Integrate the CR3BP equations of motions backwards in time

## Outcome

The impacts would come from:

1. $\mathcal{W}^{s}\left(\mathcal{W}_{L_{2}}^{c}\right)$
2. The Moon (we can hypothesize ejecta deriving from high-energy collisions with the Moon): Double Collision Orbits



## The effect of the Sun: The Bicircular Restricted 4 - Body Problem

The infinitesimal mass affected by the gravitational attractions of Earth, Moon and Sun moving in circular orbits.


We use the same initial conditions as the ones uniformly distributed inside $\mathcal{W}^{s}\left(\mathcal{W}_{L_{2}}^{c}\right)$ for the CR3BP

## Distributions of impacts

$$
d_{E M}=232400 \mathrm{~km}
$$



$$
\theta_{0}=108^{\circ}
$$



$$
\theta_{0}=252^{\circ}
$$

## Outcome

1. The effect of the Sun reduces the number of impacts on the trailing side of the Moon (anti-apex)
2. As $d_{E M}$ decreases, the apex concentration increases
3. The percentage of impact depends on $d_{E M}$ and on the initial Sun phase $\theta_{0}$
4. The highest density of impact oscillates in lunar longitude in the range $\left[50^{\circ} \mathrm{W}, 100^{\circ} \mathrm{W}\right]$
5. Some trajectories collide with the Earth

## 2. Mass transport from Mars to the Earth (martian meteorites)

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## Motivation

Fig. 4. Evolution of 200 particles launched from Mars with $v_{\infty}=1 \mathrm{~km} / \mathrm{s}$. See the caption to Fig. 2. The curve in the upper right of each panel marks aphelion at Jupiter (5.2 AU).



B.J. Gladman, J.A. Burns, M. Duncan, P. Lee, H.F. Levison: The Exchange of Impact Ejecta Between Terrestrial Planets, Science 271 (5254) 1387-1392 (1996)

## Manifods of $L_{1}$ of Mars and $L_{2}$ of the Earth



Branches of stable (blue) and unstable (red) invariant manifolds of periodic orbits around $L_{1}$ in the Sun-Mars and $L_{2}$ in the Sun-Earth CR3BPs (superimposing the respective synodic reference frames)

## Maximum and minimum distances to the Sun



Branches of the stable and unstable invariant manifolds of the $L_{1}$ and $L_{2}$ libration points in a CR3BP. The minimum and maximum distances from the center of mass are $R\left[W_{L_{1}}^{u}\right]_{\text {min }}$ and $R\left[W_{L_{2}}^{u}\right]_{\text {max }}$

## Minimum and maximum heliocentric distances (in $A U$ )

| PCR3BP | $R\left[W_{L_{1}}^{u(s)}\right]$ |  | $R\left[W_{L_{2}}^{u(s)}\right]$ |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $\min$ | $\max$ | $\min$ | $\max$ |
| Sun-Mercury | 0.37547 | 0.38562 | 0.38858 | 0.39918 |
| Sun-Venus | 0.67152 | 0.71658 | 0.73011 | 0.78024 |
| Sun-Earth | 0.92328 | 0.98998 | 1.01008 | 1.08488 |
| Sun-Mars | 1.46684 | 1.51644 | 1.53094 | 1.58331 |
| Sun-Jupiter | 3.02493 | 4.85550 | 5.56589 | $\mathbf{9 . 4 6 4 0 2}^{*}$ |
| Sun-Saturn | $\mathbf{6 . 6 3 4 6 7}^{*}$ | 9.12494 | 9.99818 | 14.04464 |
| Sun-Uranus | 15.87518 | 18.75222 | 19.69233 | $\mathbf{2 3 . 4 9 5 7 4}^{*}$ |
| Sun-Neptune | $\mathbf{2 3 . 3 5 7 3 4}^{*}$ | 29.33805 | 30.89615 | 37.25573 |

## In a 5 bodies model (Sun, Earth, Mars and Jupiter)



Initial states that impacted with Mars in a backward simulation considering the family of Lyapunov orbits about the Sun-Earth $L_{2}$

## In a 5 bodies model (Sun, Earth, Mars and Jupiter)

- 70 Lyapunov orbits and 100 initial states on the stable manifold of each one
- 59 particles impacted with Mars ( $<1 \%$ ), one test particle impacted with Jupiter and another one impacted with the Sun
- The average time of transport was 122904 years and the fastest case need of 20622.5 years with a departure (arrival) velocity of $5.87693 \mathrm{~km} / \mathrm{s}$ from the surface of Mars.
- The minimum departure velocity found was of $5.22369 \mathrm{~km} / \mathrm{s}$ ( $200 \mathrm{~m} / \mathrm{s}$ larger than the escape velocity) and this transport needed 853153 years.


## Jupiter's influence



## Perihelion and aphelion distances in the Sun-Jupiter CR3BP

For a fixed value of $C$ and $y=0$, we determine, for each value of $(x, \dot{x})$, the first perihelion and aphelion distances


With these results we distinguish several regions in the $(x, \dot{x})$ plane

## $(x, \dot{x})$ regions for a fixed value of $C$



Region 1: I.C. of inertial orbits that do not intersect the orbits of the Earth and Mars
Region 2: I.C. of orbits which only intersect the orbit of Mars
Region 3: I.C. of orbits which intersect the orbit of the Earth

Poincaré map behaviour for $C=3.14$, 3.09, 3.06, 3.03 ( $x_{E} \approx-0.2, x_{M} \approx-0.3$ )


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## Some periodic orbits for $C=3.03$

| Order | 4 | 4 | 5 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| $x$ | -0.4432 | -0.2240 | -0.2412 | -0.3388 | -0.1591 |
| $\dot{x}$ | $+0.0000$ | -0.6839 | $+0.0000$ | $+0.7269$ | +0.3938 |
| $E^{u}$ | -1.3040 | -6.2870 | +11.730 | -2.7930 | +4.8820 |
| $E^{s}$ | -0.7666 | -0.1590 | $+0.0852$ | -0.3579 | $+0.2049$ |
| Period | 25.1 | 12.6 | 18.9 | 31.5 | 25.2 |

Position $(x, \dot{x})$, stable and unstable eigenvalues ( $E^{s}$ and $E^{u}$ ) and period (in adimensional time units, $1 \mathrm{TU} \approx 12 / 2 \pi$ years $=1.9$ years) for some of the detected saddle points in the Sun-Jupiter

Orbits of order 4 and 5 and Poincaré map representation of their manifolds


## Heteroclinic intersections

1. The manifolds of one of the order- 4 saddle points have intersections with the orbit of the Earth
2. The manifolds of the order- 5 saddle point have intersections with the orbit of Mars
3. These manifolds have intersections with each other
4. The transport between regions of the phase space can be completely described by the dynamical evolution of the lobe determined by the region enclosed by the above stable and unstable manifolds.

## Lobe evolution providing natural transport between regions 2 and 3



T represents the initial states near the intersection of the manifolds of the order- 4 and order- 5 saddle points. After 21 iterations in the forward direction, some of these states move from "region 2 " to "region 3 ".

Example of transport orbit in the CR3BP for $C=3.03$ (synodic and inertial)


Integration time $=1283$ years

## 3. From the outer to the inner Solar System

## Motivation


M.W. Lo, S.D. Ross: Personal communication

## Dynamical substitutes of $L_{1,2}$ in the bicircular Sun-Jupiter-planet model

Substituts dinamics de L1 i L2 per Saturn, Ura i Neptu al sistema sinodic Sol-Jupiter


Substituts dinamics de L1 i L2 per Venus, Terra i Mart al sistema sinodic Sol-Jupiter


Sun-Jupiter reference system

## Dynamical substitutes of $L_{1,2}$ in the bicircular Sun-Jupiter-planet model








## Behaviour of $W^{u}\left(L_{1}^{N e p}\right)$ and $W^{s}\left(L_{2}^{U r a}\right)$



20 orbits integrated during 10000 Sun-Jupiter time units

## Intersection of $W^{u}\left(L_{1}^{N e p}\right)$ with $W^{s}\left(L_{2}^{U r a}\right)$



Semimajor axis (a) and eccentricity (e) of the points of the manifolds at their intersection withthe circles $R=4.7$ and $R=3.3$


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## Delta-v and time for the refined connections




## Can we rely in the computations ?



Evolution with time of two initial conditions on the unstable manifold initially separated $10^{-10}$ distance units


[^0]:    Y. Ren, J.J. Masdemont, G. Gómez, E. Fantino: Two mechanisms of natural transport in
    the Solar System. To appear in Communications in Nonlinear Science and Numerical
    Simulation, 2011.

